INSTYTUT FIZYKI JĄDROWEJ
INSTITUTE OF NUCLEAR PHYSICS
ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ

KRAKÓW

RAPORT No 1179/AP

ENERGY CORRECTIONS IN PULSED NEUTRON MEASUREMENTS FOR CYLINDRICAL GEOMETRY

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KRAKÓW 1982

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POPRAWKI ENERGETYCZNE W IMPULSOWYCH POMIARACH NEUTRONOWYCH DLA GEOMETRII CYLINDRYCZNEJ

ЭНЕРГЕТИЧЕСКИЕ ПОПРАВКИ В ИМПУЛЬСНЫХ НЕЙТРОННЫХ ИЗМЕРЕНИЯХ ДЛЯ ЦИЛИНДРИЧЕСКОЙ ГЕОМЕТРИИ

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Paper submitted to
the Journal of Physics D:
Applied Physics

Kraków, May 1982

NAKŁADEM INSTYTUTU FIZYKI JĄDROWEJ W KRAKOWIE
UL. RADZIKOWSKIEGO 152

Kopię kserograficzną, druk i oprawę wykonano w IFJ Kraków

Wydanie I *Zam. 89/82* *Nakład 120 egz.*

ABSTRACT

A solution of the thermal neutron diffusion equation for a two-region concentric cylindrical system, with a constant neutron flux in the inner medium assumed, is given. The velocity-averaged dynamic parameters for thermal neutrons are used in the method. The corrections due to the diffusion cooling are introduced into the dynamic material buckling and into the velocity distribution of the thermal neutron flux. Detailed relations obtained for a hydrogenous moderator are given. Results of the measurements of the thermal neutron macroscopic absorption cross-sections for the samples in the two-region cylindrical systems are presented.

STRESZCZENIE

W rozwiązaniu rownania dyfuzji neutronów termicznych dla układu dwóch walców koncentrycznych, przy założeniu stałego strumienia neutronów w wewnętrznej objętości, użyto dynamicznych parametrów neutronowych średniowanych po prędkości. Do dynamicznego bucklingu materiałowego i do rozkładu prędkości strumienia neutronów termicznych wprowadzono poprawki uwzględniające ochładzanie dyfuzyjne. Szczegółowe zależności podano dla moderatora zawierającego wodór. Przedstawione zostały wyniki pomiarów makroskopowego przekroju czynnego absorpcji neutronów termicznych próbek w dwustrefowych układach cylindrycznych.

РЕЗЮМЕ

В решении уравнения диффузии тепловых нейтронов для системы двух концентрических цилиндров, в случае постоянной величины потока нейтронов во внутренном обёме, были использованы динамические нейтронные параметры усредненные по скорости. В динамическом материаловом параметре и в распределении потока тепловых нейтронов были введены поправки по учету диффузионного охлаждения. Подробные зависимости представлены для водородосодержащего модератора. Показаны результаты измерений макроскопического сечения поглощения тепловых нейтронов для образцов в системе концентрических цилиндров.

1. INTRODUCTION

The value of the measured thermal neutron absorption cross-section of a small sample is determined, in the method given by Czubek (1981), from the intersection of experimental and so-called 'theoretical' curves. The experimental curve is the result of measuring thermal neutron decay in the investigated volume after a neutron burst. The decay constant λ_o is obtained as a function of geometrical size $\tilde{R}$ of the following system: an unknown sample of constant size $\tilde{R}_1$ is surrounded by the external moderator of varying size $\tilde{R}_2$. The thermal neutron parameters of the moderator are well known. The theoretical curve λ_o^* vs $\tilde{R}_2$ is calculated within the one-group diffusion approximation under the assumption that the neutron flux in the sample is constant. The accurate knowledge of the theoretical curve is important because the angle of intersection between the two curves is small. Satisfactory results of the calculation can be obtained, if some energy corrections are introduced to the one-group approximation. A general approach to this problem has been given by Drozdowicz (1981) (together with an example for the spherical geometry of measurement). The solution for the cylindrical geometry (the theoretical curve λ_o^* vs $\tilde{R}_2$) in the one-group one-velocity approximation by the perturbation calculation has been given by Woźnicka (1981). Here the energy corrections, resulting from the diffusion cooling effect and due to the thermal neutron energy distribution, are introduced to that solution.

2. ENERGY CORRECTIONS IN THE SOLUTION OF THE THERMAL NEUTRON DIFFUSION EQUATION FOR A TWO-REGION CYLINDRICAL SYSTEM

The departure point is the dynamic diffusion equation (e.g. Grosshög and Rönnberg 1971) in the form:

$$\nabla^2\phi(\vec{r}) + B_d^2\,\phi(\vec{r}) = 0 \quad , \qquad (1)$$

where $\phi(\vec{r})$ is the thermal neutron flux at point $\vec{r}$ of a medium. The cooling-corrected dynamic material buckling B_d^2 given by Drozdowicz (1981) is:

$$B_d^2 = -\frac{\bar{\Sigma}_{ad}}{\bar{D}_d} \frac{2}{1 + \sqrt{1 + 4 \frac{\bar{\Sigma}_{ad}}{\bar{D}_d^{\,2}} C \cdot \overline{1/v}}} , \qquad (2)$$

where:

Σ_{ad}, D_d - dynamic macroscopic absorption cross-section and diffusion coefficient,

$$\Sigma_{ad}(v) = \Sigma_a(v) - \lambda_o/v \quad , \qquad (3)$$

$$D_d(v) = \frac{1}{3[\Sigma_{tr}(v) + \Sigma_a(v) - \lambda_o/v]} \quad , \qquad (4)$$

v - velocity of thermal neutrons,
$\Sigma_a(v)$, $\Sigma_{tr}(v)$ - velocity-dependent macroscopic absorption and transport cross-sections for thermal neutrons,
C - thermal neutron diffusion cooling coefficient,
λ_o - fundamental decay constant of the thermal neutron flux in the medium.

The bar over any symbol q denotes the value averaged over the normalized velocity distribution $\phi(v)$ of the thermal neutron flux:

$$\bar{q} = \int_0^\infty q(v)\ \phi(v)\ dv \quad . \qquad (5)$$

The Maxwellian flux distribution is used as the $\phi(v)$ (thus, the integration over velocity v up to infinity is admissible for thermal neutrons):

$$\phi(v)\ dv = 2\ V^3\ e^{-V^2}\ dV \quad , \qquad (6)$$

where

$$V = \frac{v}{v_{oc}} \tag{7}$$

and v_{oc} is the corrected velocity:

$$v_{oc} = \left(1 - \frac{C}{D_o} B_d^2 \right) v_o \quad , \tag{8}$$

where:

v_o - the most probable velocity of the Maxwellian density distribution $v_o = 2kT/m$ (k - Boltzmann constant, T - absolute temperature, m - mass of the neutron),

D_o - pulsed diffusion coefficient of thermal neutrons.

