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NON DETERMINISTIC METHODS FOR CHARGED PARTICLE TRANSPORT

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Communication présentée à : LANL -CEA Meeting on recent applications and developments of Monte-Carlo method
Cadarache (France) 22-26 Apr 1985

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NON DETERMINISTIC METHODS FOR CHARGED

PARTICLE TRANSPORT

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ABSTRACT

The coupling of Monte-Carlo methods for solving Fokker Planck equation with ICF codes requires them to be economical and to preserve gross conservation properties. Besides, the presence in FPE of diffusion terms due to collisions between test particles and the background plasma challenges standard M.C. techniques if this pnenomenon is dominant. We address these problems through the use of a fixed mesh in phase space which allows us to handle highly variable sources, avoiding any Russian Roulette for lowering the size of the sample. Also on this mesh are solved diffusion equations obtained from a splitting of FPE. Any non linear diffusion terms of FPE can be handled in this manner. Another method, also presented here is to use a direct particle method for solving the full FPE.

INTRODUCTION

Monte-Carlo methods have been under development for many years. Quite recently, they have been used for solving Fokker Planck Equation, with many possible applications, such as α-particle transport [1,6], thermal electron transport, and suprathermal electron transport [2,3]. Any of these problems presents specific difficulties. Concerning the first example, one problem pertains to the fact that there is energy deposition from α-particles to the background plasma, which in turn allows for the propagation of the combustion in the DT plasma. This nonlinear coupling between FPE for fast ions and hydrodynamics may induce highly variable α-particles sources, then lead to some difficulties in the sampling procedure. To address this problem, we present in section II a sampling method, which preserves conservation properties of the particles distribution fonction [4]. This technique has been implemented in hydro codes FCII and FCI2 and gives very good results [5]. Both fast ions and suprathermal electrons usually encounter small deflections due to collisions with the particles constituting the background plasma. In this case, a standard explicit M.C. method can be used for a numerical simulation [2]. However, whenever exists a dense region in the plasma (i.e. the shell enclosing the DT plasma in the fast ions transport problems, or the core of the D.T. pellet in ICF problems), the collision terms are dominant, and have to be treated implicitely. In section III, an implicit method for collision terms is described. It is based on a splitting of FPE. A standard finite differences scheme is used for solving the resulting diffusion equations. We then sample the obtained distribution function. This technique permits the treatment of general nonlinear terms, such as e-e collision terms. It shows that a possibility for solving collision terms with a M.C. method is to use a variable weight method, and to work in the phase space of all independent variables. We present here numerical results in the case of fast ions transport. This technique is strongly supported by its simplicity, but one might want to avoid the necessary calculation of the distribution fonction at each time step. To skip this step one can use a direct variable weight particle method for solving FPE. This technique has been used for other equations,

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such as hydrodynamics equations [8], and its convergence properties have been studied for the diffusion equation in [10]. Numerical applications of this method are under investigation at Limeil [9].

The method itsef is based on a approximation of any variable by a sum of Dirac measures. From this approximation, a system of ordinary differential equations is deduced, which has to be solved implicitely.

A short presentation of this method, as well as current numerical results are presented in section IV. Numerical results for collision terms are expected soon, to be presented elsewhere [11].

II - DISTRIBUTION FUNCTION SAMPLING

We start here with the Fokker-Planck Equation which can be written [12]

$$\frac{\partial f}{\partial t} + v \cdot \nabla_r f - \frac{e}{m}\left(E + \frac{v \times B}{c}\right) - \left(\frac{df}{dt}\right)_s$$

$$= \sum_{\alpha} Y_{\alpha} \left[4\pi \frac{m}{M_{\alpha}} F_{\alpha} f + \frac{M_{\alpha}-m}{M_{\alpha}+m} \nabla_v \mathcal{H}_{\alpha} \cdot \nabla_v f + \frac{\nabla\nabla \mathcal{G}_{\alpha} : \nabla\nabla f}{2} \right] \qquad (1)$$

In Eq.(1), f is the distribution function for the population of interest, and the F'_a s are the distribution functions for the different species existing in the background plasma. The functions $_{\alpha}$ and $_{\alpha}$ are defined by the following expressions

$$\mathcal{H}_{\alpha}(v) = \frac{M_{\alpha}+m}{M_{\alpha}} \int \frac{F_{\alpha}(u)}{\vartheta} d^3u ,$$

$$\mathcal{G}_{\alpha}(v) = \int F_{\alpha}(u)\, \vartheta \, d^3u , \qquad (2)$$

$$\vartheta = | v - u | .$$

where m is the mass of the test particles and the M'_{α} s are the masses of the different particles existing in the background plasma.

Presently neglecting in this section the diffusion terms exhibited in the rhs of Eq.(1), we can use a standard M.C. method for solving Eq.(1). However, in the case of highly variable sources, and due to the fact that this F.P. equation is coupled to a hydro code (FCI1 and FCI2), we cannot allow the size of the sample to grow too large.

We consider a 6 dimensional grid in phase space (x,v), which is fixed in time. The total nomber of cells in this grid is NMAX.

The source term is then approximated by a sum of δ functions products.

$$\left(\frac{df}{dt}\right)_S = \sum_{\ell=1}^{N} \omega_\ell \underset{\sim}{\delta}(p-p_\ell), \tag{3}$$

where

$$\underset{\sim}{\delta}\left(p-p_\ell\right) = \prod_{\substack{i=1,3\\ j=1,3}} \delta(x_i - x_i^0)\,\delta(v_j - v_j^0) \quad ,$$

and w_ℓ is the weight associated to the position (x^0, v^0) in the phase space. We thus define a sample of size N. After this sampling, we use any particle pusher to obtain positions of the particles at the advanced time $t + \Delta t$ [1].

At the end of the time step, some particles may have been the malized, and the sample size is now Q. At the begining of the new time step, M new test particles are created. The sum M + Q may be much larger than the maximum sample size allowed NMAX.

Each test particle is now replaced in the phase space grid with a CIC or NGP technique. For example, all the particles found in a given cell are merged into a new one. Its weight is equal to the sum of the old particles weights located in the cell.

In other words, an screening through the phase space grid of the distribution function f gives us an approximation f. The corresponding sample of test particles is then advanced during the new time step.

