TRAPPING IN STOCHASTIC MECHANICS AND APPLICATIONS TO COVERS OF CLOUDS AND RADIATION BELTS §

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I. INTRODUCTION

There exists a lot of physical situations where a great number of particles are travelling in a medium which exerts rapidly varying forces on them (in space and time). First observations on particles in static fluids were made in the last century by R. Brown, leading to the discovery of Brownian motion [1]. One can also think of "particles" in a turbulent flow, e.g. the dispersion of smoke emitted by a stack in the lower atmosphere, or a cover of clouds. A different example is given by charged particles which are trapped in the magnetic field of planets. This magnetic field changes very rapidly on small scale. Other applications have been considered in different fields : Apes in a territory around some food source [2], colonies of Esterichia Coli in a Petri box [3]. Astronomical situations have been also treated in this spirit as for instance the formation of jet streams in the protosolar nebula [4] [5] [6] [7] and the morphology of Galaxies [8].

Statistical models are quite natural in such a situation although it is very hard to justify them from physical principle (the basic principles of fluid dynamics), for instance in the case of clouds in the atmosphere (see [9] and references therein).

In the situation previously considered, the forces acting on the particles have a deterministic smooth component e.g. gravitation or dipole like component of magnetic field around the earth.

This suggests, following [2-8] to use a Newtonian stochastic model which originally was initiated by E. Nelson to give an alternative description of quantum mechanics [10] [11] [12]. According to Nelson, it is possible to assign a stochastic acceleration to conservative stochastic diffusion processes. As a basic assumption, this stochastic acceleration is set equal to the deterministic smooth component of the external force acting on the particle, whereas the influences of the remainder is modelled by a diffusion coefficient. In some cases, it is possible to reduce the problem of solving the Fokker-Planck equation to a Schrödinger-like problem. Furthermore, we are interested in stationary situations which correspond to stationary solutions of the Schrödinger-like equation.

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These stationary solutions, in general, have nodal surfaces. It has been shown that these nodal surfaces correspond to impenetrable barriers for the diffusion process [6] [7] [13] [14] [15] [16] [17]. One of the basic physical assumption is that this barrier can be in some situation observed. In this paper, we shall make no attempt to justify the model on a deeper ground, but try to see whether it can account for the observation in two cases : the cover of clouds of planets and the radiation belts in the planetary magnetic field.

The paper is organized as follows.

In Section 2, we describe the basic properties of Newtonian Diffusion Stochastic Processes and indicate their connection with Schrödinger-like equations. Furthermore we give a heuristic interpretation of the nodal surfaces as impenetrable barriers for Newtonian Stochastic Diffusion Processes.

Section 3 concerns the possible applications to the observed average cloud covering in the planetary atmosphere, whereas in Section 4 we discuss the radiation belts (Van Allen Belts) along the previous ideas.

2. NEWTONIAN STOCHASTIC DIFFUSION PROCESSES

As we remarked in the introduction, we want to describe an assembly of "particles", which feel both an external deterministic field of forces and perturbations on a much lower scale (a more detailed description will follow). The net result of these influences is that the trajectory of an individual "particle" cannot be predicted in a precise way. Since we are not interested in the precise behaviour of a particle but only in the mean properties, it is tempting to use a probabilistic model : typical trajectories are trajectories of a stochastic process, i.e. one assigns a probability to the set of trajectories. This probability allows to compute all the statistical properties of the assembly of particles.

Equivalently, one can restrict the type of stochastic processes which are considered. The first restriction is that the probability is supported by continuous trajectories, which is a rather natural assumption. Furthermore one assumes that the stochastic process is a Markov process. This amounts to saying that the future of the system does not depend on its past but only on the present. The usual interpretation of these results is that on a short time scale a "particle" experiences a lot of perturbations and looses the memory of its past history. As a consequence of these two assumptions the stochastic process is a diffusion stochastic process X_t (see e.g. [18]).

