Yu. S. Kalashnikova, I. M. Narodetskii,

NUCLEON – NUCLEON FORCES
IN THE QUARK COMPOUND BAG MODEL
AND FEW – NUCLEON SYSTEMS

MOSCOW 1984
Role of quark-gluon degrees of freedom is discussed in nucleon-nucleon scattering at low and intermediate energies. It is shown that the existence of six-quark bags fixes the form of NN potential at small distances, which leads to the P-matrix satisfying the criterion of Jaffe and Low. The dynamical model of three-nucleon system is discussed taking into account the contribution of six-quark bags.
1. Introduction

There is no doubts now that the nucleon-nucleon interaction is to be constructed taking into account quark and gluon degrees of freedom. The central problem of nuclear physics is to deduce the NN potential directly from QCD. The space-time scale in nuclear physics being not small and corresponding to the region, where the nonlinear and non-perturbative effects of QCD are essential, the problem of constructing of the NN potential is closely connected to the problem of theoretical calculation of the screening radius of coloured forces.

The problems of QCD at large distances still await their resolution. Therefore the existing approaches to the problem of calculation of the short-range nuclear forces are based on using the QCB-motivated quark or chiral models \(^{1,2}\). We discuss in the present paper NN interaction in the quark compound bag model (QCB) \(^{3-5}\) which connects in natural way the quark-gluon dynamics in the region where the nucleon bags overlap and well established peripherical interaction of nucleons due to meson exchanges. In contrast to the nonrelativistic quark models, the QCB method does not use the adiabatic hypothesis, and hence, does not contain the RGM ambiguities due
to the spurious van der Waals forces /6/.

The main assumption of the QCB model is that the interaction of nucleons as the three-quark bags is characterized by sharp phase transition at the distance $r \sim 6$, where the two bags merge into the common QCB. It was shown /3/, that even in the simplest version the model explains the nature of the repulsion between nucleons at small distances and leads to the P-matrix, which satisfies the criterion of Jaffe and Low /7/ (in fact, the QCB model gives the foundation of the P-matrix method). The further analysis made by using the quark cluster model /4/, has justified the existence of surface effects in NN interaction and enabled one in addition to surface terms, to calculate also the volume terms in the NN potential. Refs. /8-10/ contain the first qualitative efforts to apply the model for describing nucleon dynamics, including the effects of parity nonconservation /8,9/. The present paper, where we discuss the general properties of QCB potentials, is the first of the series of papers, dealing with the application of the model to the calculation of dynamical characteristics of deuteron and to the description of three-nucleon systems.

The paper is organized into seven Sections. In Sect. 2 we give a brief review of the QCB model, which is necessary for understanding. In Sect. 3 we use the results of existing P-matrix analysis of NN scattering /11-13/ to discuss the choice of the QCB parameters. In particular, we show that the probability of nucleon clusters in the QCB wave function required by data agrees well with the simple algebraic estimations of refs. /14/. A further limitation on this probability one may obtain by considering the asymptotics of the QCB poten-
tial eigenvalues /15/, this question is briefly discussed in Sect. 4. In Sect. 5 we consider the formal consequence of the cluster method which states that if the coefficient at the linearly dependent term in the potential is equal to unity then the Schroedinger equation has the solution at $E = -\infty$. We refer to this solution as a "phantom" because, as it is shown in Sect. 5, the six-quark function constructed at this solution vanishes upon antisymmetrization and thus is forbidden by the Pauli exclusion principle. Therefore the formal existence of the "phantom" is the artefact of cluster ansatz, used in the QCB model, not corresponding to any physical reality - the six-quark system always has the finite energy.

One is to have in mind that the nucleon-nucleon wave function can always be redefined inside the bag; it is clear that such redefinition does not influence the phase shifts, but requires the modification of the initial form of QCB potential. In particular, one can always exclude the term of the potential linear with energy, in this way the "phantom" solution disappears. It is possible to give different recipes of determination of the wave function inside the bag, each being equally natural; an example based on the using the completeness condition is given in the end of Sect. 5.

The consideration of NN scattering alone does not permit to distinguish between different phase-equivalent potentials. This difference, however, became clear when considering nucleon-nucleon forces in the nucleus. Therefore the subsequent choice of the QCB potential is to be based on the analysis of equations for the system of several particles.
In Sect. 6 we develop the cluster methods for the nine-quark system and deduce integral equation for the bound state of three nucleons. This equation has the form of the usual Faddeev equation the kernel of which, however, is expressed in terms of the two-body T-matrices that correspond to the modified NN potential differing from the initial QCB one. We discuss the important physics that lies behind this modification. Note that our method gives an unambiguous definition of the two-body QCB potential in any nucleus with the number of nucleons \( N \geq 3 \). Sect. 7 contains a few concluding remarks.

The major part of the paper, with a few exceptions, uses an example of one QCB level, which parameters will be traditionally denoted as \( E_y \), \( C_y \), and \( \lambda_y \).

