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ELECTRONS IN A STRONG MAGNETIC FIELD

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1. INTRODUCTION

The quantized Hall effect is a spectacular macroscopic manifestation of quantum mechanics[1]. It is therefore not inappropriate to present some work motivated by this discovery in a Conference dedicated to the memory of Niels Bohr.

We first describe the average one-particle spectrum in the presence of a strong magnetic field together with random impurities obtained by F. Wegner[2] for a Gaussian distribution, and generalized in collaboration with E. Brézin and D. Gross[3], using a supersymmetric method.

We then study the effect of Coulomb interactions on an electron gas in a strong field, within the approximation of a projection on the lowest Landau level. At maximal density (or filling fraction ν equal to unity) the quantum mechanical problem is equivalent to a soluble classical model for a two-dimensional plasma[4]. As ν decreases, more states come into play. Laughlin[5] has guessed the structure of the ground state and its low lying excitations for certain rational values of the filling fraction. A complete proof is however missing, nor is it clear what happens as ν becomes so small that a "crystalline" structure becomes favoured.

Our presentation shows a link with functions occurring in combinatorics and analytic number theory, which seems not to have been fully exploited.

2. ONE PARTICLE SPECTRUM AND IMPURITIES

Let B be a strong magnetic field perpendicular to an x-y plane in which electrons are confined. In this section we measure lengths and energies in units $R_0 = \left(\frac{2\hbar}{eB}\right)^{1/2}$ and $E_0 = \frac{eB\hbar}{4m}$ respectively, in such a way that a circle of radius R_0 encloses a unit quantum flux. Neglecting spin, the Schrödinger equation reads, in the so called symmetric gauge,

$$(H_0 + V - E)\psi = 0 \qquad H_0 = 2(2a^+a+1)$$
$$a = \bar{\partial} + \frac{1}{2} z \qquad a^+ = -\partial + \frac{1}{2}\bar{z}$$
$$[a,a^+] = 1 \quad . \tag{2.1}$$

We use a complex notation $z = x + iz$.

The eigenstates are characterized by the Landau quantum number, i.e. the eigenvalue of $a^{+}a$. The states pertaining to the lowest value, zero, to which we henceforth limit ourselves are solutions of $a\psi = 0$, i.e. of the form[6]

$$\psi(z,\bar{z}) = e^{-\frac{1}{2} z\bar{z}} \varphi(z) \qquad E_0 = 2 \tag{2.2}$$

with $\varphi(z)$ analytic. An orthonormal basis is

$$\psi_n(z,\bar{z}) = \frac{e^{-\frac{1}{2} z\bar{z}}}{\sqrt{\pi}} \frac{z^n}{\sqrt{n!}} \quad . \tag{2.3}$$

The projector on the (infinitely degenerate) lowest level is

$$P(z',\bar{z}) = \sum_{n=0}^{\infty} \psi_n(z')\bar{\psi}_n(\bar{z}) = \frac{1}{\pi} \exp\{-\frac{1}{2}(z'\bar{z}'+z\bar{z}) + z'\bar{z}\} \quad . \tag{2.4}$$

The density of states per unit area is normalized to π^{-1} with our units. In the unperturbed case it is

$$\rho_0(E) = \mathrm{Im}\, \frac{1}{\pi} \langle \underline{x} | \frac{1}{E-H_0-i\epsilon} | \underline{x} \rangle = \delta(E-E_0)P(z,\bar{z}) = \frac{1}{\pi}\delta(E-E_0) \quad . \tag{2.5}$$

In the presence of impurities we substitute $H = H_0 + V$ for H_0 in (2.5) and average over V.