For each t, X_t is a random variable viz. it depends on an event $\omega \in \Omega$ such that $X_t(\omega) = \omega(t)$ is the trajectory. The process X_t satisfies a stochastic differential equation of diffusion type

$$dX_t^i = \beta_+^i(X_t, t)\, dt + \sigma dW_t^i \tag{2.1}$$

where β_+ is called the forward drift, σ the diffusion constant and W_t is the standard Brownian motion in three dimensions.

The intuitive meaning of this equation is clear. If for instance $\sigma \equiv 0$ then we have to do with a purely deterministic equation, whereas if $\beta_+ \equiv 0$ and $\sigma = 1$, X_t is the standard Brownian motion.

This is not the most general stochastic differential equation of diffusion type however for the sake of simplicity we shall only consider this case. The general case can be treated along the same line (see e.g. [16]).

Under mild assumptions on β_+ the previous stochastic differential equation has a unique solution (see e.g. [18]). It suffices to say that a Lipschitz condition as in the classical theory of differential equations is sufficient to ensure both the existence and uniqueness of the solution of the stochastic differential equation at least for sufficiently small t (given β_+ and the initial condition $X_{t_0} = x_0$ a.s).

As in the case of the Wiener process, the trajectories of stochastic diffusion processes are continuous but nowhere differentiable with probability one. This makes it difficult to write a dynamical equation to constrain the forward drift as in classical mechanics. However it is possible to define substitutes for the total time derivative.

Let $E[\cdot|X_t = x]$ be the conditional expectation given by $X_t = x$. Let furthermore F be a smooth function then

$$D_\pm F(x,t) = \lim_{t \downarrow 0} \pm (\Delta t)^{-1} E\left[F(X_{t\pm\Delta t}, t \pm \Delta t) - F(X_t,t) | X_t = x\right] \qquad (2.2)$$

defines respectively the forward or backward substitute for total time derivatives respectively and in general they differ. Indeed one has

$$D_\pm F(x,t) = \frac{\partial F}{\partial t}(x,t) + (\beta_\pm \cdot \nabla) F(x,t) \pm \frac{1}{2} \sigma^2 \quad F(x,t) \qquad (2.3)$$

If $F(x,t) \equiv x$

$$D_\pm X_t = \beta_\pm (X_t,t) \qquad \left(D_\pm x = \beta_\pm(x,t) \right) \qquad (2.4)$$

$\beta_+(x,t)$ (resp. $\beta_-(x,t)$) is interpreted as the mean velocity of outgoing (resp. ingoing) "particles" at point x at time t.

Forward and backward velocity $\beta_\pm$ are not unrelated. Indeed if we assume that the process X_t has a smooth density $\rho_t(x)$ in the sense that the expectation of a smooth function $F(X_t)$ is given at each time t by

$$E\left[F(X_t)\right] = \int F(x) \, \rho_t(x) \, dx \qquad (2.5)$$

then it is well known that $\rho_t(x)$ satisfies Fokker-Planck equation viz.

$$\frac{\partial}{\partial t}\rho_t = -\nabla.(\beta_\pm \rho_t) \pm \frac{\sigma^2}{2}\Delta\,\rho_t \tag{2.6}$$

which is in the classical case ($\sigma \equiv 0$) a continuity equation.
Comparison of these two equations shows that one can define the current and osmotic velocity v and u by

$$v = \frac{1}{2}(\beta_+ + \beta_-) \tag{2.7}$$

$$u = \frac{1}{2}(\beta_+ - \beta_-) \tag{2.8}$$

so the Fokker-Planck equations can be rewritten

$$\frac{\partial\rho}{\partial t} = -\nabla.(\rho v) \qquad \text{(continuity equation)} \tag{2.9}$$

$$\frac{\sigma^2}{2}\Delta\rho = \nabla.(\rho u) \qquad \text{(osmotic equation)} \tag{2.10}$$

