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L'ÉNERGIE ATOMIQUE DU CANADA LIMITÉE

POLE SHIFTING WITH CONSTRAINED OUTPUT FEEDBACK: AN OPTIMIZATION PROBLEM

Positionnement de pole avec retroaction sous contrainte: un problème d'optimization

D. HAMEL, S. MENSAH and J. BOISVERT

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Le concept de positionnement de poles joue un rôle important dans la commande de systèmes multivariables linéaires. Depuis son avènement, le positionnement de pôles a retenu l'attention de nombreux chercheurs, et de nombreux algorithmes sont maintenant disponibles.

Ce rapport présente une nouvelle approche au positionnement de pôles. En formulant le problème comme un problème d'optimisation avec contraintes, il devient possible d'introduire à l'étape de conception du contrôleur des contraintes physiques pratiques, telles la limitation de gains et la structure du contrôleur. Ces contraintes délimitent une région admissible pour la matrice de rétroaction.

La configuration désirée des valeurs propres en boucle fermée se traduit par la définition d'une fonction de coût appropriée offrant une description mathématique du concept de positionnement de poles. Le problème d'optimisation avec contraintes peut alors être résolu par des algorithmes d'optimisation.

La méthode décrite est implantée dans un module interactif de MVPACK, un progiciel de conception assistée par ordinateur pour la commande de systèmes.

L'utilisation de la méthode est illustrée par la conception de régulateurs pour un aéronef et un évaporateur. Les résultats démontrent l'importance de la structure du contrôleur sur la performance globale d'un système.

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ABSTRACT

The concept of pole placement plays an important role in linear, multivariable, control theory. It has received much attention since its introduction, and several pole shifting algorithms are now available.

This work presents a new method which allows practical and engineering constraints such as gain limitation and controller structure to be introduced right into the pole shifting design strategy. This is achieved by formulating the pole placement problem as a constrained optimization problem. Explicit constraints (controller structure and gain limits) are defined to identify an admissible region for the feedback gain matrix.

The desired pole configuration is translated into an appropriate cost function which must be closed-loop minimized. The resulting constrained optimization problem can thus be solved with optimization algorithms.

The method has been implemented as an algorithmic interactive module in a Computer-Aided Control System Design Package, MVPACK.

The application of the method is illustrated to design controllers for an aircraft and an evaporator. The results illustrate the importance of controller structure on overall performance of a control system.

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PARTIAL NOMENCLATURE

Symbol	Description	Defining Equation
A	Plant dynamics matrix	(1)
A	Plant closed-loop matrix	(5)
B	Plant input matrix	(1)
C	Plant output matrix	(1)
d(K)	Minimization cost function	(4)
BD	Plant disturbance matrix	(18)
d	Plant disturbance vector	(18)
K	Feedback matrix	(2)
K*	Feedback matrix minimization solution	(6)
m	Number of inputs	(1)
n	Number of states	(1)
p	Number of outputs	(1)
u	Plant input vector	(1)
x	Plant state vector	(1)
y	Plant output vector	(1)
F(K)	Constraint mapping function	(3)
Greek letters		
α, α_i	Cost function weight factors	(4)
κ	Feedback matrix subspace	(3)
λ_i	Actual closed-loop system eigenvalues	(4)
μ_i	Desired closed-loop system eigenvalues	(4)

1. INTRODUCTION

The concept of pole placement or pole shifting plays an important role in the design of control and regulating systems. Since the fundamental work of Wonham [1], pole shifting using only system outputs has received much attention [2-11] because of its importance from a practical standpoint. Indeed, all the states of a given system are not always measurable, and the use of observers or Kalman filters to reconstitute the non-available states often leads to unnecessarily complex feedback laws.

The objectives of any process or plant regulator are stability augmentation, sensitivity reduction, and good steady state tracking. Often the control system must also comply with practical engineering and physical constraints. The aim of pole shifting techniques is to shape the dynamic response of a system by assigning its eigenstructure. Unfortunately, most design techniques cannot incorporate practical engineering and physical constraints right into the design strategy. Thus the designed controller must go through time consuming, manual tuning with a possible degradation in the overall performance. Optimization-based design techniques have the potential to circumvent this type of problem. The underlying principles of these techniques are:

- define cost functions or performance indices to describe the control system specifications,

- select the practical engineering constraints which must not be violated, and

- derive an algorithm to solve the associated optimization problem and compute the control law.

This report describes such a flexible and powerful technique. The underlying theory of the method is briefly developed in Section 2. The implementation of the resulting design algorithm is discussed in Section 3. The general design procedure recommended for efficient use of the technique is outlined in Section 4, and the application of the technique to design control systems for an aircraft and an evaporator is given in Section 5. The capability of the method to handle large systems, for example a nuclear steam generator, is also investigated in Section 5.

2. FORMULATION AND THEORY

2.1 Objective

Consider the linear, time-invariant system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \qquad (1)$$

where

$x(t)$ is the state vector, of dimension n

$u(t)$ is the input vector, of dimension m

$y(t)$ is the output vector, of dimension p

The system is assumed to be completely controllable and observable and cyclic [12,13].

The objective is to find an output feedback control law

$$u(t) = Ky(t) \tag{2}$$

which, when coupled to the system of equations (1), will assign to the closed-loop system an eigenvalue

spectrum $(\lambda_1, \lambda_2, \ldots \lambda_n)$ arbitrarily close to a desired

spectrum $(\mu_1, \mu_2, \ldots \mu_n)$. The feedback matrix is obtained so that

$$K \in \kappa \tag{3}$$

where

κ is a given admissible region defined by a constraint mapping function $F(K)$.

The constraint mapping function $F(K)$ determines the structure (i.e. active gains) and the amplitude range of the feedback matrix gains.

Given the feedback control law (2) and the constraints $(F(K), \kappa)$, it is natural to deal with this constrained pole placement problem via optimization techniques. This approach allows the method to be derived in two steps. In the first step, called "Problem Definition", the control problem is stated in terms of an optimization problem. In the second step, "Problem Solution", general or specialized algorithms can be used to find the solution.

2.2 Definition of the Optimization Problem

Given the constraints $(F(K), \kappa)$, the optimization problem is well posed if an appropriate cost or performance function can be defined. In general, the definition of a cost function is difficult. A cost function has to describe mathematically a practical performance criterion. In addition, for the existence of a solution, the cost function must have continuity property over at least a subspace of the admissible region.

In this study, the cost function has to include a mathematical description of the pole shifting concept. The following cost function defined by Shapiro et al. [14] has all the desirable properties:

$$d(K) = \sum_{i=1}^{n} \alpha_i |\lambda_i - \mu_i|^2 \qquad (4)$$

where

μ_i are the ordered desired closed-loop eigenvalues

λ_i are the ordered actual closed-loop eigenvalues

$\alpha_i > 0$ are weight factors which can be selected by the designer.

Now the definition of the optimization problem can be stated as follows:

Given the system described by equation (1), find the feedback law (2) so that the closed-loop matrix

$$\tilde{A} = A + BK^*C \qquad (5)$$

has its eigenvalue spectrum arbitrarily close to a desired spectrum, and where the feedback matrix K* is determined by

$$d(K^*) = \min_{K \in \kappa \,:\, F(K)} d(K) \qquad (6)$$

2.3 Solution of the Optimization Problem

2.3.1 Continuity of the Cost Function

For the existence of a solution, the cost function must have the property of continuity over at least a subspace of the admissible region.

The continuity property of d(K) can be established if the two sets $(\lambda_1, \lambda_2, \ldots. \lambda_n)$ and $(\mu_1, \mu_2, \ldots.. \mu_3)$ are ordered in a consistent manner. The following criterion is used to order the eigenvalues:

i < j if

(i) $\operatorname{Re} \lambda_i > \operatorname{Re} \lambda_j$ (7)

or

(ii) $\operatorname{Re} \lambda_i = \operatorname{Re} \lambda_j$ and $\operatorname{Im} \lambda_i \geq \operatorname{Im} \lambda_j$ (8)

Shapiro [14] has demonstrated, with an equivalent ordering criterion, that d(K) is continuous if the closed-loop system matrix has distinct eigenvalues.

