INSTITUTE OF PLASMA PHYSICS

NAGOYA UNIVERSITY

Catastrophe in Plasma Focus Evolution

Kazunari Ikuta

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Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics Nagoya University, Nagoya 464, Japan.

Synopsis

A theory of generating strong electric field in a dense plasma column in plasma focus is established by applying the formula for the electron thermal conductivity in destroyed magnetic surfaces like those in tokamaks. The origin of the electric field may be from abrupt rise of plasma resistivity when the irregularity of magnetic field is weak. However, the electric field can be from the inductive origin in case the irregularity attains a certain level. Both origin should be mixed up depending on the magnitude of the irregularity.

§1 *Introduction*

The phenomena occuring in plused intense discharge in rarefied gases have still been the object of a great number of experimental investigations.

The plasma focus is an improved version of the linear pinch in order to optimize the efficiency of energy transfer from the energy source to the plasma by relying on the development of self-consistent phenomena after maximum compression during the cylindrical re-expansion of plasma column[1)2)].

One of the curious phenomena in the plasma focus is the creation of energetic ions in the course of plasma collapse, where ion energy reaches about 200 times the charging voltage of the capacitor bank[3)4)]. In a typical case ion energy is a few MeV. Such high energy can be obtained by ions in a very strong electric field having duration sufficient for ion acceleration. It is not a simple matter to explain the mechanism of generating such an intense electric field in a thin plasma column.

If the magnetic configuration of the plasma column surface is strongly disturbed by an asymmetric current distribution along the plasma column, the axisymmetry of the magnetic configuration are easily broken up in the discharge chamber and the surface of the plasma column may be surrounded by the braided lines of force like that in the tokamak configuration. In this case, the electrons from the pinched plasma column could escape to the chamber wall with the speed of the order of electron thermal speed. If this is the case, we may explain the observed high energy of the ions from a simple view point of the lack of equilibrim.

The purpose of the present work is to establish a theory of the strong electric field due to the rapid cooling of the plasma column described by the formula for the electron thermal conductivity associated with the major disruptions in tokamaks[5]. In §2 the magnetic configuration of the pinched plasma column with axisymmetry is discussed. In §3 possible magnetic configuration due to the asymmetrically distributed surface current in the pinched plasma surface is demonstrated. The evolution of plasma temperautre is considered in §4, where the abrupt rise of the plasma resistivity will be shown due to the anomalous heat loss along the braided magnetic lines to the chamber wall. This heat loss can explain the strong electric field in the plasma column.

§2 Magnetic Configuration of the Pinched Plasma Column

Even if the axial magnetic field of the magnitude of the earth's magnetic field is present along the discharge, the field intensty becomes sufficiently high after the compression phase of the plasma focus evolution. Actually, the field strength can be of order 1 Tesla after the compression if the initial and the final radii of the current channel are about 10 cm and 1 mm respectinely, i. e. the compression ratio may be said 10^4. If this is the case in all of fast high-power discharges, the voltage between the electrodes of the discharge chamber must be the product of the total plasma current and the plasma resistance along the spiral field lines in the plasma column. In other words, the voltage should not be the product of the plasma current and the perpendicular plasma resistivity to the lines of force.

We, therefore, assume throughout in this work that the pinched plasma column of the present interests in plasma focus has axial magnetic field even though it might be weak.

A very simple, illustrative example of the initial plasma model is as follows. A constant electric current I is flowing uniformly through the plasma column. During the compression the plasma gathers the axial magnetic field eventually present in the discharge chamber and compresses it inside the equilibrium column. Therefore we suppose that we have in the plasma column an axial field B_0. Out side of the equilibrium column the axial field is considered to be zoro.

§3 Magnetic Structure of Discharge Chamber

In plasma focus experiments a quasi-stationary equilibrium situation can be observed for a few nanoseconds after the compression phase of the plasma column with radius b. Since this equilibrium is not the minimum energy state, the plasma column will be destroyed anyway causing relaxation phenomena which lead to a final relaxed state of the plasma[6]. If the dense plasma column is produced in a perfectly conducting, axisymmetric chamber, the final relaxed state will be the one described in the previous work[7].