Using the following integration by parts formula for functions f and g with compact support in time

$$\int E\left[D_+ f.g\right] dt = -\int E\left[F.D_- g\right] dt \tag{2.11}$$

and the definition of D_+, D_- and ρ it can be shown that the osmotic equation can always be integrated

$$u = \frac{\sigma^2}{2}\,\nabla\,\mathrm{Log}\,\rho = \frac{\sigma^2}{2}\,\frac{\nabla\rho}{\rho} \tag{2.12}$$

At this stage there is still no dynamical constraint on the drift. Following E. Nelson [10] [11] we introduce the mean stochastic acceleration

$$a = \frac{1}{2}\,(D_+D_- + D_-D_+)\,X_t \tag{2.13}$$

and we assume as a basic dynamical law, Newton's law in mean i.e.

$$\mu\, a = F \tag{2.14}$$

where F is an external force and μ the mass of the "particle".
A stochastic diffusion process which satisfies such an equation is called a Newtonian stochastic diffusion process.

It is easy to rewrite the Newton's law in the mean in terms of the velocities u and v

$$\frac{\partial v}{\partial t} + (v.\nabla)v - (u.\nabla)u - \frac{\sigma^2}{2}\,\Delta u = \frac{F}{\mu} \tag{2.15}$$

In the following we are interested in the case where the external force is of the form

$$F = -\nabla U + q(E + v \times B) \tag{2.16}$$

that is we allow for a force of Lorentz type, q being the charge of the particle, U is a scalar potential, E and B the external electromagnetic fields. We denote by ϕ and A respectively the scalar and vector electromagnetic potentials, so that

$$E = -\nabla\phi - \frac{\partial}{\partial t} A \tag{2.17}$$

$$B = \nabla \times A \tag{2.18}$$

We assume furthermore that there exists a function S(x,t) such that

$$\mu\, v(u,t) = S(x,t) - q\, A(x,t) \tag{2.19}$$

Under these hypotheses and for the Coulomb gauge

$$\nabla . A = 0$$

the stochastic acceleration can be rewritten

$$\mu\, a = \nabla\left[\frac{\partial S}{\partial t} - \frac{\sigma^4\mu}{8}\nabla \operatorname{Log}\rho . \nabla \operatorname{Log}\rho + (\nabla S - qA)(\nabla S - qA)\frac{1}{2\mu} - \frac{\sigma^4\mu}{4}\Delta \operatorname{Log}\rho\right] + q\left(-\frac{\partial A}{\partial t} + v \times B\right) \tag{2.20}$$

We have at our disposal a couple of non linear coupled equations, Newton's law and the continuity equation

$$\frac{\partial S}{\partial t} - \frac{\sigma^4\mu}{2}\Delta(\operatorname{Log}\rho^{1/2}) + \frac{1}{2\mu}(\nabla S - qA)^2 - \frac{\sigma^4\mu}{2}(\nabla(\operatorname{Log}\rho^{1/2})^2 + U(x) + q\phi(x) = 0 \tag{2.21}$$

$$\frac{\partial \rho^{1/2}}{\partial t} + \nabla\rho^{1/2} . \nabla S + \rho^{1/2}\Delta S + qA . \nabla\rho^{1/2} = 0 \tag{2.22}$$

In order to solve these equations we set [10]

$$\psi(x,t) = \rho^{1/2}(x,t)\, e^{\frac{i}{\mu\sigma^2} S(x,t)} \tag{2.23}$$

and it can be shown that ψ solves the linear equation

$$i\mu\sigma^2 \frac{\partial\psi(x,t)}{\partial t} = \frac{1}{2\mu}(-i\mu\sigma^2\nabla - qA)^2\psi(x,t) + (U(x,t) + q\phi(x,t))\,\psi(x,t) \tag{2.24}$$

which is of the Schrödinger type.