Let us consider a cylindrical system. The cylinder No 1 with height H_1 and radius R_1 is surrounded by cylinder No 2 with external geometric size of H_{2g} and R_{2g} (Fig.1). The dynamic material buckling B_{d1}^2 of the inner medium '1' (say, sample) is assumed to be zero:

$$B_{d1}^2 = 0 \quad . \tag{9}$$

Then the solution of Eq.(1) for that two-region system determines the so-called theoretical value λ_o^* of the decay constant. It is finally a function of the thermal neutron parameters of the external moderator '2' and of the geometric size of the system:

$$\lambda_o^* = \lambda_o^*(\Sigma_{a2}, \Sigma_{tr2}, C_2; R_1, H_1, R_{2g}, H_{2g}).$$

The perturbation calculation in this case (Woźnicka 1981) gives the formula for the n-order approximation value of the dynamic material buckling of the external medium:

$$(B_{d2}^{*(n)})^2 = (\alpha^{*(n)})^2 + (W^{*(n)})^2 \quad , \tag{10}$$

where the asterisk denotes the values corresponding to assump-

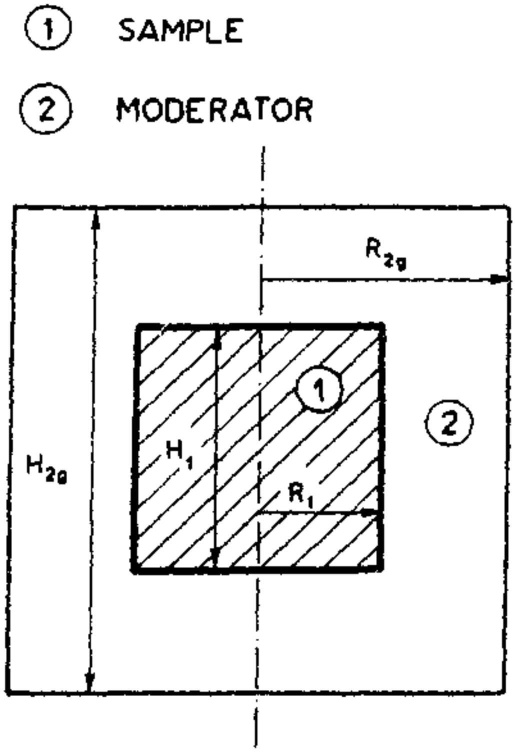

FIG. 1. SCHEME OF THE CONCENTRIC CYLINDRICAL SYSTEM

tion (9) and

$$(\alpha^{*(n)})^2 = \alpha_e^2 \, K^{(n)} \quad , \tag{11}$$

$$(W^{*(n)})^2 = W_e^2 \, K^{(n)} \quad , \tag{12}$$

where

$$\alpha_e = \frac{\pi}{H_2} \quad , \tag{13}$$

$$W_e = \frac{j_o}{R_2} \quad . \tag{14}$$

In Eqs (13) and (14) the extrapolated dimensions are used, i.e.:

$$\left.\begin{aligned} R_2 &= R_{2g} + d_d \quad , \\ H_2 &= H_{2g} + 2d_d \quad , \end{aligned}\right\} \tag{15}$$

where d_d is the dynamic extrapolated distance and $j_o \approx 2.405$ is the first zero of the Bessel function of the first kind of the order zero $J_o(x)$. The term $K^{(n)}$ is defined as:

$$K^{(n)} = 1 + \frac{\int_{V_1} \phi(z,\rho) \, \phi_1^{*(n)}(z,\rho) \, dV}{\int_{V_2} \phi(z,\rho) \, \phi_2^{*(n)}(z,\rho) \, dV} \quad , \tag{16}$$

where $\phi(z,\rho)$ is the neutron flux in the homogeneous cylindrical system and $\phi_j^{*(n)}(z,\rho)$ is the n-order approximation of the perturbed flux in the j-th part of the volume of the two-region cylindrical system $(j = 1,2)$.

In Eq.(10) all the terms are λ_o^*-dependent. The dynamic material buckling $(B_2^{*(n)})^2$, defined in Eq.(2), contains the average dynamic cross-section $\bar{\Sigma}_{ad2}$ and the average dynamic diffusion coefficient $\bar{D}_{d2}$ which depend upon the decay constant

[Eq.(3), (4) with Eq.(5)]. A weak dependence upon the decay constant appears also in the average value $\overline{1/v}$ resulting from Eqs (5) ÷ (7) with Eq.(8) which contains the dynamic material buckling. In the expressions on the right hand side of Eq.(10) the dynamic extrapolated distance d_d is used. It is proportional to the dynamic diffusion coefficient $d_d \sim \bar{D}_{d2}$ which in turn is dependent upon the decay constant as mentioned above. Finally, Eq.(10) can be written in the following form:

$$-\frac{\bar{\Sigma}_{ad2}(\lambda_o^{*(n)})}{\bar{D}_{d2}(\lambda_o^{*(n)})}\cdot\frac{2}{1+\sqrt{1+4\,\dfrac{\bar{\Sigma}_{ad2}(\lambda_o^{*(n)})}{\bar{D}^2_{d2}(\lambda_o^{*(n)})}\,C_2\cdot\overline{1/v}_2(\lambda_o^{*(n)})}} =$$

$$= \{[\alpha_e(\lambda_o^{*(n)})]^2 + [w_e(\lambda_o^{*(n)})]^2\}\cdot K^{(n)}(\lambda_o^{*(n)}) \qquad (17)$$

and it gives the n-order approximation $\lambda_o^{*(n)}$ of the decay constant λ_o^* for a given size $\tilde{R}$ of the two-region system. The general calculation procedure giving the sequential-order approximations of the decay constant in Eq.(17) is the same as that described by Woźnicka(1981). Some additional inconvenience is due to the interdependence between the dynamic material buckling [Eq.(2) with Eqs (3) + (7)] and the corrected velocity v_{oc} given by Eq.(8) but an iterative procedure gives in this case the rapidly convergent result.

The whole theoretical curve λ_o^* vs $\tilde{R}_2$ is obtained by solving Eq.(17) for the varying external dimensions R_{2g} and H_{2g} of the system.

3. DETAILED SOLUTION FOR THE CYLINDRICAL HYDROGENOUS MODERATOR

The integrals in Eq.(16) for the cylindrical system are calculated in Appendix according to the method presented by Woźnicka (1981). The coefficient $K^{(n)}$ gets now the form:

$$K^{(n)} = 1 + \frac{a_1^{(n)}}{a_2^{(n)} + a_3^{(n)} + a_4^{(n)}} \quad , \tag{18}$$

where the terms $a_1^{(n)}$ in the n-order approximation of the perturbation calculation are expressed by the quantities obtained in the (n-1)-order approximation. Some new symbols are introduced:

$$M^{(n)} = \frac{\cos[\alpha^{(n)}(h_2 - h_1) - \alpha_e h_1]}{2\,[\alpha^{(n)} + \alpha_e]} + \frac{\cos[\alpha^{(n)}(h_2 - h_1) + \alpha_e h_1]}{2\,[\alpha^{(n)} - \alpha_e]} \quad , \tag{19}$$

$$N^{(n)} = W^{(n)} R_1 J_o(W_e R_1) T^{(n)} - W_2 R_1 J_1(W_e R_1) \quad , \tag{20}$$

$$S^{(n)} = \frac{J_o(W^{(n)} R_2)}{Y_o(W^{(n)} R_2)} \quad , \tag{21}$$

$$T^{(n)} = \frac{J_1(W^{(n)} R_1) - S^{(n)} Y_1(W^{(n)} R_1)}{J_o(W^{(n)} R_1) - S^{(n)} Y_o(W^{(n)} R_1)} \quad , \tag{22}$$

where:

$J_k(x)$, $Y_k(x)$ - the Bessel functions of the first and second kind, respectively, of the k order,

α_e, W_e - are given in Eqs (13), (14),

$$\left.\begin{array}{l} h_1 = H_1/2 \quad , \\ h_2 = H_2/2 \quad . \end{array}\right\} \tag{23}$$

The terms $a_i^{(n)}$ are now expressed as:

$$a_1^{(n)} = \frac{H_2 R_1}{W_e} \sin(\alpha_e h_1) \, J_1(W_e R_1) \quad , \tag{24}$$

$$a_2^{(n)} = \frac{H_2 \sin(\alpha_e h_1)}{W_e^2 - (W^{(n-1)})^2} \, N^{(n-1)} \quad , \tag{25}$$

$$a_3^{(n)} = \frac{\pi R_1}{W_e} \; \frac{J_1(W_e R_1)}{\sin[\alpha^{(n-1)}(h_1 - h_2)]} \, M^{(n-1)} \quad , \tag{26}$$

$$a_4^{(n)} = \frac{\pi}{W_e^2 - (W^{(n-1)})^2} \cdot \frac{1}{\sin[\alpha^{(n-1)}(h_1 - h_2)]} \, M^{(n-1)} \, N^{(n-1)} . \tag{27}$$

Here the asterisk at $\alpha^{*(n)}$ and $W^{*(n)}$ symbols is omitted for the sake of simplicity.