In the course of the relaxation the turbulent, magnetic field may come out because of the plasma activity which satisfies the boundary condition. If the perfectly conducting metal casing of the plasma column is a cylinder with radius a and length 2L, the boundary condition for the magnetic field $B=B(B_r, B_\theta, B_z)$ may be

$$B_r=0 \quad \text{at } r=a \text{ and at } z=\pm L \qquad (1)$$

We consider the perturbed surface current in the plasma-vacuum interface. As long as the magnetic field thus generated satisfies the boundary condition, such surface current can flow both in the plasma surface and in the chamber wall. The relaxation phenomena leading to the final relaxed state may be induced by the perturbud surface currents.

The boundary condition can be satisfied if both the plasma current and the surface currents is described by the magnetic scalar potential Φ given by

$$B = \nabla\Phi \tag{2}$$

where

$$\Phi = \frac{\mu_0 I}{2\pi}\,\theta + \sum_{\ell} \frac{2\varepsilon_\ell}{k}\left[\frac{I_\ell(hr)}{C_1} - \frac{K_\ell(kr)}{C_2}\right]\sin\ell\theta\cos kz + \sum_{\ell} \frac{2\delta_\ell}{h}\left[\frac{I_\ell(hr)}{D_1} - \frac{K_\ell(hr)}{D_2}\right]\cos\ell\theta\cosh z\ . \tag{3}$$

Here, ε_ℓ comes from the surface currents with the helical wave number $\pm$ k and δ_ℓ does the one with that of $\pm$ h. The functions $I_\ell(s)$ and $K_\ell(s)$ are the modified Bessel functions of first and second kinds respectivity. And $k \equiv (\pi/2L) + (n\pi/L)$, $h \equiv (3\pi/2L) + (n\pi/L)$, $C_1 \equiv I_\ell'(ka)$, $C_2 \equiv K_\ell'(ka)$, $D_1 \equiv I_\ell'(ha)$ and $D_2 \equiv K_\ell'(ha)$ and n is an integer. The ' sign means the derivative with respect to the product ka.

A useful way to look over the magnetic topology is to obtain the average magnetic surface. After Morozov and Solv'ev[8], the average magnetic sirface, Ψ , induced by (3) becomes

$$\Psi = \frac{4\, r^3}{\mu_0 I} \sum_{\ell} \frac{\varepsilon_\ell \delta_\ell}{\ell} \Big[\frac{\cos kz \sinh hz}{K} \Big\{ \frac{I_\ell(hr)}{D_1} - \frac{K_\ell(hr)}{D_2} \Big\} \times$$

$$\frac{\partial}{\partial r} \Big\{ \frac{I_\ell(kr)}{C_1} - \frac{K_\ell(kr)}{C_2} \Big\}$$

$$- \frac{\cosh hz \sin kz}{h} \Big\{ \frac{I_\ell(kr)}{C_1} - \frac{K_\ell(kr)}{C_2} \Big\} \frac{\partial}{\partial r} \Big\{ \frac{I_\ell(hr)}{D_1} - \frac{K_\ell(hr)}{D_2} \Big\} \Big] \quad (4)$$

The isobar of this magnetic surface function Ψ means the projection of the average magnetic line of force on r-z plane. The change of topology of the magnetic lines of force is easily demonstrated if the case of $\ell = 1$ perturbed current with $n = 0$ is employed as an example. For $a^2/L^2 << 1$ and $\ell=1$, the functions Ψ is reduced to

$$\Psi = \frac{2\pi^2 \varepsilon_1 \delta_1}{\mu_0 I L} (r^4 - a^4) \cos^3 f \sin f \quad (5)$$

where $f \equiv \pi z/2L$.

The isobar of this function (5) is shown in Fig.1 which suggests that the electrons ejected along the averaged field lines from the surface of the plasma column can occupy the full volume of the discharge chamber.

That is tha expansion of plasma in the full volume of discharge chamber.

