ABSTRACT

The transition probabilities between various states of superfluid helium 4 are found by using the approximation method of Bogolubov, making use of the canonical transformations for different states of transitions.

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March 1985

* To be submitted for publication.
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condensed bosons, in the zero momentum state. Accordingly, the higher order corrections for his spectra gave divergent results. To remedy this anomaly, the fluctuations of the condensed bosons were incorporated into the Hamiltonian of the system. According to this, we say that, as the zero-momentum states are in oscillation for $v = 0$, the ground state consists of $N$ particles of the condensed state, which are under small oscillations.

To this end the zero momentum single-particle operators $a_n^\dagger$ and $a_n^\dagger$ taken equal to a c-number constant by Bogolubov, have been replaced as

$$a_n \rightarrow a_n^c, \quad a_n^\dagger \rightarrow a_n^{c\dagger}$$

where $c_n$ is a new annihilation operator intended to describe small oscillations for the condensate. These oscillations are independent oscillations with random amplitude obeying the usual boson commutation relations

$$\left\{ C_n, C_m^\dagger \right\} = \delta_{nm}$$

$$\left\{ C_n, C_m \right\} = \left\{ C_n^\dagger, C_m^\dagger \right\} = 0$$

$$\left\{ C_n^\dagger, C_m^\dagger \right\} = 0$$

However it is approximated that the amplitude of the oscillations represented by $c_n$ remains finite and small even when the system becomes infinitely large. Further, the operator $c_n$ may be of quantum origin or it can be due to the interaction of the zero momentum particles with the walls of the container.

