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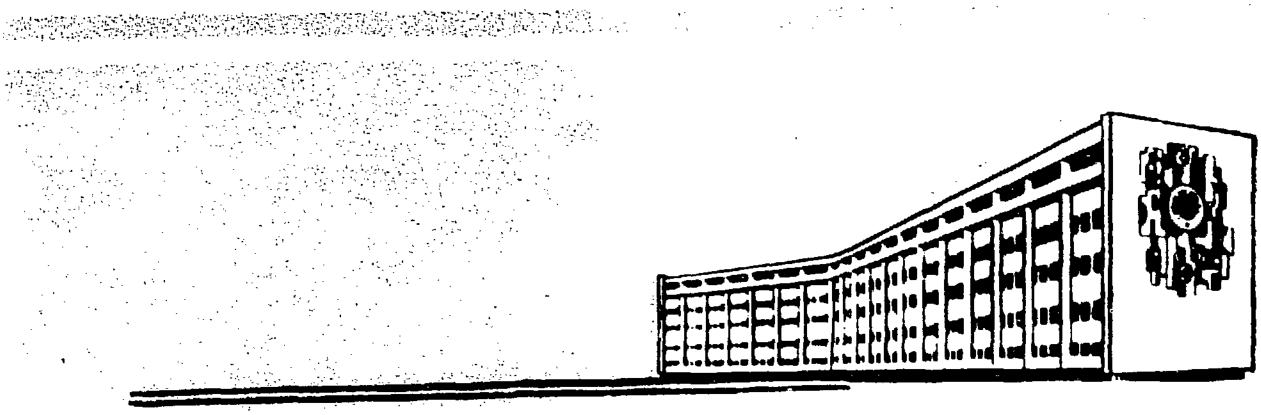

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А.Н.Махлин, Ю.М.Синюков

Релятивистская матрица плотности в диагональном импульсном представлении. Бозе-газ

Получена релятивистски-инвариантная формулировка теории идеального Бозе-газа, основанная на диагональном импульсном представлении для матрицы плотности. Найдены средние числа заполнения и корреляционные функции для статистических систем в произвольных инерциальных системах отсчета на поверхностях постоянного времени. Для получения термодинамических характеристик газов, движущихся как целое, развит метод релятивистской статистической суммы.

A.N.Makhlin, Yu.M.Sinyukov

Relativistic Density Matrix in the Diagonal Momentum Representation.Bose-Gas

The relativistic-invariance treatment of the ideal Bose-system arising from the diagonal momentum representation for the density matrix is developed. The average occupation members and their correlators for statistical systems in arbitrary inertial frames are found on the equal-time hypersurfaces. The relativistic partition function method for the calculation of thermodynamic properties of gases moving as a whole is constructed.

The relativistic-invariance formulation of the ideal gases statistical theory has been presented in the previous papers of this cycle [1,2]. It allowed us to give a description of statistical systems in an arbitrary inertial frame on the equal-time surfaces. The peculiarity of this theory implies a due regard for the non-closedness of the statistical system arising from its interaction with external bodies (e.g. the walls of the box retaining gas). Such an interaction guarantees the equilibrium of the statistical system and leads to nontrivial phenomena in the relativistic case [3].

The papers [1] and [2] have accounted for non-closedness of the system via imposing the relativistic-invariance boundary conditions on the equations for the boson or fermion fields in order to ensure the absence of particles flow off the system volume. The density matrix in the momentum representation appears to be nondiagonal under these conditions. It follows from the rigid phase coupling between the plane waves constituting the proper standing waves of the system in the rest frame. A specific character of boundary conditions (for the scalar field they are simply $\varphi(x)=0$ on the volume border) is revealed also in correlations between differentenergy particles of a gas moving as a whole [1,2].

It is weel-known in the thermodynamical limit the majority of ideal gas properties should not depend upon the choice of boundary conditions. But it does not concern the correlation functions as they arise from the identity of particles. Description of the correlations based on the periodic boundary conditions seems to be more adequate for most of the applications [4]. In this case only correlators with equal momenta of particles differ from zero.