For any choice of the grid, local and global conservation properties are preserved. Moreover, the mesh may be adapted to any special physical problem of interest.

Let us consider the application of this method to fast ions transport in a plasma. In a 1.D case, and for spherical geometry, Eq.(1) becomes

$$\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial r} + \frac{1-\mu^2}{r} v \frac{\partial f}{\partial \mu} + F\left(\mu \frac{\partial f}{\partial v} + \frac{1-\mu^2}{v}\frac{\partial f}{\partial \mu}\right) \tag{4}$$

$$= \frac{1}{v^2}\frac{\partial}{\partial v}\left(Cf + D_{\parallel}\frac{\partial f}{\partial v}\right) + D_{\perp}\frac{\partial}{\partial \mu}\left((1-\mu^2)\frac{\partial f}{\partial \mu}\right) + \left(\frac{df}{dt}\right)_S$$

with

$$C = \sum_\alpha \Gamma_\alpha\, g(x_\alpha)\ ,$$

$$D_{\parallel} = \sum_\alpha \Gamma_\alpha\, g(x_\alpha)\,\theta_\alpha / mv\ ,$$

$$D_{\perp} = \frac{1}{2v\rho_\alpha^2}\left[\left(1-\frac{1}{2x_\alpha^2}\right) g(x_\alpha) + \frac{2}{\sqrt{\pi}}\, x_\alpha\, e^{-x_\alpha^2}\right] .$$

Because diffusion terms and fields are neglected, Eq.(4) reduces to

$$\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial f}{\partial \mu} = \frac{1}{v^2}\frac{\partial}{\partial v}(Cf) + \left(\frac{df}{dt}\right)_S . \tag{5}$$

In this case, the distribution function f, is approximated at the beginning of each time step by

$$f(r,\mu,v) = \sum_\ell \hat{\omega}_\ell\, \underset{\sim}{\delta}(\rho - \rho_\ell) .$$

After the screening procedure, we obtain

$$f(r,\mu,v) = \sum_{j,g,m} N_{jgm}\, \delta(r-r_j)\, \delta(v-v_g)\, \delta(\mu-\mu_m) \quad . \tag{6}$$

At this point we must recall that, when this method is coupled to a hydro-code like FCII (or FCI2 - 2.D) the simplest choice for the projection of the grid on the physical space is the lagrangian mesh used in the code itself. Thus, there are possible coarse parts in this mesh, which can alter the precision of the approximation given in Eq.(6), if the position of the new test particles is not chosen carefully. For a new particle created in cell (j,g,m), a good location in the physical space is the centre of mass of the positions of the old particles found in the cell (j,g,m). In energy space, a weighted average of the old particles energies preserves energy conservation and gives good results, even with a rather coarse grid in 1.D [5], and 2.D calculations [16].

For simple circonstances when FPE for the fast ions is decoupled from hydrocodes FCII or FCI2, we compared solutions of equation (5) given by this method with exact solutions. We assumed the source term to be given by

$$\left(\frac{df}{dt}\right)_s = S_0\, \delta(v-v_0)\, \delta(t)\, R(r) \tag{7}$$

We define the new variables $X = r\mu$ and $Y = r(1-\mu^2)^{\frac{1}{2}}$

Through this change of variables, Eq.(5) becomes

$$\frac{1}{v}\frac{\partial \bar{f}}{\partial t} + \frac{\partial \bar{f}}{\partial X} = \frac{1}{v^2}\frac{\partial}{\partial v}(Cf) + S_0\, \delta(v-v_0)\, \delta(t)\, \bar{R}(X,Y). \tag{8}$$

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Solving Eq.(8) with the method of characteristics, we obtain

$$\bar{f}(X,Y,v,t) = S_0 \frac{C(v_0)}{C(v)} \delta\left(t - \int_v^{v_0} \frac{v'}{C(v')} dv'\right) \bar{R}\left(X - \int_v^{v_0} \frac{v'^2}{C(v')} dv' \ , \ Y\right) . \tag{9}$$

In terms of the variables v, r and μ, f is expressed as follows

$$f\,(r,\mu,v,t) = S_0 \frac{C(v_0)}{C(v)} \delta\left(t - \int_v^{v_0} \frac{v'}{C(v')} dv'\right) R\left\{\left[\left(r\mu - \int_v^{v_0} \frac{v'^2}{C(v')} dv'\right)^2 + r^2(1-\mu^2)\right]^{1/2}\right\} . \tag{10}$$

We deduce from Eq.(10) the energy deposition E(r,t) in the plasma, which writes

$$E(r,t) = \iint_{[0,t]\times(e_{th},e_0)} \tilde{C}(e)\, \delta\left(t' - \int_e^{e_0} \frac{1}{m\tilde{C}(e')} de'\right) A(r,e')\, dt' \quad , \tag{11}$$

where

$$e = \frac{1}{2} m v^2 \quad , \quad \tilde{C}(e) = C(v) \quad ,$$

and

$$A(r,e) = 2\pi S_0 \frac{\tilde{C}(e_0)}{\tilde{C}(e)} \int_{-1}^{1} R\left\{\left[\left(r\mu - \int_e^{e_0} \frac{(2e'/m)^{1/2}}{m\tilde{C}(e')} de'\right)^2 + r^2(1-\mu^2)\right]^{1/2}\right\} d\mu . \tag{12}$$

Defining $H(e) = \int_e^{e_0} \frac{1}{m\tilde{C}(e')} de'$ and inserting this expression in Eq.(12), we obtain

$$E(r,t) = \int_0^t m\, \tilde{C}(H^{-1}(t'))\, A(r, H^{-1}(t'))\, dt' \ ,$$

or, with the change of variables $t' \longrightarrow e^* = H^{-1}(t')$,

$$E(r,t) = \int_{e_t}^{e_0} A(r,e^*)\, de^* \quad , \qquad H(e_t) = t . \tag{13}$$

Investigating the case of 1 MeV protons slowing-down in a 50 KeV BDT plasma, and choosing $R(r) = exp(-r^2/d^2)$ where d was taken to be equal to 10^{-2} times the thermalization length of the protons, we compared numerical results and exact solutions for the energy deposition of the fast ions to the background plasma. Results are displayed on Fig. (1) and show the very good agreement between the two curves at any time.

