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BASIC PRINCIPLE OF CONSTANT q_a CURRENT BUILD-UP IN TOKAMAKS

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ABSTRACT

An analytic expression is derived such that the current profile shape is kept constant during the current build-up phase in tokamaks. The required conductivity profile is parametrized by two externally controllable parameters, $\dot{I}_p$ and $\dot{a}_p$ in the case of the Gaussian current profile. It is shown that a Gaussian current profile can be maintained for a realistically broad conductivity profile by using the constant q_a current build-up method even under the condition of a high $\dot{I}_p$.

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I. INTRODUCTION

In tokamak discharges, plasma current build-up and termination which avoids MHD instabilities is preferable to decrease the plasma-wall interaction and the consequent impurity contamination as well as to avoid sudden termination due to disruptions. In many tokamaks, MHD oscillations and disruptive phenomena have been observed during the initial stage.[1-3] A rapid drop of the thermal energy has been observed during disruptive activity[3] in the current build-up phase. MHD mode development and the mixing process[4] are possible causes of these disruptive phenomena and also the possible cause of the anomalous current diffusion occurring during the current build-up phase. In order to avoid unfavorable MHD instabilities during the current build-up phase, control of the current density profile would be most desirable from the point of view of improving MHD stability.[5]

Control of the current density profile by a combination of gas puffing and current build-up has been well demonstrated in PLT,[6] JIPPT-II,[7] and ALCATOR-A[8] experiments. The time evolution of the current density profile is determined by the magnetic field diffusion equation: the formation of a hollow current density profile is expected for a high temperature, large tokamak plasma under the condition of classical resistivity. However, in general, the current build-up phase in tokamaks is subject to nonclassical resistive current diffusion and anomalous current diffusion typically prevents the formation of a hollow current profile. The effectiveness of minor radius control during the current rise to avoid a hollow current profile has been shown by Düchs, Furth, and Rutherford.[9] Some experiments have been conducted on T-10.[10] However, a detailed understanding of the current density profile evolution under minor radius control has not yet been obtained.

In the following section, we derive, using the magnetic field diffusion equation and assuming classical resistivity, the conditions for separable current profile solutions in which the current profile shape is kept fixed during the current rise. It is shown that during the current build-up phase the requirement on the conductivity profile is relaxed by constant q_a operation and that the current density profile in tokamak experiments may be controlled.

II. CONDITION FOR QUASISTEADY CURRENT PROFILE

The time evolution of the current profile in a large aspect ratio, circular tokamak is determined by the following equation,

$$\frac{\partial}{\partial t} B_\theta = \frac{1}{\mu_o} \frac{\partial}{\partial r} \left[\frac{\eta}{r} \frac{\partial}{\partial r} (rB_\theta)\right] . \tag{1}$$

We define the expanding coordinates as follows,

$$\begin{aligned} x &\equiv \frac{r}{a(t)} \\ t &\equiv \tau \end{aligned} \tag{2}$$

Differential operators are rewritten as follows,

$$\begin{aligned} \frac{\partial}{\partial t} &= \frac{\partial}{\partial t} - x \frac{\dot{a}}{a} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial r} &= \frac{1}{a} \frac{\partial}{\partial x} . \end{aligned} \tag{3}$$

After converting the magnetic field diffusion Eq. (1) into an equation for q^{-1}, the equation is expressed in the expanding coordinates as follows,

$$\frac{\mu_o}{J_o} (a^{\dot{2}} J_o) xF = \frac{\partial}{\partial x} \left(\frac{\eta}{x} \frac{\partial}{\partial x} (x^2 F) + \mu_o \, a \, \dot{a} \, x^2 F \right) \tag{4}$$

where,

$$\frac{1}{q} = \frac{RB_\theta}{rB_T} = \frac{1}{q_o(t)} F(x) \quad \text{and} \quad J_o = \frac{2 \, B_T}{\mu_o \, R \, q_o} \; .$$

Here we assume that the q profile is expressible as the product of functions of t and x (separable solution) so that the shape of the q profile does not change in time.