From the relativistic point of view an implication of periodic boundary conditions contrary to the retaining ones, is however less consistent step. Indeed the periodicity in space directions of fundamental volume can be shown to be compatible with the relativistic field equation only in the rest frame of this volume. In this paper we give the relativistic-invariant description of quantum Bose gas on the equal-time hypersurfaces. The boundary conditions are set covariantly in such a way that in the rest frame they reduce to the usual periodic conditions in space directions of the volume containing the system. This approach allows us to get the relativistic invariant generalization of the statistical theory of ideal gases based on the eigenfunctions of momentum operator.

2. Let us treat the equilibrium gas of free uncharged bosons of mass m in a rectangular region of volume V_* =abc. The asterisk denotes the quantities measured in a rest frame of gas. Equation of motion for the field of these particles reads as

$$(\Box - m^2)\,\varphi(x) = 0. \tag{1}$$

The periodic boundary conditions on the hypersurface Σ_{t_*} : t* =const are imposed.

$$\varphi(x^* + a\,\xi^*_{(1)}) = \varphi(x^* + b\,\xi^*_{(2)}) = \varphi(x^* + c\,\xi^*_{(3)}) = \varphi(x^*), \tag{2}$$

where

$$\xi^*_{(1)} = (0,1,0,0)\ ,\ \xi^*_{(2)} = (0,0,1,0)\ ,\ \xi^*_{(3)} = (0,0,0,1)$$

are four-vectors defining directions of periodicity in the Minkowski space. As φ is the scalar field the boundary conditions in the (t,x)-frame, where the gas moves with 4-velocity $u^\mu = (u^0, u^1, 0, 0)$, are to be

$$\varphi(x + a\,\xi_{(1)}) = \varphi(x + b\,\xi_{(2)}) = \varphi(x + c\,\xi_{(3)}) = \varphi(x)\ , \tag{3}$$

$$\xi_{(1)} = L(\xi^*_{(1)}) = (u^1, u^0, 0, 0)\ .\ \xi_{(2),(3)} = L(\xi^*_{(2),(3)}) = \xi^*_{(2),(3)}\ .$$

These relations are the covariant form of periodic boundary conditions on the hypersurface Σ_{t^*} : t* =const. In terms of $(t,\vec{x})$-frame the hypersurface has the form

$$\Sigma_{t^*}(t,\vec{x}) = t + v x^1 = const'\ ,\quad v = u^1/u^0. \tag{4}$$

The plane waves are taken as the basic set of c-number solutions of Eq. (1) with boundary conditions (3):

$$\varphi(x) = \sum_n \left[a_n \varphi_n(x) + a_n^+ \overline{\varphi}_n(x) \right] = \varphi^{(+)}(x) + \varphi^{(-)}(x), \tag{5}$$

where

$$\varphi_n = (2p_n \cdot u\, V_*)^{-1/2}\ e^{-ip_n^0 t + i\vec{p}_n \cdot \vec{x}} \tag{6}$$

and

$$n = (n_1, n_2, n_3)\ ,\quad n_i = 0, \pm 1, \ldots$$

$$p_n^1 = u^1 \sqrt{m^2 + \left(\frac{2\pi n_1}{a}\right)^2 + \left(\frac{2\pi n_2}{b}\right)^2 + \left(\frac{2\pi n_3}{c}\right)^2} + u^0 \frac{2\pi n_1}{a}$$

$$p_n^2 = \frac{2\pi n_2}{b}\ , \quad p_n^3 = \frac{2\pi n_3}{c}\ , \quad p_n^0 = \sqrt{m^2 + \vec{p}_n^{\,2}}\ . \tag{7}$$

The functions $\varphi_n(x)$ form the orthonormal set with scalar product

$$(\varphi_1, \varphi_2) = i \int_{\Sigma_{t^*}} (\overline{\varphi}_1 \overleftrightarrow{\partial_\mu} \varphi_2)\, d\sigma^\mu \tag{8}$$

defined on the part of hypersurface $\Sigma_{t^*}(t, \vec{x})$ encircled by the world lines of the box bounds:

$$-\frac{a}{2u_0} + vt < X^1 < \frac{a}{2u_0} + vt\ , \quad -\frac{b}{2} < X^2 < \frac{b}{2}\ , \quad -\frac{c}{2} < X^3 < \frac{c}{2}\ . \tag{9}$$