A interesting feature of this method is that is smooths out noise in the results.

Let us consider here a spherical plasma of radius $R = 1.1\ L_{th}$, and a constant density source of α-particles at $t = 0$. The radius of the source itself is 4 R/5 on Figure (2) and R on Figure (3).

These figures give the energy deposition of α-particles to the background plasma. Due to the fact that this method is to be coupled with FCII code, we want to use a small sample. Initially we chose here to take 10 particles per cell. Compared to a standard M.C. method P_2, our method P, gives smoother results, being still as robust and economical as P_2.

III - NUMERICAL TREATMENT OF THE COLLISION TERMS OF FPE

The approximation of FPE used in section II relies on the assumption that the background plasma is transparent to the fast particles. Whenever exists a dense region in the plasma, we have to include collision terms in the numerical treatment of FPE. Moreover the collisions have to be taken into account in an implicit manner, because we want to describe adequately cases of large gradients in density, such as may happen in ICF micro pellets. The use a direct MC method is a very difficults task, because of the integro-differential form of the collision terms in FPE.

An easier technique, however, is to use a standard finite-element or finite differences scheme for solving this problem. We thus split Eq.(1) into its hyperbolic part which is solved using the technique described in section II, and diffusion part, for which we choose a finite differences scheme. The resulting equations cannot be solve independently, because test particles slow down along straight lines only on distances small compared to the 90° deflection length (for angular diffusion terms). In a similar manner, the velocity diffusion terms in Eq.(1) spread the velocity distribution function of the particles and induce its asymptotic Maxwellian form when all the particles are thermalized in the background plasma. Thus the test particles cannot be advanced more than a thermalization length.

Thus for a given time step Δt, we have to compare three different lengths, the straight line trajectory of the test particle $L_{\Delta t}$ during the time step, the thermalization length L_{th} and the mean free path L_d. Combining $L_{\Delta t}$, L_{th} and L_d in a harmonic mean, we define the following straight line path

$$L_s = 1/\left[L_{\Delta t}^{-1} + (\alpha L_{th})^{-1} + (\beta L_d)^{-1}\right] \quad , \qquad (14)$$

where α and β are random numbers which take into account the "Random Walk" of the test particle.

We can define equivalently the time step we use to advance α-particle of velocity v,

$$\Delta t_s = v^{-1} L_s \quad .$$

The complete procedure for tho treatment of collision terms can be now described, as follows. At the begining of the time step, Δt_s is calculated and the particles are advanced during Δt_s. Then, using the phase space grid defined in Section II, we calculate the distribution function of the particles, which allows us to solve the diffusion equations obtained by splitting Eq.(1). For this, we use a finite differences scheme. The solution is then sampled, using the same grid as before.

Although very simple, the approximation chosen for Δt_s calculation can be supported by a detailed analysis, which will be presented elsewhere [4].

We consider again the example of fast ions slowing down, and solve Eq.(4), in which we neglect the electromagnetic field term. In this case, we choose to split it as follows

$$\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial r} + v \frac{(1-\mu^2)}{r} \frac{\partial f}{\partial \mu} - \frac{1}{v^2}\frac{\partial}{\partial v}(C f) = \left(\frac{df}{dt}\right)_S , \qquad (15)$$

$$\frac{\partial f}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v}\left(D_\parallel \frac{\partial f}{\partial v}\right) , \qquad (16)$$

$$\frac{\partial f}{\partial t} = D_\perp \frac{\partial}{\partial \mu}\left((1-\mu^2)\frac{\partial f}{\partial \mu}\right) . \qquad (17)$$

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In this special case, such a choice is valid, because the main result we are interested in is the energy deposition of the fast ions to plasma. However, whenever it is important to precisely describe the distribution function, (e.g. transition towards a Maxwellian), we would have to retain in Eq.(16) the full collision term and write

$$\frac{\partial f}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} \left(C f + \frac{\partial}{\partial v} \left(D_{\parallel} \frac{\partial f}{\partial v} \right) \right) . \tag{18}$$

Equation (18) would then be solved with a scheme giving an accurate asymptotic solution, such as in [14].

Considering the case presented in section II of 1 MeV protons in a 50 KeV BDT plasma, in which the diffusion is important, we used the splitting method for solving equation (4). The spherical source radius is 1/10 of the thermalization length. On Figure (4) is displayed energy deposition from the protons to ions and electrons of the BDT plasma, with and without angular diffusion term. These results are in good agreement with [13]. The small oscillations are due to the small initial number of particles per cell, here 10. The velocity diffusion term in then taken into account, and results are presented on Figure (5), for which the same comments can be done.

Applications to suprathermal electrons transport are also being devellopped and results are soon expected.

This method is a close interpretation of the physical processes. This makes it flexible, and allows other physical phenomena to be implemented in the same manner. This feature can be considered as crucial for the treatment of FPE coupled with codes like FCI1 and FCI2.

IV - "VARIABLE WEIGHT" PARTICLE METHOD

We have seen in section III that the effect of the collision terms is to redistribute the weights of the test particles. There is a generalization of this idea, which consists in approximating the distribution function with a sum of Dirac measures, of time dependent coefficients, and solving directly a system of ordinary differential equations ror these coefficients [7,8].

Although much less flexible that the method presented in the above section, it should be of great interest for including collision terms in standard particle codes. We devote this section to the description of this method which has been developed in [1,8]. Theoretical extension to diffusion is presented in [10]. Application to Vlasov Poisson equations is presented in [9], with numerical results. A description of the method, applied to FPE, can be found in [15]. Also numerical applications to electron/ion transport are under investigation [11]. Following [10] and [15], we can describe this method as follows.

Let us rewrite Eq.(1) as follows

$$\frac{\partial f}{\partial t} + \sum_{1}^{6} \frac{\partial}{\partial x^i}(a^i f) + \sum_{3}^{6} \frac{\partial}{\partial x^i}\left(b^i \frac{\partial f}{\partial x^i}\right) = \left(\frac{df}{dt}\right)_s , \tag{19}$$

where $x = (x^i)$ is the phase space vector (x,y,z,v_x,v_y,v_z).