2. The definition of QCB potential

The QCB model is based on the physical picture, where the nucleon-nucleon interaction at small distances is defined by the wave function of the six-quark bag and by the amplitude of the transition \( NN \rightarrow \text{QCB} \) at the boundary of the QCB. The explicit description of this transition would require the knowledge all the variables describing dynamics of deformed bags in the region corresponding to the regime of strong coupling in the QCD. In the QCB model this transition is considered phenomenologically using the following ansatz for the six-quark wave function /4/:

\[
\psi = \psi_q + \psi_{\text{cl}}
\]

(1)

where \( \psi_q \) is the wave function of the QCB, while \( \psi_{\text{cl}} \) describes the relative motion of the nucleons as the three-quark
clusters and can be written as

$$
\psi_{c\ell} = A \left\{ \left[ \psi^c_N(123)\psi^c_N(456) \right]_{ST} \psi_{ST} \left( \vec{r} \right) \right\}
$$

(2)

$$
A = \frac{1}{10} \left( 1 - K \right)
$$

In Eq. (2) $K$ is the operator permutating quarks from different nucleons. The function $\psi(\vec{r})$ depends on inter-nucleon distance $\vec{r}$. This function is nonzero both inside and outside QCB. On the contrary, $\psi_q$ is nonzero only inside QCB and can be decomposed in this region into the complete set of eigenfunctions $\psi_\nu$ of the six-quark Hamiltonian $H$ with boundary conditions corresponding to the quark confinement:

$$
\psi_q = \sum_\nu \alpha_\nu \psi_\nu \quad H \psi_\nu = E_\nu \psi_\nu.
$$

Using standard methods of cluster decompositions /16/ it is possible to eliminate quark coordinates from the equation

$$(H-E)\psi = 0$$

and to obtain the Schrödinger equation which determines $\psi(\vec{r})$

$$
(H_0 + V_{NN} + V_{NQ})\psi = E\psi
$$

(4)

where $H_0 = -\frac{1}{m_N} \Delta$, $V_{NN}$ includes quark and gluon exchanges between nucleons, and $V_{NQ}$ describes the nucleon interaction inside the QCB. For $\varepsilon > \ell$, $V_{NN}$ contains the exchange of colour singlet objects which can be identified with physical mesons. For $\varepsilon < \ell$, the usual approximation /14/ is to neglect $V_{NN}$ compared with $V_{NQ}$, i.e. to set

$$
V_{NN} = V_{OBE} \Theta(\varepsilon - \ell).
$$

The kernel of the operator $V_{NQ}$ after separating angular variables can be written in the separable form
\[ V_{Nq_d}(\tau, \varepsilon') = \frac{1}{E - E'} \sum_j f_j(\tau, E) f_j(\varepsilon', E) \]  

where \( E_j \) is an eigenvalue in Eq. (3) and \( f_j(\tau, E)/\tau \) is the radial part of the overlap integral.

The further simplification of this expression may be obtained by projecting \( \psi_\nu \) onto the nucleon clusters. Denoting

\[ \int \psi_\nu(\tau - E) \left\{ \psi_N^*(\varepsilon') \psi_N^*(\varepsilon) \right\} \delta(\varepsilon - \varepsilon') i \prod d\varepsilon_i \]  

we obtain

\[ f_\nu(\tau, E) = -C_\nu \delta(\tau - E) + \chi_\nu(E - E) \bar{\nu}(\tau) \]  

The first term in the right-hand side of Eq. (8) corresponds to the nucleon interaction at the surface of the QCB, and is given by (6) with \( E \) substituted by \( E_\nu \). Note that this term yields the pole of the P-matrix \( P(\kappa, \xi) \) at \( E = E_\nu \). The second term in (8) corresponds to the volume interaction of nucleons inside the QCB. This term leads to the linear energy-dependent part of the NN potential. The coefficient \( \chi_\nu \) at energy-dependent term measures the probability of nucleon clusters in the QCB. Clearly, \( \chi_\nu \neq 0 \), because

* Effects of smearing the surface interaction have been discussed in /8/.
the quark content of the two-nucleon system coincides with
that of the QCB. An algebraic estimate yields \( x_\nu^2 = 10/g_{NN}^4 \)
where \( g_{NN}^4 \) is the corresponding fractional parentage coefficient;
for the \( \Lambda^6_{1/2} \) configuration \( g_{NN}^4 = 1/3/14 \)
and \( x_\nu^2 = 1.4 \). This estimate, however, should not be
taken too seriously, because it neglects the contribution of
other quark configurations and gluon corrections. We consider
\( x_\nu \) as a parameter which must be determined from experimen-
tal data. In the next Section we shall establish the limits
for \( x_\nu \) which follow from the P-matrix analysis of NN
scattering with \( J^P = 0^+ \) and \( 1^+ \).