Eq.(18), together with Eqs (24) ÷ (27), is valid in the case of the cylindrical system for any kind of material, because it represents the geometrical dependences. The extrapolated dimensions H_2, R_2 (Eq.(15)) used in the calculation are, however, dependent upon the kind of the moderator.

Here a hydrogenous moderator is considered. In such a moderator the velocity dependence of the thermal neutron absorption and transport cross-sections can be assumed as being:

$$\left. \begin{aligned} \Sigma_{a2}(v) &\sim \frac{1}{v} \quad , \\ \Sigma_{tr2}(v) &\sim \frac{1}{v} \quad . \end{aligned} \right\} \tag{28}$$

Then, the dynamic extrapolated distance d_d is expressed by Drozdowicz (1981) as:

$$d_d \approx 2.28\ \overline{D}_{d2} \quad . \tag{29}$$

The average dynamic diffusion coefficient obtained under assumption given by Eq.(28) is defined as:

$$\overline{D}_{d2} = \frac{\sqrt{\pi}}{4\left[\frac{v_{oc2}(\lambda_o^{*(n)})}{2\ D_{o2}} - \frac{\lambda_o^{*(n)}}{v_{oc2}(\lambda_o^{*(n)})}\right]} \quad . \tag{30}$$

The dependence of the corrected velocity v_{oc2} upon the decay constant $\lambda_o^{*(n)}$ is through the dynamic material buckling $(B_{d2}^{*(n)})^2$ in Eq.(8).

The dynamic material buckling $(B_{d2}^{*(n)})^2$ for a hydrogenous moderator, calculated from Eq.(2) under the assumptions given by Eqs (28) and (6), inserted into the $\lambda_o^{*(n)}$-determining Eq.(17) yields:

$$\frac{4\ \frac{\lambda_o^{*(n)} - \overline{v\Sigma}_{a2}}{v_{oc2}(\lambda_o^{*(n)})}\cdot\left[\frac{v_{oc2}(\lambda_o^{*(n)})}{2D_{o2}} - \frac{\lambda_o^{*(n)}}{v_{oc2}(\lambda_o^{*(n)})}\right]}{1 + \sqrt{1 - \frac{16\ C_2}{v_{oc2}(\lambda_o^{*(n)})}\cdot\frac{\lambda_o^{*(n)} - \overline{v\Sigma}_{a2}}{v_{oc2}(\lambda_o^{*(n)})}\cdot\left[\frac{v_{oc2}(\lambda_o^{*(n)})}{2D_{o2}} - \frac{\lambda_o^{*(n)}}{v_{oc2}(\lambda_o^{*(n)})}\right]^2}} =$$

$$= \left\{[\alpha_e(\lambda_o^{*(n)})]^2 + [W_e(\lambda_o^{*(n)})]^2\right\}\cdot K^{(n)}(\lambda_o^{*(n)}) \quad , \tag{31}$$

where $K^{(n)}(\lambda_o^{*(n)})$ is given in Eq.(18) and $\alpha_e(\lambda_o^{*(n)})$ and $W_e(\lambda_o^{*(n)})$ are defined by Eqs (13), (14) with (15) and (29) as:

$$\alpha_e(\lambda_o^{*(n)}) = \frac{\pi}{H_{2g} + 4.56\ \overline{D}_{d2}(\lambda_o^{*(n)})} \quad , \tag{32}$$

$$w_e(\lambda_o^{*(n)}) = \frac{j_o}{R_{2g} + 2.28\,\bar{D}_{d2}(\lambda_o^{*(n)})} \quad , \qquad (33)$$

where $\bar{D}_{d2}(\lambda_o^{*(n)})$ is given in Eq.(30). The thermal neutron parameters of the external moderator $\overline{v\Sigma}_{a2}$, D_{o2} and C_2 should be well known, e.g. from the classic pulsed measurements based on the method of the varying geometrical buckling for a homogeneous material.

Eq.(31) permits us to obtain the values of the consecutive approximations of the decay constant λ_o^*. The n-order approximation $\lambda_o^{*(n)}$ is calculated using the values of the quantities obtained in the previous (n-1)-order approximation:

$$\lambda_o^{*(n)} = \lambda_o^{*(n)}(K^{(n-1)}[\alpha^{(n-1)},\ W^{(n-1)},\ \lambda_o^{*(n-1)}]) \quad . \qquad (34)$$

It results from the perturbation calculation that

$$\lim_{n\to\infty} \lambda_o^{*(n)} = \lambda_o^* \quad . \qquad (35)$$

Here, two points should be taken into account:

I. The departure values for the zero-order approximation do not result from this calculation procedure. They are obtained from the method of separation of variables in the solution of the diffusion equation (1) for the considered cylindrical system (Woźnicka 1981):

$$\alpha^{(o)} = \frac{\pi}{H_2 - H_1} \quad , \qquad (36)$$

and $W^{(o)}$ as being the solution of the equation:

$$\frac{J_1(W^{(o)}R_1)}{Y_1(W^{(o)}R_1)} = \frac{J_o(W^{(o)}R_2)}{Y_o(W^{(o)}R_2)} \quad . \qquad (37)$$

II. Of course, the sequence of $\lambda_o^{*(n)}$ must be truncated for a finite value of n. It is connected with an assumed accuracy of the calculation. E.g. the procedure ends when the difference between the two sequential values of the decay constant is smaller than the accuracy ε:

$$| \lambda_o^{*(n)} - \lambda_o^{*(n-1)}| < \varepsilon \quad . \tag{38}$$

4. EXPERIMENTAL RESULTS AND CONCLUSIONS

A series of measurements has been performed in the cylindrical geometry to observe the influence of the corrections on the final experimental result, i.e. the thermal neutron macroscopic absorption cross-section of the sample. The schema of the measurement geometry is shown in Fig.2. The external cylinders have been made of pure Plexiglass (hydrogenous moderator). Water solutions of boric acid have been used as the internal samples. The ratios $2R_1/H_1 = 1$ and $2R_{2g}/H_{2g} = 1$ have been always preserved. The dimensions of the internal sample have been fixed and the external dimensions of the outer moderator have been a variable.

The system was irradiated by bursts from a fast neutron generator. The thermalized neutrons were registered by a He-3 detector and the counts were stored in a time analyser. The decay constant λ_o of the fundamental mode of the thermal neutron flux was determined. The description of the experimental set-up and of the measurement conditions and the detailed results are given in another paper (Czubek et al. 1982).