We define as the solution of the differential equation first $x=x(t,\xi)=F_t(\xi)$

$$\frac{d}{dt}(x(t)) = a(x(t),t) \quad , \quad x(0) = \xi \tag{20}$$

and $J(t,\xi)$ as the jacobian of the transformation F_t.

Then J is a solution of the differential equation

$$\frac{d}{dt}\left(J(t)\right) = J(t)\ \nabla \cdot a\left(x(t),t\right)\ , \qquad J(0) = 1 . \tag{21}$$

New considering a function f, for any fonction ϕ, we can define the integral

$$\int f \varphi \, dx = \int f\left(x(t,\xi)\right) \varphi\left(x(t,\xi)\right) J(t,\xi)\, d\xi \ . \tag{22}$$

Any numerical integration of (22) gives a expression of the following type

$$\int f \varphi \, dx = \sum_{1}^{n} \omega_j\, f\left(x(t,\xi_j)\right) \varphi\left(x(t,\xi_j)\right) J(t,\xi_j) \ . \tag{23}$$

Defining $f_j(t) = f\left(x\left(t,\xi_j\right)\right)$, $x_j(t) = \left(x_j^n\right) = x(t,\xi_j)$, and $J_j(t) = J(t,\xi_j)$, Eq.(23) becomes

$$\int f \varphi \, dx = \sum_{1}^{N} \omega_j\, J_j(t)\, f_j(t)\ \varphi\left(x_j(t)\right) \ . \tag{24}$$

From Eq.(24), we deduce a "weak" approximation of f, $\tilde{f}$, which is expressed as

$$\tilde{f}(x) = \sum_{1}^{N} \omega_j\ J_j(t)\ f_j(t) \prod_n \delta\left(x^n - x_j^n\right) \ . \tag{25}$$

After regularization of the Dirac mesures δ through a convolution with a C^∞ function ζ_n such that $\int \zeta_n(x^n)\, dx^n = 1.$, the approximation f can be defined. We obtain

$$\hat{f}(x) = \sum_{1}^{N} \omega_j\ J_j(t)\, f_j(t) \prod_n\ \zeta_n\left(x^n - x_j^n\right) \ . \tag{26}$$

A detailed discussion of the suitable choice for the weights ω_j can be found in [15] Also in [15] is presented possible choices for the shape functions (e.g hat-function, gaussian, super-gaussian...).

The transport term in phase space of Eq.(19) is

$$\frac{\partial f}{\partial t} + \sum_{i}^{6} \frac{\partial}{\partial x^i}(a^i f) \qquad (21)$$

Multiplying expression (27) by ϕ, and integrating over phase space, then inserting the approximation given by Eq.(25), we obtain

$$\frac{\partial f}{\partial t} + \sum_i \frac{\partial}{\partial x^i}(a^i f) \simeq \sum_j \frac{d}{dt}\left(\omega_j J_i(t) f_j(t)\right) \prod_n \delta\left(x^n - x_j^n\right). \qquad (28)$$

The treatment of the diffusion terms in Eq.(19) is more delicate [10,15]. If we define $q^i = \frac{\partial f}{\partial x^i}$, we obtain

$$\frac{\partial}{\partial x^i}\left(b^i \frac{\partial f}{\partial x^i}\right) = b^i \frac{\partial q^i}{\partial x^i} + q^i \frac{\partial b^i}{\partial x^i} . \qquad (29)$$

Now, applying the approximations given by Eq.(25), (26) to b^i and q^i,

we can write

$$\tilde{b}_1^i = \sum_j \omega_j J_j b_j^i \prod^6 \delta\left(x^n - x_j^n\right),$$

$$\hat{b}^i = \sum_k \omega_k J_k b_k^i \prod^6 S_n\left(x^n - x_j^n\right). \qquad (30)$$

From Eq.(30), we can deduce

$$\frac{\partial \hat{b}^i}{\partial x^i} = \sum_k \omega_k J_k b_k^i S'_i\left(x^i - x_k^i\right) \prod_{n \neq i}\left(x^n - x_k^n\right) . \qquad (31)$$

The approximate diffusion terms can now be obtained through the use of Eqs.(29-31). We have

$$\frac{\partial}{\partial x^i}\left(b^i\frac{\partial f}{\partial x^i}\right)=\sum_j\sum_k \omega_j J_j \omega_k J_k\left(b_j^i q_k^i + b_k^i q_j^i\right)\zeta_i'(x_j^i - x_k^i)\prod_{n\neq i}\zeta_n(x_j^n - x_k^n) \qquad (32)$$

The function ζ_n are of finite extension ε_n. Assuming that $|x_j - x_k| < \varepsilon$ with $\varepsilon = \max_n(\varepsilon_n)$, we approximate the term $b_j^i q_k^i + b_k^i q_j^i$ as follows

$$b_j^i q_k^i + b_k^i q_j^i = \left(b_j^i + b_k^i\right)\left(f_j - f_k\right)\frac{x_j^i - x_k^i}{(x_j^i - x_k^i)^2 + \varepsilon^2} \qquad (33)$$

From Eqs.(32-33) we deduce

$$\frac{\partial}{\partial x^i}\left(b^i\frac{\partial f}{\partial x^i}\right) = \sum_j\sum_k D_{jk}^i f_k \prod_n \delta(x^n - x_j^n),$$

where

$$D_{jk}^i = \omega_j J_j\left(b_j^i \sum_\ell N_{j\ell}^i \omega_\ell J_\ell + \sum_\ell N_{j\ell}^i \omega_\ell J_\ell b_\ell^i\right)\delta_{jk} \qquad (34)$$

$$- \omega_j J_j \omega_k J_k \left(b_j^i + b_k^i\right) N_{jk}^i ,$$

and

$$N_{jk}^i = \frac{(x_j^i - x_k^i)\,\zeta_i'(x_j^i - x_k^i)}{(x_j^i - x_k^i)^2 + \varepsilon^2}\prod_{n\neq i}\zeta_n(x_j^n - x_k^n)$$

It has been shown [10] that D_{jk}^i is a symmetric matrix, and positive definite, and that this numerical scheme is conservative.