The function \( \eta_j(z) \) in (8) has the threshold behaviour
\( z^{\ell+1} \) as \( z \to 0 \) and vanishes at \( z \sim \xi \). In calculations
this function is parametrized as

\[
\eta_j(z) = N (\beta z) \delta_{\ell} (\beta z)
\]

(9)

where \( N \) is the normalization constant and the parameter \( \beta \) is
fixed by requiring \( \eta_j(\xi) = 0 \), that is \( \beta = \frac{z_{\ell}}{\beta} \) \( \xi \)
being the first zero of the spherical Bessel function \( j_\ell(\xi) \). In
particular, for \( \ell = 0 \) one finds in the momentum space

\[
\eta_j(p) = \pi \sqrt{2\xi} \frac{\sin \phi \phi}{p (\xi^2 - p^2)^{3/2}}
\]

(10)

This function falls as \( 1/p^3 \) as \( p \to \infty \). This behaviour
is sufficient to ensure the Fredholm properties of the Fed-
deev equations discussed in Sect. 6. It should be emphasized
that the most of our results does not depend on a specific
choice of \( \eta_j(z) \) in Eq. (9).
3. Further discussion of the QCB potential. Choice of the parameters

As was stated above, in applications one usually neglect the part of $V_{NN}$ inside the QCB /4,5/. In this case the short range NN interaction is completely determined by $V_{NqN}$. In the simplest case of one QCB level $V_{NqN}$ depends on four parameters: $\ell$, $E_v$, $c_v$, and $x_v$. The radius $\ell$ of the equivalent hadronic bag can be connected to that of the six-quark state in the MIT bag model: $\ell = 1.42 R_9$ /11/, so $\ell ({}^1S_0) \approx \ell ({}^3S_1) \approx 1.4 \text{ fm}$. In practice, however, the values of $\ell$ used in recent P-matrix analysis of NN scattering /11-13/ are varied within the interval $1.2 - 1.6 \text{ fm}$. It is important to note that the radius $\ell$ being known, $E_v$ is not the independent fitting parameter but is determined through $\ell$. Indeed, the P-matrix calculations with various realistical external potential show, that the value $E_v/\ell$ for $^1S_0$ and $^3S_1 - ^3D_1$ channels do not depend on the model choice of external potential to a good accuracy /13/. The parameter $c_v$, that determines the residue of the P-matrix at the pole $E_v$, is also model independent with the accuracy 1-2%.

Let us consider now the choice of the parameter $x_v$. For the simplicity we restrict ourselves to the case of central forces. First we note that the experimental $^3S_1$ and $^1S_0$ P-matrices have only one pole in the energy region $T_L < 1 \text{ GeV}$, where the solutions of phase-shift analysis are now available, this pole corresponding to the QCB. The potential poles of the P-matrix /10/ are absent. This experimental fact imposes strong restrictions at the value $x_v$. To see how this
happens we shall write the expression for the S-wave P-matrix in the QCB model. When restricting to the case of one pole in (5) and choosing \( \eta_v(z) \) as (9), we obtain
\[
P = P_\circ (\kappa, b) + \frac{m_N c^2}{E - E_v}
\]
where
\[
P_\circ (\kappa, b) = \kappa \cot \beta \kappa b +
\]
\[
\frac{m_N c_v x_v (2 N \beta - m_N c_v x_v) + m_N x_v^2 (E - E_v) N^2 b^2 / (\beta^2 - \kappa^2)}{\beta^2 - \kappa^2 + m_N x_v^2 (E - E_v)}
\]
\[\beta = \pi/6 \quad \text{and} \quad N = \sqrt{\frac{2}{b}}.
\]
It is easy to check, that \( P_\circ (\kappa, b) \) does not possess a compensating pole at \( \kappa_c = \pi/6 \) for all \( x_v \), though other poles of \( \kappa \cot \beta \kappa b \) are still present. Besides, \( P_\circ (\kappa, b) \) may have extra poles which are determined by roots of the equation
\[
\beta^2 - \kappa^2 + m_N x_v^2 (E - E_v) = 0
\]
We obtain for \( x_v \neq 0 \)
\[
E_\rho = 2 m_N x_v^2 \left( 1 + \sqrt{\left( 1 - \frac{1}{x_v} \right)^2 - \frac{2 v / m_N x_v^2 + z_c / m_N x_v^2}{x_v} \right)}
\]
where \( z_v = E_v - 2 m_N \), \( z_c = \kappa_c^2 / m_N \) is the non-relativistic compensation energy. The pole position \( E_\rho \) depends on \( z_v \) and \( x_v \).

We recall that \( z_v \) when calculating for the matching radius \( b \) corresponding to the MIT bag satisfies the condition \( z_v > z_c \). With this choice of \( b \) Eq. (13) has only complex solutions for \( x_v = 1 \). One can check that these solutions do not influence the Wigner behaviour of
However, considering $E_\rho$ as a function of we observe that when $\chi_\nu$ varying $E_\rho$ may approach the real axis of energy or even become real giving unobserved poles of the $P$-matrix. A little numerical experience shows that only for

$$0.35 \leq \chi_\nu \leq 1.15$$

extra poles $E_\rho$ do not influence the causality behaviour of the $P$-matrix. The values of Eq. (14) coincide with the algebraic estimates of refs. /14/ within 20% accuracy. One may, however, obtain even more strong limitation on $\chi_\nu$ than given in Eq. (14). To this end we shall consider the behaviour of the QCB potential /15/.

4. The discussion of the trajectories of eigenvalues

The energy of the QCB potential leads to some specific properties which distinguish it from the commonly used static potentials. To illustrate these peculiarities we consider the simplest QCB model, where the interaction outside the QCB is neglected. This model ignores an important role of pion exchanges at $\rho > 6$, which lead to considerable corrections to the $P$-matrix, especially at very low energies. The model under consideration, however, possesses the advantage of having analytical solution; besides the inclusion of pion corrections gives only quantitative variations.

