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SUMMARY OF THE WORKING GROUP ON FEL THEORY*†

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I. Introduction

The working group on FEL theory dedicated most of its discussions to topics relevant to the high gain regime in a free electron laser. In addition the area of interst was mainly restricted to FELs for the production of XUV radiation (< 1000 Å). A list of the topics that were felt to be relevant is:

1) characterization of the FEL high gain regime;
2) the amplified spontaneous emission mode of operation (ASE);
3) superradiance in FELs;
4) diffraction effects for high gain FELs;
5) noise and start-up;
6) coherence properties of the radiation for the ASE and superradiant FELs.

Other important problems, like for instance quantum effects for short wavelength FELs, had been considered in the Brookhaven Workshop on Free Electron Generation of XUV Coherent Radiation; a summary of the discussions held at that meeting can be found in reference 1.

Because of the limited time available it was not possible to discuss and reach a consensus on all of these problems. For some of them, in particular for topics 5 and 6, it was only possible to conclude that more work is needed. On topics 1 to 4 much work has been done during the last year or two and new results are reported in papers appearing in these proceedings[2-5] and will be summarized here.

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2. High gain regime

The energy transfer between the electron beam and the radiation in an FEL can be enhanced by a collective instability producing an exponential growth of the radiation. When this instability becomes important the FEL is said to operate in the High Gain Regime. The existence of this regime is very important for the FEL operation in the XUV region where we do not have optical components with sufficiently high reflectivity and small absorption.

The FEL performance can be characterized by one parameter[6], ρ:

$$\rho = (K\Omega_p/4\omega_o)^{2/3} \tag{1}$$

where the plasma and undulator frequencies are

$$\Omega_p = (4\pi r_e c^2 n_o/\gamma^3)^{1/2} \tag{2}$$

$$\omega_o = 2\pi c/\lambda_o \ . \tag{3}$$

The quantities introduced in (1), (2), (3) are: the classical electron radius r_e; the light velocity c; the electron beam density n_o; the electron energy, γ, in unit of rest mass mc^2; the undulator period, λ_o, magnetic field, B_o, and parameter $K = eB_o\lambda_o/2\pi mc^2$. We will also use the radiation wavelength, λ, related to the electron resonant energy, γ_R by

$$\lambda = \lambda_o(1 + K^2)/2\gamma_R^2 \ .$$

Using the parameter rho, and assuming zero detuning, i.e. $\gamma = \gamma_R$ the gain per unit undulator period is given by

$$G_p = 3^{1/2}2\pi\rho \ . \tag{4}$$

Equation (4) is valid as long as the condition that the electron beam relative energy spread, σ_E, be less than ρ is satisfied.

The evolution of the radiation field amplitude, A, or of the electron beam bunching, is described by an exponential if the system starts from a small initial field or an equivalent noise level:

$$A = A_0 \exp(G_p N_w) \tag{5}$$

where N_w is the number of undulator periods. The high gain regime is defined by the condition

$$G_p N_w > 1 \; . \tag{6}$$

The exponential growth slows down and saturates when the electrons are trapped in the potential well formed by the radiation. This condition can be approximately extimated by requiring that the electron oscillation period in the wave potential well be of the order of the exponential growth rate. This gives for the wave electric field amplitude a value

$$E_s = cB_0 (2\rho\gamma/K)^2 \tag{7}$$

Using (7) we can calculate the radiation field peak power, P_L, at saturation, obtaining the simple result that P_L is simply ρ times the electron beam peak power, $P_B = mc^2\gamma I_p$, where I_p is the electron peak current:

$$P_L = \rho P_B \; . \tag{8}$$

Hence, ρ measures also the efficiency of energy transfer from the electron beam to the radiation field. Notice also that at saturation the radiation power, given by (8), is proportional to $N_e^{4/3}$, N_e being the number of electrons.

3. The Amplified Spontaneous Emission mode of operation of a FEL

If we let one electron traverse an undulator, it will produce spontaneous undulator radiation. What happens when we consider an electron beam with N_e particles in it? In doing the sum of the field amplitudes, we must attribute to each electron a phase factor $\exp(i\,\phi_0)$, ϕ_0 being the initial phase of the electron relative to the wave. To determine the distribution of the phases we must study the electron longitudinal density distribution on the scale of the radiation wavelength. For a beam with a uniform longitudinal density distribution, the sum over the initial phase factors would give zero, and no radiation would be observed. A random distribution of the phases, like is observed in a real beam, gives an intensity proportional to the number of electrons, N_e.