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Equations (28) and (34) determine a system of N ordinary differential equations. The numerical technique is then to sample the initial distribution function, then to solve this N-system.

Current numerical results are displayed on Figures (6) and (7). Figure (6) gives energy deposition from α-particles to a 50 KeV spherical DT plasma with a source located at the centre of the sphere. Figure (7) shows a typical output obtained investigating the two beam plasma instability [9].

This technique, although solving diffusion terms in the framework of a particle method, does not have the flexibility of the method described in section III. Its cost seems higher than standard MC methods. However, its advantages outweight these drawbacks. It allows in effect the precise complete treatment of any non linear diffusion term, which should be of great interest in the case of electron transport problem.

V - CONCLUSION

The utility of M.C. methods is greatly enhanced by designing them to be more robust and economical, so that they are applicable to more demanding physical situations, such as transport in non homogeneous plasmas, and that they can be coupled to transport hydro-codes. Besides the importance in many problems of the collision terms suggests that any particle method should easily handle such terms.

Economy requires that the interaction of the particles with a plasma in which large differences of temperature and spatial gradient lengths arise can be represented with a minimum size sample. In the same way, it is likely that the algorithm reproduces important limiting cases and preserves gross conservation properties even with coarse zoning of the background plasma, or large time step.

The method presented in section I, II and III accomplishes this goal and strongly supports the use of non deterministic particle methods for solving generalized transport problems, such as those described by Fokker Planck Equation.

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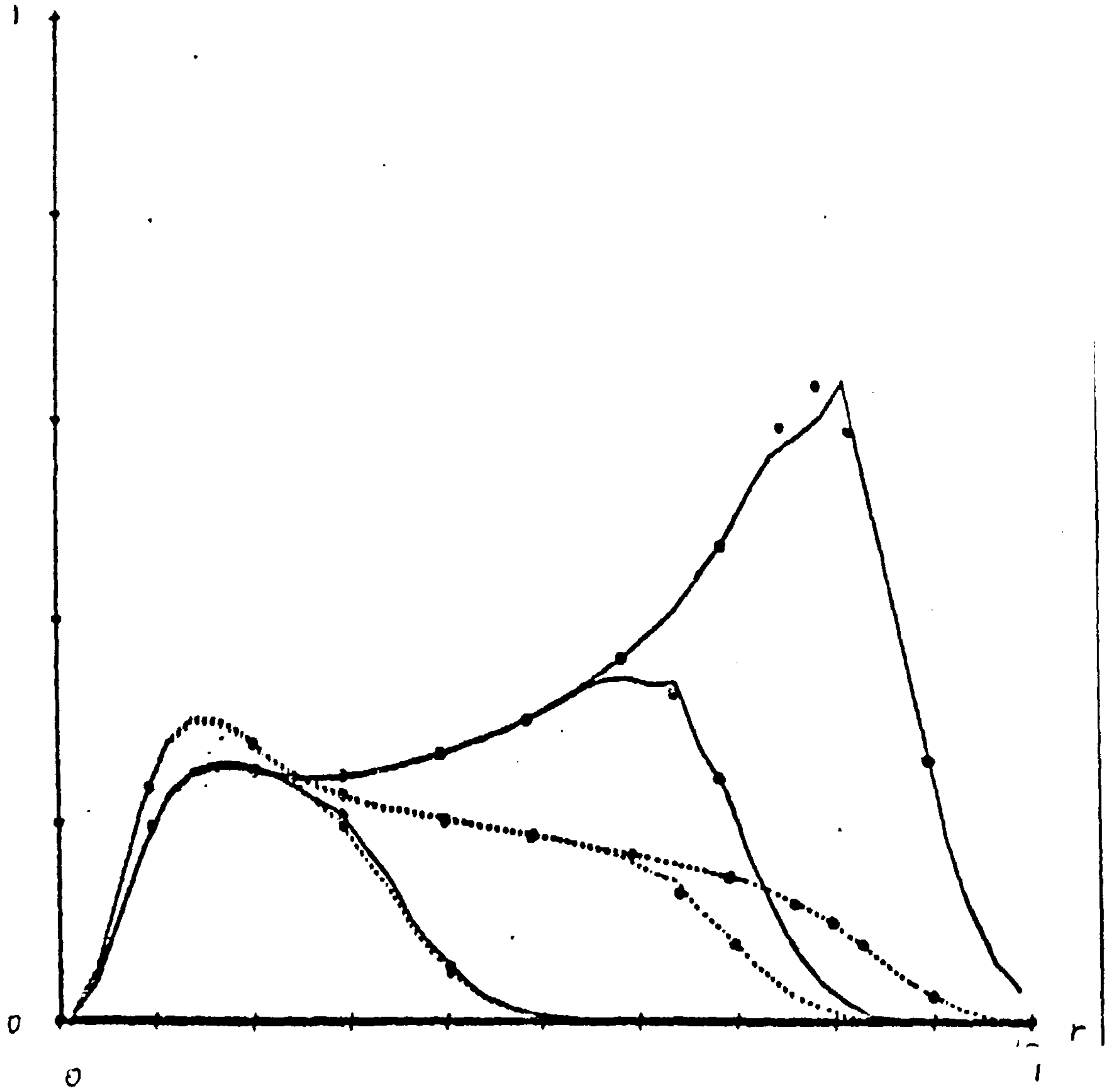

Figure 1. 1 Mev protons slowing down in a 50 Kev DT plasma
Energy deposition to the ions ———
to the electrons . . . .
exact solution . . .