So the ground state of the superfluid helium 4 may be defined as

$$\left| \psi_0 \right> = \psi_0 \left| \psi_0 \right>$$

and the ground state of the condensed bosons is defined as

$$\left| \psi_0 \right> = \psi_0 \left| \psi_0 \right>$$
Accordingly, the true vacuum state is related to the vacuum state of \( a_0 \) as
\[
\langle \pi | a_0 | \pi \rangle = \langle a_0 | \pi \rangle \quad (7)
\]
The suffix \( a \) is in \( \pi \) with the vector \( \langle \pi | a_0 \rangle \) because it is a superposition of the states with different occupation numbers. Now to compute the transition probability between the states given in Eq. (7) we first find the coupling coefficient between the state of \( \pi \)-oscillating bosons and the state of \( \pi \)-oscillating condensed bosons, defined by
\[
| \pi \rangle \equiv \sum_{\{n\}} | n \rangle_{\pi} \quad (8)
\]
Using the Eqs. (6), (9) and (10), we get for \( k \) and \( l \), both non-increasing, the following recursion formula
\[
\begin{align*}
| \pi \rangle & = \sum_{\{n\}} | n \rangle_{\pi} \\
& = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] | n \rangle_{\pi} \\
& = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] \left[ \frac{1}{n!} \right] | n \rangle_{\pi} \\
& = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] \left[ \frac{1}{n!} \right] \left[ \frac{1}{\pi} \right] | n \rangle_{\pi} \\
& = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] \left[ \frac{1}{n!} \right] \left[ \frac{1}{\pi} \right] | n \rangle_{\pi} \\
& = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] \left[ \frac{1}{n!} \right] \left[ \frac{1}{\pi} \right] | n \rangle_{\pi}
\end{align*}
\]
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& = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] \left[ \frac{1}{n!} \right] | n \rangle_{\pi} \\
& = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] \left[ \frac{1}{n!} \right] | n \rangle_{\pi} \\
& = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] \left[ \frac{1}{n!} \right] | n \rangle_{\pi} \\
& = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] \left[ \frac{1}{n!} \right] | n \rangle_{\pi}
\end{align*}
\]
with \( M(\pi) \neq 0 \) for \( k \) and \( l \), both odd or even.
\[
\begin{align*}
| \pi \rangle & = \sum_{\{n\}} | n \rangle_{\pi} \\
& = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] \left[ \frac{1}{n!} \right] | n \rangle_{\pi} \\
& = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] \left[ \frac{1}{n!} \right] | n \rangle_{\pi} \\
& = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] \left[ \frac{1}{n!} \right] | n \rangle_{\pi} \\
& = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] \left[ \frac{1}{n!} \right] | n \rangle_{\pi}
\end{align*}
\]
Let the generating function be defined by
\[
\begin{align*}
| C_\pi(a, \beta, \gamma) \rangle & = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] \left[ \frac{1}{n!} \right] | n \rangle_{\pi} \\
& = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] \left[ \frac{1}{n!} \right] | n \rangle_{\pi} \\
& = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] \left[ \frac{1}{n!} \right] | n \rangle_{\pi}
\end{align*}
\]
Its differentiation with respect to \( \alpha \) and subsequently using Eqs. (10) and (11) gives us
\[
\begin{align*}
| C_\pi(a, \beta, \gamma) \rangle & = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] \left[ \frac{1}{n!} \right] | n \rangle_{\pi} \\
& = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] \left[ \frac{1}{n!} \right] | n \rangle_{\pi} \\
& = \sum_{\{n\}} \left[ \frac{1}{k!} \right] \left[ \frac{1}{l!} \right] \left[ \frac{1}{n!} \right] | n \rangle_{\pi}
\end{align*}
\]
The states of oscillation of condensed bosons are of harmonic type of vibrations. These independent oscillations, executed by the condensed bosons, are with random amplitude and direction but with the same frequency, which are denoted by the index \( \nu \) in the operator \( c^{\dagger}_\nu \). Such a motion can be imagined as the collective state that gives rise to the gradient of oscillation. For the states of oscillation by zero momentum condensed bosons, a state having such a collective characteristic can be obtained by choosing the appropriate linear combination of the degenerate states. As this system of oscillating condensed bosons has motions of the harmonic oscillator type, different types of correlations are obtained by choosing different linear combinations. The lowest state may have no collective dilatational form. However, the highest levels in the system of the oscillating condensed bosons with \( k = 0 \), form the excitation spectrum on this intrinsic fundamental state. This collective behaviour finally leads to the formation of the phonon spectrum of the superfluid helium. This phonon spectrum is the cause of superfluidity in helium at low temperature. So the collective excited states \( \{|\Omega\rangle\} \) of phonon states can be thought to have formed as a result of the different frequencies of the oscillating condensed bosons. In this way, we visualize a breakdown of a symmetry condition. Mathematically we can say that by imposing an asymmetric condition on the ground state, we can have phonons, as mixed states composed of parts with broken symmetries.

The discussion on the subject of broken symmetries in the field theory had led to the remarkable theorem of Goldstone according to which there is always a zero mass particle associated with the broken symmetry of the continuous group. In mathematical terms, it means that if the continuous symmetry of the Hamiltonian is not dealt with by the ground state, a gapless excitation is produced \( \{|\Omega\rangle\} \); this is a state whose excitation energies \( E(k) \) vanish for \( k = 0 \).

The continuous symmetry group under question can be represented by the unitary operator \( U(x) = e^{ix\mathbf{A}} \) in the N-particle Hilbert space:

\[ c^{\dagger}_\nu \text{ and } c^{\dagger}_\nu \text{ are the annihilation and creation operators for the oscillating condensed bosons obeying the commutation relations as given by Eq. (2). If the true ground state is degenerate in the sense that oscillating condensed bosons have the same frequencies, then the underlying Hamiltonian becomes invariant to the above symmetry operators, giving rise to the conservation law } E(\nu) = H. \]