For large systems, the solution of equation (6) is one of the most difficult mathematical as well as numerical problems [15,16,17]. However, the problem has received much attention and many algorithms are available [18,19,20].

MVPACK, a Computer-Aided Control System Design (CACSD) package [21], has an optimization module based on the Rosenbrock hill climbing technique [18], and mainly for this reason, it was decided to use this algorithm to solve the optimization problem. Among other available optimization algorithms, the hill climbing technique was selected and implemented in MVPACK for its efficiency in solving unconstrained as well as constrained minimization problems. The memory space required by this algorithm is also less than that required by many other algorithms.

2.3.2 Effects of the Constraints

The domain of the cost function d(K) consists of the feedback matrices K such that

$$K \in \kappa : F(K)$$

where

κ is the admissible region defined by the constraint mapping function F(K).

In the case of output feedback, the specification of structural constraints (selected elements of the feedback matrix set to zero) defines κ as a subspace of R^{mxp}:

$$\kappa \subseteq R^{mxp}$$

The specification of gain limits defines hyperplane boundaries in the subspace, thus reducing even more the admissible region.

Given all these restrictions on the domain of d(K), it may become impossible to achieve an exact pole placement (i.e. the solution K* does not lie in κ). In such a case, the computed minimum of the cost function is located on an edge or in a "corner" of κ.

The domain may be enlarged by relaxing active constraints, in order to include the minimum in the domain κ.

2.3.3 Weight Factors

Considering equation (4), the weight factors α_i determine the shape of the slope of d(K) in the corresponding directions $\lambda_i(K)$. Thus, the weight factors directly affect the search for d(K*) by determining the direction of best possible progress.

However, if there exists at least one matrix K* such that d(K*)=0 (exact positioning) in the admissible region, the weight factors do not affect the exact location of K*, but determine the rate at which the closed-loop eigenvalues will converge to their desired location.

When the minimum lies outside the permissible region, the weight factors can be used to drive the solution towards the exact assignment of some eigenvalues.

3. IMPLEMENTATION IN MVPACK

3.1 Algorithm

The proposed method for pole shifting with constrained output feedback leads to a structured design algorithm. The algorithm proceeds in two stages.

Stage 1 - Design Specifications

Step 1 Module Initialization

- determine state of module execution and get system matrices

Step 2 specification/modification of Feedback Matrix Structure

- selection and/or deletion of active gains

Step 3 specification/modification of Gain Limits

Step 4 Initialization of Feedback Matrix

- specification/modification of initial values for gains

Step 5 Specification/modification of Desired Closed-Loop Spectrum

Step 6 Specification/modification of Weight Factors

Stage 2 - Minimization Search

Step 1 Gain Boundaries Update

- real and implicit boundaries are reset for the requirement of the minimization algorithm

Step 2 Feedback Matrix Modification

- feedback matrix is updated with the result of minimization search step

Step 3 Closed-Loop Matrix Computation

Step 4 Closed-Loop Eigenvalues Computation

Step 5 Computation of d(K)

Step 6 Minimization Search Step

Step 7 Analyze Search Status

- if not converged, go back to step 1.
- if converged, return to stage 1 for display of final results and modification of design parameters

This algorithm offers the designer many degrees of freedom, i.e.,

- selection of the structure of the feedback matrix,
- selection of limits on the feedback gains,
- selection of the closed-loop eigenvalues,
- selection of the weight factors.

3.2 Implementation

The entire algorithm is implemented in MVPACK [21] as two modules: MVPPCO and MVMIND.

MVPPCO is a highly interactive, numerically robust module, and it provides the inexperienced user with an extensive help mechanism. It prompts the user to get the design specifications and calls MVMIND, the minimization module.

MVMIND activates the minimization of the cost function. On demand, the user may view intermediate results during the minimization search, to monitor the search progress. Assigned eigenvalues and the computed feedback matrix are displayed. Thus, the designer has the option to return to MVPPCO, the master module, where modifications can be made to the design parameters.

When the result is satisfactory, MVPPCO may upon request call MVGRL to add an integral controller and MVRSIM to perform closed-loop simulation [22].

4. GENERAL DESIGN PROCEDURE

4.1 General

The MVPPCO design specification module allows the designer to input the following input parameters:

- feedback matrix structure
- feedback gain boundaries

- feedback matrix initialization
- desired closed-loop spectrum
- cost function weight factors

The design of a controller using this optimization-based algorithm is an interactive and iterative process. Between iterations, the designer has the option to view the current results and modify design parameters. He can use his theoretical background and engineering experience in defining the design specifications.

4.2 Specification of the Feedback Matrix Structure

For a given control system, the structure of the feedback matrix can be defined to achieve input/output decoupling and reduction of disturbance propagation. Modal analysis and engineering standards can be used to identify the possible structures.

The designer may start with some basic structure to accelerate convergence and subsequently modify it to achieve the desired pole shifting.

The structure can also be selected to make some stable and very fast modes uncontrollable during the pole shifting process. Fine tuning through simulation can then close these loops with a static or dynamic compensator to improve overall performance.

4.3 Specification of Gain Limits

Gain limits can be specified on active gains. The permissible gain value is then limited to the range

$$-|\text{limit}| \leq \text{gain value} \leq +|\text{gain limit}|$$

Gain limits are specified to

(a) respect physical limitations of available amplifiers,

(b) keep all the gains within some ranges, to prevent an ill-conditioned feedback system,

(c) discriminate between the importance of output variables in the control actions.

Initially, there may be no definite rule for specifying gain boundaries. The use of this design feature must be based on engineering judgement, or to force the module to locate another minimum solution in the vicinity of K*.

4.4 Feedback Matrix Initialization

The starting point of the minimization search can be selected by specifying the initial feedback matrix.

Since the feedback matrix is always preserved by the module, automatic re-initialization of the search from the previous final search point can be activated. This feature is used when no minimum of d(K) has been located after a run of the minimization search module.

The starting point feature is chiefly used to accelerate the minimization search, or to look for local minima.

4.5 Selection of Desired Closed-Loop Eigenvalues

Contrary to most existing pole shifting methods, the optimization method described in this report can assign the entire closed-loop spectrum.

The desired closed-loop eigenvalues are selected to meet control system requirements. The closed-loop system must be stable and have sufficient phase and gain margins. Transients must be sufficiently damped with minimal overshoot and acceptable settling time.

The desired closed-loop eigenvalues should all be distinct.

Fine tuning of spectrum configuration can be achieved through analysis of closed-loop simulation.

4.6 Selection of Weight Factors

The effects of weight factors on the minimization search were discussed in Section 2.3.3.

Experience gained with MVPACK showed that initial weight factors, selected as the inverse of the norm of the corresponding desired closed-loop eigenvalues, distribute the shifting effort on the eigenvalues.

The rate of convergence of a given closed-loop eigenvalue can be speeded up by increasing the value of the corresponding weight factor. The weight factors which should be modified can be identified after only a few minimization steps.

5. APPLICATIONS

5.1 Introduction

To illustrate the capability of the technique of pole shifting with constrained output feedback, controllers for an aircraft, an evaporator, and a nuclear steam generator are studied in this section.

5.2 Aircraft

5.2.1 Description of the Model

For a simplified flight control problem, dealing with the inner loop lateral axis, the aircraft dynamics can be described by the differential equations [14]

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) \qquad (12)$$

where

vector x defines the state variables:

p_s	stability axis roll rate	rad/s
r_s	stability axis yaw rate	rad/s
β	angle of sideslip	rad
δ_a	aileron deflection	rad
δ_r	rudder deflection	rad
θ	bank angle	rad

The input and output vectors are, respectively,

$$u = \begin{bmatrix} \delta_{rc} \\ \delta_{ac} \end{bmatrix} \quad \begin{matrix} \text{rudder command} \\ \text{aileron command} \end{matrix} \qquad y = \begin{bmatrix} p_s \\ r_s \\ \beta \\ \delta_a \\ \delta_r \end{bmatrix}$$

The elements of matrices A, B and C are given in Appendix A.