To illustrate the main result it is sufficient to consider the one-level QCB potential, in this case we obtain for the off-shell $T$-matrix:

*) For simplicity we consider S-wave scattering. Here and below $Z$ is the kinetic energy in the center-of-mass frame.
where \( z_\nu = E_\nu - \frac{1}{2} m_N \), \( \kappa^2 = m_N^2 \), and

\[
\Psi_j(p, z) = \frac{1}{p} \int_0^\infty \Psi_j(z, z, \lambda) \sin p \, \lambda \, dz = -c_j \tilde{\varepsilon}(\rho) - \lambda_j(E-E_\nu)\Psi_j(p) \tag{16}
\]

\[
\tilde{\varepsilon}(\rho) = \sin \rho \rho^2, \quad \Psi_j(p) \text{ is given by Eq. (9).}
\]

The function \( \Delta(z) \) defines the energy dependence of the eigenvalue \( \lambda(z) \) of the QCB potential

\[
\lambda(z) = \frac{1}{2 - z_\nu} \tag{17}
\]

The analysis of the P-matrix solutions for \( ^1S_0 \) and \( ^3S_1 \) channels shows that for \( \lambda = 1.2 \pm 0.6 \) \( \lambda(0) > 1 \) and, as it was stated above, \( z_\nu > z_c \). We are interested in the behaviour of the trajectory \( \lambda(z) \) for \( |z| \to \infty \).

As it was shown in /15/, for \( \rho = \pi / 6 \) the asymptotical behaviour of \( \lambda(z) \) for \( |z| \to \infty \) has the form

\[
\lambda(z) = \chi_j^2 \left( 1 - \frac{z^2}{2} \right) + O\left( \frac{1}{z^3} \right) \tag{18}
\]

The fact that \( \lambda(z) \to \chi_j^2 \) for \( |z| \to \infty \) trivially follows from the form of the QCB potential /4/. We are interested in the non-leading term in (18) which shows that for \( z \to -\infty \) the trajectory \( \lambda(z) \) tends to its asymptotic value from the upper side *.

The typical behaviour of

*Absence of monotonic term in Eq. (18) which usually leads to the non-leptonic behaviour of \( \lambda(z) \) is due to our choice of \( \varepsilon_0 \).
the trajectory \( \lambda(\bar{z}) \) is shown in Fig. 1. Recall that for static potentials \( \lambda \to 0 \) for \( |z| \to \infty \).

It immediately follows that for singlet potentials \( x^2 \) must be greater or equal unity, otherwise the potential supports a bound state. It is clear also that the low energy \(^1S_0\) scattering cannot be described by one QCB level, because if \( \lambda(0) > 1 \), the singlet scattering length is positive. So, at least one extra QCB level is necessary to obtain the experimental value of \( a(\ ^1S_0) \). A minimal modification of the one-level QCB potential is to add the second level with \( x_2 = 0 \); such the two-level potential has now two eigenvalues. The explicit position of the second level is not essential, provided it lies far enough, at low energies its effect reduces to an additive constant \( \frac{\gamma}{2} = -\frac{m_N^2}{2} \) to the P-matrix. So one can put \( z_2 \to \infty \) and \( c_2 \to \infty \) provided that \( \gamma \) would remain constant. Such procedure, which had already been adopted in [10], means that we introduce only one extra parameter to compared to the case of the one-level potential. The behaviour of the trajectories \( \lambda_1, \lambda_2(\bar{z}) \) for the two-level potential is shown in Fig. 2. Qualitatively the form of the trajectory \( \lambda_1 \) is the same as for one-level potential, and \( \lambda_1 \to 0 \) for \( |z| \to \infty \), and its behaviour is similar to that for the energy independent potentials.

When describing the \(^3S_1\) channel the situation is nearly the same. For \( x^2_1 > 1 \) the second QCB level is necessary to obtain the deuteron bound state (Fig. 3a). Now, however, we cannot exclude the value \( x^2_1 < 1 \) because of the pos-
sibility of the situation depicted in Fig. 3b*).

It is rather evident that the inclusion of the external interaction at \( z > \beta \) does not change the asymptotic behaviour of \( \lambda_1(z) \). It is instructive, however, to verify this statement explicitly. Considering again only one QCB level and denoting the residual interaction as \( V \), we obtain for the off-shell T-matrix

\[
 t(p', \rho'; z) = t_V(p', \rho'; z) + \frac{\psi_V(p', 2) \psi_V(p, 2)}{2 - 2V - \Delta(2)}
\]  

(19)

where

\[
\psi_V(p, 2) = f_V(p, 2) + m \int \frac{t_V(p, p', 2) f_V(p', 2)}{k^2 - p^2 + i\varepsilon} \frac{d^3p'}{(2\pi)^3}
\]  

(20)

\[
\Delta(2) = m \int \frac{f_V(p, 2) \psi_V(p, 2)}{k^2 - p^2 + i\varepsilon} \frac{d^3p}{(2\pi)^3}
\]  

(21)