Conversely, given a square integrable solution of the previous equation, one can recover solution of the coupled equations(2.21), (2.22) and ultimately a stochastic diffusion process (see e.g. [19] [20]), the associated probability density being

$$\rho(x,t) = |\psi(x,t)|^2 \tag{2.25}$$

the current and osmotic velocities being given by

$$v(x,t) = \sigma^2 (\nabla \operatorname{Im} \operatorname{Log}(\psi))(x,t) \tag{2.26}$$

$$u(x,t) = \frac{\sigma^2}{2} \nabla \left(\mathrm{Log} |\psi|^2(x,t) \right) \tag{2.27}$$

Now let us consider the case where the electromagnetic field varies in time. Among the solutions of the Schrödinger-like equations, we are interested in the stationary ones, namely those which satisfy :

$$\psi_E(x,t) = \psi_E(x)\, e^{-i\frac{E}{\mu\sigma^2} t} \tag{2.28}$$

for some constant E , $\psi_E(x)$ is solution of the stationary Schrödinger equation

$$\frac{1}{2\mu}(-i\mu\sigma^2\nabla - qA)^2\psi(x) + (U(x)+q\phi(x))\psi(x) = E\psi(x) \tag{2.29}$$

The corresponding density ρ and velocities u and v are time independent, i.e. the associated processes are stationary.

These solutions, except the one of lowest energy, have nodes, i.e. zeros. For a given ψ_E solution of (2.29), we consider the nodal surfaces N_ρ of the corresponding density

$$N_\rho = \left\{ x \in R^3 / |\psi_E(x)| = 0 \right\}$$

In general, nodal surfaces divide the space into disjoint subsets and it can be shown [13] [14] [15] [16] [17] that the stochastic process never crosses these nodal surfaces.

Intuitively, one can think of such phenomena as follows. In the neighbourhood of the nodal surface, the osmotic velocity u increases to infinity and is directed away from the surface, see Fig. 1.

3. APPLICATION TO CLOUD COVERING OF THE PLANETS [17]

In this section we make an attempt to apply the previous model to the large scale features of the covers of clouds of the planets. Indeed the now available pictures of planets with a substantial atmosphere exhibit regular structures, namely zonal bands on a large scale. This was already known by the end of the Seventeenth Century for Jupiter [22] when the first telescopic observations were made. Now direct exploration of the solar system is possible by means of automatic space crafts. (Pioneer and Voyagers encounters with Jupiter and Saturn [23] [24] [25] [26]).

We propose a mechanism as general as possible, which does not depend too much on different parameters of the planetary atmospheres but accounts for the general features of these large scale structures.

Think of clouds as being composed of "particles" either droplets or icy flakes (with typical size 1 μm). Apart from the gravitational forces, these "particles" feel very complicated forces from the surrounding turbulent atmosphere. We do not intend to take into account the detail of these influences but assume that it can be replaced by a diffusion mechanism. Turbulent diffusion is known to be a more efficient mechanism of diffusion than molecular diffusion [9].

Furthermore we shall make no precise statement about the overall force only assuming it is spherical symmetric and derives from a potential U(r). Under these assumptions, the associated stationary Schrödinger-like equation :

$$- \mu \frac{\sigma^2}{2} (\Delta \psi_E)(x) + U(r) \psi_E(x) = E \psi_E(x) \quad ; \quad r = |x| \ . \qquad (3.1)$$

Furthermore we assume that the potential u(r) is "sufficiently strong" to ensure the existence of bound states.

These eignestates, expressed in terms of spherical coordinates (r, θ, φ), have the form

$$\psi_{E_{nl}}(r, \theta, \varphi) = R_{nl}(r) \ P_l^m(\cos \theta) \exp\{ \ im \varphi \} \qquad (3.2)$$

for a discrete set of values of $E = E_{nl}$ labelled by a pair of positive integers n and l, P_l^m, $(|m| \leq \ell)$ is the associated Legendre function, which is real and such that $P_\ell^m = P_l^{-m}$.

Nodal surfaces are either spheres around the origin corresponding to the zeros $r_1, r_r, \ldots$ of the radial part R_{nl} or cones defined by the zeros $\theta_1, \theta_2 \ldots$ of P_ℓ^m . Possible zones of confinement are the anuli which are depicted in Fig. 2.