To perform this average, denoted by a bracket, it is convenient to represent a matrix element of the resolvent $(E-H)^{-1}$, with a vanishingly small negative imaginary part implied for E, by a path integral over a complex scalar field $\varphi(\underline{x})$. The normalizing factor of this integral is replaced by a similar integral over a Grassmanian field $\psi(\underline{x})$. Consequently

$$\begin{aligned} \pi\rho(E) &= \langle \mathrm{Im}\; i \int D(\varphi,\bar{\varphi},\psi,\bar{\psi})\, \bar{\varphi}(\underline{x})\varphi(\underline{x}) \exp -iA \rangle \\ A &= -i \int d^2x\, \{\bar{\varphi}(E-H_0)\varphi + \bar{\psi}(E-H_0)\psi - V(\bar{\varphi}\varphi + \bar{\psi}\psi)\} \quad . \end{aligned} \tag{2.6}$$

For the random potential $V(\underline{x})$ we assume zero range correlations, with generating function

$$\langle \exp i \int d^2x\, V(\underline{x})\alpha(\underline{x}) \rangle = -\int d^2x\, g(\alpha(\underline{x})) \quad . \tag{2.7}$$

This means that $g(\alpha)$ is related to the single site probability distribution $P(V)dV$ through

$$\int dV\, P(V) \exp iV\alpha = \exp - g(\alpha) \; ; \quad g(0) = 0 \quad . \tag{2.8}$$

In the Gaussian case

$$g(\alpha) = -\frac{W}{2}\alpha^2 \quad . \tag{2.9}$$

A Poissonian model due to Friedberg and Luttinger[7] of zero range scatterers with uniform distribution of density $\frac{f}{\pi}$, leads to

$$g(\alpha) = f(1-e^{i\lambda\alpha}) \tag{2.10}$$

where λ is the intensity of the potential.

For a magnetic field B strong enough we project all states on the lowest Landau level, neglecting transitions to excited levels. This is implemented by restricting the fields in the path integral (2.6) to the space generated by the basis (2.3). We choose

$$\varphi = \frac{e^{-\frac{1}{2}z\bar{z}}}{\sqrt{\pi}}\, u(z) \qquad \bar{\psi} = \frac{e^{-\frac{1}{2}z\bar{z}}}{\sqrt{\pi}}\, \bar{v}(z) \tag{2.11}$$

with u (bosonic) and $\bar{v}$ (fermionic) analytic in z. Both are combined in a superfield Φ analytic in the combined (z,θ) variables, where θ is an independent Grassman variable

$$\phi(z,\theta) = u(z) + \theta\, \bar{v}(z) \quad . \tag{2.12}$$

The crux of the method is the identity[3]

$$\frac{1}{n}\int d\bar{\theta}\, d\theta\, (e^{-\bar{\theta}\theta}\, \bar{\phi}\phi)^n = (\bar{u}u + \bar{v}v)^n \quad . \tag{2.13}$$

In terms of the function $(g(0) = 0)$

$$h(\alpha) = \int_0^{\alpha} \frac{d\beta}{\beta}\, g(\beta) \tag{2.14}$$

the average over V leads to

$$\begin{aligned} \pi\rho(E) &= \text{Im}\, i \int \mathcal{D}(u,\bar{u},v,\bar{v}) \left[\frac{e^{-\bar{z}z}}{\pi}\, \bar{u}(\bar{z})\, u(z)\right] \exp{-A} \\ A &= \int d^2z\, d\bar{\theta}\, d\theta \left\{ i(E-E_0-i\varepsilon)\, \frac{e^{-(\bar{z}z+\bar{\theta}\theta)}}{\pi}\, \bar{\phi}\phi + h\left(\frac{e^{-(\bar{z}z+\bar{\theta}\theta)}}{\pi}\, \bar{\phi}\phi\right)\right\} \end{aligned} \tag{2.15}$$

The action A displays explicitly the invariance under superspace rotations, and translations in the form

$$\phi(z,\theta) \to \phi(z-a,\theta-\omega)\, \exp\{\bar{a}z + \bar{\omega}\theta - \frac{1}{2}(\bar{a}a + \bar{\theta}\theta)\} \quad . \tag{2.16}$$

This enables one to substitute in (2.15) the invariant form $[\pi^{-1}\exp\{-(\bar{z}z+\bar{\theta}\theta)\}\bar{\phi}\phi]$ for $[\pi^{-1}\exp\{-\bar{z}z\}\bar{u}u]$, since both lead to the same c-number result. The last step is a perturbative expansion of (2.15). One recognizes that the propagator is the exponential of a supersymmetric quadratic form in z and θ, thanks to the reproducing kernel

$$\int d^2z' \, d\bar{\theta}' \, d\theta' \, \frac{1}{\pi} \exp\{-(\bar{z}'z + \bar{\theta}'\theta) + \bar{z}'z + \bar{\theta}'\theta\} \, \phi(z',\theta') = \phi(z,\theta) \ . \quad (2.17)$$