In doing this calculation one assumes that the beam intensity is small and that as a consequence also the intensity of the radiation is small so that the radiation field does not influence in any significant way the particle dynamics. If we let the beam intensity grow, or, alternatively we consider a very long undulator, this condition is violated and we must study the evolution of the system, starting from noise, including the action of the radiation field on the particle motion. This leads to amplification of the radiation, and to the Amplified Spontaneous Emission (ASE), mode of operation in which $P_L \sim N_e^{4/3}$. The study of the ASE has been done in ref. 6,7, 8. To be in this mode one must clearly satisfy the condition (6) for high gain. Furthermore to reach saturation in this mode one needs a value of the exponential gain factor, $G_p N_w$, of the order of 10, if the beam has an energy spread smaller than ρ.

The main advantage of operating an FEL in the ASE mode, as compared to an FEL oscillator using an optical cavity, is that one does not need any optical elements. This advantage is important at short wavelength, where good mirrors are not available. Compared to a TOK the ASE can provide a much higher efficiency of energy transfer from the electrons to the radiation. The disadvantage of the ASE is that it requires a large value of the gain. $G_p N_w \approx 10$, while for an oscillator it is sufficient to have a gain on the order of one and for TOK less than one.

4. Superradiance in FELs

As we discussed in the previous sections the maximum laser power obtainable in an FEL is limited by saturation effects and is proportional to $N_e^{3/4}$. This is correct under the condition that the undulator has constant period and magnetic field and that slippage effects are negligible.

Tapering of the undulator[9] is a way to remove this limitation. Another possibility has been discussed at this meeting by Bonifacio and Casagrande[3]. They study the ASE when the electron bunch is shorter than the slippage distance $s \approx \lambda N_w$; in this case some of the radiation produced will escape from the bunch during the transversal of the undulator. In this way one can remove radiation from the bunch and avoid saturation. On the other hand the bunch must have a large enough density to be in he high gain regime, so one can define a region in the parameter space of the FEL where ASE is not limited by saturation and the radiation intensity can grow up to the point where it is proportional to the square of the electron number.

This superradiant regime is very exciting not only as an interesting aspect of the FEL physics but also because it can provide the capability of producing very short radiation pulses with large peak power.

5. <u>Diffraction effects for high gain FEL</u>

The expression for the gain given in Section 2, is valid only in the limit of a one dimensional problem, when the radiation is described by a plane wave.

Using simple geometrical optics, for an electron beam of radius "a", radiation wavelength λ, radiation waist w_0 and undulator length L_w, we can expect diffraction effects to be important when

$$\frac{\lambda}{w_0} L_w > a \; . \tag{9}$$

In most cases one can assume $a \approx w_0$ and introduce the Raileigh range $Z_R = \pi a^2/\lambda$. The condition for diffraction to be negligible and the one dimensional theory to be applicable can then be written as

$$\frac{Z_R}{L_w} > 1. \tag{10}$$

For small emittance beams and long undulator, both needed for high gain, this conditon can be easily violated.

This problem has been studied in the papers by Moore[4] and by Scharlemann, Sessler and Wurtele[5] presented at this meeting. Thes authors show that the electron beam can produce a guiding action on the radiation, similar to that provided by an optical fiber. The electron beam has an index of refraction, n, whose real and imaginary parts are related, in the one dimensional model, to the phase and amplitude derivative of the radiation field and thus to the beam bunching. The real and imaginary part of the index of refraction describes refractive guiding when Re(n) > 1, and gain focusing where Im (n) > 1. For an FEL both terms are present and can play a dominant role in different conditions.

A simple and qualitative of describing this effect is to introduce an "effective Raileigh range", Z_R^*, larger than the geometrical optics Z_R. The ratio Z_R^*/Z_R is a measure of the beam guiding action, and is again given by the FEL parameter ρ:

$$Z_R^*/Z_R = 4\pi\rho N_w \ . \tag{11}$$

One can rewrite the condition (10) substituting Z_R^* to Z_R, as

$$4\pi\rho \frac{Z_R}{\lambda_w} > 1 \ . \tag{12}$$

This condition can be used to estimate the applicability of the one dimensional model. If (12) is violated diffraction effects are important and the gain of the system is less than that given in the one dimensional model.

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