In tables 1, 2 and 3, there is an attempt to fit the observed zones of covers of clouds with the previous model.

The agreement of the model with observations is very good if one keeps in mind the following facts.

i) there are few free parameters involved, namely the integers n, l, m and it is nice that one can make a fit with relatively low numbers.

ii) the physical parameters in planetary atmosphere vary on a large range as far as the composition, temperature and pressure are concerned,

that means that one cannot hope to get a more precise fit to the observationsin this model.

Actually the model is too crude in the sense that it does not incorporate the basic mechanisms which are responsible for the physics of the atmosphere, e.g. temperature gradient. But it is a good indication that Newtonian diffusion in a sense is compatible with the geometry of the problem.

4. APPLICATION TO RADIATION BELTS [21]

In this section we apply our model to the confinement of charged particles in the terrestrial magnetic field. By the end of the fifties it was discovered that at very high altitude, typically between two and five earth radii there are electrically charged particles trapped in the earth magnetic field [27]. Their density is rather high and they display a toroïdal shape with a typical crescent shape section. It is obvious that the magnetic field is responsible for the trapping of these charged particles and the theory for the movement of charged particles in the dipole like earth magnetic field has been developed a long time ago [28]. However it seems difficult to ignore the rapid variation of the magnetic field which certainly perturbs the previous classical pictures [29]. This is the reason why it is tempting to apply the previous stochastic model to explain the main features of this confinement zone.

We assume that the magnetic field around the earth is dipole like. namely of the form

$$\begin{aligned} B_x(\underline{x}) &= -x \frac{\partial \mathcal{A}}{\partial z}(r,z) \\ B_y(\underline{x}) &= -y \frac{\partial \mathcal{A}}{\partial z}(r,z) \\ B_z(\underline{x}) &= 2\mathcal{A}(r,z) + r\frac{\partial \mathcal{A}}{\partial r}(r,z) \end{aligned} \tag{3.1}$$

where $r = (x^2+y^2)^{1/2}$.

This "Ansatz" corresponds to the most general magnetic field which has cylindrical symmetry and whose lines of forces are contained in meridian plane. $\mathcal{A}$ is a priori an arbitrary function. In Coulomb gauge, the vector potential has the following form

$$A_x(\underline{x}) = -y\,\mathcal{A}(r,z)$$
$$A_y(\underline{x}) = x\,\mathcal{A}(r,z) \qquad , \quad A_z(\underline{x}) = 0$$

It is important to notice that the lines of forces are defined by the equations

$$\alpha(r,z) = r^2 \mathcal{A}(r,z) = c^{te}$$

and

$$y + cx = 0$$

where c is an arbitrary constant.

As discussed previously, we look for stationary solutions of the Schrödinger-like equations

$$\frac{1}{2\mu}(-i\mu\sigma^2\nabla - qA)^2 \psi_E(\underline{x}) = E\,\psi_E(\underline{x})$$

where μ is the mass of particles, q the charge and σ a diffusion constant which models the random character of the environment.

In cylindar coordinates, the Schrödinger-like equation assumes the form

$$-\frac{\mu\sigma^4}{2}\left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial \varphi^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\phi_i(r, z, \varphi)$$

$$+ \, i\left\{ q\sigma^2 \mathcal{A}(r,z)\frac{\partial}{\partial \varphi} + \frac{q^2 r^2}{2\mu}\mathcal{A}^2(r,z) - E_i\right\}\phi_i(r,z,\varphi) = 0$$

where

$$\phi_i(r,z,\varphi) = \psi_{E_i}(\underline{x})$$

the solutions of this equation are linear combinations of solutions $\phi_l^{\pm}(r,z,\varphi)$ of the form

$$\phi_l^{\pm}(r,z,\varphi) = r^{-1/2}\, P_l(r,z)\, \exp\{\pm i\, l\, \varphi\}$$

the P_l satisfying the following equation

$$-\frac{\mu\sigma^4}{2}\left\{\frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial z^2}\right\} P_l(r,z)$$