As a result each Feynman integral in this expansion is identically one, as in a zero-dimensional model, leaving only combinatorial factors. This yields the following result

$$\boxed{\pi\rho(E) = -\frac{1}{\pi} \operatorname{Im} \frac{d}{dE} \left\{ \ln \int_0^\infty d\alpha \, e^{-i(E-E_0-i\varepsilon)\alpha - h(\alpha)} \right\}} \quad (2.18)$$

where $h(\alpha)$ is related to the single site distribution through (2.8) and (2.4). Special cases are

(i) Gaussian - we recover Wegner's formula[2]

$$g(\alpha) = w\frac{\alpha^2}{2} \qquad h(\alpha) = w\frac{\alpha^2}{4}$$

$$\pi\rho(E) = \frac{2}{\pi\sqrt{\pi w}} \; \frac{e^{\frac{1}{w}(E-E_0)^2}}{1 + \frac{4}{\pi}\left(\int_0^{(E-E_0)/\sqrt{w}} dt \, e^{t^2}\right)^2} \quad (2.19)$$

(ii) Lorentzian - self reproducing

$$P(V) = \frac{\lambda}{\pi} \frac{1}{V^2+\lambda^2} \qquad g(\alpha) = \lambda|\alpha|$$

$$\pi\rho(E) = \frac{\lambda}{\pi} \frac{1}{(E-E_0)^2+\lambda^2} \quad . \quad (2.20)$$

For a discussion of the Poissonian model we refer to[3]. The conclusion is that short range impurities are easily handled as far as obtaining the average (broadened) one particle spectrum. In general it does not seem to reveal very specific features. To study the transport properties which involve averages of the modulus square of matrix elements of the resolvent, one needs a generalization of the above method which does not seem to yield easily manageable expressions.

In the sequel we shall use (2.4) as a projector on one-body states pertaining to the lowest Landau band and absorb the exponential factor in the measure

$$d\mu(z) = \frac{1}{\pi} e^{-\bar{z}z} d^2z \quad (2.21)$$

3. INTERACTING FERMIONS[8]

We ignore impurities and spin and study the N-body Hamiltonian

$$\hat{H} = \sum_{1\le i\le N} \frac{1}{2m} (\underline{p}_i - e\underline{A}_i)^2 + \sum_{1\le i<j\le N} V(|\underline{x}_i - \underline{x}_j|) \qquad (3.1)$$

with $V(r)$ the Coulomb potential in r^{-1}. Projecting on the lowest band with $P = \prod_{1\le i\le N} P_i$ we find that the hamiltonian is a sum of $N e\hbar B/2m$ plus an operator

$$H = \sum_{1\le i<j\le N} P\, V_{ij}\, P \quad . \qquad (3.2)$$

Henceforth we only deal with H and antisymmetric wave functions analytic in the z_i's. We keep $R_o = (2\hbar/eB)^{1/2}$ as a unit of length, but adopt the convenient unit of energy

$$E_o = \frac{e^2}{4\pi\varepsilon} \left(\frac{2\hbar}{eB}\right)^{1/2} \qquad (3.3)$$

in such a way that the Coulomb potential is $V^c(r) = 1/r$. For $N=2$, the eigenvalues are discrete, even for a repulsive potential, with

$$e_n = \frac{1}{n!} \int_0^\infty dr^2\, e^{-r^2}\, r^{2n}\, V(\sqrt{2}r) \qquad (3.4)$$

for n odd, leading in the Coulomb case to

$$e_n^c = \frac{1}{\sqrt{2}} \frac{\Gamma(n+1/2)}{n!} = \int_0^1 \frac{d\lambda}{\sqrt{2\pi\lambda(1-\lambda)}} \lambda^n \underset{n\to\infty}{\sim} \frac{1}{\sqrt{2n}} \qquad (3.5)$$

corresponding to the representation of the potential as a Gaussian superposition

$$V^c(|\underline{x}|) = \frac{1}{|\underline{x}|} = \int_0^\infty \frac{d\lambda}{\sqrt{2\pi\lambda(1-\lambda)}} V^\lambda(|\underline{x}|)$$

$$V^\lambda(|\underline{x}|) = \lambda^{-1} \exp - \frac{1-\lambda}{2\lambda} \underline{x}^2 \qquad e_n^\lambda = \lambda^n \quad . \qquad (3.6)$$