Equation (4) is easily solved for the resistivity η as follows,

$$\eta(x,t) = \frac{2x}{d/dx \, (x^2 F)} \left\{ \eta_o(t) + \frac{\mu_o}{2 \, J_o} (a^{\dot{2}} J_o) \int_o^x xF(x) dx - \frac{\mu_o a \, \dot{a}}{2} x^2 F(x) \right\} \; . \tag{5}$$

This equation gives the requirement on the resistivity (or equivalently conductivity) profile to maintain an arbitrary quasisteady current profile.

As an example, we choose the Gaussian current profile,

$$J_z(x,t) = J_o(t) \exp(-\alpha_J x^2) \; , \tag{6}$$

$$q(x,t) = \frac{q_o \, \alpha_J x^2}{1 - \exp(-\alpha_J x^2)} \; , \tag{7}$$

$$I_p = \int_o^\infty J_z \; 2\pi r \; dr = \frac{\pi a^2}{\alpha_J} J_o \; , \qquad (8)$$

where α_J is a constant. Substituting the expression for F(x) into Eq. (5), we obtain,

$$\eta(x,t) = \eta_o(t) \; \exp(\alpha_J x^2) \; \Big\{ 1 + \frac{\mu_o a^2 \dot{I}_p}{4 \; \alpha_J I_p \eta_o(t)} \int_o^{\alpha_J{}^2} \frac{1-e^{-y}}{y} \, dy - \frac{\mu_o a \; \dot{a}}{2\alpha_J \eta_o(t)} \left[1 - \exp(-\alpha_J x^2) \right] \Big\}$$

$$\equiv \eta_o(t) \; \exp(\alpha_J x^2) \; \{ 1 + \mu \; \zeta_1(\alpha_J, x) + \delta \; \zeta_2(\alpha_J, x) \} \qquad (9)$$

where

$$\mu \equiv \frac{\mu_o a^2 \; \dot{I}_p}{4 \; \alpha_J I_p \eta_o(t)} = \frac{\mu_o a^2 / 4\alpha_J \eta_o(t)}{I_p / \dot{I}_p} \quad \frac{\text{(Magnetic Skin Time)}}{\text{(Current Build-up Time)}} \; ,$$

$$\delta \equiv \frac{\mu_o \; a \; \dot{a}}{2 \; \alpha_J \; \eta_o(t)} = \frac{\mu_o a^2 / 4 \; \alpha_J \eta_o(t)}{a^2 / \dot{a}^2} \quad \frac{\text{(Magnetic Skin Time)}}{\text{(Expansion Time)}} \; ,$$

$$\zeta_1(\alpha_J, x) = \int_o^{\alpha_J x^2} \frac{1 - \exp(-y)}{y} \, dy \; ,$$

$$\zeta_2(\alpha_J, x) = - \left[1 - \exp(-\alpha_J x^2) \right] \; .$$

Radial profiles of the two functions ζ_1 and ζ_2 are shown in Fig. 1.

For the steady-state case ($\dot{I}_p = \dot{a} = 0$), the conductivity σ ($= \eta^{-1}$) profile coincides with the current profile. However, for a changing plasma current ($\dot{I}_p \neq 0$), the difference between the conductivity profile and the current profile becomes significant for $\mu \gtrsim 1$.

Typical μ values for PLT,[6] JIPPT-II,[7] ALCATOR-A,[3] and TFTR[10] are shown in the ($a^2\dot{I}_p/I_p$, T_e) diagram under the Spitzer resistivity condition in Fig. 2. As described in Ref. 11, the current rise in TFTR is divided into two phases: a rapid current penetration phase and a current build-up phase. For the rapid current penetration phase ($t \leq 0.2$ sec), much higher μ values are deduced; typically $I_p/\dot{I}_p = 0.1$ sec, whereas $I_p/\dot{I}_p = 0.5 \sim 1.0$ sec in the current build-up phase. However, reliable temperature measurements at that stage are not yet obtainable. The control requirement for the conductivity profile for several μ values is shown in Fig. 3. A fairly peaked conductivity profile is needed for high μ values, and it may be more difficult to maintain such a peaked conductivity profile only by gas puffing in larger tokamaks.