However, if any dynamical effect is taking place between any two particles or subsystems, in the true ground state, then the above conservation law seems to be violated thereby breaking the above symmetry. This results in the creation of phonon states:

\[ E(\nu) = H \]

where \( |0\rangle \) is the vacuum phonon state. The phonon operators, \( a^{\dagger}_\nu \) which are the transforms of the oscillating condensed boson operators \( c^{\dagger}_\nu \) and \( c^{\dagger}_\nu \) are given by:

\[ a^{\dagger}_\nu = \lambda \mathbf{C} \mathbf{U}^\dagger \mathbf{C}_\nu \mathbf{U} \mathbf{C}_l \]

\[ a^{\dagger}_\nu = \mathbf{C} \mathbf{U}^\dagger \mathbf{C}_\nu \mathbf{U} \mathbf{C}_l \]

where \( S = i \mathbf{C} \mathbf{U} \mathbf{C}_\nu ; \mathbf{C} = \cosh x, v = \sinh x; |s|, |a| = |s^\dagger, a^\dagger| = 0 \)

Adopting the same procedure as in Sec. II, we have the coupling coefficient of the \( \nu \) phonon state to zero phonon state (\( \{|\Omega\rangle\} \) vacuum phonon state as follows:

\[ \gamma^m_{\nu} (\nu^m_{\nu} \nu_{\nu}^m) = \frac{\lambda^m_{\nu} x^m_{\nu} x_{\nu}^m}{e^{\frac{x_{\nu}^m}{x}}} \sum \left( \frac{\lambda^m_{\nu} x^m_{\nu} x_{\nu}^m}{e^{\frac{x_{\nu}^m}{x}}} \right) \]

For even \( m \)
Thus the transition probability between the above mentioned states is given by

\[ |C_{m}^{(n)}(\lambda)|^2 = \frac{1}{\pi} \left| \sum_{\mu} \left( \frac{1 - \frac{\lambda}{\lambda_0}}{1 - \frac{x}{x_0}} \right)^{\frac{1}{2}} \right|^2 \]

for odd \( m \).

\[ |C_{m}^{(n)}(\lambda)|^2 = \frac{1}{\pi} \left| \sum_{\mu} \left( \frac{1 - \frac{\lambda}{\lambda_0}}{1 - \frac{x}{x_0}} \right)^{\frac{1}{2}} \right|^2 \]

for even \( m \).

\[ |C_{m}^{(n)}(\lambda)|^2 = \frac{1}{\pi} \left| \sum_{\mu} \left( \frac{1 - \frac{\lambda}{\lambda_0}}{1 - \frac{x}{x_0}} \right)^{\frac{1}{2}} \right|^2 \]

for even \( m \).

IV. TRANSITION FROM PHONON TO ROTON STATES

According to the Landau hydrodynamic theory, for the superfluid He \(^4\) the phonon region ranges from \( p > 0 \) upwards and the roton region falls around \( p < p_0 \). In this way the whole spectrum of excitation of liquid helium \(^4\) can be imagined as two parts of one and the same excitation spectrum. The phonon branch of the spectrum is rotation free, while the roton branch of it consists of rotation motion. The roton branch at very low temperatures, may be assumed to have a characteristic short wavelength elastic mode form, which is totally degenerate. But as the temperature of the system increases, the roton spectrum predominates and exerts its characteristics.

Now in order to correlate the phonon states with the roton states, one of us \(^1\) had chosen the bilinear operators \( a_1 a_j^\dagger \) due to the phonon spectrum in the Bogolubov approximation and constructed from them the angular momentum operator \( L \) given by

\[ L = \frac{i}{\hbar} (\alpha_1 a_j^\dagger - a_j \alpha_1^\dagger) \]

In this formalism of liquid He \(^4\), the roton states are well defined angular momentum states formed from the eigenvalue of the operators of \( J \) and \( S \).

Here we use the operator

\[ L = \Theta \left( L (i, \theta) - i \right) \]

for the transformation from phonon to roton states. In this \( \Theta \) denotes a variation of phase, as we go from phonon states to roton states.