In the general case we have then

$$H\varphi(z_1,\dots,z_N) = \sum_{i<j} e\left[\frac{1}{2}(z_i - z_j)(\partial_i - \partial_j)\right] \varphi(z_1,\dots,z_N) \qquad (3.7)$$

which specializes in the Coulomb case to

$$H^c\varphi(z_1,\dots,z_N) = \int_0^1 \frac{d\lambda}{\sqrt{2\pi\lambda(1-\lambda)}} H^\lambda\varphi(z_1,\dots,z_N)$$

$$H^\lambda\varphi(z_1,\dots,z_N) = \sum_{i<j} \varphi\left(z_1,\dots,z_i \frac{1+\lambda}{2} + z_j \frac{1-\lambda}{2},\dots,z_i \frac{1-\lambda}{2} + z_j \frac{1+\lambda}{2},\dots,z_N\right) \quad . \qquad (3.8)$$

For any potential, H commutes with N, the number of particles, with the angular momentum

$$L = \sum_i z_i \, \partial_i \tag{3.9}$$

which indicates the total degree of φ, larger or equal to $L_o = \frac{1}{2} N(N-1)$, due to antisymmetry, and with

$$M = \frac{1}{N} \sum_{i,j} z_i \, \partial_j \quad . \tag{3.10}$$

We define the filling fraction (in the limit $N \to \infty$) as

$$\nu = \frac{L_o}{L} \tag{3.11}$$

so that the antisymmetric wavefunction φ_A can be written $\varphi_A = \Delta \varphi_S$, with $\Delta = \prod_{i<j} (z_i - z_j)$ the Vandermonde determinant of the z_i's, and φ_S a symmetric function of degree $f = L - L_o = L_o \frac{1-\nu}{\nu}$. States with $M = 0$ will only depend on the relative coordinates $z_i - z_j$ and multiplying such a wavefunction by a power of Σz_i will increase L and M but not affect the energy. The lowest value of L at which a given level will appear is therefore such that $M = 0$.

In the Coulomb case the energy scales as $N^{3/2}$, since an extra particle will interact with N other ones, with an energy (see (3.5)) of order $N^{-1/2}$ due to Pauli's principle. To obtain thermodynamic quantities, one has to subtract this leading contribution by assuming a neutralizing continuous background. For maximal packing, $\nu = 1$, the unique normalized eigenfunction is equal to $\left(\prod_1^N n! \right)^{-1/2} \Delta(z)$, and a generating function for the energies is

$$\sum_{2 \le N} y^N E_N^c = \frac{y}{(1-y)^{3/2}} \int_0^1 d\lambda \left[\frac{2\lambda}{\pi(1-\lambda)} \right]^{1/2} \cdot \frac{1}{1-\lambda^2} \left[\frac{1}{(1-y)^{1/2}} - \frac{1}{(1-\lambda^2 y)^{1/2}} \right]$$

$$\underset{y \to 1}{\sim} \frac{2}{\sqrt{\pi}} \, \frac{1}{(1-y)^{5/2}} \left[1 + O((1-y)\ln(1-y))\right] - \frac{\sqrt{\pi}}{2} \frac{1}{(1-y)^2} \quad . \tag{3.12}$$

Consequently

$$E_N^c \underset{N \to \infty}{\sim} \frac{8}{3\pi} N^{3/2} \left[1 + O\left(\frac{\ln N}{N} \right) \right] - \frac{\sqrt{\pi}}{2} (N+1) \tag{3.13}$$

in agreement with our expectations.