Significant difference between the conductivity profile and the current profile can be reduced by minor radius expansion during current build-up. The effect of minor radius control on the requirement for the conductivity profile is shown in Fig. 4. The difference is substantially reduced for the condition $\mu \simeq \delta$. Attention should be paid to the fact that the $\mu = \delta$ condition coincides with constant q_a operation during the current build-up phase. Thus we can control the current profile to maintain a nonhollow shape (typically Gaussian) by using constant q_a current build-up even with a normal broad conductivity profile.

III. CONCLUSION

By solving the magnetic field diffusion equation in the expanding coordinates, a general control requirement on the conductivity profile for maintaining the current profile in a quasisteady state is derived analytically.

Application of the formula to the Gaussian current profile case shows us that the required conductivity profile is parametrized by μ and δ which are related to the externally controllable parameters $\dot{I}_p$ and $\dot{a}$.

For a constant minor radius, we must obtain highly peaked conductivity profiles for high μ values which are typical for large tokamaks. However, by the application of a_p control, the difference between the required conductivity profile and the current profile can be reduced. The best compensation by a_p control is obtained for the case $\mu = \delta$, which corresponds to constant q_a operation during the current build-up phase.

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FIGURE CAPTIONS

FIG. 1 Radial profiles of the functions ζ_1 and ζ_2 for the $\alpha_J = 3.0$ case.

FIG. 2 Current build-up diagram $(a^2\dot{I}_p/I_p, T_e(0))$ for typical current density profile control experiments. Classical resistivity is assumed ($Z_{eff} = 2.0$, $\ell n\Lambda = 16.0$, $\alpha_J = 3.0$ are also assumed for equi-μ contours).

FIG. 3 Control requirement on the conductivty profile for various μ values (solid line). The dotted line is a reference Gaussian current density profile. The difference between the current density profile and the conductivity profile becomes larger as the μ value increases.

FIG. 4 Effects of minor radius expansion on the conductivity profile control requirement for the $\mu = 5$ (Fig. 4a) and $\mu = 12$ (Fig. 4b) cases. Application of minor radius control results in a wider conductivity profile, which is easier to obtain.