In the secondary quantization picture the operators a_n and a_n^+ contributing to the field operator $\varphi(x)$ expansion (5) obey the commutation relations

$$[a_n, a_{n'}^+] = \delta_{n,n'}\ , \quad [a_n, a_{n'}] = [a_n^+, a_{n'}^+] = 0 \tag{10}$$

The basic wave functions $\varphi_n(x)$ are invariantly defined in any reference frame. The corresponding operators are used to construct the invariant Fock space of states:

$$|0\rangle\ , \quad a_n^+|0\rangle\ , \quad \prod_\kappa (N_\kappa !)^{-1/2} (a_\kappa^+)^{N_\kappa} |0\rangle\ . \tag{11}$$

The locally conserved dynamical variables are expressed via the field operator $\varphi(x)$ in a usual manner:

$$j_\mu(x) = \varphi^{(-)}(x)\, i \overleftrightarrow{\partial_\mu}\, \varphi^{(+)}(x) \quad , \quad \partial_\mu j^\mu(x) = 0 \qquad (12)$$

$$T^{\mu\nu}(x) = \partial^\mu \varphi(x) \partial_\nu \varphi(x) - g^{\mu\nu} \mathcal{L}(x) \; ; \; \partial_\mu T^{\mu\nu} = 0 \qquad (13)$$

$$\mathcal{L}(x) = \frac{1}{2}\left[\partial^\mu \varphi(x) \partial_\mu \varphi(x) - m^2 \varphi^2(x) \right].$$

The integral quantities,namely,the operator $N(\Sigma)$ of a total number of particles crossing the hypersurface Σ and the energy-momentum operator $P^\mu(\Sigma)$ on this surface are defined in a standard way:

$$N(\Sigma) = \int_\Sigma j^\mu(x) d\sigma_\mu \quad ; \quad P^\mu(\Sigma) = \int_\Sigma T^{\mu\nu}(x) d\sigma_\nu . \qquad (14)$$

In accordance with the observers equivalence pinciple we study the physical properties of the system on hypersurfaces of equal time in an arbitrary inertial (x,t)-frame. So we take $\Sigma = \Sigma_t$: t=const in Eqs. (14). It is easy to verify that only in the case of $\Sigma_t = \Sigma_{t^*}$, the number of particles $N(\Sigma_t)$ and energy-momentum $P(\Sigma_t)$ operators are represented as simple mode sums:

$$N(\Sigma_{t^*}) = \sum_n a_n^+ a_n \quad ; \quad P^\mu = \sum_n p_n^\mu a_n^+ a_n . \qquad (15)$$

On the surfaces $\Sigma_t \neq \Sigma_{t*}$ in the moving frames such a representation doesn't take place. Moreover the Fock state vectors (11) being the eigenvectors of $N(\Sigma_{t*})$ operator with the integer eigenvalues appear not to be even eigenvectors of the $N(\Sigma_t)$ -operator if $\Sigma_t \neq \Sigma_{t*}$. The latter arises just from non-closedness of the system of particles in a cell of periodicity.

Visually it looks as follows. In the rest $(t^*, \vec{x}^*)$-frame of a gas the equal-time surface $\Sigma_t(t^*, \vec{X}^*)$ of moving $(t,\vec{x})$-frame reads as $\Sigma_t(t^*, \vec{X}^*)$: $t^* - v X_*^1$ =const (See Fig.1). Operator $N(\Sigma_t)$ for the fixed surface Σ_t is invariant and can be found in the $(t_*, \vec{x}_*)$-frame on the surface $\Sigma_t(t_*, \vec{X}_*)$. The local conservation of current (12) permits to bind $N(\Sigma_t)$ and $N(\Sigma_{t*})$ (see Fig. 1):

$$N(\Sigma_t) - N(\Sigma_{t*}) = \int_{\Sigma_R} d\sigma_\mu j^\mu(x) - \int_{\Sigma_L} d\sigma_\mu j^\mu(x) \qquad (16)$$