\( t_V \) is the T-matrix for the residual interaction, \( t_V = V + V G_0 t_V \). The eigenvalue \( \lambda_1(z) \) we are interested in is now \( \Delta(2)/2 - 2V \). It is easy to see, that for \( |z| \to \infty \)

\[
\Delta(2) \to \Delta(2) \quad \text{indeed, in this limit the integral in the right side of (20) is proportional to}
\]

\[
\int e^{i p \vec{z}} V(\vec{z}) \psi_V(\vec{z}) d^3z
\]

and, evidently, is equal to zero. Therefore for \( |z| \to \infty \)

\[
\psi_V(p, 2) \to f_V(p, 2) \quad \text{and} \quad \lambda_1(z) \to \lambda_1^2.
\]

\* Note, that we could not find the satisfactory \( 3S_1 \) solution with \( \lambda_1^2 < 1 \). Several examples of the two-level QCB potentials with \( \lambda_1^2 \geq 1 \) are given in [8]. The necessity of considering eigenvalues was firstly proposed by I.L.Graham.
5. The states forbidden by the Pauli principle and the problem of continuation of the on-shell T-matrix

We continue our discussion of the QCB potential. In the previous section we have seen, that \( \lambda \to x_{\lambda}^2 \) when \( |\lambda| \to \infty \) So the Schroedinger equation for \( x_{\lambda}^2 = 1 \) has the bound state at infinity. This result can be seen directly from equation (4). Putting \( |\lambda| \to \infty \) and omitting finite terms, we obtain (for one QCB level)

\[
x_{\lambda}^2 \int_{0}^{\infty} \eta_{\nu}(\gamma) u(\gamma) d\gamma = u(\zeta)
\]

(22)

This equation for \( x_{\lambda}^2 \) has the evident solution \( u(\zeta) = \eta_{\nu}(\zeta) \).

This conclusion is not rigorous, however, because it is based on the neglecting of operator \( K \) that enter the antisymmetrizer \( A \) in Eq. (2). The problem of the existence of the "phantom" is closely connected to the properties of this operator; the equation (22) being substituted by

\[
(1 - K) \psi - \sum_{\lambda} x_{\lambda}^2 \langle y_{\lambda} | \psi \rangle y_{\nu} = 0
\]

(23)

When putting \( K = 0 \) and in the case of one QCB level we again return to (22). In the general case the problem of the existence of the solution of (23) is determined by the spectrum of the operator \( K \). If, for example, \( K \) would be the first rank separable operator: \( K = \chi |d \rangle \langle d | \), then the condition of the existence of the solution would have the form: \( \chi x_{\lambda}^2 \langle y_{\lambda} | d \rangle \langle d | y_{\nu} \rangle = (1 - x_{\lambda}^2)(l - y) \).

It is easy to show that, even if the solution of (23)
exists, the norm of the six-quark wave function \( \psi \) constructed at this solution is equal to zero independently of the properties of the operator \( K \). Indeed, let \( \psi \) in (2) be normalized, then

\[
\| \psi \|^2 = \sum_v a_v^2 + 2 \sum_v a_v (\psi_v, \psi_{\text{el}}) + (\psi_{\text{el}}, \psi_{\text{el}}) =
\]

\[
= -\sum_v a_v^2 + (\psi_{\text{el}}, \psi_{\text{el}}) = -\frac{i}{\hbar} \sum_v a_v^2 \langle \psi_v | \psi \rangle^2 + \frac{i}{\hbar} \frac{1}{Z} \left[- \langle \phi | K | \phi \rangle \right] = 0
\]

because of equation (23). So the solution under consideration is forbidden by the Pauli exclusion principle: the corresponding wave function is zero.

This situation reminds the situation in the usual RGM scheme, where, however, the wave functions of forbidden states coincide with the eigenfunctions of kernel \( K \), corresponding to eigenvalue equal to one /18/. Another point is that the forbidden states in RGM have the finite energy, while in QCB - the infinite one.

The formal reason of these states to appear is due to nonorthogonality of \( \psi_v \) and \( \psi_{\text{el}} \) in Eq. (1), \( (\psi_v, \psi_{\text{el}}) \neq 0 \). Physically, such nonorthogonality means that the quark content of nucleons and bag coincides. If we redefined the nucleon component inside the bag, so, that the condition \( (\psi_v, \psi_{\text{el}}) = 0 \) is satisfied, then the bound state at infinity disappears. Such a redefinition does not influence the asymptotics of \( \psi(z) \) and therefore does not influence scattering shifts and finite binding energies, but leads to another phase-equivalent QCB potential. The difference between phase-equivalent potentials can be seen only when considering systems with the nucleon number \( N \geq 3 \). So the problem is maintained here in determination of the QCB poten-
tial which is able to describe the nucleon interaction in nucleus. In the next Section we shall construct the modified two-body QCB potential for three-nucleon system treating nucleons as three-quark bags. Before discussing the nine-quark system it is instructing to consider one possible definition of phase-equivalent potential which, as it will be seen later, has a direct relation to the three-nucleon problem.

