RESONANT DEPOLARIZATION IN ELECTRON STORAGE RINGS
EQUIPPED WITH "SIBERIAN SNAKES"

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J. BUON

Laboratoire de l'Accélérateur Linéaire, 91405 ORSAY - France

Résumé - Il est montré que les "serpents sibériens" ne permettent pas de réduire les effets dépolarisants dans les anneaux de stockage à électrons, contrairement aux synchrotrons à protons.

Abstract - Resonant depolarization induced by field errors and quantum emissions in an electron ring equipped with two "siberian snakes" is investigated with a first order perturbation calculation. It is shown that this depolarization is not reduced by the snake when the operating energy is set out of the depolarization resonances.

"Siberian snakes" have been proposed [1] for avoiding the crossing of depolarization resonances during the acceleration of polarized protons in synchrotrons. These siberian snakes are magnetic devices which rotate the spin by 180° around an horizontal axis. They produce an overall spin precession of 180° per turn around the vertical line. The spin tune is half-integer whatever is the energy, avoiding the depolarization resonances. However, at high energies, residual effects of these resonances are still troublesome and schemes with several snakes are needed [2].

Siberian snakes have also been proposed for electron storage rings as remedies to the strong depolarizing effects which occur at high energies [3]. However, no detailed study of their ability has been presented in this case, although the situation is very different from that of the proton synchrotrons. In an electron storage ring the energy is fixed and depolarization resonances are not crossed. Normally the energy is set such that the spin tune is half-integer, midway from the closest resonances. However, at high energies depolarizing effects, driven by quantum fluctuations of the synchrotron radiation in presence of field errors, are so strong that they depolarize significantly even outside the resonances. The question is the ability of siberian snakes to reduce these depolarizing effects. It has already been quoted by E. Courant [2] that they may not be so efficient outside the resonances.

Moreover, at very high energies the beam energy dispersion becomes comparable with the resonance spacing. It results a strong spin tune modulation which excites synchrotron satellite resonances. It has been recognized that siberian snakes allow to avoid this spin tune modulation with energy, and suppress this depolarization enhancement due to large energy dispersion.

Besides, siberian snakes counteract the Sokolov-Ternov polarization mechanism in electron storage rings. In a ring with one siberian snake the spin closed solution, along which the polarization can only be maintained, lies in the horizontal plane. In this case the polarizing mechanism vanishes and one is
left with the depolarizing effect of the spin-flip synchrotron radiation. The only way to avoid this fast depolarization is to have a vertical spin closed solution, parallel to the magnetic field of magnets. That can be achieved by installing in the ring two siberian snakes [4]. If they are symmetrically located and if they rotate the spins around two orthogonal and horizontal axes, the spin tune is half-integer and energy-independent as expected. The spin closed solution is upward in one half of the ring and downward in the other half. The Sokolov-Ternov mechanism is polarizing in the former half and depolarizing in the latter, with a null balance. Some asymmetry between the two halves is needed for obtaining a non-zero polarization equilibrium. Wiggler magnets, or kink magnets, can provide for such an asymmetry.

Here is presented the result of a first order perturbation study of the resonant depolarizing effects enhanced by synchrotron radiation, in the case of an electron storage ring equipped with two such siberian snakes. The result is given in terms of the spin-orbit coupling functions with and without snakes respectively, the field errors being the same. The depolarizing effects of the snakes themselves are assumed negligible as could be obtained with spin matching applied to these snakes. The length of the snakes is also neglected, assuming a very large ring and relatively short snakes. Finally higher order effects like spin tune variation with fields errors are ignored in this first order computation.

**FIRST-ORDER CALCULATION OF THE SPIN-ORBIT COUPLING FUNCTION**

The general analytical method of first-order perturbation developed previously [5] is applied here. Schematically the ring consists of two arcs and two snakes at azimuths 0 and π respectively. In each arc the spin precesses by ν around the vertical line in absence of field errors (ν = E(GeV)/.44065 is the spin tune). The rotation axes of the snakes are at angles Φ₁ and Φ₂ respectively in the horizontal plane, and are orthogonal (Φ₂ - Φ₁ = π/2).

Within the 2 × 2 matrix formalism, the spin transfer matrix of one arc, followed by a snake, is:

\[
S_{1,2} = -i[(v_+ e^{-i(\phi_1,2 + \text{hermitian conjugate})})
\]

where \(a_{1,2} = \nu/2 - \Phi_{1,2}\) and \(v_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\)

The one-turn spin transfer matrix is:

\[
T = S_{1,2} S_{2,1} = i\sigma_3 = e^{i\sigma_3 \pi/2}
\]

independently of the origin. It exhibits the half-integer spin tune.
The unperturbed spin transfer matrix \( D \) between an azimuth \( \Theta \) (0 < \( \Theta < \pi \)) and an azimuth \( \theta \) (\( n\pi < \theta < (n+1)\pi \)) is written, with respect to an uniform-spin-precession frame:

\[
D_0(\Theta, \theta) = e^{-i\theta/2} e^{i\theta_0/2} D_0(\Theta, n\pi) e^{i\theta_0/2}
\]

with

\[
D_0(\Theta, 2n\pi) = T^k \quad \text{and} \quad D_0(\Theta, (2k+1)\pi) = S_1 T^k
\]