The flows of particles through the time-like surfaces Σ_R and Σ_L do not equal zero and each other:

$$\langle \alpha | \int_{\Sigma_{R,L}} d\sigma_\mu j^\mu(x) | \alpha \rangle \neq 0, \qquad (17)$$

where $|\alpha\rangle$ is an arbitrary state vector from Fock space (11) except $|0\rangle$, and this arises from the non-closedness of the system. Only when Σ_t differs from $\Sigma_{t*} : t_* = 0$ by the time shift excludingly (v =0): Σ_t $(t^*, \vec{x}^*)$: t^*=const, the right-hand side of (16) becomes zero, because Σ_R and Σ_L have equal areas and constitute a space period apart. This leads to the compensation of incoming and outgoing flows and the numbers of particles with any momentum whose world lines cross the surfaces $t_* = 0$ and t_* =const $\neq 0$ coincide as well as the very operators. But this compensation of flows through the time-like surfaces fails when the surface $\Sigma_t(t^*,\vec{x}^*)$ doesn't coincide with Σ_{t*}: t^* =const; i.e. when the equal-time surface is given in the frame of reference where the gas velocity $v \neq 0$. So the equality of $N(\Sigma_t)$ and $N(\Sigma_{t*})$ breaks down. Hence the eigenvectors (11) of the operator $N(\Sigma_{t*})$ cannot be those of $N(\Sigma_t)$. The latter gives rise to nondiagonal structure of the $N(\Sigma_t)$ in the basis of invariant state space (11):

$$N(\Sigma_t) = \sum_n \frac{p_n^0}{u^0(p_n \cdot u)} a_n^+ a_n + \sum_{n \neq n'} \lambda_{nn'} a_n^+ a_n, \tag{18}$$

where

$$\lambda_{nn'} = \frac{(p_n^0 + p_{n'}^0)\,\delta_{n_2}^{n_2'}\delta_{n_3}^{n_3'}}{2u^0\sqrt{p_n \cdot u\, p_{n'} \cdot u}} \frac{\sin\frac{a}{2u^0}(p_n^1 - p_{n'}^1)}{\frac{a}{2u^0}(p_n^1 - p_{n'}^1)} e^{i(p_n \cdot u - p_{n'} \cdot u)\frac{t}{u^0}}. \tag{19}$$

In order to clarify the situation and introduce the operators $N(k, \Sigma_t)$ of number of particles with the given momentum k the Wigner representation of bilinear operators happens to be suitable [5]. This representation implies the density of particles on the surface t=const to be written as

$$N(x) = \int d^4k\, N(x,k), \tag{20}$$

where

$$N(x,k) = (2\pi)^{-4} \int d^4\xi\, \varphi^{(+)}(x+\tfrac{\xi}{2})\, i\overleftrightarrow{\partial_t}\, \varphi^{(-)}(x-\tfrac{\xi}{2})\, e^{-ik\xi} = \tag{21}$$

$$= \frac{1}{V_*} \sum_{nn'} \frac{a_n^+ a_{n'}}{\sqrt{p_n \cdot u\, p_{n'} \cdot u}} \frac{p_n^0 + p_{n'}^0}{2} e^{i(p_n - p_{n'})\cdot x}\, \delta\!\left(k - \frac{p_n + p_{n'}}{2}\right).$$