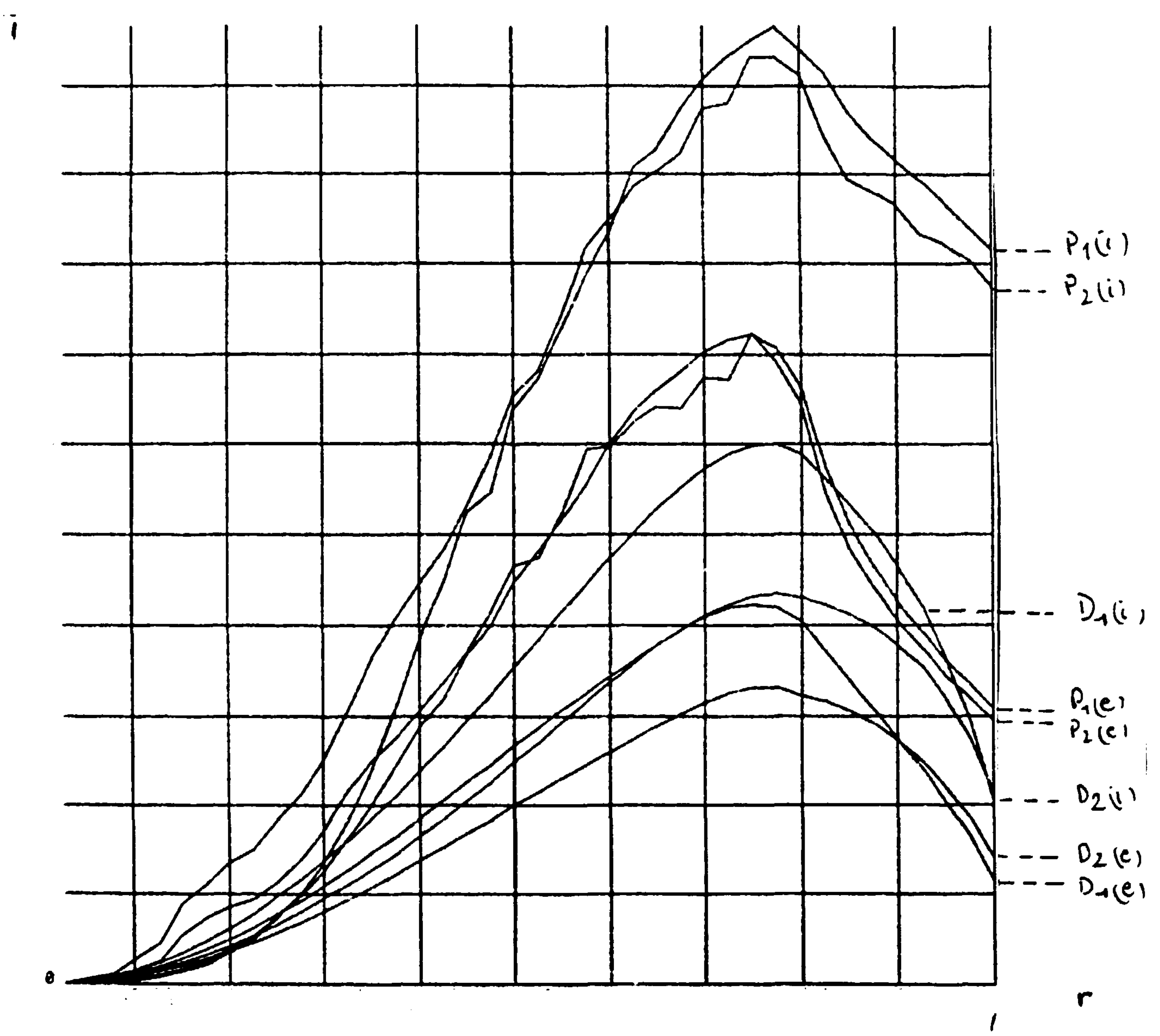

Figure 2. Energy deposition of 3.5 MeV α-particles in a spherical 50 KeV DT plasma.

Source. $0 \leq r \leq 4R/5$

P_1 M.C method 1

P_2 M.C. method 2

D_1 Diffusion 1

D_2 Diffusion 2.