§4 Energy Balance of Focussed Column

The current along the Z pinch plasma generates the toroidal field which surrounds the plasma column. If the distribution of plasma current is exactly axisymmetric, every line of force closes itself and each line of force settles coaxially about the z axis. Once the axisymmetry of current distribution is perturbed, for example, by the asymmetric emission of electrons from the cathode, the spiral out of the lines of force from the plasma column is eventually resulted everywhere on the plasma surface. Then the whole surface of the plasma column can emit electrons in the radial direction. As for the case (5) in the preceding section, the separatrix is generated at z=0 by the perturbed current. By the separatrix the plasma surface is directly connected with the chamber wall. This separatrix enables the electrons to propagate freely to the chamber wall. This enhanced electron penetration may induce both the cooling of plasma temprature and the abrupt change of the inductance thus generating enhanced resistivity of plasma together with high voltage spike along the plasma column.

If the heat conductivity across the plasma surface at r=b is originated from the magnetic field irregularities by the surface current discussed in the preceding section, the heat conductivity A may be written by, after Rechester and Rosenbluth[9],

$$A = n\pi a v\ (\delta B_r/B)^2 \qquad (6)$$

where n is the density of plasma, v the electron thermal velocity and the ratio $\delta B_r/B$ is a prescribed level of magnetic irregularity due to the surface current.

In the dense focussed plasma the ion and the electron templetures should be equal to each other. Then the energy balance equation per unit length of plasma becomes, for uniform plasma density,

$$3n \frac{dT}{dt} = \sigma E^2 - \frac{AT}{b^2} - \frac{3nT}{\tau_E}, \tag{7}$$

where T is the temperature of plasma, E the constant electric field, σ the Spitzer conductivity of plasma along field lines and τ_E is the axial energy confinement time.

Before proceeding further, we should note the energy confinement time of plasma in the discharge between electrodes. Since both end of the plasma column is in contact with the electrodes, the energy loss to the electrodes would be dominated by the electron-electron collision process along the magnetic field lines. In this situation we might say that τ_E is in proportion to the electron collision time. One of the simplest assumptioms, therefore, is that the axial energy confinement time τ_E is equal to the electron-electron collisions time.

In order to proceed our discussions further more we need an important assumption on the temperature dependence of the field irregularity in the energy balance equation (7). The level of the irregular magnetic field determines the anomalous heat conductivity. We employ a simple form for the level as follows.

$$\left(\frac{\delta B_r}{B} \right)^2 = \frac{\gamma}{B^2} \tag{8}$$

where the constant γ corresponds to the irregularity of the magnetic field due to the asymmetry of the current distribution of the plasma column and $B \equiv \mu_0 I/2\pi b$.

Then, the temperature for the steady state is obtained from

$$\sigma E^2 - \frac{n\pi a\gamma\nu T}{b^2B^2} - \frac{3nT}{\tau_E} = 0 \tag{9}$$

To facilitate the analysis it is useful to rewrite Eq. (9) to a dimensionless form. We introduce the dimensionless temperature x by

$$T = T_0 x \quad . \tag{10}$$

And Eq. (9) is reduced to

$$\frac{6\pi^2}{\Lambda}\frac{n}{a}\left(\frac{u}{w}\right)^4 \left[\frac{(24\pi)^{1/2}}{\Lambda}\ \frac{K\varepsilon_0 E^2 a}{mw^2} - \frac{\gamma a^2}{B^2b^2}\right]x^2 - 1 = 0 \tag{11}$$

where K is the Boltzmann constant, Λ the coulomb logarithm, $u \equiv (KT_0/m)^{1/2}$ and $w^2 \equiv (ne^2/mE_0)$. Here the notations m and e are the mass and the change of a single electron and ε_0 is the dielectric constant of the vacuum. We see that the case of no real roots occurs in the quadratic equation of the dimensionless temperature x when

$$\frac{\gamma}{B^2} > \frac{(24\pi)^{1/2}}{} \frac{b^2}{a} \frac{K\varepsilon_0 E^2}{mw^2} \tag{12}$$