The aircraft movements are defined from three axes: Normal (or Vertical), Lateral (or Horizontal) and Longitudinal.

5.2.2 <u>Open-Loop Modal and Simulation Analysis</u>

Modal analysis, performed on the open-loop system to identify the relationship between modes and state variables, shows that the system is unstable. The open-loop eigenvalues are:

Mode	Eigenvalue
1,2	$3.176 \times 10^{-5} \pm j0.7$
3	-0.2
4	-0.778
5	-5.0
6	-10.0

Modes 5 and 6 are associated with the aileron actuator and the rudder actuator, respectively. Simulation analysis was required to identify the distribution of the remaining modes.

Figure 1 shows that the initial transient of the yaw rate is dominated by a damped mode, with a time constant of approximately 5 seconds. This corresponds to mode 3, the aircraft spiral mode.

Figure 2 shows that the initial transient of the roll rate is heavily damped, with a time constant of approximately 1.25 seconds. This corresponds to mode 4, the roll subsidence mode.

In Figure 3, the period of oscillation is approximately 9 seconds. Thus complex modes 1 and 2 may be identified as the "dutch" roll mode. These modes are certainly associated with sideslip. Furthermore, Figures 1 to 3 show that the dynamic coupling of the system is mainly due to the unstable complex pair.

5.2.3 Selection of a Proper Feedback Matrix Structure

For this aircraft model, equation (12), the most general output feedback law is given by

$$\begin{bmatrix} \delta_{rc} \\ \delta_{ac} \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} \end{bmatrix} \begin{bmatrix} P_s \\ r_s \\ \beta \\ \delta_a \\ \delta_r \end{bmatrix} \qquad (13)$$

The rudder command signal should be a function of the sideslip angle, the rudder angle, and the yaw rate. To reduce interaction between control actions, the rudder control should not respond directly to changes in the roll rate and aileron deflection.

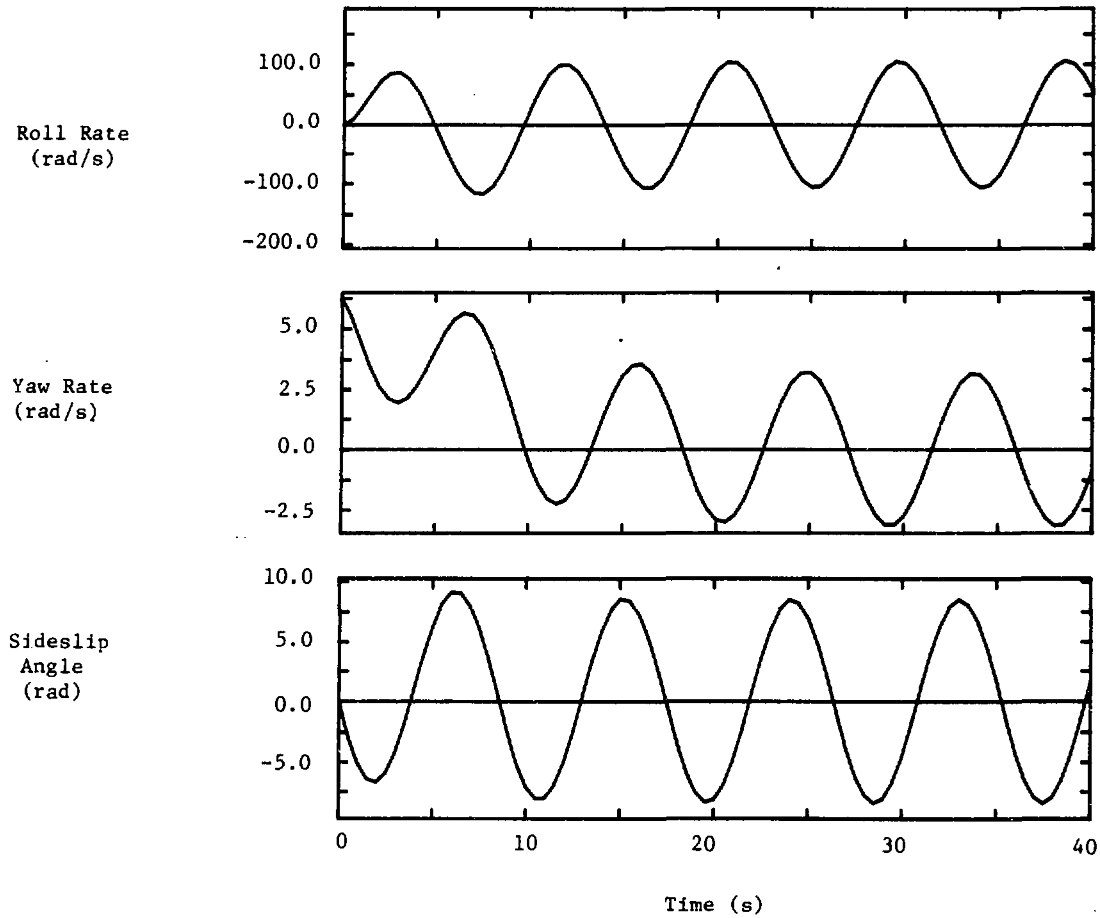

FIGURE 1 Aircraft Open-Loop Response to an Initial Yaw Rate Error of 6.283 rad/s

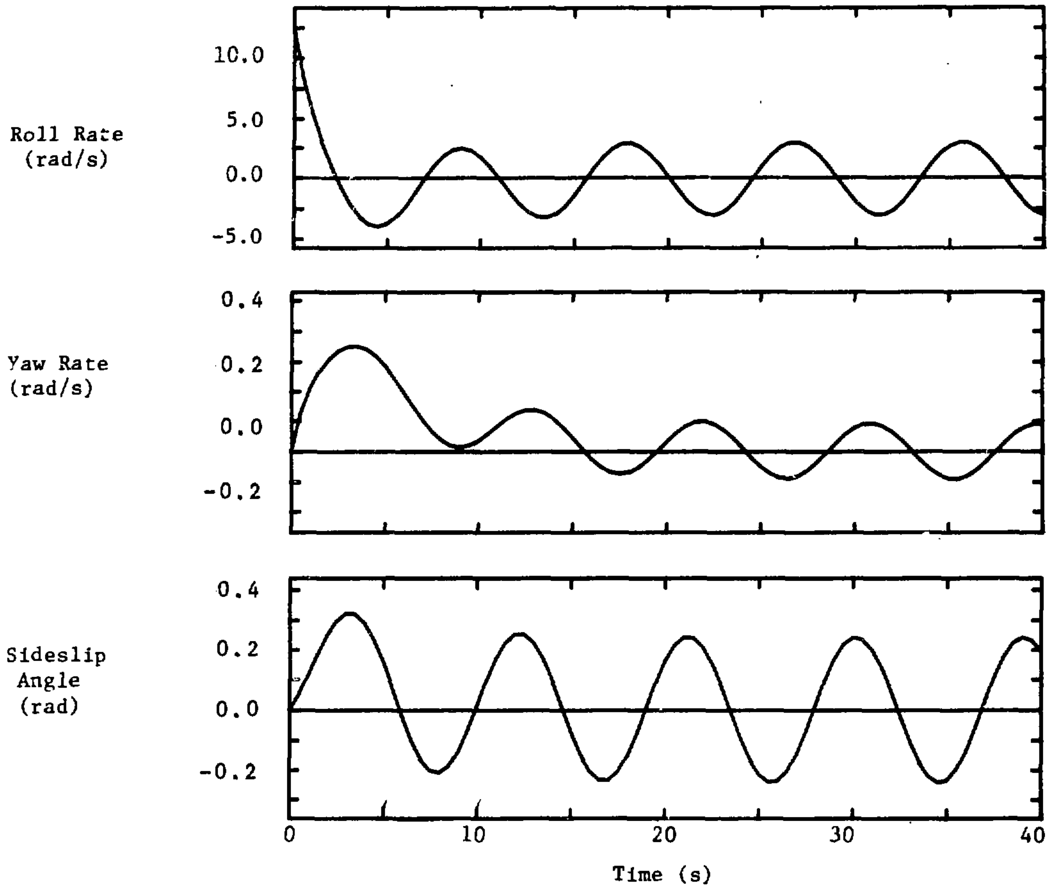

FIGURE 2 Aircraft Open-Loop Response to an Initial Roll Rate Error of 12.566 rad/s