The operator of number of particles of momentum $\vec{k}$ in the volume $V = V_*/u^\circ$ on the hypersurface Σ_t : t=const is defined as

$$N(k,\Sigma_t) = \int_V d^3\vec{x} \int_{-\infty}^{\infty} dk^\circ \, N(x,k), \tag{22}$$

where the bounds of integration are given by (9). It is easy to verify that

$$N(\vec{k},\Sigma_t) = \sum_{nn'} a_n^+ a_{n'} \lambda_{nn'} \delta\left(\vec{k} - \frac{\vec{P}_n + \vec{P}_{n'}}{2}\right), \tag{23}$$

where $\lambda_{nn'}$ are defined by Eq. (19). As far as only discrete values $\vec{k}_\ell = \frac{\vec{P}_n + \vec{P}_{n'}}{2}$, $\ell = \{n,n'\}$ of momenta are allowed the delta-function is implied to be the Kronecker symbol. The total number of particles operator reads as

$$N(\Sigma_t) = \sum_{\vec{k}_\ell} N(\vec{k}_\ell, \Sigma_t) \tag{24}$$

The off diagonal elements of operators $N(\vec{k}_\ell, \Sigma_t)$ and $N(\Sigma_t)$ are due to the boundary conditions. They do not contribute to one-particle distribution function $\langle N(\vec{k})\rangle$ but are significant for the correlation function $\langle N(\vec{k}_1) N(\vec{k}_2)\rangle$. Their contribution remains even in the thermodynamical limit.

3. The statistical averages are calculated by means of density matrix. It is defined on the invariant state space (11) and is also invariant, i.e. it can be given in any frame. In the rest frame it is the matrix of statistical operator

$$\rho = \frac{1}{Z} e^{-\frac{H}{T}} \; ; \; Z = Sp\, e^{-\frac{H}{T}} \; ; \; H = P_*^\circ(\Sigma_{t_*}) = \sum_n p_{*n}^\circ a_n^+ a_n . \tag{25}$$

So all the thermodynamical and statistical properties of the system on the equal-time surface in an arbitrary inertial frame can be obtained by means of averaging the operators, defined in the basis (11) on the Σ_t -surface.

The average number of particles with momentum p_n on the Σ_t surface in the Bose-gas moving as a whole with the 4-velocity u^μ reads as

$$\langle N_{p_n}\rangle_{\Sigma_t} = Sp\,\rho N(p_n,\Sigma_t) = \frac{p^0}{u^0 p u}\left[z^{-1}\exp\frac{p_n\cdot u}{T} - 1\right]^{-1}, \quad (26)$$

where the fugacity $z = 1$. The correlation function in momentum space is defined like

$$\nu(N_{p_n}, N_{p_{n'}})_{\Sigma_t} = Sp\,\rho N(p_n,\Sigma_t)N(p_{n'},\Sigma_t) - \langle N_{p_n}\rangle\langle N_{p_{n'}}\rangle.$$

In virtue of thermodynamical Wick theorem in the large volume approximation we get

$$\nu(N_{p_n}, N_{p_{n'}}) = \frac{1}{2u^0}\,\frac{p_n^0}{p_n\cdot u}\left[ch\,\frac{p_n\cdot u}{T} - 1\right]^{-1}\delta_n^{n'}. \quad (27)$$

The anisotropy is peculiar to the expressions (26) and (27). The average occupation numbers of the given p_n (or given n) -level are not invariants, and this is also due to non-closedness of the system. But the total number of particles is surely invariant.

$$\langle N\rangle_{\Sigma_t} = Sp\,\rho N(\Sigma_t) = \sum_n \langle N_{p_n}\rangle_{\Sigma_t} = inv. \quad (28)$$

In the rest frame of reference the correlation function depends upon the energies of particles only and is isotropic with respect to the momenta directions. In a moving frame, as seen from Eq. (27), the correlation function of two particles with the same momentum (the same quantum number n) changes and acquires the anisotropy. The correlation increases for the momenta with positive projections on the direction of motion and decreases for the negative ones. Contrary to the case of retaining boundary conditions only the particles of equal 4-momentum P_n correlate.