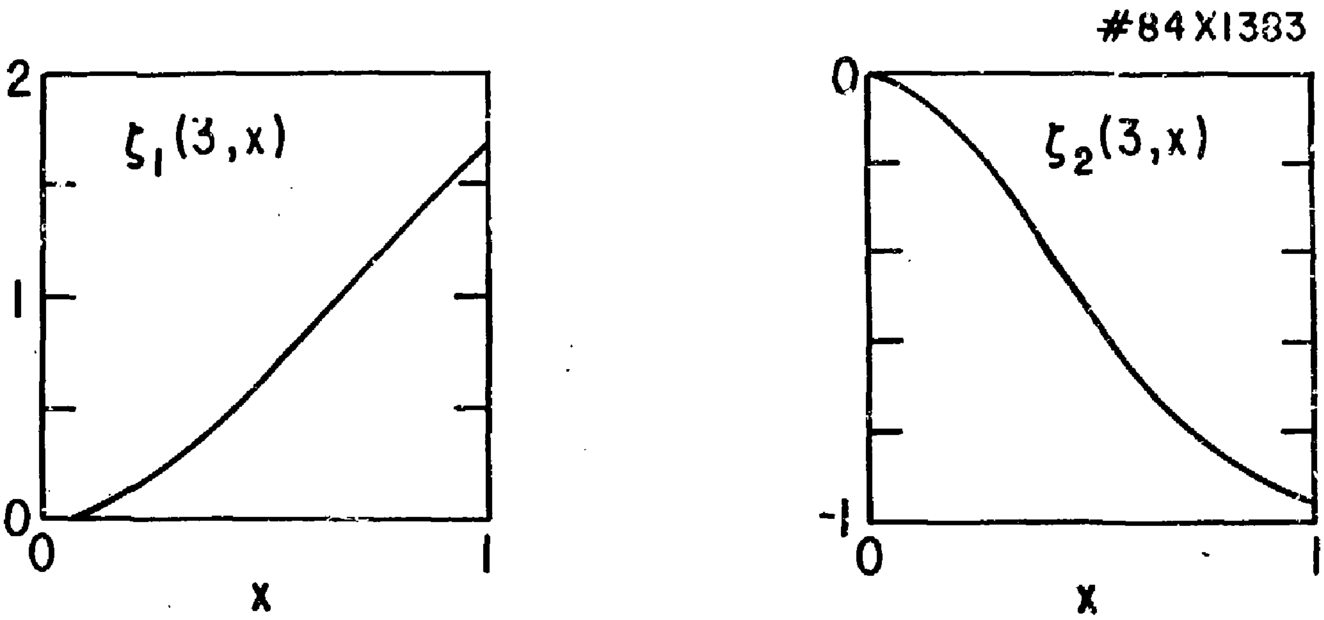
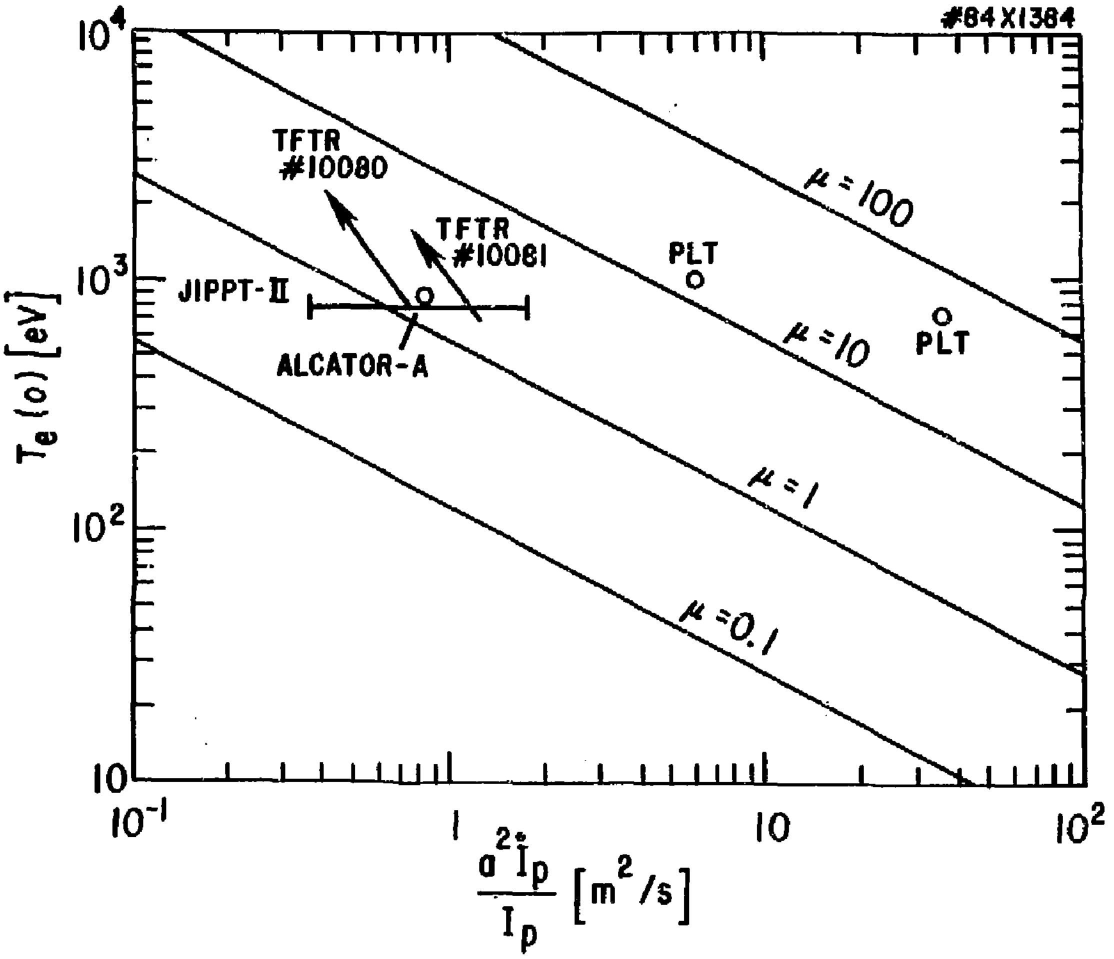