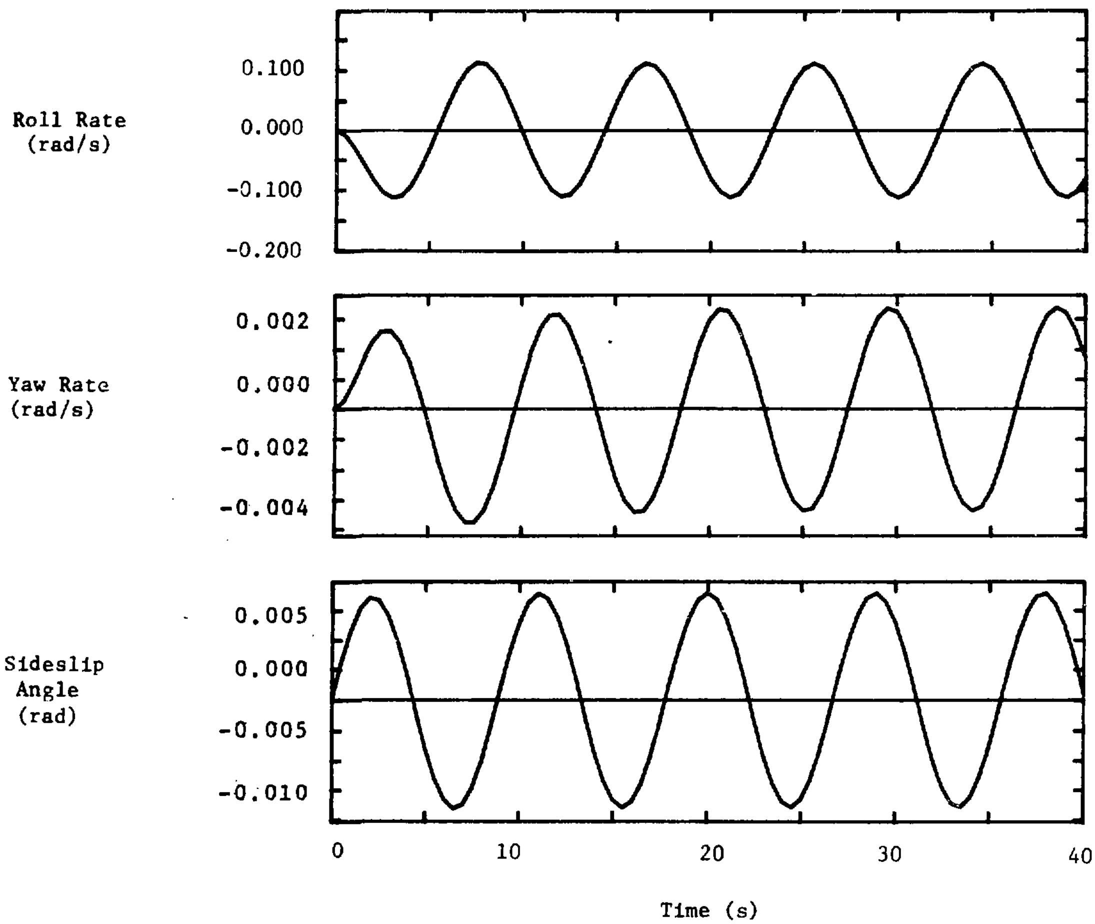

FIGURE 3 Aircraft Open-Loop Response to an Initial Bank Angle Error of 10°

The main objective of the aileron command is to control the bank angle via the sideslip angle (the bank angle is not an output), the roll rate, and the aileron deflection.

To meet these practical engineering requirements, the following structure was first selected for the feedback matrix:

$$K = \begin{bmatrix} 0 & K_{12} & K_{13} & 0 & K_{15} \\ K_{21} & 0 & K_{23} & K_{24} & 0 \end{bmatrix} \qquad (14)$$

The aileron feedback can be further simplified by removing the sideslip angle feedback. Indeed, sideslip control should remain the responsibility of the rudder. However, depending on the type of the aircraft, the aileron movement can induce a sideslip as a secondary effect, due to aileron drag. This problem is also investigated by selecting

$$K = \begin{bmatrix} 0 & K_{12} & K_{13} & 0 & K_{15} \\ K_{21} & 0 & 0 & K_{24} & 0 \end{bmatrix} \qquad (15)$$

5.2.4 Desired Closed-Loop Eigenvalues

To achieve satisfactory performance, the desired closed-loop poles are selected as:

$$-0.05$$

$$-1.77 \pm j1.77$$

$$-4.0$$

$$-100.0$$

$$-200$$

5.2.5 Minimization Results

For the feedback structure of equation (14), MVPPCO was activated with the following input parameters:

- Weight factors: α_i = [20, 0.4, 0.25, 0.25, 0.01, 0.005]
- Initial feedback matrix: all gains set to zero
- Gain limits: no gain limits

The computed feedback matrix after 600 iterations is

$$K = \begin{bmatrix} 0 & 18.14 & -37.18 & 0 & -9.66 \\ -5.59 & 0 & 4.17 & -9.84 & 0 \end{bmatrix} \qquad (16)$$

and the assigned closed-loop eigenvalues are:

$$\begin{array}{l} -0.05 \\ -1.77 \pm j1.77 \\ -4.0 \\ -100 \\ -200 \end{array}$$

showing that exact pole shifting was achieved.

For the feedback structure of equation (15), MVPPCO was activated with the same input parameters, except for the first weight factor, which was set to 60.0.

The computed feedback matrix after 950 iterations is

$$K = \begin{bmatrix} 0 & 17.82 & -37.03 & 0 & -9.66 \\ -5.76 & 0 & 0 & -9.85 & 0 \end{bmatrix} \qquad (17)$$

and assigned closed-loop eigenvalues are:

$$-0.06$$

$$-1.77 \pm j1.77$$

$$-4.0$$

$$-100$$

$$-200$$

showing that all the closed-loop eigenvalues were correctly assigned, except for the first pole.

The performance of both feedback matrix structures was assessed via closed-loop simulation. Figure 4 shows that the transients are practically identical. Control of the roll rate by controller (16) is slightly better. Figure 5 also shows equal performance for both controllers. In Figure 6, however, controller (17) achieves slightly better control on the sideslip. For both controllers, the roll rate response is independent of the slowest mode. This indicates that dynamic decoupling is achieved by both controllers.

Controller (17) can also be viewed as controller (16) with a failure in the connection between sideslip signal and aileron command. The simulation results show that such a failure does not induce any degradation in the overall performance of the aircraft.

5.3 Evaporator

5.3.1 Description of the Model

The differential equations for an evaporator [23], linearized about a certain operating point, can be written as

$$\dot{x}(t) = Ax(t) + Bu(t) + BDd(t) \qquad (18)$$

$$y(t) = Cx(t)$$

The elements of vectors x, u, d and y are defined as normalized perturbation variables. Their definitions in terms of the process variables and the elements of matrices A, B, C and BD are given in Appendix B.