The rule for transition from the discrete to continuous momenta in the limit of $V_* \to \infty$ follows from formulas (7) (7) and reads as

$$\sum_n \longrightarrow \int d^3\vec{p}\, \frac{V_*}{(2\pi)^3} \frac{u\cdot p}{p_0}. \tag{29}$$

The density of particles in the momentum space transfers from (26) to

$$\left.\frac{dN}{d^3\vec{p}}\right|_{\Sigma_t} = \frac{V}{(2\pi)^3}\left(z^{-1} \exp\frac{p\cdot u}{T} - 1\right)^{-1}, \tag{30}$$

where $V = V_*/u^0$ is the Lorentz-contracted volume. Eq. (27) for density correlations transfers to

$$\nu(\vec{p},\vec{p}')_{\Sigma_t} = \frac{V}{2(2\pi)^3}\left[ch\frac{p\cdot u}{T} - 1\right]^{-1} \delta(\vec{p}-\vec{p}'). \tag{31}$$

In order to define thermodynamical energy, momenta and pressure on the Σ_t surface the averages of local energy-momentum densities are to be found. The direct calculation gives

$$\langle T_{00}(x)\rangle = (\epsilon + \Pi^1)u_0^2 - \Pi^1 \;;\; \langle T_{22}(x)\rangle = \Pi^2$$

$$\langle T_{11}(x)\rangle = (\epsilon + \Pi^1)u_1^2 + \Pi^1 \;;\; \langle T_{33}(x)\rangle = \Pi^3 \tag{32}$$

$$\langle T_{01}(x)\rangle = \langle T_{10}(x)\rangle = (\epsilon + \Pi^1)u_0 u_1 .$$

The rest components are zero. Here

$$\epsilon = \frac{1}{V_*}\sum_n p^0_{*n}\langle N_{p_{*n}}\rangle_{\Sigma_{t_*}}\ ,\ \Pi^j = \frac{1}{V_*}\sum_n \frac{(p^j_{*n})^2}{p^0_{*n}}\langle N_{p_{*n}}\rangle_{\Sigma_{t_*}} \tag{33}$$

are invariants. In the limit of great volume

$$\epsilon = \int d^3\vec{p}\, p^0 \frac{dN}{d^3p}\Big|_{\Sigma_{t_*}}\ ;\ \mathcal{P} = \Pi^j = \int d^3\vec{p}\, \frac{\vec{p}^{\,2}}{3p_0}\frac{dN}{d^3p}\Big|_{\Sigma_{t_*}} \tag{34}$$

because of (29). Then

$$\langle T^{\mu\nu}\rangle = (\epsilon + \mathcal{P})u^\mu u^\nu - \mathcal{P}g^{\mu\nu}, \tag{35}$$

where invariant quantities ϵ and $\mathcal{P}$ in the averaged tensor (35) stand for the energy density and pressure in the rest frame. This leads to the following total thermodynamical energy and momenta of the system on the Σ_t hypersurface in the moving inertial frame

$$\langle P^\mu\rangle_{\Sigma_t} = \int_{\Sigma_t} d^3x\,\langle T^{\mu 0}(x)\rangle =$$
$$= u^\mu\left[\langle P^0_*\rangle_{\Sigma_{t_*}} + (1 - \delta^0_\mu u_0^{-2})\mathcal{P}V_*\right]. \tag{36}$$

The method of partition function for the calculation of thermodynamic quantities on the hypersurface Σ_t in an arbitrary frame will be given for the conclusion of this section. The partition function Z defined according to (25) is an invariant and can be written in the next form:

$$Z=\sum_{\{N_n\}} z^{\sum_n N_n} \exp\left\{-\frac{1}{T}\sum_n p_n\cdot u\, N_n\right\} =$$

$$=\prod_n\left(1-z_n e^{-\frac{p_n\cdot u}{T}}\right)^{-1}=\prod_{n=(n_1,n_2,n_3)}\left(1-z_n e^{-\frac{p_n\cdot u}{T}}\right)^{-\lambda_n}, \tag{37}$$

where $z_n = z = 1$ and λ_n are such numbers that $\lambda_{n_1,n_2,n_3} + \lambda_{-n_1,n_2,n_3} = 2$. These numbers can be chosen in such a way that the classical prescription for the evaluation of $\langle N_{p_n}\rangle_{\Sigma_t}$ will be valid:

$$\langle N_{p_n}\rangle_{\Sigma_t} = z_n\frac{d}{dz_n}\ln Z = \lambda_n\frac{z_n e^{-\frac{p_n\cdot u}{T}}}{\left(1-z_n e^{-\frac{p_n\cdot u}{T}}\right)}\Bigg|_{z_n=z=1}. \tag{38}$$