Once this inequality is satisfied in the discharge chamber of plasma focus, the temperature of plasma drops abruptly because of no equilibrium in the plasma. This means that there is a threshold of the irregular magnetic field for the existence of the equilibrium. As a typical set of plasma paramenters in plasma forcus we may take $b=10^{-3}$m, $a=10^{-1}$m, $n=10^{22}m^{-3}$ and $E \simeq 10^4$volt. Then, the condition (12) becomes

$$\frac{\Upsilon}{B^2} \gtrsim 0.3 \times 10^{-27} \tag{13}$$

This means that the threshold amplitude of irregular magnetic field necessary to break down the equilibrium is as small as

$$\frac{\delta B_r}{B} \gtrsim 10^{-14} \tag{14}$$

It is clear from the above discussions that slight irregularity of the magnetic field easily leads to the collapse of plasma equilibrium.

Once this catastrophe occurs in the plasma column the plasma resistivity rises abruptly, And at the same time, the radial expansion of electrons along the braided magnetic field lines will dominate the periphery of thin plasma column. In this situation the change of inductance of the plasma column should be

$$\frac{dL^*}{dt} = \frac{dL^*}{db}\frac{db}{dt}$$

$$\simeq \frac{dL^*}{db} v \left(\frac{\delta B_r}{B} \right) \tag{15}$$

where L^* is the inductance of the plasma housed coaxially in a cylinder. We may write, in the present work,

$$L^* = \frac{\mu_0}{\pi} L \ln\frac{a}{b} \tag{16}$$

The expression (16) and the relation (15) lead to

$$\frac{dL^*}{dt} \simeq - \frac{\mu_0 L}{\pi b} v(\frac{\delta B_r}{B}) \qquad (17)$$

The voltage V_{in} induced by this rapid change of inductance is

$$V_{in} = I\frac{dL^*}{dt}$$

$$\simeq - \frac{\mu_0 L}{\pi b} v\ I(\frac{\delta B_r}{B}) \qquad (18)$$

For the plasma with $b=10^{-3}$m, $L-10^{-1}$m, $v \simeq c$ (the speed of light) and $I \simeq 10^5$amperes, we have

$$V_{in} \simeq - 4\times10^9(\frac{\delta B_r}{B}) \qquad (19)$$

This means that the threshold of the field irregularity does not induce strong inductive electric field in the plasma. We need the magnitude of field irregularity of order 1% in order to explain the observations[4). However, even if the plasma temperature drops abruptly to 1/20 of the initial temperature, this temperature drop is sufficiently large to explain the observations[4) of energetic particles from the plasma, provided that the plasma current is externally supplied constantly in the tine by any means.

§5 Discussion and Conclutions

In this work we took a liberty to choose the simplest temprature dependence of the field irregularity, i. e. $(\delta B_r/B)^2= \gamma/B^2$. In general we may extend our consideration to the case of

$$\left(\frac{\delta B_r}{B} \right)^2 = \frac{\gamma}{B^2} x^s \tag{20}$$

where s is as arbitrary, real number. We can easily demonstrate the existence of the threshold of the field irregularity for the equilibrium in any case, although the threshold value depends on the number s.

The decisive conclusion of the origin of the strong electric field in the plasma column is not straightforward. As long as the level of the field irregularity is in the vicinity of the threshold value, we may say that the strong electric field is induced by the abrupt rise of plasma resistivity due to the catastraphic collapse of equilibrium. However, in the case of having the field irregularuty of order 10^{-3}, i. e. $(\delta B_r/B) \gtrsim 10^{-3}$, we may have a reason to explain the strong electric field from the inductive origin. Both origin should be mixed depending on the magnitude of the field irregularity.

We can calculate the amplitude of the field irregularity under an assumption that the helicity, $K = \int A \cdot B dV$, is a constant in time[6] just before the occurence of the catastrophe and that the field pattern of the irregular field is given. We easily see that the constant γ is of the order of the product $\delta_\ell \varepsilon_\ell$.

We have described the field patterns in the discharge chamber only for the case of n=0. We can have many separatrixes depending on the value of n. For larger n the particle excursions to the wall along the separatrix becomes easier and easier. This effectively increases the channels of electron loss to the wall. The inductive explanation of the electric field can get more chances for larger n.