5.3.2 Open-Loop Modal Analysis

Modal analysis is performed on the open-loop system to identify the relationship between the modes and the state variables. The open-loop eigenvalues are

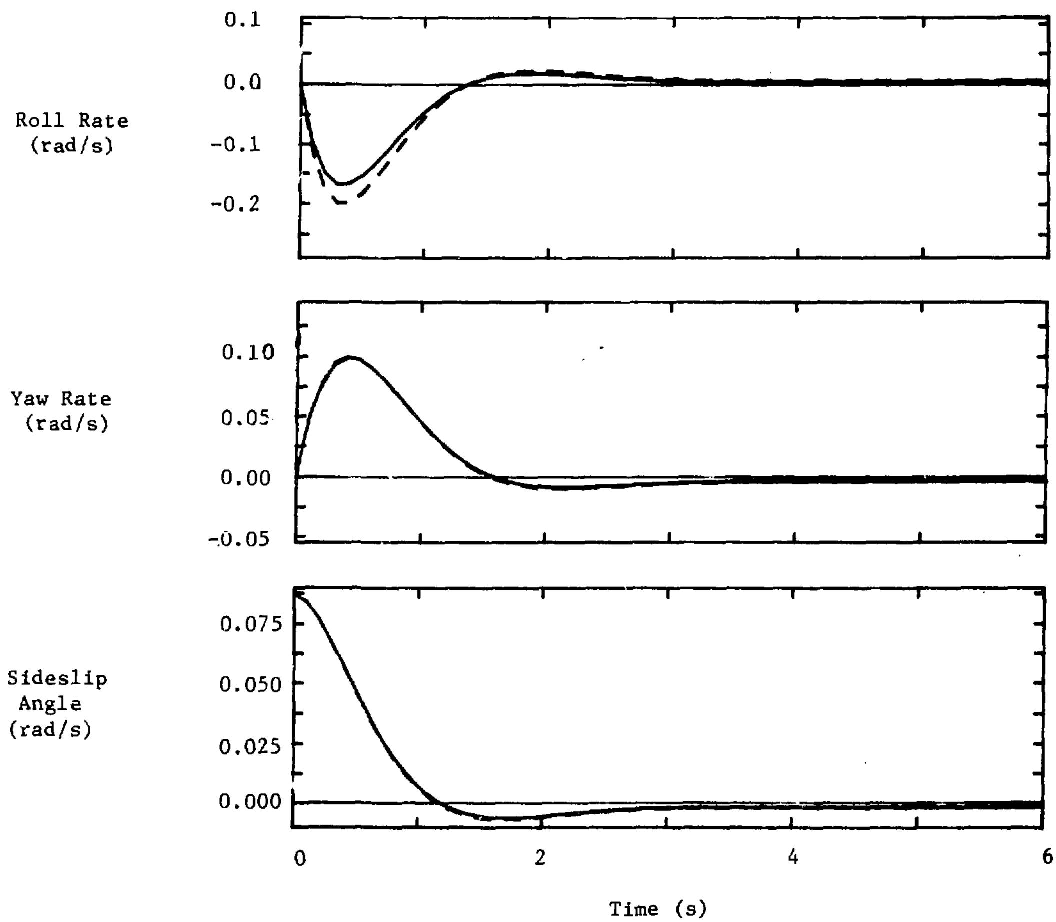

FIGURE 4 Aircraft Closed-Loop Response to an Initial Sideslip Error of 5°

——————————— Controller Structure of Equation (16)

- - - - - - - - - - - - - Controller Structure of Equation (17)

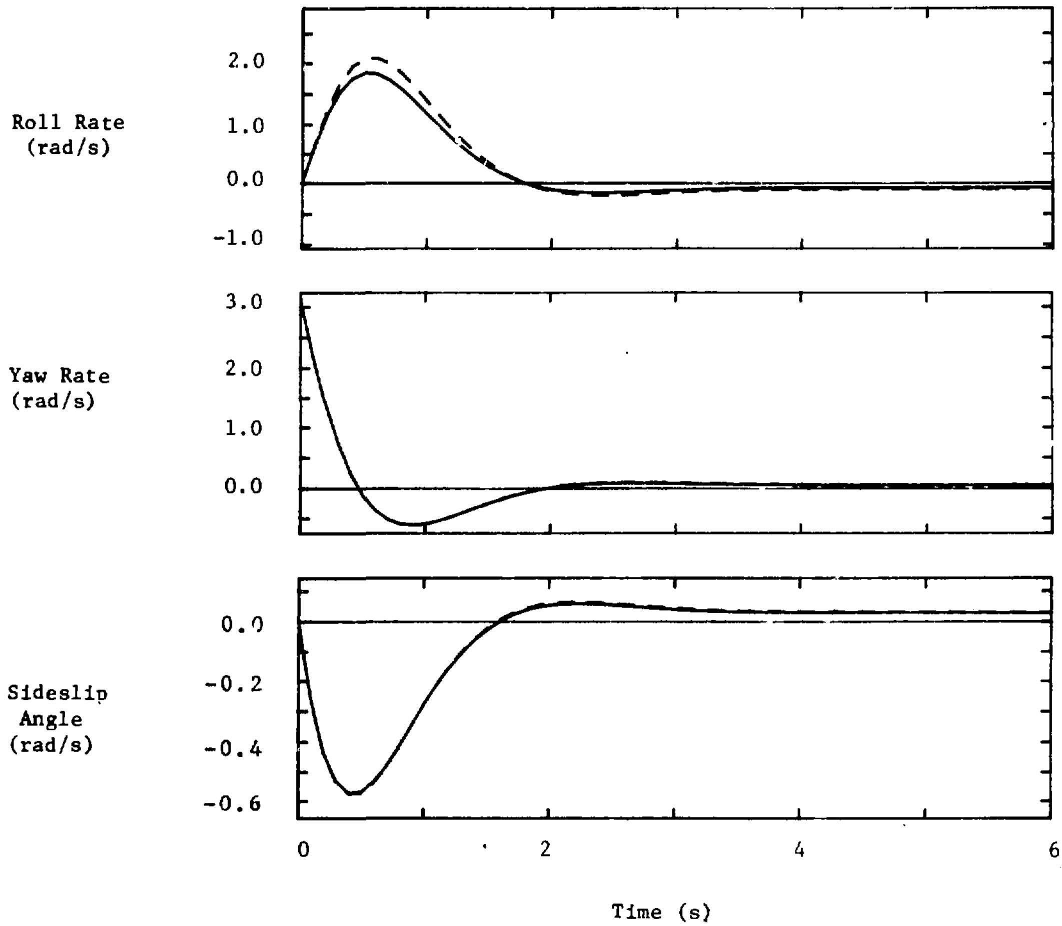

FIGURE 5 Aircraft Closed-Loop Response to an Initial Yaw Rate Error of 3.14 rad/s

———————— Controller Structure of Equation (16)

-------------------- Controller Structure of Equation (17)