The corrected curves were calculated from Eq.(31) with the use of a computer program (Drozdowicz and Woźnicka 1982). The thermal neutron parameters of Plexiglass $\overline{v\Sigma}_{a2}$, D_{o2}, C_2 were taken from Drozdowicz et al.(1980). The curves without the corrections were obtained when the zero value of the diffusion cooling coefficient of the external moderator was assumed, $C_2=0$ [then the terms due to the corrections vanish in Eqs (2) and (8)

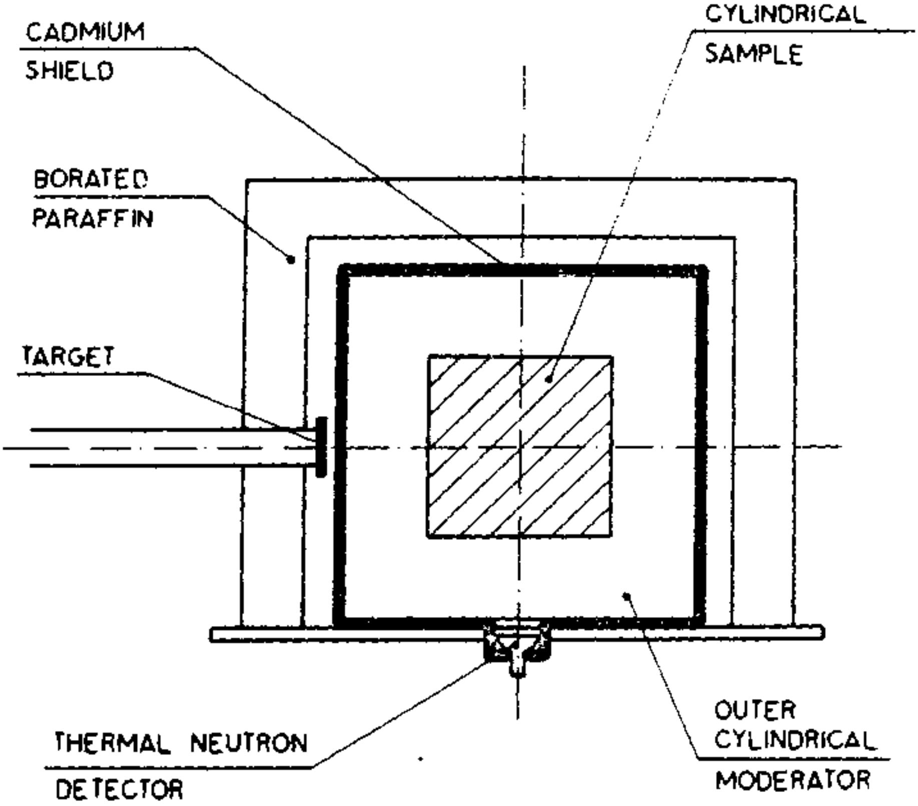

FIG. 2. GEOMETRY OF MEASUREMENT

and, consequently, in the final equation (31), too].

The two solutions of boric acid H_3BO_3 in water were prepared: 1 % and 2 % by weight. The value of $\overline{v\Sigma}_{a1\,calc}$ for these solutions was calculated starting from the knowledge of the elemental compositions. The calculated and experimental values were compared. The results are presented in Figs 3, 4, 5 and in Table 1, where:

$\overline{v\Sigma}_{a1\,meas}$ – the value obtained from the measurement when the uncorrected theoretical curve was used,

$\overline{v\Sigma}^{c}_{a1\,meas}$ – the value obtained when the corrected theoretical curve was used,

$$\Delta = \frac{\overline{v\Sigma}_{a1\,meas} - \overline{v\Sigma}_{a1\,calc}}{\overline{v\Sigma}_{a1\,calc}} \cdot 100\ \% \quad ,$$

$$\Delta^{c} = \frac{\overline{v\Sigma}^{c}_{a1\,meas} - \overline{v\Sigma}_{a1\,calc}}{\overline{v\Sigma}_{a1\,calc}} \cdot 100\ \% \quad .$$

The results obtained here for cylindrical systems and those from measurements performed in spherical geometry (Drozdowicz 1981) are consistent. Similarly, one can find that the proposed corrections are more significant if:

i) the absorption cross-section of the sample is higher (cf. the results for the samples No 1 and No 2);

ii) the dimensions of the sample are smaller (cf. the results for the samples No 2 and No 3).

Moreover, for the same medium different results are obtained from the samples of different dimensions, if the uncorrected theoretical curves are used. On the other hand, one obtains very good agreement between the measurement results ($\overline{v\Sigma}^{c}_{a1\,meas}$) when the corrected theoretical curves are used (cf. the values $\overline{v\Sigma}^{c}_{a1\,meas}$ for the samples No 2 and No 3).

In general, the experimental values $\overline{v\Sigma}_{a1\,meas}$ obtained with use of the uncorrected curves are always too low. Here they

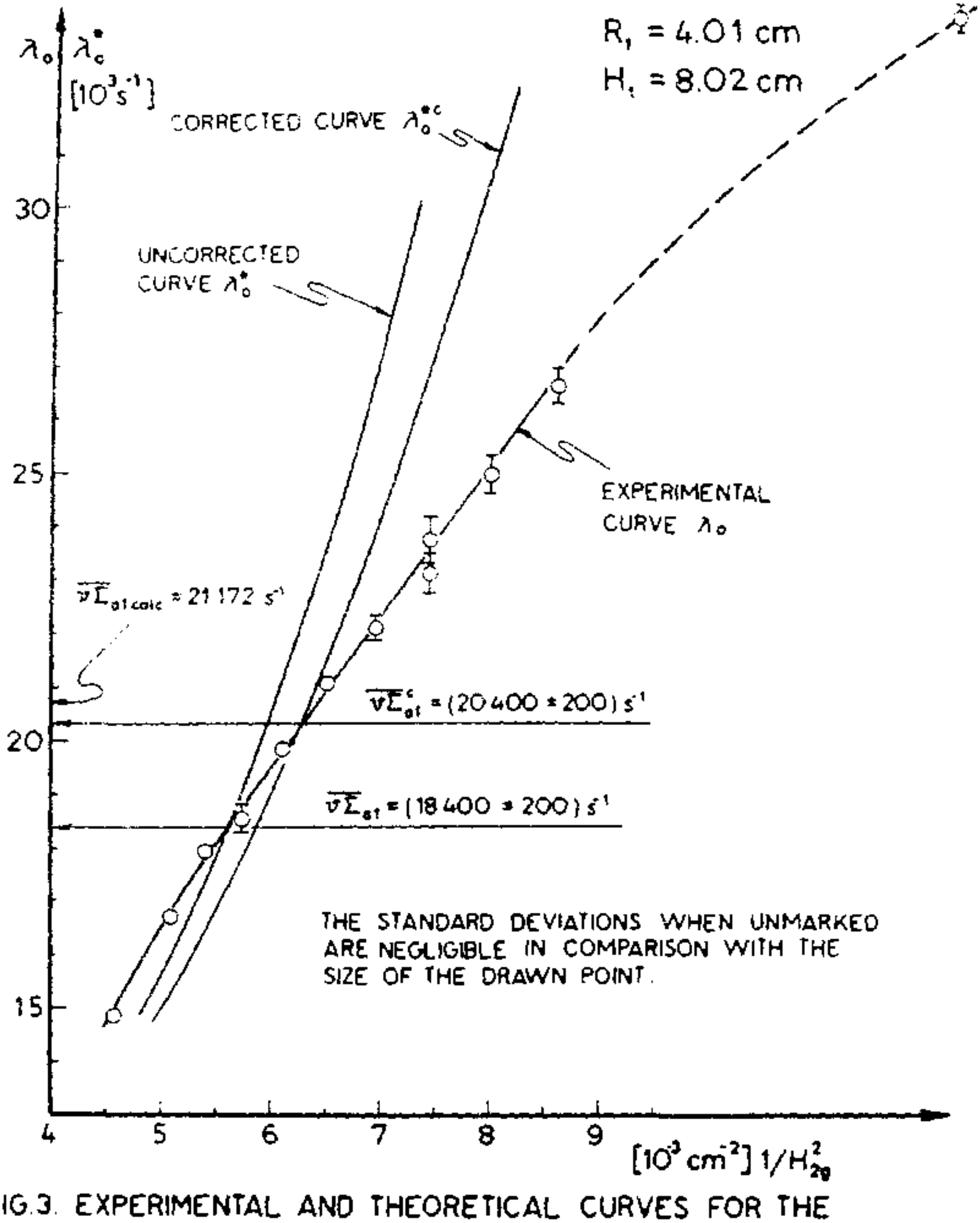

FIG.3. EXPERIMENTAL AND THEORETICAL CURVES FOR THE SAMPLE No 1.