Accordingly the Bogolubov transformation is \( e^L \) and the transforms \( c \) and \( c^\dagger \) of phonon operators \( a_1 \), \( a_j^\dagger \) are given by

\[ c_k = e^L a_k^\dagger c^\dagger = \sqrt{\epsilon} \alpha_k^\dagger c^\dagger = \sqrt{\epsilon} \alpha_k^\dagger c^\dagger + \epsilon \alpha_k^\dagger c^\dagger \]

where \( |\alpha_k^\dagger, \alpha_j^\dagger \rangle = \delta_{kj} \).

\[ U = U_1 = (c \Theta), \quad V = V_1 = S \Theta \]

Thus

\[ |\psi_{\text{rot}}\rangle = S(\epsilon) |\psi_{\text{photon}}\rangle \]

\[ = S(\epsilon) |\alpha_k^\dagger, \alpha_j^\dagger \rangle \]

\[ |\alpha_k^\dagger, \alpha_j^\dagger \rangle \]

in the vacuum roton state.

Here the coupling coefficient between two possible roton states is given by

\[ \langle \alpha_k^\dagger, \alpha_j^\dagger | \gamma \rangle \]

\[ = \frac{1}{\hbar} \langle \gamma | a_j^\dagger a_j a_j^\dagger a_j^\dagger | \alpha_k^\dagger, \alpha_j^\dagger \rangle \]

\[ = \frac{1}{\hbar} \langle \gamma | a_j^\dagger a_j a_j^\dagger a_j^\dagger | \alpha_k^\dagger, \alpha_j^\dagger \rangle \]

(25)
Following the procedure adopted in the previous page, we obtain the following relations with indices all non-increasing:

\[
\begin{align*}
\langle \Phi | \mathcal{A}_0 | 0 \rangle &= -\mathcal{A}_0 \langle \Phi | \mathcal{A}_0 | 0 \rangle + \mathcal{A}_0^{1/2} \\
\langle \Phi | \mathcal{A}_0 | 1 \rangle &= -\mathcal{A}_0 \langle \Phi | \mathcal{A}_0 | 1 \rangle + \mathcal{A}_0^{1/2} \\
\langle \Phi | \mathcal{A}_0 | 2 \rangle &= -\mathcal{A}_0 \langle \Phi | \mathcal{A}_0 | 2 \rangle + \mathcal{A}_0^{1/2} \\
\langle \Phi | \mathcal{A}_0 | 3 \rangle &= -\mathcal{A}_0 \langle \Phi | \mathcal{A}_0 | 3 \rangle + \mathcal{A}_0^{1/2} \\
\langle \Phi | \mathcal{A}_0 | 4 \rangle &= -\mathcal{A}_0 \langle \Phi | \mathcal{A}_0 | 4 \rangle + \mathcal{A}_0^{1/2}
\end{align*}
\]  

(26)

As a consequence of momentum conservation, and from the definition of \( H \)'s we conclude that

\[
H_{p,q,r,s}(0) = 0 \quad \text{unless} \quad p + q = r - s.
\]

(27)

Here we define the generating functions by

\[
H(\theta, \phi, \psi, \chi) = \int \frac{d^4 x}{(2\pi)^4} \frac{e^{i\theta x}}{x} \frac{e^{i\phi y}}{y} \frac{e^{i\psi z}}{z} \frac{e^{i\chi t}}{t}.
\]

(30)

Differentiation of this with respect to \( \theta \) and the use of Eqs. (28) and (29) gives:

\[
H_{0,0,0,0}(\theta) = \int \frac{d^4 x}{(2\pi)^4} \frac{e^{i\theta x}}{x} H(\theta, \phi, \psi, \chi) = \int \frac{d^4 x}{(2\pi)^4} \frac{e^{i\theta x}}{x} \frac{e^{i\phi y}}{y} \frac{e^{i\psi z}}{z} \frac{e^{i\chi t}}{t}
\]

(31)

\[
H_{0,0,0,0}(\theta) = \int \frac{d^4 x}{(2\pi)^4} \frac{e^{i\theta x}}{x} H(\theta, \phi, \psi, \chi) = \int \frac{d^4 x}{(2\pi)^4} \frac{e^{i\theta x}}{x} \frac{e^{i\phi y}}{y} \frac{e^{i\psi z}}{z} \frac{e^{i\chi t}}{t}
\]

(32)

The value of \( H_{0,0,0,0}(\theta) \) is obtained from its definition and Eq. (26).