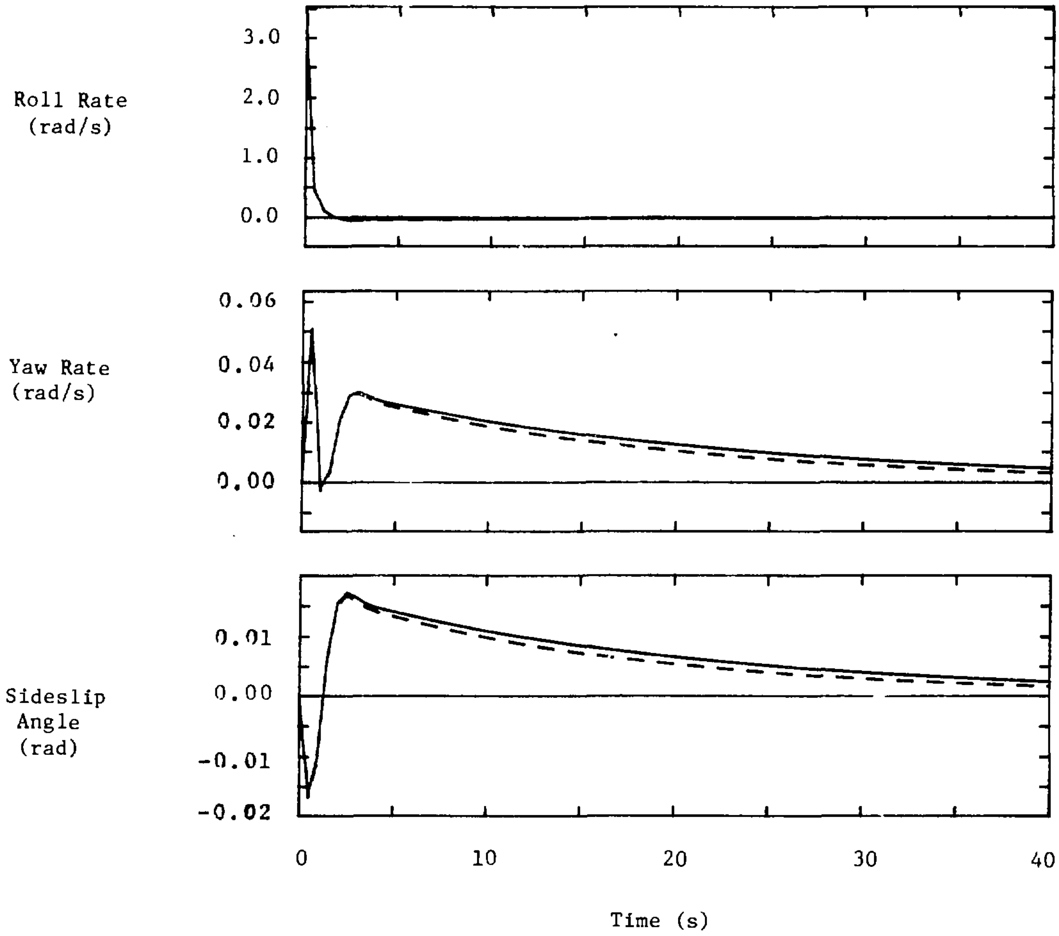

FIGURE 6 Aircraft Closed-Loop Response to an Initial Roll Rate Error of 3.14 rad/s

——————— Controller Structure of Equation (16)

- - - - - - - - Controller Structure of Equation (17)

Mode	Value
1	0.0
2	-0.015
3	-0.038
4	-0.0766
5	-0.773

The first two modes are associated with liquid levels. Modes 3 and 4 involve concentrations. The last mode is the internal coupling mode through the enthalpy.

5.3.3 Selection of a Proper Feedback Matrix

The main objective of the regulating system is to control the second-effect concentration, while maintaining the tank levels around an acceptable range. It is desired to leave the steam flowrate signal under the control of the second effect concentration.

The evaporator is a highly coupled system. The first-effect bottom flow discharges directly into the second-effect tank. In turn, the second-effect level has a direct effect on the second-effect concentration. It is therefore important to include both the second-effect level and concentration in the control of first-effect bottom flowrate.

Since the second-effect level and second-effect concentration are intimately related, it is natural to use the second-effect concentration in the control of second-effect bottom flowrate.

To meet these practical considerations, the following structure is first selected for the feedback matrix:

$$K = \begin{bmatrix} 0 & 0 & K_{13} \\ K_{21} & K_{22} & K_{23} \\ 0 & K_{32} & K_{33} \end{bmatrix} \qquad (19)$$

To investigate the importance of second-effect concentration on second-effect level, the following feedback matrix structure is selected:

$$K = \begin{bmatrix} 0 & 0 & K_{13} \\ K_{21} & K_{22} & K_{23} \\ 0 & K_{32} & 0 \end{bmatrix} \quad (20)$$

5.3.4 Desired Closed-Loop Eigenvalues

The desired closed-loop eigenvalues (-0.04, -0.06, -0.2, -0.4, -0.8) are selected to achieve stability with acceptable time response.

5.3.5 Minimization Solution

For the feedback matrix structure of equation (19), MVPPCO was activated with the following input parameters:

- Weight factors: α_i = (25, 16.7, 5, 2.5, 1.25)
- Initial feedback matrix: all gains set to zero
- Gain limits: no gain limits

The computed feedback matrix after 390 iterations is

$$K = \begin{bmatrix} 0 & 0 & -1.225 \\ 11.096 & 9.064 & 11.108 \\ 0 & 0.2127 & -1.133 \end{bmatrix} \quad (21)$$

and the assigned eigenvalues are

-0.04

-0.06

-0.2

-0.4

-0.8

showing that exact pole shifting was achieved.

For the feedback matrix structure of equation (21), MVPPCO was activated with the same input parameters.

The computed feedback matrix after 1,300 iterations is

$$K = \begin{bmatrix} 0 & 0 & -0.168 \\ 12.165 & 16.7 & 22.4 \\ 0 & 1.075 & 0 \end{bmatrix} \qquad (22)$$

and the assigned eigenvalues are

$$\begin{array}{l} -0.041 \\ -0.062 \\ -0.2 \\ -0.4 \\ -0.772 \end{array}$$

showing that the first two modes are partially assigned, and the last mode was made "uncontrollable" by the elimination of gain K_{33}.

The performance of both feedback matrix structures was assessed via closed-loop simulation. Figures 7 to 9 show closed-loop responses to setpoint changes.

Considering that the main objective of the regulating system is to control the second effect concentration, controller structure (22) shows poor performance compared to controller (21). On the second effect concentration it induces slower response with severe overshoots and larger steady-state errors. It is interesting to note that these differences in the performance of both controllers could not be anticipated by inspecting the assigned eigenvalues. This confirms the importance and the role of simulation analysis in the design of control systems. This example also illustrates well the effects of controller structure on the overall performance of a control system.

5.4 Steam Generator

5.4.1 Mode Description

This example is to illustrate the capability of the method to handle large systems.

The dynamics of the steam generator around a steady-state operating point can be described by

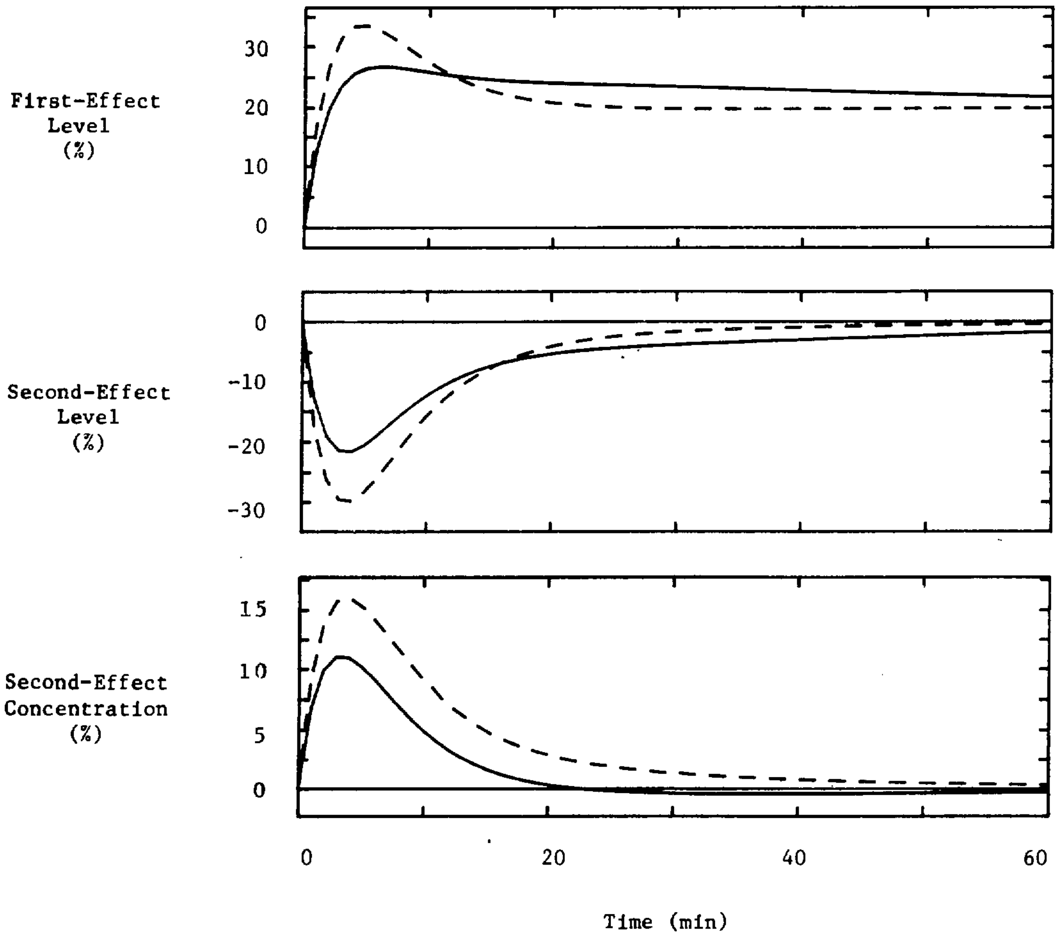

FIGURE 7 Evaporator Closed-Loop Response to a 20% Increase of First-Effect Level Setpoint