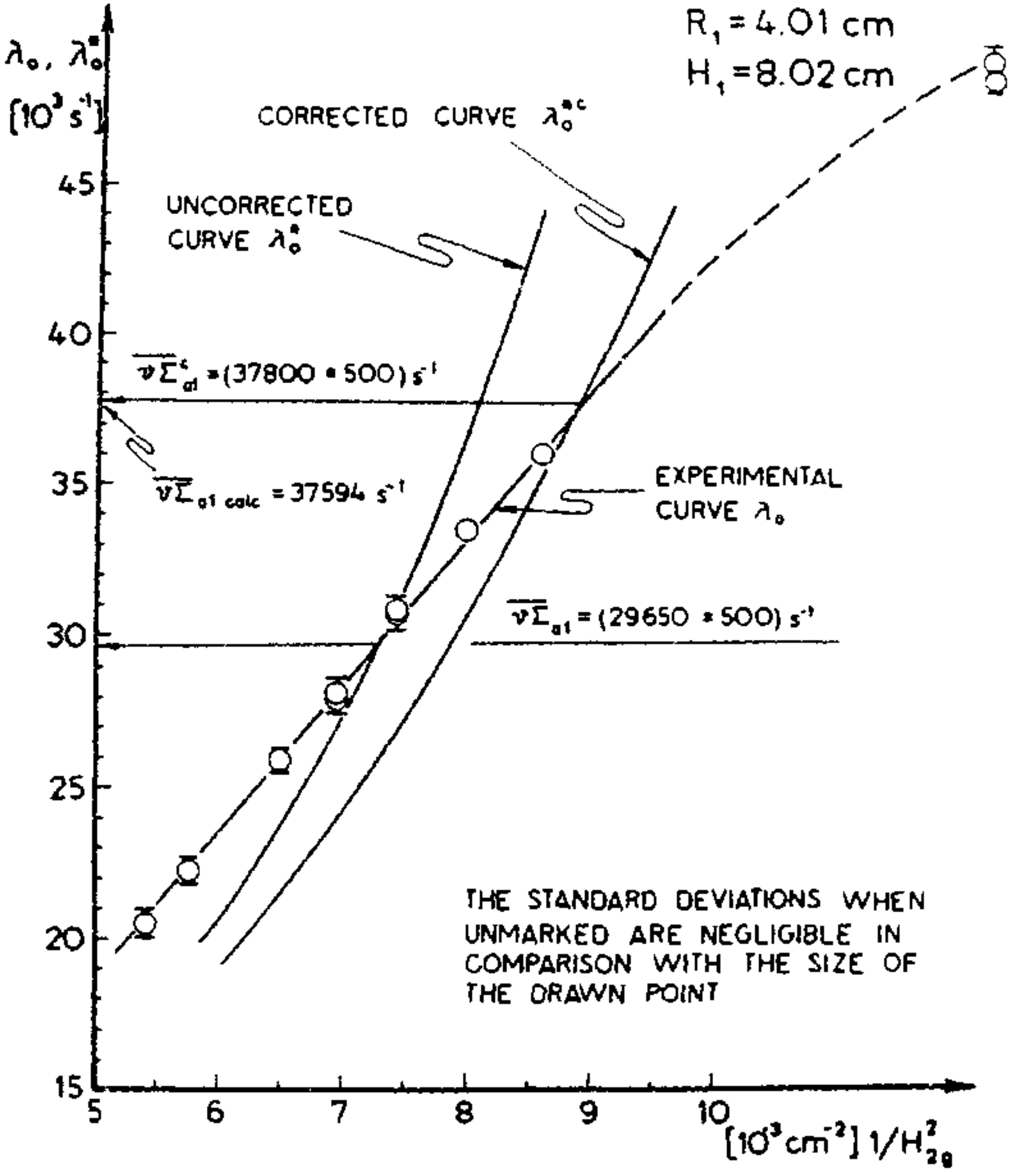

FIG. 4. EXPERIMENTAL AND THEORETICAL CURVES FOR THE SAMPLE No. 2.

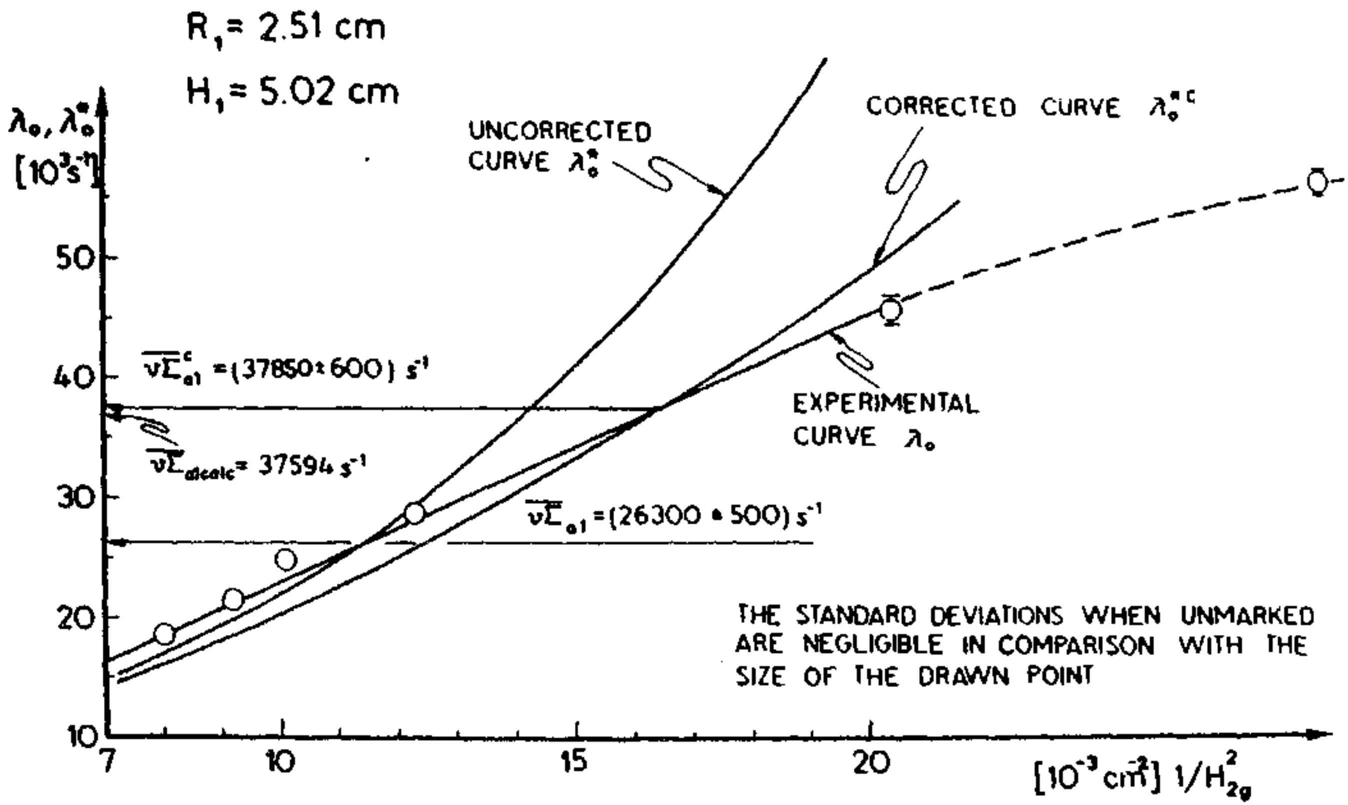

FIG.5. EXPERIMENTAL AND THEORETICAL CURVES FOR THE SAMPLE No 3.

Table 1. Calculation and measurement results.