Let us consider the condition of completeness for the six-quark functions

$$\sum_n \psi_n(\bar{\gamma}_i)\psi^{*}_n(\bar{\gamma}'_i) = \prod_i \delta(\bar{\gamma}_i - \bar{\gamma}'_i) \quad (25)$$

To rewrite this equation in terms of the inter-nucleon wave function $\psi(\bar{\xi})$ we project each of $\psi_n^*$ onto $\bar{\xi} = \psi_n(123)\psi_n(456)\delta(\bar{\xi} - \bar{\xi}')$ and integrate over quark coordinates. It is convenient to introduce the new functions

$$\tilde{\psi}_n(\bar{\xi}) = \psi_n(\bar{\xi}) + \frac{x_v}{\bar{\xi} - \bar{\xi}_v} <\psi_v|\psi_n> \psi_v(\bar{\xi}) \quad (26)$$

Then the completeness condition written in terms of $\tilde{\psi}_n$ has the simpler form

$$\sum_n \tilde{\psi}_n(\bar{\xi})\tilde{\psi}_n^*(\bar{\xi}') = \delta(\bar{\xi} - \bar{\xi}') \quad (27)$$

The potential corresponding to $\tilde{\psi}_n$ is conveniently written in the momentum representation

$$V_{N\xi\nu}(P, P'; E) = \lim_{t \rightarrow \infty} \frac{t}{t - \frac{k^2_v}{E - \bar{\xi}_v}} f_\nu(P, E) \tilde{f}_\nu(P') \quad (28)$$
This expression does not contain linear energy term, so the Schroedinger equation for \( \widetilde{\psi}_n \) does not support the "phantom" solutions. Note, that the potential being proportional to \( \frac{1}{1-x^2_y} \) does not mean that the interaction (28) increases infinity for \( x_y \to 1 \). Indeed, operating by this potential at \( \widetilde{\psi}_n \), we obtain, that the coefficient at \( \frac{1}{1-x^2_y} \) is equal to \( \frac{\gamma_y(p_y|E)(\widetilde{\psi}_y(\widetilde{\psi}_n))}{E-E_y} \), but from (26) it follows that

\[
(\widetilde{\psi}_y \widetilde{\psi}_n) = (1-x^2_y)(\widetilde{\psi}_y \psi_n)
\]

In particular, off-shell T-matrix is finite for \( x_y = 1 \)

\[
t(p_1, p_f; z) = \frac{\gamma_y(p_1, \bar{z}) \gamma_y(p_f)}{z - z_y - \Delta(z)}
\]

where the function \( \Delta \) is the same as in Eq. (15).

For \( p^2 = \kappa^2 \) the expression (31) coincides with (15).

6. Two-body forces in nuclei. Integral equations for three-nucleon bound state

In present Section a problem is discussed of modification of two-body QCB forces in nuclei due to presence of six-quark bags. To answer this question it is sufficient to consider only the three-nucleon system, because, as it will be seen later, the QCB potential in nucleus does not depend on the number of nucleons provided that \( N \geq 3 \). The discussion of many-body forces connected to the admixture of 9-q and 12-q (for \( N \geq 4 \)) bags is out of scope of the present investigation.
Below the equations are obtained for the three-nucleon component of the nine-quark wave function. To clear up the main features of our approach we again consider a "toy" model without external interaction, because we are interested only in the quark part of the potential. We also assume that there is only one QCB level with $X_y = 0$; we shall see later, that the terms in the potential corresponding to the levels with $X_y = 0$ are not changed when going from nucleons to nuclei. At last we suggest that 6-quark bags with $J^P = 0^+$ and $1^+$ have the same radius and mass, so we can avoid unessential for our purposes spin-isospin algebra. The technical part will be published in the separate paper which will include also the calculation of binding energy of $^3\text{He}$ with the QCB potential.

We denote the quark triplets as $\alpha, \delta, \zeta$; for example, $\alpha = (123), \delta = (456), \zeta = (789)$. The natural generalization of cluster ansatz (1) for the nine-quark system

$$\psi = \psi_q + \psi_{\text{cl}}$$

where

$$\psi_q = A_\alpha \psi_(\delta \varepsilon) \psi_\alpha (q_{\alpha}) + A_\delta \psi_(\delta \varepsilon) \psi_\delta (q_{\delta}) + A_\zeta \psi_(\delta \varepsilon) \psi_\zeta (q_{\zeta})$$

$$\psi_{\text{cl}} = A \left[ \psi_\alpha (a) \psi_\delta (b) \psi_\zeta (c) \lambda (\vec{r}, \vec{\theta}, \vec{\phi}) \right]$$

Here $\psi_\alpha (a), \psi_\delta (a), \psi_\zeta (a)$ are the wave functions of the six-quark bags defined by Eq. (3), $A_\alpha, A_\delta, A_\zeta$ are the antisymmetrizers defined as

$$A_\alpha = \frac{1}{4!} \left( 1 - \mathcal{K}_\alpha \right), \mathcal{K}_\alpha = \sum_{i \in \delta \zeta} \sum_{j \in \alpha} P_{ij}$$
\[ A = A_a \tilde{A}_a = A_t \tilde{A}_t = A_c \tilde{A}_c \]

where, for example,
\[ \tilde{A}_a = \frac{1}{10} \left( 1 - K_{2a} \right), \quad K_{2c} = \sum_{i \in c} \sum_{j \in c} \frac{p_i}{p_{ij}} \]  