——————— Controller Structure of Equation (21)

--------------- Controller Structure of Equation (22)

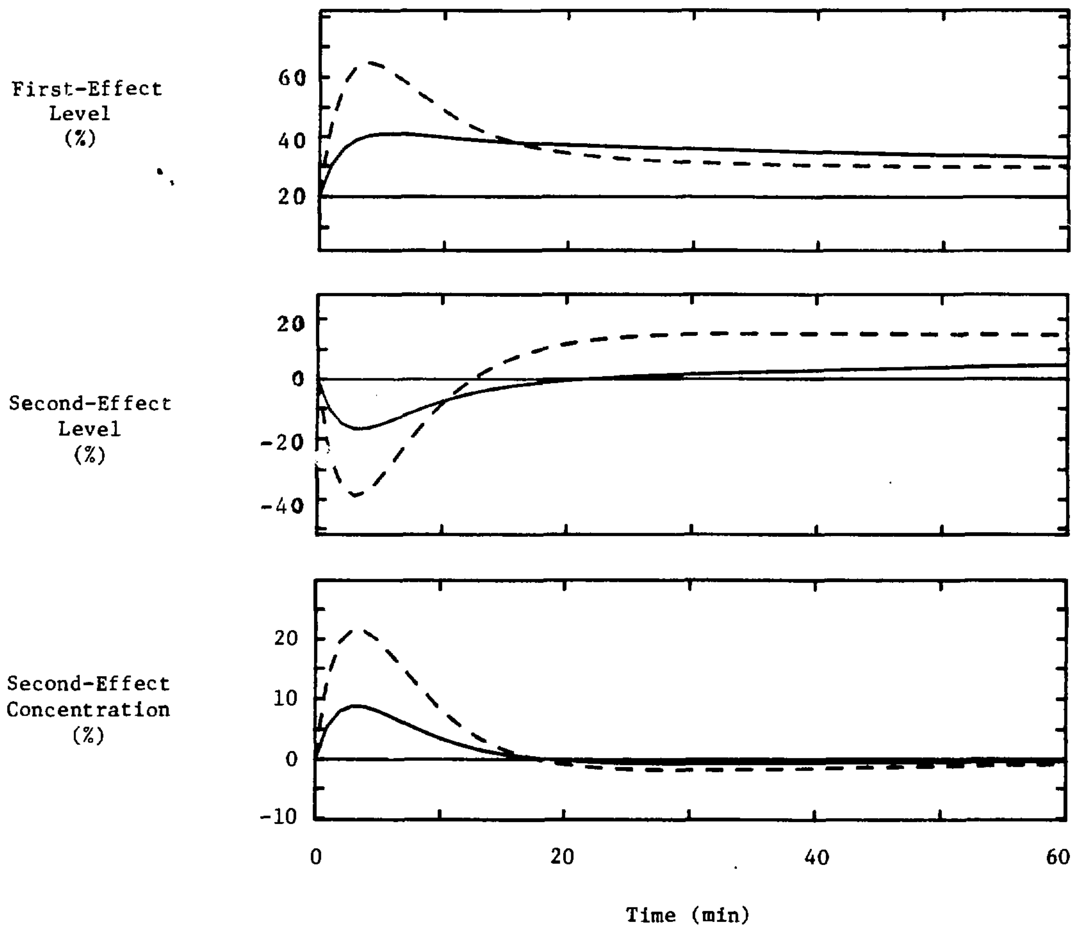

FIGURE 8 Evaporator Closed-Loop Response to a 20% Increase of Second-Effect Level Setpoint

———————— Controller Structure of Equation (21)

------------ Controller Structure of Equation (22)

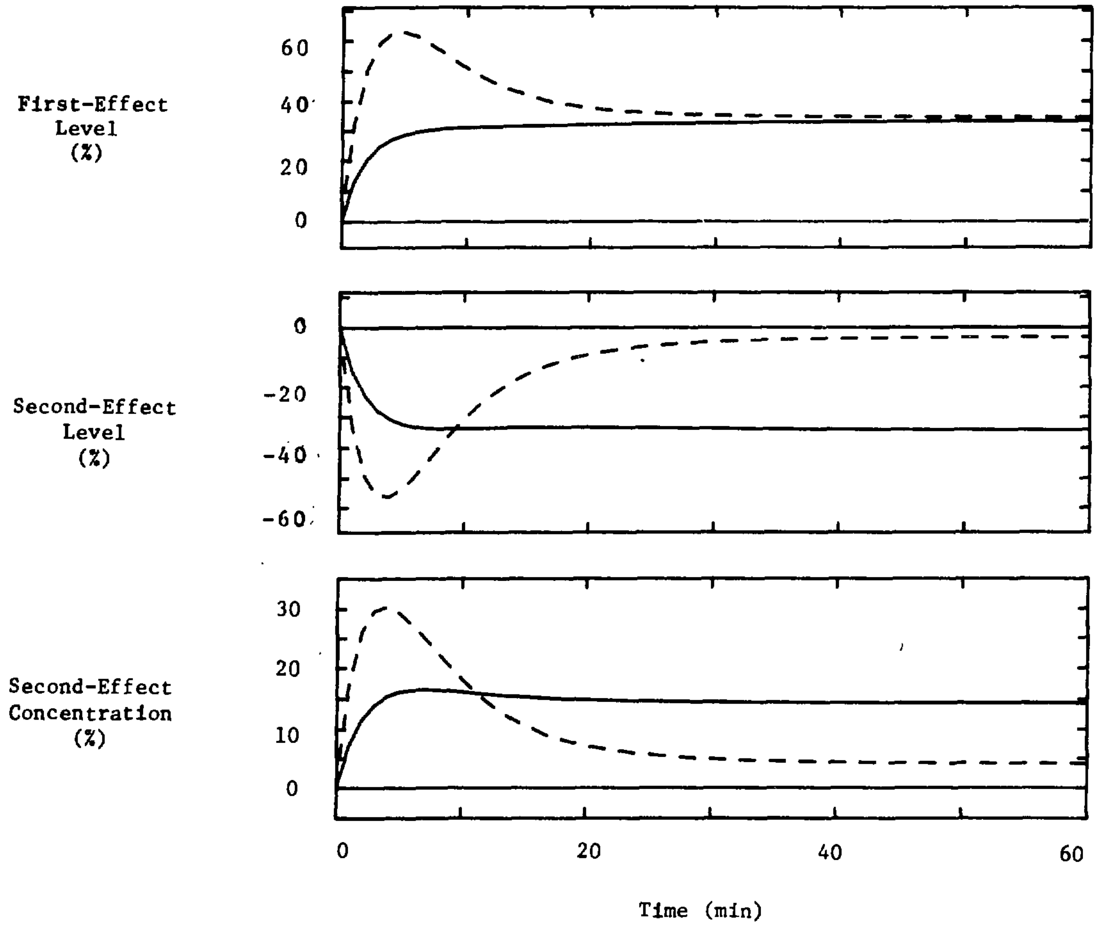

FIGURE 9 Evaporator Closed-Loop Response to a 20% Increase of Second-Effect Concentration Setpoint

———————— Controller Structure of Equation (21)

---------------- Controller Structure of Equation (22)

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) \tag{23}$$

where system matrices are given in [24]. A detailed description of this model, derived by Ali [25], is beyond the scope of this report.

Most of the state variables in the model are temperatures of internal nodes that are either inaccessible or difficult to measure. These variables appear mainly in the faster modes, while the slow modes are associated with the conventional process variables, such as level and pressure. For this reason, only the first 6, i.e. slow, modes are shown:

Mode	Eigenvalue
1	2.46×10^{-5}
2	-5.18×10^{-2}
3	-0.239
4	-0.331
5,6	$-0.886 \pm j0.255$

5.4 Feedback Matrix Structure

The system has only two inputs (steam valve lift, feedwater flow) and two outputs (steam pressure, downcomer level).