Sample	Concentration of H_3BO_3 solution	Dimensions $R_1=H_1/2$	$\overline{v\Sigma}_{a1\ calc}$ and $\sigma(\overline{v\Sigma}_{a1\ calc})$	$\overline{v\Sigma}_{a1\ meas}$ and $\sigma(\overline{v\Sigma}_{a1\ meas})$	$\overline{v\Sigma}^{c}_{a1\ meas}$ and $\sigma(\overline{v\Sigma}^{c}_{a1\ meas})$	Δ and $\sigma(\Delta)$	Δ^c and $\sigma(\Delta^c)$
	[%]	[cm]	[s^{-1}]	[s^{-1}]	[s^{-1}]	[%]	[%]
No 1	1.00	4.01	21 172 22	18 400 200	20 400 200	-13.1 1.4	-3.6 1.4
No 2	2.00	4.01	37 594 22	29 650 500	37 800 500	-21.1 1.3	+0.5 1.3
No 3	2.00	2.51	37 594 22	26 300 500	37 850 600	-30.7 1.3	+0.7 1.5

differ from the exact [1] values $v\Sigma_{a1\ calc}$ up to $\Delta \approx 30\ \%$, while the corrected ones $v\Sigma^{c}_{a1\ meas}$ are almost equal to the true values. The deviation $\Delta^{c} = 3.6\ \%$ for the sample No 1 is due to the limitations of the method, which has been discussed by Drozdowicz (1981).

ACKNOWLEDGEMENTS

We want to express our gratitude to professor Dr J.A.Czubek for helpful discussions on the form of the paper. Special thanks are due to A. Igielski and E. Krynicka-Drozdowicz for their participation in performing measurements and to J. Burda and W. Kowalik for their technical assistance.

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[1]) The values of the thermal neutron macroscopic absorption cross-sections calculated on the basis of the well-known elemental composition of a medium can be assumed as exact. They are in very good agreement with the experimental values obtained in the classic pulsed experiments by the method of the verying geometric buckling (e.g. Drozdowicz et al. 1980).

APPENDIX

CALCULATION OF THE COEFFICIENT $K^{(n)}$.

Coefficient $K^{(n)}$ has been defined in Eq.(16) as:

$$K^{(n)} = 1 + \frac{\int_{V_1} \phi(z,\rho)\, \phi_1^{*(n)}(z,\rho)\, dV}{\int_{V_2} \phi(z,\rho)\, \phi_2^{*(n)}(z,\rho)\, dV} .$$

The two-region cylindrical system where the thermal neutron flux is considered is shown in Fig.6. The cylindrical sample of height H_1 and radius R_1 (volume V_1) is surrounded by the outer cylindrical moderator of the extrapolated dimensions H_2 and R_2 [cf. Eq.(15)]. The volume of the moderator is V_2 . The origin of the cylindrical coordinate system (z, ρ, φ) is placed in the geometric center of the inner cylinder. Then the whole discussion can be carried out for only the half-space $z \geqslant 0$, because of the symetry of the system.

The assumption given by Eq.(9) is equivalent to the condition that the thermal neutron flux in the volume V_1 is constant:

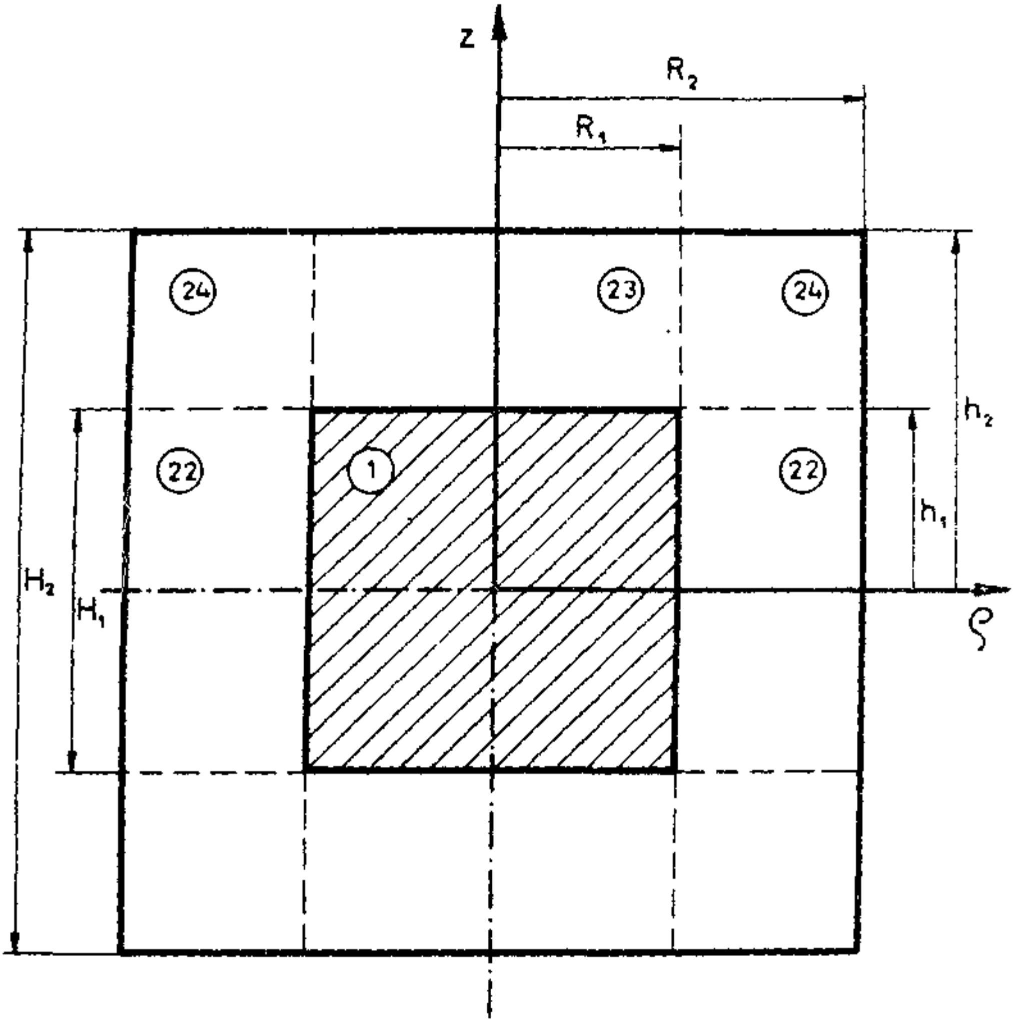
$$\phi_1^{*(n)}(z,\rho) = A_1 \qquad \text{for} \begin{cases} 0 \leqslant z \leqslant h_1 \\ 0 \leqslant \rho \leqslant R_1 \end{cases} , \qquad (A1)$$

where A_1 is a constant.

The boundary conditions in the calculation of the thermal neutron flux $\phi_2^{*(n)}$ in the external cylinder need to have the volume V_2 divided into three parts:

$$V_2 = V_{22} + V_{23} + V_{24} . \qquad (A2)$$

The flux $\phi_2^{*(n)}$ in each part of the volume is expressed (cf.