$$+ \frac{1}{2\mu r^2}\left(q\alpha(r,z) - \mu\sigma^2(l + \tfrac{1}{2})\right)\left(q\alpha(r,z) - \mu\sigma^2(l - \tfrac{1}{2})\right) P_l(r,z) = E_i P_l(r,z)$$

which is a two dimensional Schrödinger equation with an equivalent potential U_{eff} given by

$$U_{eff} = \frac{1}{2\mu r^2}\left(q\alpha(r,z) - \mu\sigma^2(l + \tfrac{1}{2})\right)\left(q\alpha(r,z) - \mu\sigma^2(l - 1/2)\right)$$

The existence of a bound state solution for this equation depends on the shape of the equivalent potential. If ql is positive the potential is purely repulsive and there is no solution. If ql is negative, then the existence of the solution depends on more precise assumptions on the behaviour of the magnetic field at infinity. We shall assume that these assumptions are satisfied. Consequently, the possible confinement zones have toroidal shape defined by the equation

$$P_\ell(r,z) = 0$$

It is rather hard to make a precise statement in this general framework on the shape of these curves. However, the following semi-classical argument shows that for the low-lying eigenstate, the confinement zone has the expected shape. Indeed the effective potential U_{eff} is zero precisely on the line of forces (see Fig. 3)

$$\alpha(r,z) = (\ell \pm 1/2)\mu\sigma^2/q$$

and for small σ the density has a strong tendency to concentrate in between these lines.

As a final remark, the form of ψ_{E_i} indicates that the particle drift is either eastward or westward according to the charges. This is indeed the case in the Van Allen belts.

REFERENCES

[1] R. BROWN, A Brief Account of Microscopical Observations made in the Months of June, July, and August 1827, on the Particles Contained in the Pollen of Plants, and on the General Existence of Active Molecule in Organic and Inorganic Bodies, Philosophical Magazine N.S. 4, 161-173 (1828).

[2] M. NAGASAWA, Segregation of a Population in an Environment, J. Math. Biology 9, 213-235 (1980).

[3] M. NAGASAWA, An Application of the Segregation Model for Septation of Esterichia Coli, J. Theor. Biol. 90, 445-455 (1981).

[4] S. ALBEVERIO, Ph. BLANCHARD, R. HOEGH-KROHN, A Stochastic Model for the Orbits of Planets and Satellites. An Interpretation of Titius-Bode Law, Exp. Math. 4, 365-373 (1983).

[5] S. ALBEVERIO, Ph. BLANCHARD, R. HOEGH-KROHN, Processus de Diffusion, Confinement et Formation de "Jet-Streams" dans la Nébuleuse Protosolaire, Preprint TH 3536-CERN, Febr. 1983.

[6] S. ALBEVERIO, Ph. BLANCHARD, R. HOEGH-KROHN, Newtonian Diffusions and Planets, with a Remark on Non-Standard Dirichlet Forms and Polymers, to be published in Lecture Notes in Mathematics, Springer Verlag, Proceedings of the London Math. Soc. Symposium, Stochastic Analysis, Swansea, 1983.

[7] S. ALBEVERIO, Ph. BLANCHARD, R. HOEGH-KROHN, Reduction of Non-Linear Problems to Schrödinger or Heat Equations : Formation of Kepler Orbits, Singular Solutions for Hydrodynamical Equations, to be published in Lecture Notes in Mathematics, Springer Verlag, Heidelberg, Proceedings CIRM, April 1983.

[8] S. ALBEVERIO, Ph. BLANCHARD, R. HOEGH-KROHN, L. FERREIRA, L. STREIT, Work in preparation.

[9] A.S. MONIN, A.M. YAGLOM, Satistical Fluid Mechanics, Mechanics of Turbulence, Volume 1, English Translation, 1977, MIT Press.

[10] E. NELSON, Derivation of the Schrödinger Equation from Newtonian Mechanics, Phys. Rev. 150, 1079-1089 (1966).