At this stage we have two functions, depending on relative momenta \( \vec{p}_{2a}, \vec{q}_c \) in the three nucleon system. We use the standard definitions:
\[ \vec{p}_{2a} \rightarrow \vec{p}_a - \frac{\vec{p}_b}{2} \quad \text{and} \quad \vec{q}_c \rightarrow -\vec{p}_c \], etc.
When projecting the Schroedinger equation
\[ (H - E) \psi = 0 \]  

we find the connection \( \mathcal{A} \) and \( \varphi \). Projecting at \( \mathcal{A}_1 \), one obtains
\[ (\Xi V - \Xi) \varphi (\Xi) + \Xi \sum_{j \neq a} \int \left( \eta (\Xi) f_j (\Xi, \Xi') \right) \varphi (\Xi) \frac{d^3 p_j}{(2\pi)^3} = 0 \]  

where \( d_j, f_j = (a b), (c a), (b c) \) and it is denoted \( \Xi = 2 - \frac{3}{4} q_{2a}^2 \) and \( \Xi' = 2 - \frac{3}{4} q_{2b}^2 \). \( \Xi \) is the three nucleon kinetic energy in the center-of-mass frame. Here and below we set \( M_\text{N} = 1 \).

Projecting onto \( \mathbf{B}_1 \) yields
\[ (\mathbf{H}_0 - \Xi) \mathbf{B}_1 (\Xi, \Xi) + \sqrt{10} \sum_{j \neq a} f_j (\Xi, \Xi') \varphi (\Xi) = 0 \]  

\[ \Xi = \frac{1}{10} \left( 1 - K_{2a} \right), \quad K_{2c} = \sum_{i \in c} \sum_{j \in c} \frac{p_i}{p_{ij}} \]
where \( \tilde{\mathcal{H}}_o = \frac{\hbar^2}{2m} \rho^2 + \frac{3}{4} \mathbf{q} \cdot \mathbf{q} \). We define now the Paddeev decomposition of function \( \mathcal{H}(\vec{p}_\alpha, \vec{p}_\beta) \), letting

\[
\mathcal{H}_x = \mathcal{H}(\vec{p}_\alpha, \vec{p}_\beta) = \frac{1}{\mathcal{Z} - \mathcal{Z}_x - \frac{3}{4} \mathbf{q}_x^2} \mathcal{Z} \mathcal{N}(\vec{p}_\alpha, \vec{p}_\beta) \mathcal{V}(\vec{p}_\alpha) \mathcal{V}(\vec{p}_\beta)
\]

(40)

Substituting \( \mathcal{H}_x \) into (38) and making use of (15) for \( \Delta(2) \), we obtain

\[
(2\mathcal{Z}_x - \mathcal{Z}_x + \Delta(2)) \mathcal{V}(\vec{q}_x) + \mathcal{N}_x \int \mathcal{V}(\vec{p}_\alpha) \mathcal{N}(\vec{p}_\alpha, \vec{p}_\beta) \frac{\alpha^2 \gamma^2}{(2\mathcal{Z}_x - \mathcal{Z}_x)^3} \mathcal{V}(\vec{p}_\beta)
\]

(41)

Performing integration in Eq. (41) and omitting channel indeces, we finally obtain

\[
(2\mathcal{Z}_x - \mathcal{Z}_x + \Delta(2)) \mathcal{V}(\vec{q}_x) - 2 \mathcal{N}_x \int \mathcal{V}(\vec{q}_x) \mathcal{N}(\vec{q}_x, \vec{q}_y) \mathcal{V}(\vec{q}_y) \frac{\alpha^2 \gamma^2}{(2\mathcal{Z}_x - \mathcal{Z}_x)^3} \mathcal{V}(\vec{q}_y)
\]

(42)

where

\[
\mathcal{V}(\vec{q}_x) = \left[ \frac{1}{2} \mathbf{q} \cdot \mathbf{q} + \frac{1}{2} \mathbf{q} \cdot \mathbf{q} \right], \quad \mathcal{V}(\vec{q}_y) = \left[ \frac{3}{4} \mathbf{q} \cdot \mathbf{q} + \frac{1}{4} \mathbf{q} \cdot \mathbf{q} \right]
\]

(43)

When immitting the first term (proportional \( \chi \)) in the right hand side of equation (42), the obtained equation coincides with the usual Paddeev equation for the wave function of spectator nucleon, the kernel of this equation being defined by the two-body off-shell T-matrix (15). So, as it was claimed above, the two-body forces in the QCB model, corresponding to the bags with \( \chi \neq 0 \), do not change in many-body systems. If \( \chi = 0 \), then the kernel of Paddeev equation can be
expressed in terms of modified off-shell T-matrix

\[ \tau \left( i, \tilde{e}', z \right) = \frac{\tau_{ij}(\tilde{e}) \tilde{e}'_i \tilde{e}'_j}{2 - 2\, \sqrt{1 - \Delta(z)}} \]  

where \( \tau_{ij} \) is defined by Eq. (31). This T-matrix is transposed with respect to the T-matrix in Eq. (30), which coincides with (15) only on energy shell.