$h_1 = H_1/2$

$h_2 = H_2/2$

FIG. 6. SPACE PARTITION OF THE TWO-REGION SYSTEM IN CYLINDRICAL COORDINATES

Woźnicka 1981) by the different relationships:

a) for $0 \leqslant z \leqslant h_1$ and $R_1 \leqslant \varrho \leqslant R_2$:

$$\phi_{22}^{*(n)}(z,\varrho) = A_1 \frac{J_o(W^{(m)}\varrho) - S^{(m)}\ Y_o(W^{(m)}\varrho)}{J_o(W^{(m)}R_1) - S^{(m)}\ Y_o(W^{(m)}R_1)} \quad ; \qquad (A3)$$

b) for $h_1 \leqslant z \leqslant h_2$ and $0 \leqslant \varrho \leqslant R_1$:

$$\phi_{23}^{*(n)}(z,\varrho) = A_1 \frac{\sin[\alpha^{(m)}\ (z - h_2)]}{\sin[\alpha^{(m)}\ (h_1 - h_2)]} \quad ; \qquad (A4)$$

c) for $h_1 \leqslant z \leqslant h_2$ and $R_1 \leqslant \varrho \leqslant R_2$:

$$\phi_{24}^{*(n)}(z,\varrho) = A_1 \frac{J_o(W^{(m)}\varrho) - S^{(m)}\ Y_o(W^{(m)}\varrho)}{J_o(W^{(m)}R_1) - S^{(m)}\ Y_o(W^{(m)}R_1)} \times$$

$$\times \frac{\sin[\alpha^{(m)}\ (z - h_2)]}{\sin[\alpha^{(m)}\ (h_1 - h_2)]} \quad ; \qquad (A5)$$

where the used symbols have been explained in Eq.(21), below Eq. (22) and in Eq.(23) and

$$m = n - 1 \qquad (A6)$$

is introduced for simplicity of the notation.

The thermal neutron flux $\phi(z,\varrho)$ for a homogeneous cylindrical system is defined as:

$$\phi(z,\varrho) = A_o \cos(\alpha_e z)\ J_o(W_e \varrho) \quad , \qquad (A7)$$

where A_o is a constant and $\alpha_e = \pi/H_2$ and $W_e = j_o/R_2$ have been introduced in Eqs (13) and (14).

Now, the integrals in Eq.(16) can be written as:

$$\int_{V_1} \phi(z,\rho)\ \phi_1^{*(n)}(z,\rho)\ dV = b_1^{(n)} \quad , \tag{A8}$$

$$\int_{V_2} \phi(z,\rho)\ \phi_2^{*(n)}(z,\rho)\ dV = b_2^{(n)} + b_3^{(n)} + b_4^{(n)} \quad , \tag{A9}$$

where

$$b_i^{(n)} = \int_{V_{2i}} \phi(z,\rho)\ \phi_{2i}^{*(n)}(z,\rho)\ dV \quad , \qquad i=2,3,4 \tag{A9a}$$

and the coefficient $K^{(n)}$ can be expressed as:

$$K^{(n)} = 1 + \frac{b_1^{(n)}}{b_2^{(n)} + b_3^{(n)} + b_4^{(n)}} \quad . \tag{A10}$$

Here, it is comfortable to mention the relationships useful during the calculation of the integrals in Eqs (A6) and (A9a):

(i) $$\int x\ J_o(x)\ dx = x\ J_1(x) \quad ,$$

[Ryżyk and Gradsztejn (1964)]

or

$$\int x\ J_o(ax)\ dx = \frac{x}{a}\ J_1(ax) \quad ,$$

(ii) $$\int x\ Z_k(ax)\ \ell_k(bx)\ dx =$$

$$= \frac{bx\ Z_k(ax)\ \ell_{k-1}(bx) - ax\ Z_{k-1}(ax)\ \ell_k(bx)}{a^2 - b^2} \quad ,$$

[Ryżyk and Radsztejn (1964)]

where $Z_k(x) = c_1 J(x) + c_2 Y_k(x)$, $\mathcal{Z}_k(x) = c_3 J_k(x) + c_4 Y_k(x)$.

One can calculate the term $b_1^{(n)}$ inserting Eqs (A7) and (A1) into Eq.(A8). Then:

$$b_1^{(n)} = 2 \int_0^{h_1} \int_0^{R_1} \int_0^{2\pi} A_o \cos(\alpha_e z)\, J_o(W_e \rho)\, A_1\, \rho\, dz\, d\rho\, d\varphi \quad , \qquad \text{(A11)}$$

i.e.

$$b_1^{(n)} = 4\pi A_o\, A_1\, I_1\, I_2 \quad , \qquad \text{(A12)}$$

where:

$$I_1 = \int_0^{h_1} \cos(\alpha_e z)\, dz \qquad \text{(A13)}$$

$$= \left[\frac{\sin(\alpha_e z)}{\alpha_e} \right]_0^{h_1} = \frac{H_2}{\pi} \sin(\alpha_e h_1) \quad ;$$

$$I_2 = \int_0^{R_1} J_o(W_e \rho)\, \rho\, d\rho \qquad \text{(A14)}$$

$$= \left[\frac{\rho}{W_e} J_1(W_e \rho) \right]_0^{R_1} = \frac{R_1}{W_e} J_1(W_e R_1) \quad .$$

Thus:

$$b_1^{(n)} = 4\pi A_o\, A_1\, \frac{H_2}{\pi} \sin(\alpha_e h_1)\, \frac{R_1}{W_e} J_1(W_e R_1) \quad , \qquad \text{(A15)}$$

or after the substitution of $a_1^{(n)}$ as defined in Eq.(24):

$$b_1^{(n)} = 4\, A_o\, A_1\, a_1^{(n)} \quad . \qquad \text{(A16)}$$

The term $b_2^{(n)}$ defined by Eq.(A9a) is equal to:

$$b_2^{(n)} = 2 \int_0^{h_1} \int_{R_1}^{R_2} \int_0^{2\pi} A_o \cos(\alpha_e z) \, J_o(W_e \rho) \, A_1 \frac{J_o(W^{(m)}\rho) - S^{(m)} \, Y_o(W^{(m)}\rho)}{J_o(W^{(m)}R_1) - S^{(m)} \, Y_o(W^{(m)}R_1)} \rho \, dz \, d\rho \, d\varphi \quad , \qquad \text{(A17)}$$

i.e.

$$b_2^{(n)} = \frac{4\pi A_o A_1}{J_o(W^{(m)}R_1) - S^{(m)} \, Y_o(W^{(m)}R_1)} \, I_1 \, (I_3 - S^{(m)} \, I_4) \quad , \qquad \text{(A18)}$$

where:

$$I_3 = \int_{R_1}^{R_2} J_o(W_e \rho) \, J_o(W^{(m)}\rho) \, \rho \, d\rho \qquad \text{(A19)}$$

$$= \left[\frac{W^{(m)} \, \rho \, J_o(W_e \rho) \, J_{-1}(W^{(m)}\rho) - W_e \, \rho \, J_{-1}(W_e \rho) \, J_o(W^{(m)}\rho)}{W_e^2 - (W^{(m)})^2} \right]_{R_1}^{R_2}$$

$$= \left[\frac{- W^{(m)} \, J_o(W_e \rho) \, J_1(W^{(m)}\rho) + W_e \, J_1(W_e \rho) \, J_o(W^{(m)}\rho)}{W_e^2 - (W^{(m)})^2} \, \rho \right]_{R_1}^{R_2}$$

$$= \frac{j_o J_1(j_o)\, J_o(W^{(m)}R_2) - W_e R_1 J_1(W_e R_1)\, J_o(W^{(m)}R_1) + W^{(m)} R_1 J_o(W_e R_1)\, J_1(W^{(m)}R_1)}{W_e^2 - (W^{(m)})^2} \;;$$

and

$$I_4 = \int_{R_1}^{R_2} J_o(W_e\varrho)\, Y_o(W^{(m)}\varrho)\, \varrho\, d\varrho \tag{A20}$$

$$= \left[\frac{-W^{(m)} J_o(W_e\varrho)\, Y_1(W^{(m)}\varrho) + W_e J_1(W_e\varrho)\, Y_o(W^{(m)}\varrho)}{W_e^2 - (W^{(m)})^2}\, \varrho \right]_{R_1}^{R_2}$$

$$= \frac{j_o J_1(j_o)\, Y_o(W^{(m)}R_2) - W_e R_1 J_1(W_e R_1)\, Y_o(W^{(m)}R_1) + W^{(m)} R_1 J_o(W_e R_1)\, Y_1(W^{(m)}R_1)}{W_e^2 - (W^{(m)})^2} \;.$$

