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A NON LINEAR HALF SPACE PROBLEM FOR RADIATIVE TRANSFER EQUATIONS. APPLICATION TO THE ROSSELAND APPROXIMATION

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Half space problems for radiative transfer equations

Application to the Rosseland approximation

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It is known that the radiative transfer equations may be approximated by a non linear diffusion equation (called Rosseland equation) when the mean free paths of the photons are small with respect to the size of the medium. Recall the general outlines of this approximation in a monodimensionnal medium $\mathcal{O} =]0, X_m[$. Denote by X the space variable ($X \in \mathcal{O}$), by ν the frequency variable ($\nu \in F = R^+$), and μ the angular variable ($\mu \in V = [-1,+1]$). If $I_\varepsilon(X,\mu,\nu,t)$ is the intensity of radiative energy and $\varphi_\varepsilon(X,t)$ the fourth power of the material temperature (up to a multiplicative constant) at time t, the radiative transfer equations are :

$$\frac{\partial I_\varepsilon}{\partial t} + \frac{1}{\varepsilon}\mu\frac{\partial I_\varepsilon}{\partial X} + \frac{1}{\varepsilon^2}\left[HI_\varepsilon + \sigma(\varphi_\varepsilon)(I_\varepsilon - b(\varphi_\varepsilon))\right] = 0$$

(1)

$$\frac{\partial E(\varphi_\varepsilon)}{\partial t} + \frac{1}{\varepsilon^2} \langle \sigma(\varphi_\varepsilon)(b(\varphi_\varepsilon) - I_\varepsilon)\rangle = 0$$

where we set for any function $f = f(x,\mu,\nu,t)$:

$$\langle f \rangle = \int_0^{+\infty}\int_V f(\mu,\nu)\frac{d\mu}{2}\,d\nu$$

$$(H f)(\mu,\nu) = \tau f(\mu,\nu) - \tau\int_V f(\mu',\nu)\frac{d\mu'}{2}$$

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Here, $E = E(\varphi)$ (the internal energy of the material) is an increasing function from R^+ into R^+, which is smooth ;

τ is a non negative constant and $\sigma = \sigma_\nu(\varphi)$ is a positive bounded function whith good properties of monotony, [τ and σ are the scattering and absorption cross section] .

The function $B = B_\nu(\varphi)$ called the Plank function is an universal positive bounded function, which is strictly increasing with φ .

Of course we have to write the initial conditions for $I_\varepsilon(x,\mu,\nu,0)$ and $\varphi_\varepsilon(x,0)$ and the boundary conditions related to (1) in order to have a well-posed problem for $(\varphi_\varepsilon, I_\varepsilon)$.

With this formulation, in the case where the boundary conditions are of the following type (Plankian equilibrium) :

$$(2) \qquad \begin{aligned} I_\varepsilon(0,\mu,\nu,t) &= B_\nu(\alpha(t)) \qquad && \mu > 0 \\ I_\varepsilon(x_m,\mu,\nu,t) &= B_\nu(\beta(t)) \qquad && \mu < 0 \end{aligned}$$

Larsen-Badham-Pomraning [1] have shown that $\varphi_\varepsilon(X,t)$ may be approximated (when ε goes to 0) by $\varphi(X,t)$ which satisfies the following Rosseland equation :

$$\frac{\partial}{\partial t}(\varphi + E(\varphi)) - \frac{\partial}{\partial X}\left(\frac{1}{3\sigma_R}\frac{\partial \varphi}{\partial X}\right) = 0$$

with the boundary conditions

$$(3) \quad \text{i)} \quad \varphi(0,t) = \alpha(t) \qquad \text{ii)} \quad \varphi(x_m,t) = \beta(t)$$

and some initial condition.

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Of course it is necessary to make some technical assumptions, namely about the initial conditions to avoid any problem of initial layer terms.

. In this paper , we first study the previous diffusion approximation, when the boundary conditions are not in a Plankian equilibrium like (2). Then we show that it appears boundary layer terms at $X = 0$ and $X = X_m$, in a natural way. In order to write these terms near $X = 0$ (for instance), we use the functions $\phi(x)$ and $u(x, \mu, \nu)$ (where x varies in R^+ and has to be changed by X/ε) which satisfy :

$$(*)\quad \begin{cases} \mu \dfrac{du}{dx} + \sigma(\phi)(u - b(\phi)) + Hu = 0 \\[2ex] \langle \sigma(\phi)(u - b(\phi)) \rangle = 0 \end{cases}$$

with a boundary condition for $u(0, \mu, \nu)$ taking into account the data $I_\varepsilon(0, \mu, \nu, t)$ (for $\mu > 0$).

. Afterwards we study the non linear "half-space problem" (*). We show that there exists a unique bounded solution and that $\phi(x)$ converge to a limit ϕ_∞ when x goes to infinity. This limit yields the boundary condition of the Rosseland equation, that is to say that (3-i) has to be changed into :

$$\phi(0, t) = \phi_\infty .$$

For studing existence and uniqueness of the half-space problem we write the system (*) in the following form :

$$\mu \frac{\partial u}{\partial x} + Q\, u + Hu = 0$$

....

where Q is a non linear operator which is accretive in $L^1(V \times R^+_y)$ and we use monotony technics.

REFERENCES

1. Larsen LW, Pomraning GC, Badham VC.. Asymptotic analysis of radiative transfer problems, J. Quant. Spect. Radiat. Transfer 29 (1983)