For $\nu \to 1$ and $N \to \infty$, the operator H is to be diagonalized in a space of $p(N,f)$ dimensions, the number of homogeneous polynomials in N variables of total degree f. This is also the number of partitions of f into at most N parts

$$p(N,f) = \exp S(N,f) \tag{3.14}$$

with the "entropy" S(N,f) given asymptotically in parametric form as a truncted Fermi distribution

$$S(N,f) = fo - \int_0^N du \, \ln (1-e^{-\rho u})$$

$$f = \int_0^N du \, \frac{u \, e^{-\rho u}}{1-e^{-\rho u}} \qquad . \tag{3.15}$$

In terms of ν

$$\frac{1}{N} S(N,\nu) = \begin{cases} \ln e^2/2\nu - \nu + \dots & \nu \to 0 \\ \pi \left(\frac{1-\nu}{3\nu}\right)^{1/2} \left[1 - \frac{3}{\pi^2} e^{-\frac{\pi}{\sqrt{3}}\left(\frac{\nu}{1-\nu}\right)^{1/2}} + \dots\right] & \nu \to 1 \end{cases} \tag{3.16}$$

This corresponds to the first term in Ramanujan's expansion for $p(\infty,f)$. In a grand canonical formalism, with y the activity and t a similar variable conjugate to L, one writes

$$Z_0(y,t) = \sum_{N=0}^{\infty} \sum_{L \geq L_0} y^N t^L \, p(N,L-L_0) = \sum_{N=0}^{\infty} \frac{y^N \, t^{\frac{N(N-1)}{2}}}{(1-t)(1-t^2)\dots(1-t^N)}$$

$$= \prod_{\ell=0}^{\infty} (1+y\,t^{\ell}) \qquad |t| < 1 \qquad . \tag{3.17}$$

In the thermodynamic limit, setting $t = e^{-\rho}$

$$\langle N\rangle = y \frac{\partial}{\partial y} \ln Z_0 = \frac{1}{\rho} \ln (1+y)$$

$$\langle L\rangle = - \frac{\partial}{\partial \rho} \ln Z_0 = \frac{1}{\rho} \ln Z_0 = \frac{1}{\rho^2} g(y) \tag{3.18}$$

$$\nu = \frac{[\ln(1+y)]^2}{2\, g(y)}$$

with g(y) the dilogarithm function

$$g(y) = \int_0^y \frac{dx}{x} \ln (1+x) \tag{3.19}$$

and

$$\frac{1}{N} \ln Z_0 \sim \begin{cases} 1 + \frac{1}{2}\nu + \dots & \nu \to 0 \\ \frac{\pi}{\sqrt{12(1-\nu)}} + \dots & \nu \to 1 \end{cases} \tag{3.20}$$

Similarly we can study the energy levels through a partition function, using a second quantized formalism

$$Z(y,t;\beta) = \mathrm{Tr}\,(y^N\, t^L\, e^{-\beta H}) \tag{3.21}$$

with fermionic fields

$$f(z) = \sum_0^\infty f_\ell \frac{z^\ell}{\sqrt{\ell!}} \qquad f^+(\bar z) = \sum_0^\infty f_\ell^+ \frac{\bar z^\ell}{\sqrt{\ell!}} \qquad \{f_\ell, f_{\ell'}^+\} = \delta_{\ell\ell'}$$

such that

$$N = \int d\mu(Z) f^+(\bar z) f(z) \qquad\qquad L = \int d\mu(z) \left(\frac{df}{dz}(z)\right)^+ \frac{df(z)}{dz}$$

$$H^c = \int_0^1 \frac{d\lambda}{\sqrt{2-\lambda(1-\lambda)}}\; H^\lambda \tag{3.22}$$

$$H = \frac{1}{2}\int d\mu_1 d\mu_2\; f^+(\bar z_1) f^+(\bar z_2) f\left(\frac{1+\lambda}{2} z_2 + \frac{1-\lambda}{2} z_1\right) f\left(\frac{1+\lambda}{2} z_1 + \frac{1-\lambda}{2} z_2\right) .$$

If instead of a Coulomb we have a harmonic potential, corresponding to

$$H^a = 2\frac{d}{d\lambda}\lambda H^\lambda \Big|_{\lambda=1} \tag{3.23}$$

then the partition function is obtained as the series

$$Z^a(y,t,\beta) = 1 + \frac{y}{1-t} + \frac{1}{1-t}\sum_{N=2}^\infty \frac{y^N\, t^{\frac{N(N-1)}{2}}}{(e^{2N\beta}-t^2)(e^{3N\beta}-t^3)\dots(e^{N^2\beta}-t^N)} \quad . \tag{3.24}$$