To find the expression for QCB potential, corresponding to the T-matrix (44), we again return to the coupled system (36), (39) and try to express \( \Phi \) through \( \tau \). To perform this we at first rewrite the equation (38) in the form

\[ \left( \nu - \lambda \right) \left( \tilde{e} - \frac{1}{2} \lambda \tilde{e} \right) \Phi(i, \tilde{e}, z) + \lambda \sum_{\alpha} \int_{\Omega} \Phi_{i, \alpha}(\tilde{p}, \tilde{e}, \tilde{z}, x) \Phi_{\alpha, j}(\tilde{p}, \tilde{e}, \tilde{z}, x) \frac{d^3 p_{ij}}{(2\pi)^3} \]

For the second term in (45) one can substitute Eq. (39), and, after solving (45) for \( \Phi \), obtain

\[ \Phi(i, \tilde{e}, z) = \frac{1}{\lambda - \nu} \frac{1}{\tilde{e} - \sqrt{1 - \Delta(z)}} \int_{\Omega} \tau_{ij}(\tilde{p}, \tilde{e}, \tilde{z}, x) \Phi_{i, \alpha}(\tilde{p}, \tilde{e}, \tilde{z}, x) \frac{d^3 p_{ij}}{(2\pi)^3} \]

where \( \tau_{ij} \) is defined by (31). Substituting (46) again into (39), we obtain

\[ \left( \tilde{p}^2 - \frac{\tilde{e}}{2} + 2 \right) \Phi(\tilde{p}, \tilde{e}) + \sum_{\alpha} \int_{\Omega} \Phi_{\alpha}(\tilde{p}, \tilde{e}, \tilde{z}, x) \Phi_{i, \alpha}(\tilde{p}, \tilde{e}, \tilde{z}, x) \frac{d^3 p_{ij}}{(2\pi)^3} \]

where

\[ \Phi(i, \tilde{e}, z) = \frac{1}{\lambda - \nu} \frac{1}{\tilde{e} - \sqrt{1 - \Delta(z)}} \int_{\Omega} \tau_{ij}(\tilde{p}, \tilde{e}, \tilde{z}, x) \Phi_{i, \alpha}(\tilde{p}, \tilde{e}, \tilde{z}, x) \frac{d^3 p_{ij}}{(2\pi)^3} \]

The potential (48) is transposed to the potential in (28), as expected.
Eqs. (47), (48) are the main result of the present Section. In particular, Eq. (48) shows how the QCB interaction modifies in nucleus. It is clear, that our conclusions can be applied for nuclear system with arbitrary number \( N \) of nucleons; the difference would be in appearance of the factor \( N - 1 \) instead of 2 in the right side of equation (42).

It can be easily verified by direct counting the powers of momenta that the kernel \( K \) of modified Faddeev equation is an \( L^2 \)-operator for \( Z \leq 0 \). Indeed, \( \sum \kappa(z) \kappa^T(z) \) converges at the upper limit as \( \frac{1}{1 + z^2} \) for \( \kappa \neq 1 \) and as \( \frac{1}{1 + z} \) for \( \kappa = 1 \).

The reason of QCB potential to be modified in nucleus is connected to the physical picture lying in the cluster ansatz (1). At small distances three-quark bags are contained either in QCB (in this case they are included in \( \Psi_{\alpha} \)) or in nucleon component \( \Psi_{\nu} \). The kernel of the integral equation (42) contains the \( u \)-channel exchanges of the three-quark bags with nucleon quantum numbers, the bags being both in \( \Psi_{\alpha} \) and \( \Psi_{\nu} \). The exchange of three-quark bag from \( \Psi_{\alpha} \) leads to usual Faddeev kernel which is expressed in terms of NN amplitude. The exchange of three-quark bag from \( \Psi_{\nu} \) gives an additional contribution modifying the initial QCB potential. Clearly, this is a typical many-body effect.

**Conclusions**

We have shown that the parameters of QCB potential are to satisfy rather restricted conditions, imposed by existing \( P \)-matrix analysis of NN scattering. In the next papers we shall give the results of the description of NN scattering in \(^1S_0\).
and $^3S_1 - ^3D_1$ channels and the wave function of deuteron in
the QCB model. There exist many results which can be obtained
by means of QCB potential and compared with experiment. They
are the behaviour of deuteron electromagnetic form-factors for
large momentum transfer, the weight of the high-momentum com-
ponent of the deuteron wave function and that of $^3\text{He}$ and $^3\text{He}$,
connected to the nuclear processes at small distances. One of
the most serious tests of the QCB potential is using it in cal-
culation of the binding energies of light nuclei. The studying
of all these items are now in progress.

The authors are indebted to A.I. Veselov and I.L. Grach
for useful remarks and to Yu.A. Simonov and K.A. Ter-Martirosyan
for useful discussions.
Fig. 1. Eigenvalues trajectories for one level QCB potential.

Fig. 2. Eigenvalues trajectories for $^1S_0$ QCB potential.
Fig. 3. Eigenvalues trajectories for $^3S_1$ QCB potential.
References


   press).


   p. 1542.

   ITEP-93, 1983.

   p 349.


12. Veselov A.I., Grach I.L., Narodetskii I.M. Yad.Fiz.,1984,
    v. 39, p. 719.

13. Bakker B.L.G., Grach I.L., Narodetskii I.M. De Vrije
    University Preprint, 1983.

    p. 435. Harvey, ref. /1/.