Thus:

$$I_3 - S^{(m)} I_4 = \tag{A21}$$

$$= \frac{W^{(m)} R_1 J_o(W_e R_1)[J_1(W^{(m)}R_1) - S^{(m)} Y_1(W^{(m)}R_1)] - W_e R_1 J_1(W_e R_1)[J_o(W^{(m)}R_1) - S^{(m)} Y_o(W^{(m)}R_1)]}{W_e^2 - (W^{(m)})^2} \,.$$

Substituting Eqs (A13) and (A21) into Eq.(A18) one obtains:

$$b_2^{(n)} = \frac{4\ A_o\ A_1}{W_e^2 - (W^{(m)})^2}\ H_2\ \sin(\alpha_e h_1) \times$$

$$\times\ [W^{(m)} R_1\ J_o(W_e R_1)\ \frac{J_1(W^{(m)} R_1) - S^{(m)} Y_1(W^{(m)} R_1)}{J_o(W^{(m)} R_1) - S^{(m)} Y_o(W^{(m)} R_1)} - W_e\ R_1\ J_1(W_e R_1)] \quad , \qquad (A22)$$

or using the symbols defined in Eqs (22) and (20) :

$$b_2^{(n)} = 4\ A_o\ A_1 \frac{H_2\ \sin(\alpha_e h_1)}{W_e^2 - (W^{(m)})^2}\ N^{(m)} \quad ,$$

which according to Eq.(25) gives:

$$b_2^{(n)} = 4\ A_o\ A_1\ a_2^{(n)} \quad . \qquad (A23)$$

The term $b_3^{(n)}$ is following:

$$b_3^{(n)} = 2 \int_{h_1}^{h_2} \int_0^{R_1} \int_0^{2\pi} A_o\ \cos(\alpha_e z)\ J_o(W_e \rho)\ A_1\ \frac{\sin[\alpha^{(m)}\ (z - h_2)]}{\sin[\alpha^{(m)}\ (h_1 - h_2)]}\ \rho\ dz\ d\rho\ d\varphi \quad , \qquad (A24)$$

i.e.

$$b_3^{(n)} = \frac{4\pi A_0 A_1}{\sin[\alpha^{(m)} (h_1 - h_2)]} I_5 I_2 \quad . \tag{A25}$$

where I_2 is given in Eq.(A14) and

$$I_5 = \int_{h_1}^{h_2} \cos(\alpha_e z) \sin[\alpha^{(m)} (z - h_2)] \, dz \quad , \tag{A26}$$

$$I_5 = \int_{h_1}^{h_2} \cos(\alpha_e z) \, [\sin(\alpha^{(m)} z) \cos(\alpha^{(m)} h_2) - \cos(\alpha^{(m)} z) \sin(\alpha^{(m)} h_2)] \, dz$$

$$= \Bigg[\cos(\alpha^{(m)} h_2) \left\{ - \frac{\cos[(\alpha^{(m)} + \alpha_e) z]}{2 (\alpha^{(m)} + \alpha_e)} - \frac{\cos[(\alpha^{(m)} - \alpha_e) z]}{2 (\alpha^{(m)} - \alpha_e)} \right\} +$$

$$- \sin(\alpha^{(m)} h_2) \left\{ \frac{\sin[(\alpha^{(m)} - \alpha_e) z]}{2 (\alpha^{(m)} - \alpha_e)} + \frac{\sin[(\alpha^{(m)} + \alpha_e) z]}{2 (\alpha^{(m)} + \alpha_e)} \right\} \Bigg]_{h_1}^{h_2}$$

$$= \cos(\alpha^{(m)} h_2) \left\{ - \frac{\alpha_e \sin(\alpha^{(m)} h_2)}{(\alpha^{(m)})^2 - \alpha_e^2} + \frac{\cos[(\alpha^{(m)} + \alpha_e) h_1]}{2 (\alpha^{(m)} + \alpha_e)} + \frac{\cos[(\alpha^{(m)} - \alpha_e) h_1]}{2 (\alpha^{(m)} - \alpha_e)} \right\} +$$

$$- \sin(\alpha^{(m)} h_2) \left\{ - \frac{\alpha_e \cos(\alpha^{(m)} h_2)}{(\alpha^{(m)})^2 - \alpha_e^2} - \frac{\sin[(\alpha^{(m)} + \alpha_e) h_1]}{2\ (\alpha^{(m)} + \alpha_e)} - \frac{\sin[(\alpha^{(m)} - \alpha_e) h_1]}{2\ (\alpha^{(m)} - \alpha_e)} \right\} \quad ,$$

i.e.

$$I_5 = \frac{\cos[\alpha^{(m)}\ (h_2 - h_1) - \alpha_e h_1]}{2\ (\alpha^{(m)} + \alpha_e)} + \frac{\cos[\alpha^{(m)}\ (h_2 - h_1) + \alpha_e h_1]}{2\ (\alpha^{(m)} - \alpha_e)} \quad , \qquad \text{(A26a)}$$

or, using the symbol introduced in Eq.(19), simply:

$$I_5 = M^{(m)} \quad .$$

Thus:

$$b_3^{(n)} = \frac{4\pi\ A_o\ A_1}{\sin[\alpha^{(m)}\ (h_1 - h_2)]} \cdot \frac{M^{(m)}}{W_e}\ J_1(W_e R_1)$$

and finally:

$$b_3^{(n)} = 4\ A_o\ A_1\ a_3^{(n)} \quad . \qquad \text{(A27)}$$

The calculation of the term $b_4^{(n)}$ gives:

$$b_4^{(n)} = \int_{h_1}^{h_2} \int_{R_1}^{R_2} \int_{0}^{2\pi} \Big\{ A_o \cos(\alpha_e z) \, J_o(W_e \varrho) \times$$

$$\times A_1 \frac{J_o(W^{(m)}\varrho) - S^{(m)} \, Y_o(W^{(m)}\varrho)}{J_o(W^{(m)}R_1) - S^{(m)} \, Y_o(W^{(m)}R_1)} \; \frac{\sin[\alpha^{(m)} \, (z - h_2)]}{\sin[\alpha^{(m)} \, (h_1 - h_2)]} \, \varrho \Big\} \, dz \, d\varrho \, d\varphi \, , \quad \text{(A28)}$$

$$b_4^{(n)} = \frac{4\pi \, A_o \, A_1 \, I_5 \, (I_3 - S^{(m)} \, I_4)}{[J_o(W^{(m)}R_1) - S^{(m)} \, Y_o(W^{(m)}R_1)] \, \sin[\alpha^{(m)} \, (h_1 - h_2)]} \quad , \qquad \text{(A29)}$$

where I_3, I_4, I_5 are defined in Eqs (A19), (A20), (A26), and the results are given by Eqs (A21) and (A26a). Then, using the symbols defined in Eqs (19), (20) and (22) one obtains from Eq.(A29) the expression:

$$b_4^{(n)} = \frac{4\pi \, A_o \, A_1 \, N^{(m)} \, M^{(m)}}{\sin[\alpha^{(m)} \, (h_1 - h_2)] \, [W_e^2 - (W^{(m)})^2]} \quad ,$$

which after using $a_4^{(n)}$ from Eq.(27) gives:

$$b_4^{(n)} = 4\ A_o\ A_1\ a_4^{(n)} \quad . \tag{A30}$$

The substitution of Eqs(A16), (A23), (A27) and (A30) into Eq.(A10) yields:

$$K^{(n)} = 1 + \frac{a_1^{(n)}}{a_2^{(n)} + a_3^{(n)} + a_4^{(n)}} \quad ,$$

i.e. the result given in the main text in Eq.(18).

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