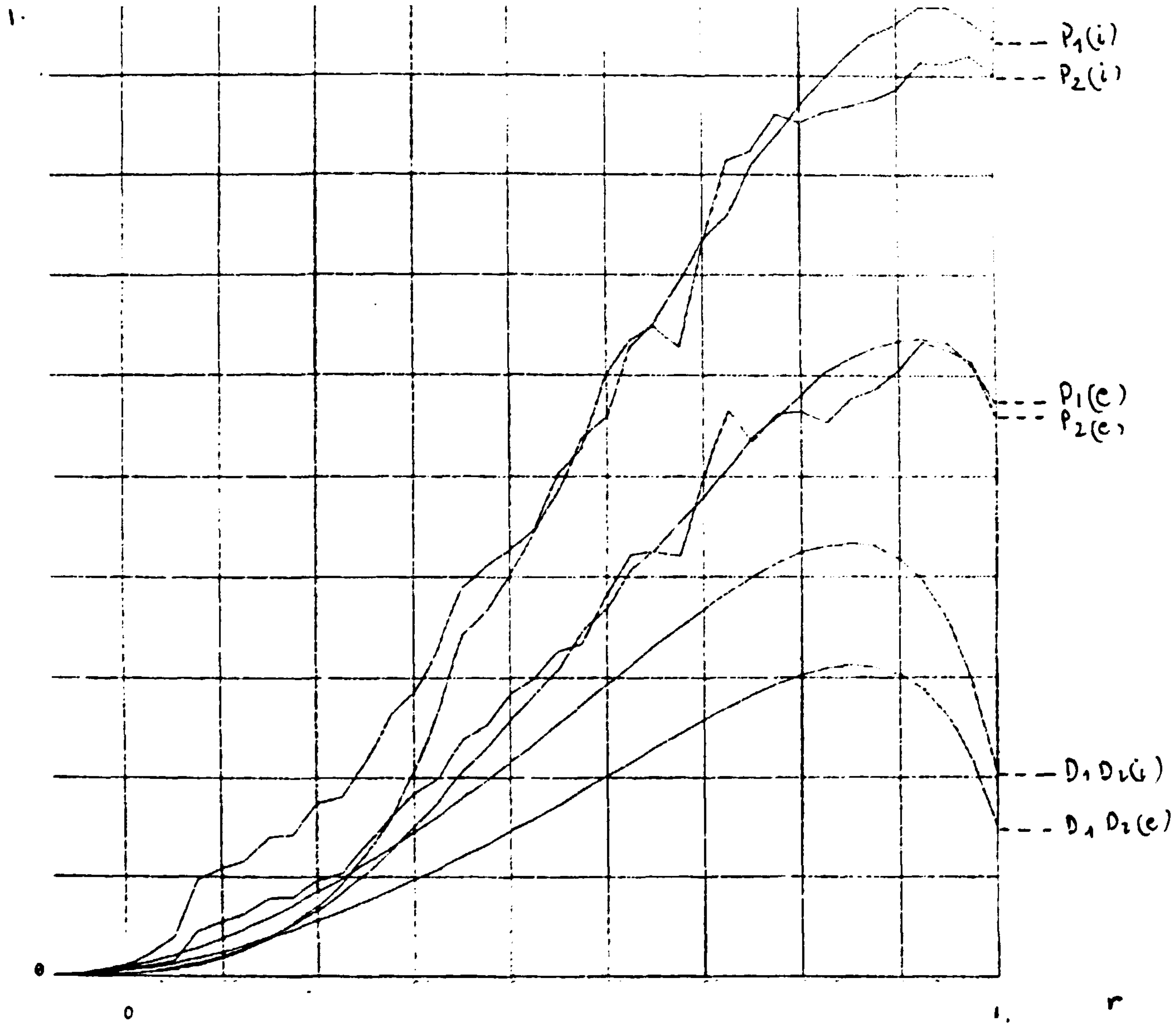

Figure 3. Energy deposition of 3.5 MeV α particles in a 50 KeV spherical DT plasma

Source $0 \leq r \leq R$

P_1 MC method 1

P_2 MC method 2

D_1 Diffusion method 1

D_2 Diffusion method 2.