V. TRANSITION FROM ROTON STATES TO THE ROTON PAIRING STATES

According to Feynman\(^7\), the roton excitations in the superfluid helium \( \alpha \) are formed as a result of high density fluctuations. By assuming that the strong interaction of high density fluctuations is necessary for the production of rotons, he obtained the excitation spectrum of the form of Landau. However his expression for the energy of excitation is approximately twice that found experimentally\(^15\). Accordingly, it became pertinent to improve upon the Feynman wave function. To this end Feynman and Cohen\(^16\) introduced the idea of back flow and visualized the roton as a modified free particle excitation which is moving in the system and is surrounded by the back flow of the helium atoms. The back flowing atom can be assumed to leave a hole there; the hole density fluctuation can be as high as the particle density fluctuation responsible for the creation of roton\(^7\). So this high density hole fluctuation can also result in the formation of roton. The two rotons, one as a result of high particle density fluctuation and the other due to their high hole density fluctuation may be regarded as independent excitations.
and treated in an identical manner. So we can consider that the roton excitations in the superfluid $^4$He, are created in pairs. Feynman and Cohen have shown that the roton-roton interaction includes a component arising from the process of one roton emitting a phonon which is then absorbed by another roton. Pitaevskii, while considering the interaction between excitations found a bending of one excitation dispersion curve, when it goes near the two-roton continuum, with a threshold energy $2\Lambda_0$, but approaches it asymptotically. Such a bend in the spectrum as an energy near $2\Lambda_0$ has been demonstrated by the neutron scattering experiments, but seems to occur at an energy larger than $2\Lambda_0$ in contrast to Pitaevskii's prediction, although his treatment neglected the influence of a finite roton lifetime. The neutron scattering data showed the existence of distinct branches in the single excitation spectrum, which remain anomalous in terms of previous theories. However, this splitting of excitation spectrum together with the anomalous peak structure in the Raman scattering experiments with the two-roton continuum were explained by the existence of bound states in the roton system by Ruvalds and Zwadowski. By assuming that the roton-roton interaction is attractive as is also suggested by phonon mediated scattering mechanisms, they showed that a bound state of a roton can split off below the continuum at an energy $2\Lambda_0 - E_b$, with $E_b$ denoting the binding energy of two rotons. By means of extremely precise techniques Greytak has measured the binding energy and consequently confirmed experimentally the existence of two-roton bound states with zero total momentum. The possibility of forming two-roton resonances with total momentum zero has been demonstrated independently by Iwamoto, using a phase shift analysis of the scattering amplitude for two rotons coupled by a separable potential. In fact the very existence of the energy gap in the energy spectrum of the superfluid helium $^4$He can be thought of as due to some kind of binding energy between the rotons in the roton region of the spectrum. This binding energy requires an effective attractive interaction among rotons which if essentially a pairing interaction would lead to the formation of some kind of bound pairs. The binding energy of the pair will be modified by the presence of all other particles and this results here in the formation of the energy gap in the spectrum of the total system. The formation of roton bound pairs takes into account Pitaevskii's singularities.

Other examples of bound states and resonances of two excitations are given by Magnon bound states in the Heisenberg ferromagnets, bound phonon pairs in crystals, and Cooper pairs in superconductivity.