In conclusion, we have established the theory for the origin of the strong electric field in the plasma using the transport theory developed for tokamaks. High energy charged particle emission should be derived from a sudden appearance of no equilibrium state in the plasma column.

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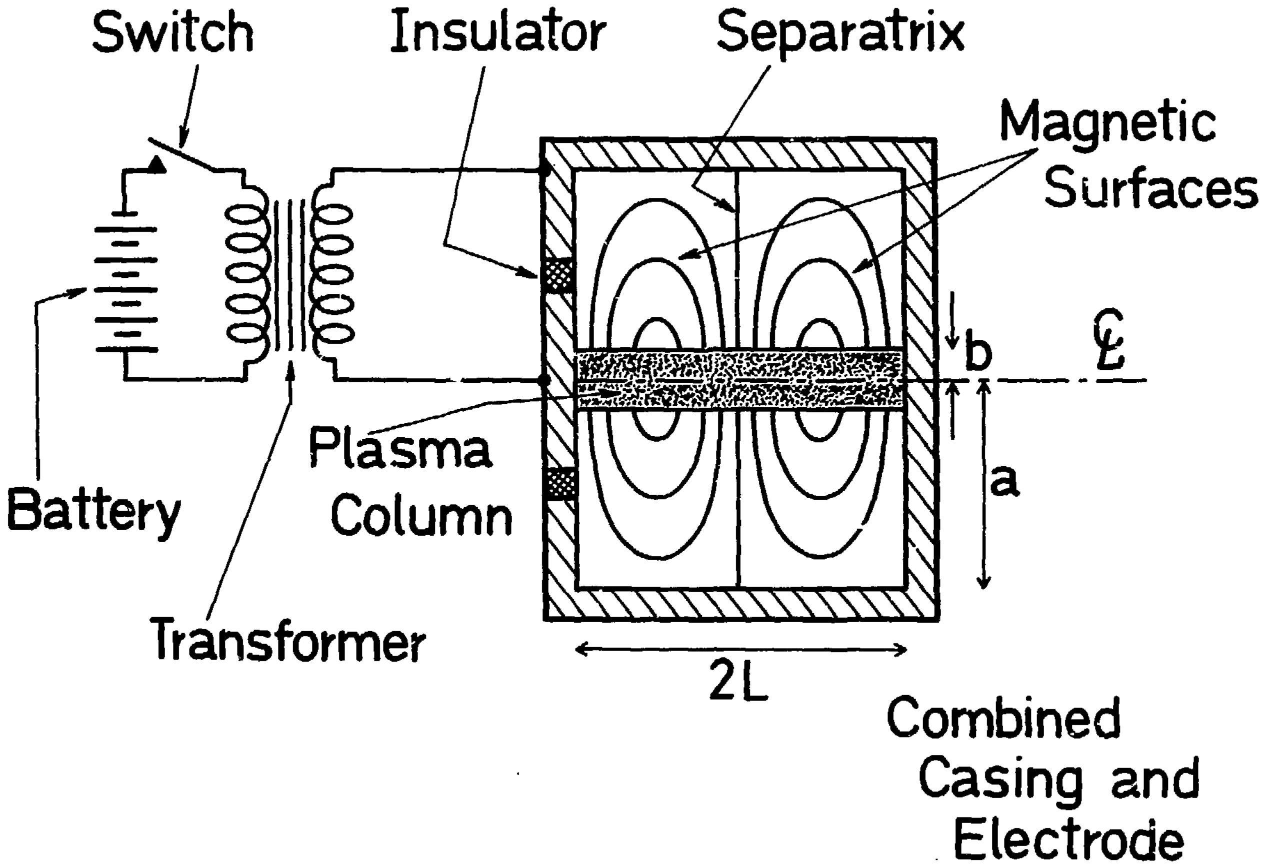
Figure Caption

Fig. 1 Averaged magnetic surfaces in the axisymmetric casing occupied by the plasma column with asymmetric surface currents. The divergent flux lines come out from the surface of the plasma column. Electrons from the hot plasma column can escape along these flux tube to the wall.



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