[11] E. NELSON, Dynamical Theories of Brownian Motion, Princeton University Press, (1967).

[12] E. NELSON, Quantum Fluctuation, an Introduction, Physica 124A, 509-520 (1984).

[13] S. ALBEVERIO, R. HOEGH-KROHN, A Remark on the Connection between Stochastic Mechanics and the Heat Equation, J.M.P. 15, 1745-1747 (1974).

[14] S. ALBEVERIO, M. FUKUSHIMA, W. KARWOSKI, L. STREIT, Capacity and Quantum Mechanical Tunneling, Commun.Math.Phys. 81, 501-513 (1981).

[15] L. STREIT, Energy Forms, Schrödinger Theory, Processes, Physics Reports 77, 363-375 (1981).

[16] S. ALBEVERIO, Ph. BLANCHARD, R. HOEGH-KROHN, Diffusion sur une variété riemannienne : Barrières infranchissables et Applications, Astérisque, Société Mathématique de France (1985).

[17] Ph. BLANCHARD, Trapping for Newtonian Diffusion Processes, in Stochastic Methods and Computer Techniques in Quantum Dynamics, H. Miller, L. Pittner Eds., Springer Verlag, 1984.

[18] I.I. GIHMAN, A.V. SKOHOROD, The Theory of Stochastic Processes I, II and III, Springer Verlag, 1974.

[19] E. CARLEN, Conservative Diffusions, prepr it Princeton University, Physics Department, January 1984.

[20] W. ZHENG, Semi-martingales dans les variétés et mécanique statistique de Nelson, Thèse, Juin 1984, IRMA Strasbourg 235/TE-25.

[21] S. ALBEVERIO, Ph. BLANCHARD, Ph. COMBE, R. HOEGH-KROHN, R. RODRIGUEZ, M. SIRUGUE, M. SIRUGUE-COLLIN, Magnetic Bottles in a Dirty Environment. A Stochastic Model for Radiation Belts, preprint ZIF Bielefeld 54 (1984).

[22] J. CASSINI, Ancient Memoir of "Académie des Sciences", tome 2, p. 104.

[23] Voyager I, Encounter with Jupiter, Science 204, n° 4396 (1979).

[24] Voyager II, Encounter with Jupiter, Science 206, n° 4421 (1979).

[25] B.A. SMITH et al., Encounter with Saturn, Voyager I, Imaging Science Results, Science 212, 163-190 (1981).

[26] B.A. SMITH et al., A New Look at the Saturn System, The Voyager II Images, Sciences 215, 504-536 (1982).

[27] J.A. VAN ALLEN, G.H. LUDWIG, E.C. RAY, C.E. MacILWAIN, Observation of High Intensity Radiation by Satellites, Jet Propulsion 28, 588-592 (1958).

[28] C. STORMER, The Polar Aurora, Oxford University Press (1955).

[29] M. WALT, T.A. FARLEY, in Handbook of Astronomy and Geophysics, vol. 1, The Earth Upper Atmosphere, Ionosphere and Magnetosphere (1978).

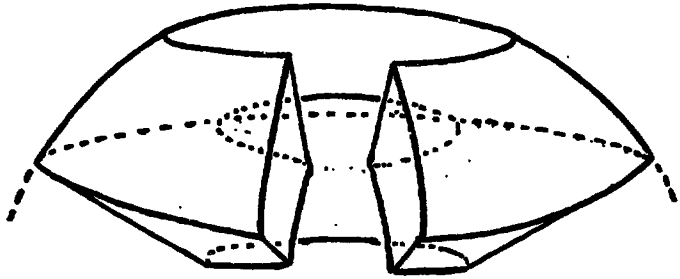

Figure 1

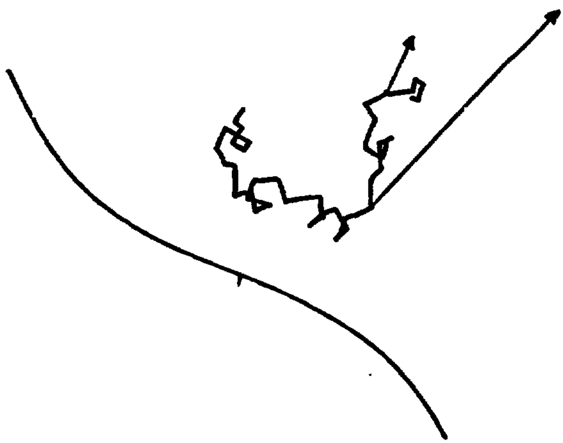

Figure 2

Table 1:

Zonal Structure of the Atmospheres of the Telluric Planets

(Venus, the Earth, Mars) North Hemisphere

Planet Mass/♁ Mass R_o km $(R_1 - R_o)$ km	Period of Revolution	Observed Latitude of the Boundaries between Zones	Tentative ℓ	 m	Position of the zeros of P_m^ℓ	General Direction of the Winds in the Zone (east or west wind)
VENUS .8 6050 65	243 d	$\Theta_1 = 0$ $\Theta_2 = 90$			$\Theta'_1 = 0$ $\Theta'_2 = 90$	east
THE EARTH 1 6378 15	24 H	$\Theta_1 = 0$ $\Theta_2 = 30$ $\Theta_3 = 60$ $\Theta_4 = 90$	6	1	$\Theta'_1 = 0$ $\Theta'_2 = 28$ $\Theta'_3 = 56$ $\Theta'_4 = 90$	east west east
MARS .1 3900 ~ 10	24 H	$\Theta_1 = 15$ $\Theta_2 = 90$	8	6	$\Theta'_1 = 15$ $\Theta'_2 = 90$	east west
MARS (inner structure) $\lesssim 5$		$\Theta_1 = 15$ $\Theta_2 = 60$ $\Theta_3 = 90$	5	1	$\Theta'_1 = 16$ $\Theta'_2 = 51$ $\Theta'_3 = 90$	east west east

Table II:

Zonal Structure of Jupiter — North Hemisphere

Planet Mass/♁ Mass R_o km $(R_1 - R_o)$km	Period of Rotation	Observed Latitude of the Boundary between the Zones	Tentative ℓ	Tentative m	Position of the Zeros of P_m^ℓ	General Direction of the Winds in the Zone
JUPITER	10 H		31	1		
318					$\theta'_1 = 2.9$	east
72000					$\theta'_2 = 8.7$	
~8500 (estimated)		$\theta_3 = 16$			$\theta'_3 = 14.5$	west
		$\theta_4 = 20$			$\theta'_4 = 20.5$	east
		$\theta_5 = 30$			$\theta'_5 = 26$	west
		$\theta_6 = 34$			$\theta'_6 = 32$	east
		$\theta_7 = 38$			$\theta'_7 = 37.6$	-id-
		$\theta_8 = 42$			$\theta'_8 = 43.6$	
		$\theta_9 = 50$			$\theta'_9 = 49.5$	
		$\theta_{10} = 55$			$\theta'_{10} = 55$	
		$\theta_{11} = 59$			$\theta'_{11} = 60.5$	
					$\theta'_{12} = 67$	
					$\theta'_{13} = 73.7$	
					$\theta'_{14} = 78.5$	

Table III:

Zonal Structure of Saturn

North Hemisphere

Planet Mass/δ Mass R_0 km $(R_1 - R_0)$km	Period of Rotation	Observed Latitude of the Boundary between the Zones	Tentative ℓ	Tentative m	Position of the Zeros of P_m^ℓ	General Direction of the Winds in the Zone
SATURN	11 H		11	1		east
95					$\Theta_1' = 8$	- id -
					$\Theta_2' = 23.6$	
60 000		$\Theta_3 = 38$			$\Theta_3' = 40$	
~ 30 000 (estimated)		$\Theta_4 = 57$			$\Theta_4' = 55$	
		$\Theta_5 = 70$			$\Theta_5' = 70$	
					$\Theta_6' = 90$	