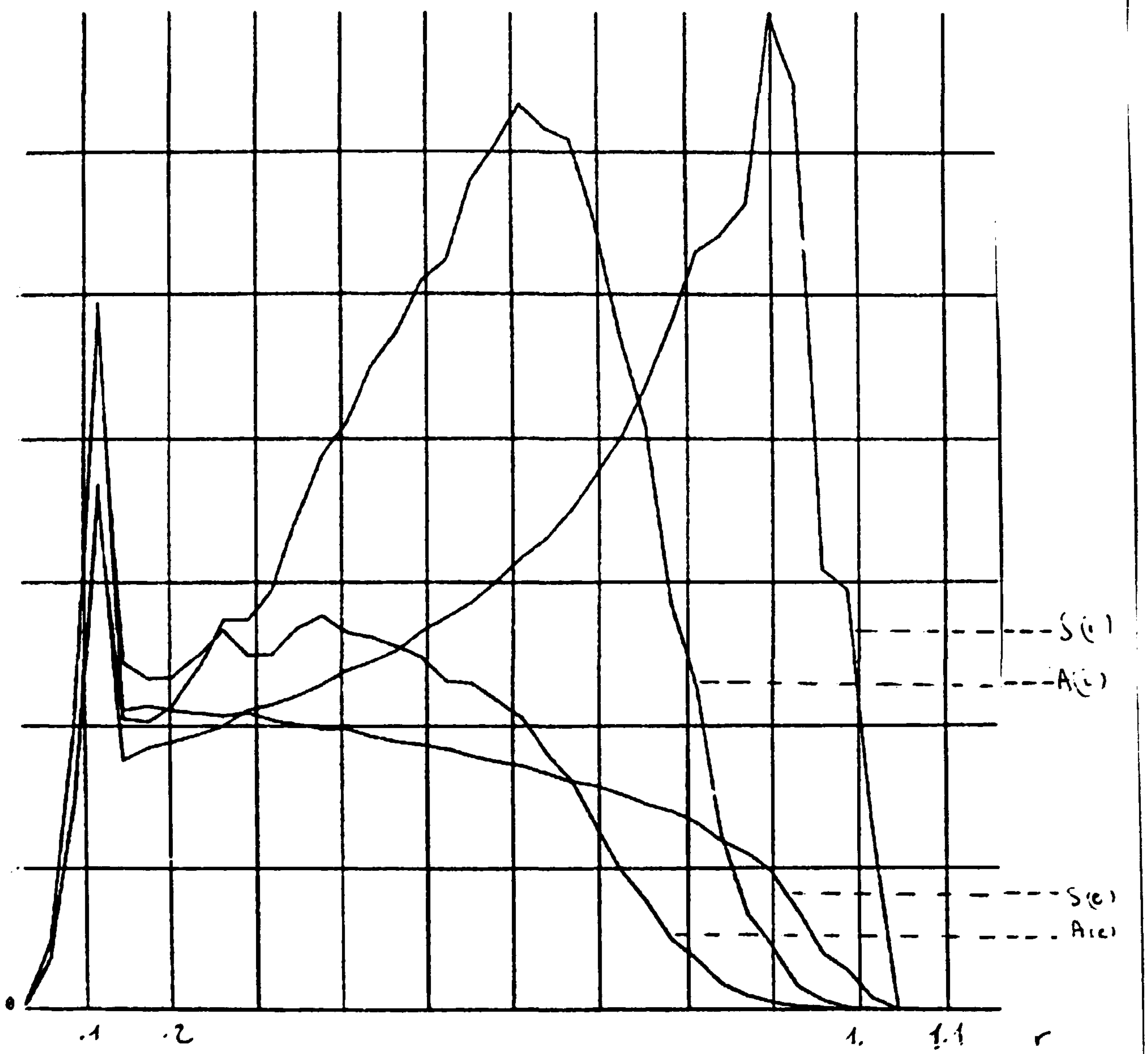

Figure 4. Energy deposition of 1 Mev protons in a 50 Kev BDT spherical plasma

(S) without angular diffusion terms

(A) with diffusion terms

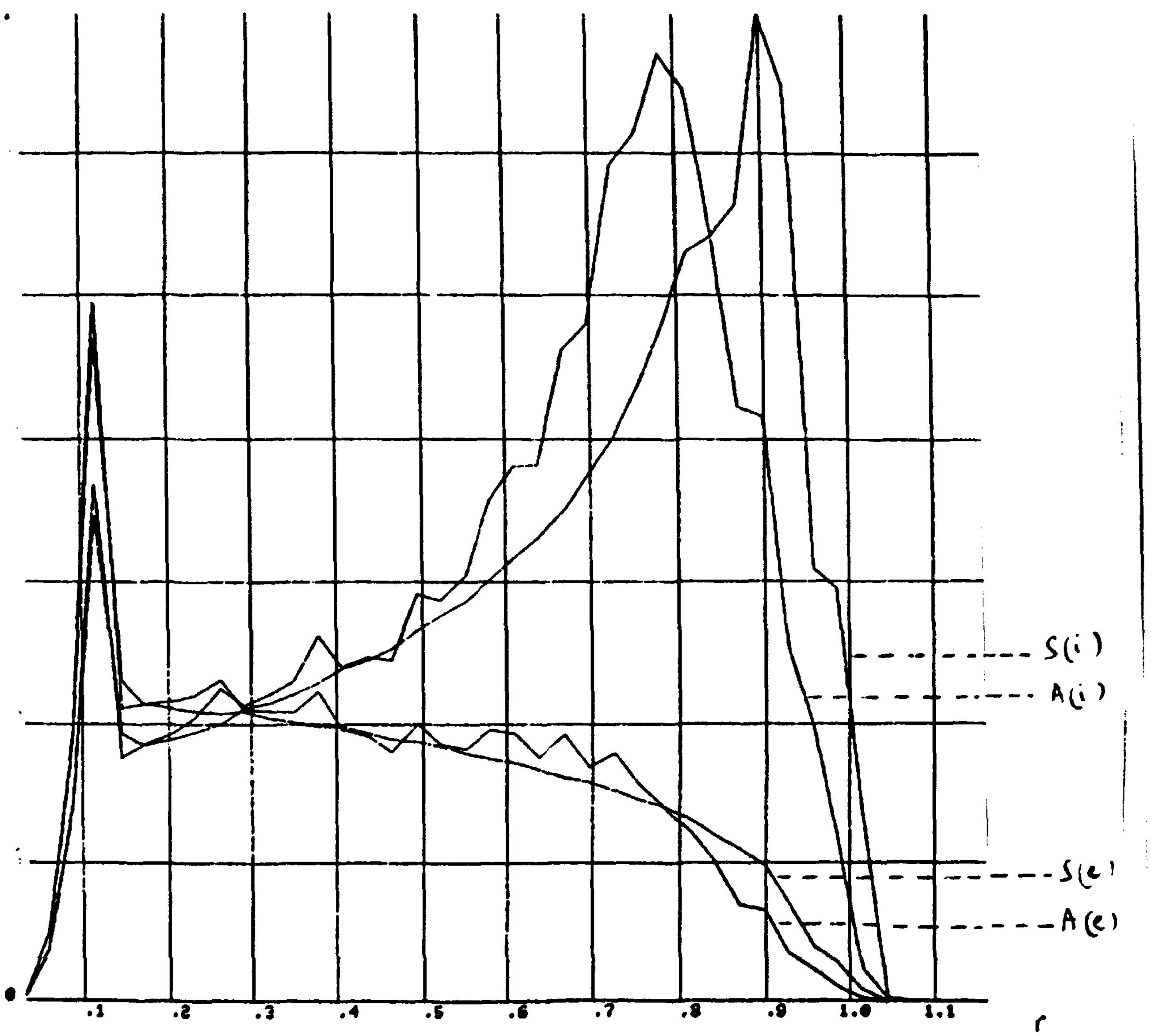

Figure 5. Energy deposition of 1 Mev. Protons in a 50Kev BDT spherical plasma.
(S) without velocity diffusion terms
(A) with velocity diffusion terms.

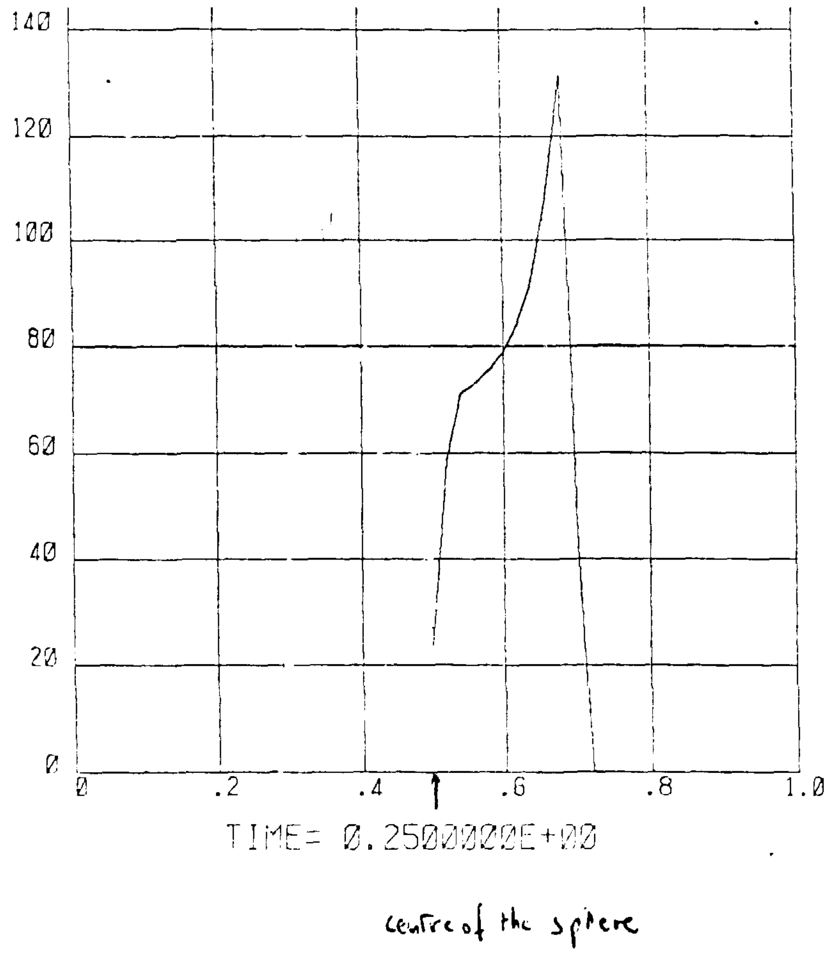

Figure 6. Energy deposition of 3.5 MeV α-particles in a 50 keV DT plasma, without diffusion, in arbitrary units.

FIG. 7

$\Delta t = 0.5 \quad \Delta x = 2$

Two beam instability - Phase space -