Considering the size of the model and the limited number of available control actions, a full structure is selected for the feedback matrix:

$$K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \tag{24}$$

5.4.3 Desired Closed-Loop Eigenvalues

The main objective is to achieve stability. Thus, only the first two unstable modes need to be reassigned. The desired closed-loop spectrum is

-0.015

-0.092

-0.239

-0.331

$-0.886 \pm j0.255$

5.4.4 Minimization Results

MVPPCO was activated with the following input parameters:

- Weight factors:	inversely proportional to desired eigenvalues
- Initial feedback matrix:	all gains set to zero
- Gain limits:	no gain limits

After eventual adjustment of weight factors to α = (1072, 87.2, 33.4, 24.2, 1,....,1) and 2000 iterations, the computed matrix for this unconstrained pole shifting problem is

$$K = \begin{bmatrix} 7.447E-3 & 2.645E-3 \\ -2.915E-3 & 1.335E-2 \end{bmatrix} \qquad (25)$$

and the assigned eigenvalues are

-0.0142

-0.0954

-0.232

-0.358

$-0.913 \pm j0.222$

This example shows that with a limited number of inputs, the proposed method is able to drive the closed-loop poles near the desired locations.

6. CONCLUSIONS

A technique to generate a proportional, multivariable controller using only output feedback has been presented. By using optimization to achieve closed-loop pole assignment, the method can incorporate practical engineering and physical constraints into the design strategy. This approach leads to a powerful design algorithm which has been implemented in the computer-aided design package MVPACK. The use of the technique has been illustrated in the design of multivariable controllers for an aircraft and an evaporator. The simulation results illustrate the importance of controller structure in the design of control systems.

The application of module MVPPCO showed that the optimization algorithm used may require a large number of iterations to converge. This problem could be solved by implementing additional optimization algorithms in MVPACK. This implementation may include an automatic switching mechanism between algorithms to accelerate convergence.

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APPENDIX A

Aircraft Model Matrices

Matrix A

-0.746000	0.327000	-12.9000	[illegible].05000	0.952000	0.000000
2.400000E-02	-0.174000	0.400000	-0.416000	-1.76000	0.000000
6.000000E-03	-0.999000	-5.800000E-0[illegible]	1.200000E-0[illegible]	9.200000E-03	3.690000E-02
0.000000	0.000000	0.000000	5.00000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	-10.0000	0.000000
1.00000	0.000000	0.000000	0.000000	0.000000	0.000000

Matrix B

0.000000	0.000000
0.000000	0.000000
0.000000	0.000000
0.000000	10.0000
20.0000	0.000000
0.000000	0.000000

Matrix C

1.00000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	1.00000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	1.00000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	1.00000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	1.00000	0.000000

APPENDIX B

Evaporator Model Matrices

Matrix A

0.000000	-1.100000E-03	-0.125500	0.000000	0.000000
0.000000	-7.550000E-02	0.125500	0.000000	0.000000
0.000000	-6.000000E-03	-0.774100	0.000000	0.000000
0.000000	-1.200000E-03	-0.144800	-1.500000E-02	1.000000E-04
0.000000	3.930000E-02	0.144800	0.000000	-3.800000E-02

Matrix B

0.000000	-7.660000E-02	0.000000
0.000000	0.000000	0.000000
0.216000	0.000000	0.000000
0.000000	7.950000E-02	-3.810000E-02
0.000000	-4.140000E-02	0.000000

Matrix C

1.00000	0.000000	0.000000	0.000000	0.000050
0.000000	0.000000	0.000000	1.00000	0.000000
0.000000	0.000000	0.000000	0.000000	1.00000

Matrix BD

0.109800	0.000000	0.000000
-3.330000E-02	7.660000E-02	0.000000
-1.880000E-02	0.000000	9.110000E-02
0.000000	0.000000	0.000000
0.000000	0.000000	0.000000

APPENDIX B (cont'd)

EVAPORATOR STATE, INPUT, DISTURBANCE AND OUTPUT VECTORS

State Vector X

W1	First-effect level
C1	First-effect concentration
H1	First-effect enthalpy
W2	Second-effect level
C2	Second-effect concentration

Control Vector u

S	Steam flowrate
B_1	First-effect bottoms flowrate
B_2	Second-effect bottoms flowrate

Disturbance Vector d

F	Feed flowrate
CF	Feed concentration
HF	Feed enthalpy

Output Vector y

W1	First-effect level
W2	Second-effect level
C2	Second-effect concentration

APPENDIX C

STEAM GENERATOR MATRICES

Matrix A

-1.43100	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	
4.94500	-5.59300	0.000000	0.000000	0.000000	0.000000	0.648600	0.000000
0.000000	0.000000	0.000000	-2.00100	0.000000	0.000000	0.000000	
0.000000	0.760700	-1.40900	0.000000	0.000000	0.000000	4.747400E-02	0.592400
-5.619300E-02	4.747400E-02	-7.020900E-02	1.97000	-2.671900E-02	-0.144000	0.222500	
0.000000	0.000000	0.760700	-1.40900	0.000000	0.000000	0.000000	0.000000
0.648600	0.000000	0.000000	0.555400	0.000000	0.000000	0.000000	
0.000000	0.000000	0.000000	4.94500	-5.59300	0.000000	-2.048200E-02	2.424300E-02
2.424300E-02	0.628100	3.029000E-02	-2.05900	1.171200E-02	6.246300E-02	-9.597700E-02	
0.000000	0.000000	0.000000	0.000000	1.43100	-10.0000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	
0.000000	2.40600	0.000000	0.000000	0.000000	0.000000	-3.78600	-9.855300E-02
-9.855300E-02	8.326100E-02	-0.125100	1.09000	6.071600E-02	-0.253900	1.12300	
0.000000	0.000000	2.40600	0.000000	0.000000	0.000000	1.200700E-02	-5.41500
-1.515900E-02	1.200700E-02	-1.894000E-02	0.290600	0.427100	-3.905600E-02	6.001200E-02	
0.000000	0.000000	0.000000	2.40600	0.000000	0.000000	-2.864600E-03	3.390700E-03
-5.39700	-2.864600E-03	4.236500E-03	-6.501100E-02	0.435000	8.730300E-03	-1.342400E-02	
0.000000	0.000000	0.000000	0.000000	2.40600	0.000000	-1.862200E-02	2.204000E-02
2.204200E-02	-3.83000	2.753900E-02	-0.427600	0.116500	5.679000E-02	0.645200	
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	1.968000E-05	2.647100E-02
2.647100E-02	1.968000E-05	-4.986400E-02	-1.282100E-02	-5.150300E-03	15.0000	9.272300E-05	
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	-4.043800E-02	4.786400E-02
4.786400E-02	-4.043800E-02	5.080400E-02	-0.417700	2.473900E-02	0.173300	-0.183500	
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	9.763300E-04	1.31300
1.31300	9.763300E-04	-5.693500E-04	-2.42700	-0.414700	-0.381900	4.575100E-03	
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	1.00000	0.000000	0.000000	0.000000	0.000000	
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	1.00000	0.000000	0.000000	

Matrix B

0.000000	0.000000
0.000000	0.000000
2.44400	0.215100
0.000000	0.000000
-1.05500	-9.280100E-02
0.000000	0.000000
4.28700	0.377200
0.659300	5.[illegible]2600E-02
-0.147500	-1.297900E-02
-0.958700	-8.437300E-02
1.06000	0.626400
-2.08200	-0.183200
-63.2800	2.31700
-4.840700E-02	-5.499800E-03
0.427900	-1.10600

Matrix C

0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	-1.133600E-03	2.368000E-03
2.368000E-03	-1.133600E-03	6.989900E-04	-2.693000E-02	3.615700E-04	-0.308400	-5.312000E-03	
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	7.946100E-06	1.068800E-02
1.068800E-02	7.946100E-06	3.053800E-03	-2.196700E-02	7.175500E-03	-5.83700	-8.284000E-02	

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