Bogolyubov and Valatin have shown that single and pair excitations of a superconductor can be treated on the same footing by means of a transformation of creation and destruction operators for quasiparticle excitations of the state. Since superfluidity and superconductivity have been considered on the same basis by many authors, we can treat the single and pair roton excitation on the same basis as that proposed by Valatin and Bogolyubov. The pair roton excitation forms a dipole. The suggestion that roton is strictly a dipole supports this viewpoint.

Thus we have accordingly the pair creation and destruction operators

$$\hat{h}_{\mathbf{k}}^\dagger = \alpha_{\mathbf{k}}^\dagger \alpha_{\mathbf{k}}^\dagger \beta_{\mathbf{k}}^\dagger = \alpha_{\mathbf{k}}^\dagger \beta_{\mathbf{k}}^\dagger$$

obeying the commutation relations:

$$\{\hat{h}_{\mathbf{k}}, \hat{h}_{\mathbf{k'}}\} = \{\mathbf{k} - \mathbf{k'} = \mathbf{q}, \mathbf{k} \cdot \mathbf{q} = 0\}$$

The operator that brings the transformations from single roton state to the roton-pairing state is given by

$$\hat{U}(\mathbf{q}) = \exp\left[\frac{i\mathbf{k} \cdot \mathbf{q}}{2} \left(\alpha_{\mathbf{k}}^\dagger \beta_{\mathbf{k}}^\dagger - \gamma_{\mathbf{k}}^\dagger \gamma_{\mathbf{k}}^\dagger\right)\right]$$

$$\hat{U}_{\mathbf{k}}(\mathbf{q}) = \exp\left[i\mathbf{k} \cdot \mathbf{q}\gamma_{\mathbf{k}}^\dagger \gamma_{\mathbf{k}}^\dagger\right]$$

The transformation of the operators $\alpha_{\mathbf{k}}$ and $\gamma_{\mathbf{k}}$ are given by

$$\alpha_{\mathbf{k}} = u_{\mathbf{k}}^{1^2} \gamma_{\mathbf{k}}^{1^3}$$

$$\gamma_{\mathbf{k}} = u_{\mathbf{k}}^{1^2} \gamma_{\mathbf{k}}^{1^3}$$
\[ u = u - \cosh a \]

\[ V_k \pm V_{-k} = \sinh a \cdot \]

Define the coupling coefficient of different possible pairs as

\[ \mathcal{C}_p \left( \psi \right) = \frac{1}{B^2} \exp \left( -\frac{1}{2} \beta \right) \]

\[ \mathcal{P} \left( \psi \right) = \frac{1}{B^2} \exp \left( -\frac{1}{2} \beta \right) \]

Hence the transition probability between the roton state and the roton pairing state is given by

\[ \left| \mathcal{P} \left( \psi \right) \right|^2 = \left( \frac{1}{B^2} \right)^2 \exp \left( -\frac{1}{2} \beta \right) \]

In actual practice \( p = q \); then we have

\[ \left| \mathcal{P} \left( \psi \right) \right|^2 = \left( \frac{1}{B^2} \right)^2 \exp \left( -\frac{1}{2} \beta \right) \]

VI. CONCLUSION

In this work we have found the forms of transition probabilities between (i) the states of condensed bosons and oscillating condensed bosons; states of oscillating condensed bosons and phonon states, phonon states to roton states and roton states to roton pairing states. We used

\[ \text{the approximation method developed by Bogoliubov}\]

\[ \text{making use of his famous canonical transformations. The study of such transformations on the roton region has been made by some authors.}\]

\[ \text{The transition probability between the condensed boson and the oscillating condensed boson state appears to be of a complex nature. This leads us to conclude that the oscillation of condensed bosons must have a quantum origin. The theoretical determination of these transition probabilities opens a new vista of experimentation in the He II Systems.}\]

ACKNOWLEDGMENT:

One of the authors (M.A.) would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for the hospitality at the International Centre for Theoretical Physics, Trieste.
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