Such an explicit result is not available in the Coulomb case, but we can instead use a small β expansion (eventhough we are of course interested in the $\beta\to\infty$ limit to obtain the ground state and its excitations) to derive moments of the spectrum. This expansion offers no more than technical difficulties. Wick's theorem applies in the form

$$Z_0^{-1}\,\mathrm{tr}\, y^N t^L f^+(\bar z_1)\dots f^+(\bar z_s) f(z_1')\dots f(z_s) = (-1)^{\frac{s(s-1)}{2}} \det \overline{\bar z_i z_j'} \quad . \tag{3.25}$$

$$\overline{\bar z z'} = \sum_0^\infty \frac{(\bar z z')^\ell}{\ell!}\,\frac{yt^\ell}{1+yt^\ell} = \sum_1^\infty (-1)^{s+1} y^s e^{t^s \bar z z'} \quad . \tag{3.26}$$

For the average trace of H^c we find

$$\langle H^c\rangle_0 = Z_0^{-1}\,\mathrm{tr}\,(y^N t^L H^c) \qquad (3.27)$$

$$= \left(\frac{\pi}{8}\right)^{1/2} \sum_{n_1,n_2=1}^{\infty} (-y)^{n_1+n_2}\left[\frac{1}{\sqrt{(1-\frac{1}{2}(t^{n_1}+t^{n_2}))(1-t^{n_1})(1-t^{n_2})}} - \frac{1}{\sqrt{1-t^{n_1+n_2}}}\right]$$

Dividing this expression by (1-t) and expanding in $y^N t^L$ one gets the mean value of the energy in the lowest sub-band corresponding to $M=0$. For low values of N we obtain as function of L a curve which compares favourably with numerical diagonalization by Girvin and Jach[9] and in particular simulates in a damped way the accidents found for the ground state. With y related to ν through (3.18) we find in the thermodynamic limit

$$\langle E^c\rangle = N^{3/2}\,E_{3/2}(\nu) - N\,E_1(\nu) \qquad (3.28a)$$

where $E_{3/2}(\nu)$ is a complicated function of ν, and

$$E_1(\nu) = \frac{\sqrt{\pi}}{2}\left(1 - \frac{y}{(1+y)\ln(1+y)}\right) \quad . \qquad (3.28b)$$

In the limiting cases

$$\begin{array}{lll} \nu\to 1 & E_{3/2}(1) = \frac{8}{3\pi} & E_1(1) = \frac{\sqrt{\pi}}{2} \\ \nu\to 0 & E_{3/2}(\nu) \sim \frac{\sqrt{\pi}}{2}\,\nu^{1/2} & E_1(\nu) \sim \frac{\sqrt{\pi}}{2}\,(\nu - \frac{\nu^2}{6} + \ldots) \ . \end{array} \qquad (3.28c)$$

The values at $\nu=1$ coïncide of course with (3.13), since in this limit the number of states reduces to one.

Performing computations to the next order yields intricate formulas[8]. However two features are worth mentioning. First in $(\Delta E^c)^2$ there are no terms of order N^3, confirming that a fixed neutralizing background is sufficient to eliminate the $N^{3/2}$ non-thermodynamic contribution to all energies for a given density or filling fraction. Rather $(\Delta E^c)^2$ can be written $N^2\,\Delta E_1(\nu)^2$

$$\begin{array}{ll} \Delta E_1(\nu) \sim \left\{2K(1/4)\nu\right\}^{1/2}\,(1+O(\nu)) & \nu\to 0 \\ \Delta E_1(\nu) \sim \left(\frac{A4\sqrt{3}}{\pi^3}\right)^{1/2}\,(1-\nu)^{1/4} + \ldots & \nu\to 1 \end{array} \qquad (3.29)$$

with K(x) the complete elliptic integral and $A=4.1553\ldots$. While $\Delta E_1(\nu)$ is vanishingly small at both limits it is quite sizeable as compared to $E_1(\nu)$ at intermediate values. Also of course none of the expected cusps at fractional values of ν do appear in these very smeared quantities. Apart from clever educated guesses what one should do to exhibit them remains an open question, at least for this author.

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