At azimuth \( \Theta \), an electron emits a photon and undergoes some betatron and synchrotron oscillations. Due to field errors, the electron experiences radial fields which rotate the spin out of the vertical line. This spin rotation is represented by a 2 \( \times \) 2 matrix \( \varepsilon \):

\[
\varepsilon = \varepsilon_+ \sigma_+ + \text{h.c.}
\]

and \( \varepsilon_+ \) has a Fourier decomposition:

\[
\varepsilon_+ = \sum_j \varepsilon_j e^{-i\nu_j \phi}
\]

where the frequency tunes \( \nu \) are linear combinations of the betatron and synchrotron tunes and of integers. A depolarization resonance occurs when the spin tune coincides with one of the \( \nu_j \) lines.

The amplitudes \( \varepsilon_j \) are proportional to the energy fluctuation:

\[
\varepsilon_j = \eta_j \delta E/E.
\]

At first order, the perturbed spin transfer matrix \( D \) between azimuths \( \Theta \) and \( \theta \) is given by [51]:

\[
D(\Theta, \theta) = D_0(\Theta, \theta)[1 - i \Lambda(\Theta, \theta)/2]
\]

with

\[
\Lambda(\Theta, \theta) = \int_{\Theta}^{\theta} D^+(\Theta, \theta') D_0(\Theta, \theta') d\theta'
\]

The 2 \( \times \) 2 matrix \( \Lambda(\Theta, \theta) \) describes the spin rotation out of the vertical line. It is decomposed into a sum:

\[
\Lambda(\Theta, \theta) = \int_0^\pi + \int_\pi^{2\pi} + \ldots + \int_{(n-1)\pi}^{n\pi} D^+(\Theta, \theta') D_0(\Theta, \theta') d\theta'
\]

A straightforward calculation leads to:

\[
\int_{\Theta}^{\theta} D^+ D_0 d\theta = \sum_j \varepsilon_j e^{-i(\nu_j - \nu_j)\theta} e^{-\frac{i}{i(\nu_j - \nu_j)} e^{-i\nu_j \Theta} - 1} \sigma_+ + \text{h.c.}
\]
Extending the final azimuth $\theta$ up to infinity, and summing up all the integrals (the convergence of the series being assured by the oscillation damping), one obtains the overall spin rotation:

$$\lambda(\theta_0, \omega) = \sigma_+ \text{SE/E} \sigma_+ + \text{h.c.}$$

with:

$$\lambda_+ = \sum_j \left( -2 \eta_j e^{i(\nu + \nu_j) \theta_0 / 2} \sin(\nu - \nu_j) \theta_0 / 2 \right)$$

$$+ \left( \eta_j e^{i(\nu + \nu_j) / 2} \sin(\nu - \nu_j) / 2 \right) \right)$$

The modulus of the spin-orbit coupling function $\gamma \partial n / \partial y$ at azimuth $\theta_0$ is just the modulus of $\lambda_+$. It can be compared to the similar expression in the ordinary case without snakes:

$$\lambda_+^{\text{without}} = \sum_j \eta_j e^{-i \nu \theta_0 \eta_j}$$

CONCLUSION

On the above expression of $\lambda_+$ one observes, as expected, the cancellation of the spin-orbit coupling function and the suppression of the depolarization on the top of the resonances $\nu = \nu_j$. However, out of these resonances the spin-orbit coupling function has about the same order of magnitude as in the ordinary case without snakes.
For instance, assuming only one resonance, the ratio \( R \) of the spin-orbit coupling functions with and without snakes, at the origin \((\theta_0 = 0)\), satisfies the inequality:

\[
\frac{\sin \pi(v - v_j)/2}{|\cos \nu_j|} < 2
\]

Figure 1 shows the variation of this maximum value of \( R \) versus the resonance tune \( v \) for different spin tunes in the arcs.

One observes also a strong depolarization for a half-integer resonance tune \((v_j = 1/2)\) whatever \( v \) is the spin tune in the arcs, as expected since the snakes produce an overall spin tune which is half-integer.

Finally, the only advantage of installing Siberian snakes in electron storage rings is to suppress the depolarization enhancement produced by the energy dispersion. This advantage must be balanced with the suppression of the polarizing Sokolov-Ternov mechanism.

Fig. 1 - Maximum value of the ratio \( R \) between the spin orbit coupling functions with and without snakes versus the tune \( v \) of an isolated depolarization resonance and for different values of the fractional part of the spin tune \( v \).

REFERENCES