Comparing with (26) we actually find

$$\lambda_n=\frac{p_n^0}{u^0 p_n\cdot u} \qquad \text{and } \lambda_{n_1,n_2,n_3}+\lambda_{-n_1,n_2,n_3}=2. \tag{39}$$

Thereby we find the partition function representation on the surface Σ_t

$$Z_{\Sigma_t}(z_n,\beta^\mu)=\prod_n\left(1-z_n e^{-\beta\cdot p_n}\right)^{-\frac{p_n^0}{u^0 p_n\cdot u}}\Bigg|_{\beta^\mu=\frac{u^\mu}{T};\ z_n=1}. \tag{40}$$

The $\langle N_p\rangle_{\Sigma_t}$ is calculated via (38). The total particles number is defined by

$$\langle N\rangle = z\frac{d}{dz}\ln Z(z_n=z,\beta)\Bigg|_{\beta^\mu=\frac{u^\mu}{T}} = inv. \tag{41}$$

The prescription for the evaluation of correlators looks like

$$\langle N(p_n)N(p_{n'})\rangle_{\Sigma_t} = Z^{-1} z_n \frac{d}{dz_n} z_{n'} \frac{d}{dz_{n'}} Z_{\Sigma_t}(z_n, z_{n'}; \beta)\Big|_{\substack{\beta = \frac{u}{T} \\ z_n = z_{n'} = 1}} \qquad (42)$$

and makes use of (38). Thermodynamical energy and momentum in the moving frame are as follows:

$$\langle P^\mu \rangle_{\Sigma_t} = -\frac{\partial}{\partial \beta_\mu} \ln Z_{\Sigma_t}(z_n, \beta)\Big|_{\beta = \frac{u}{T},\, z_n = 1}. \qquad (43)$$

Though the partition function (40) is an invariant, the total energy and momentum of the system do not make up a 4-vector. The invariant pressure is also determined by the partition function of the grand canonical ensemble in a usual way

$$\mathcal{P} V_* = -(\beta_\mu \beta^\mu)^{-1/2} \ln Z_{\Sigma_t}(z_n, \beta)\Big|_{\beta = \frac{u}{T},\, z_n = 1}. \qquad (44)$$

In the continuous limit the logarithm of the partition function (40) on the surface Σ_t reads as

$$\ln Z_{\Sigma_t} = -\frac{V}{(2\pi)^3}\int d^3p \ln(1 - z e^{-\beta\cdot p}) - \ln(1 - z e^{-\beta\cdot p_{(0)}}) = inv \Big|_{\beta^\mu = \frac{u^\mu}{T}}, \qquad (45)$$

where the contribution of the lowest level is singled out.

4. Some principal points should be underlined for conclusion. The theory of ideal gases is well known to permit the two formulations, based on the retaining (or "zero") and periodic boundary conditions for the solutions of quantum (either relativistic or not) equations of motion. It is actually shown in this paper that the statistical relativistic invariance theory describing the ideal gases in an arbitrary

inertial frame also allows two approaches. In the previous papers [1,2] the approach based on the retaining invariant boundary conditions has been used. In this case the density matrix of a system regarded at the equal-time surface appears to be nondiagonal and relativistically noninvariant in the momentum representation. Only the partition function keeps invariance. Operators of observables are diagonal in this representation.

It is shown in this paper that the relativistic-invariant formulation of ideal gases statistical theory based on the covariant generalization of periodic boundary conditions is possible also. In this approach the density matrix turns out to be relativistic invariant and diagonal but the operators physical quantities are not of this sort.

In the thermodynamical limit all the quantities calculated in both approaches by means of one-particle distributions coincide. The distinction is manifested only via the correlations due to the exchange forces. From the physical point of view it arises from the presence or absence of coherent superpositions in space of momentum eigenfunctions. Just this coherence is peculiar to the retaining boundary conditions.

The choice of periodic or retaining conditions should be appointed by the specificity of the problem.

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R E F E R E N C E S

1. Sinyukov Yu.M. On the relativistic statistical physics and thermodynamics of ideal gases. Preprint ITP-82-94E, Kiev, 1982, 33 p.
2. Sinyukov Yu.M. On the relativistic partition function of ideal gases. Phys.Lett., 1983, 127B, No 6, p.443-447. Зиновьев Г.М., Синюков Ю.М., Шелест В.П.О релятивистской статистической сумме и поляризационных свойствах ферми-газов.ТМФ,1983, т.57, №2, с.182-192.
3. Gorenstein M.I., Sinyukov Yu.M. Local anisotropy effects in the hadrodynamical theory of multiparticle production. Preprint ITP-83-128E, Kiev, 1983, 22 p. (Phys.Lett.,1984, 142B, No5,6 , p.425-429.
4. Марч Н., Янг У., Сампантхар С. Проблема многих тел в квантовой механике. Гл.I, изд-во "Мир", М., 1978, 405 с.
5. Гроот С. де, Леувен В. ван, Верт К. ван. Релятивистская кинетическая теория. Гл.3, Изд-во "Мир", М., 1983, 422 с.

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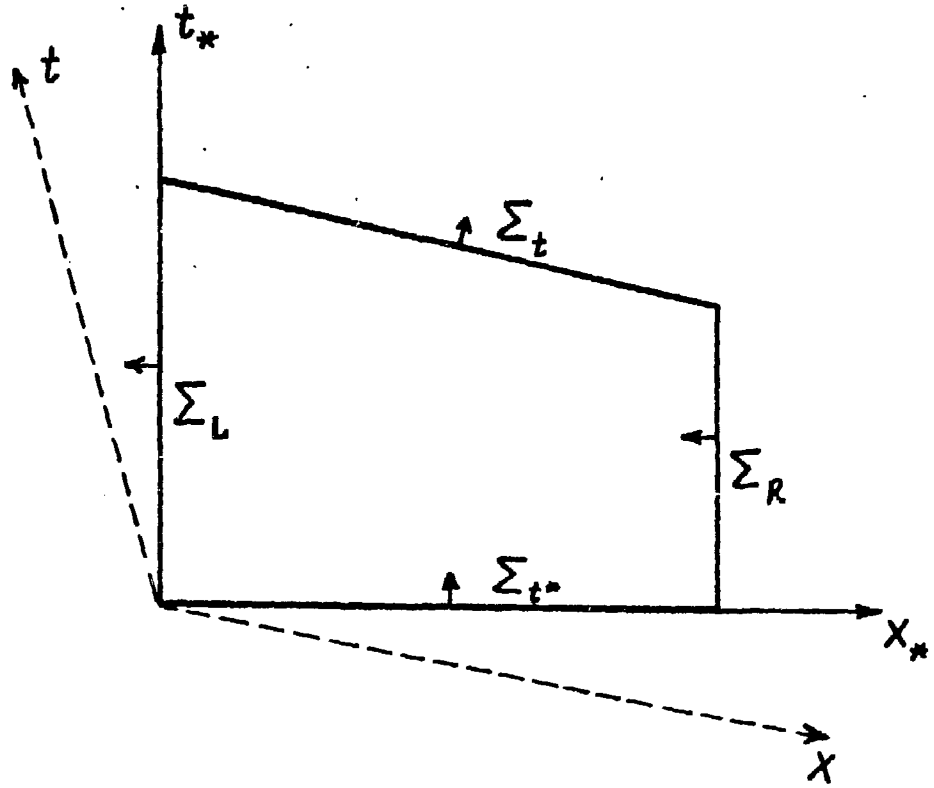

Fig.1.Hypersurfaces for the investigation of statistical system in the coordinates of the (t^*,x^*) rest frame. $\Sigma_{t_*}: t_*=0$ is the equal time surface in the rest frame. Σ_t: t=const= $= u^0 t_* - u^1 x_*$ is the surface of equal time in that frame where the thermodynamical system moves at the 4-velocity $u^\mu = (u^0, u^1, 0, 0)$.

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