Fig. 1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$T_e(o)$ [eV]
10^4
10^3
10^2
10
10^{-1}
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10
10^2
$\frac{a^2 \dot{I}_p}{I_p}$ [m^2/s]
$\mu = 100$
$\mu = 10$
$\mu = 1$
$\mu = 0.1$
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TFTR #10081
JIPPT-II
ALCATOR-A
PLT
PLT

Fig. 2

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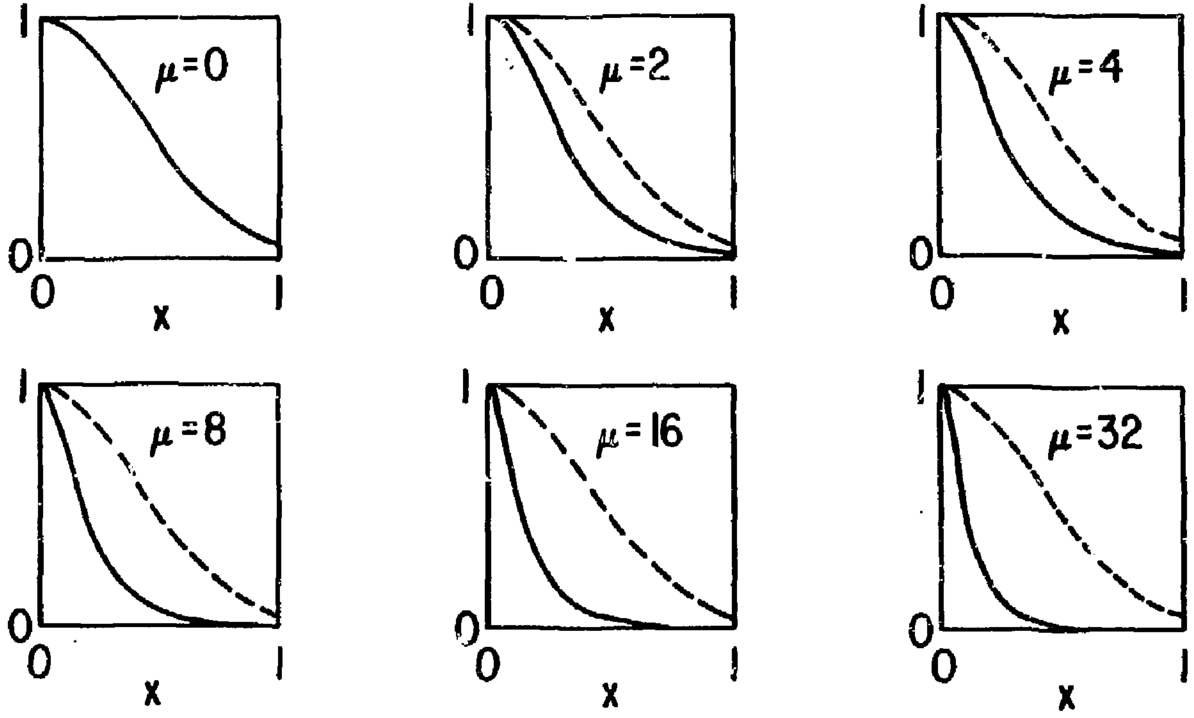

Fig. 3

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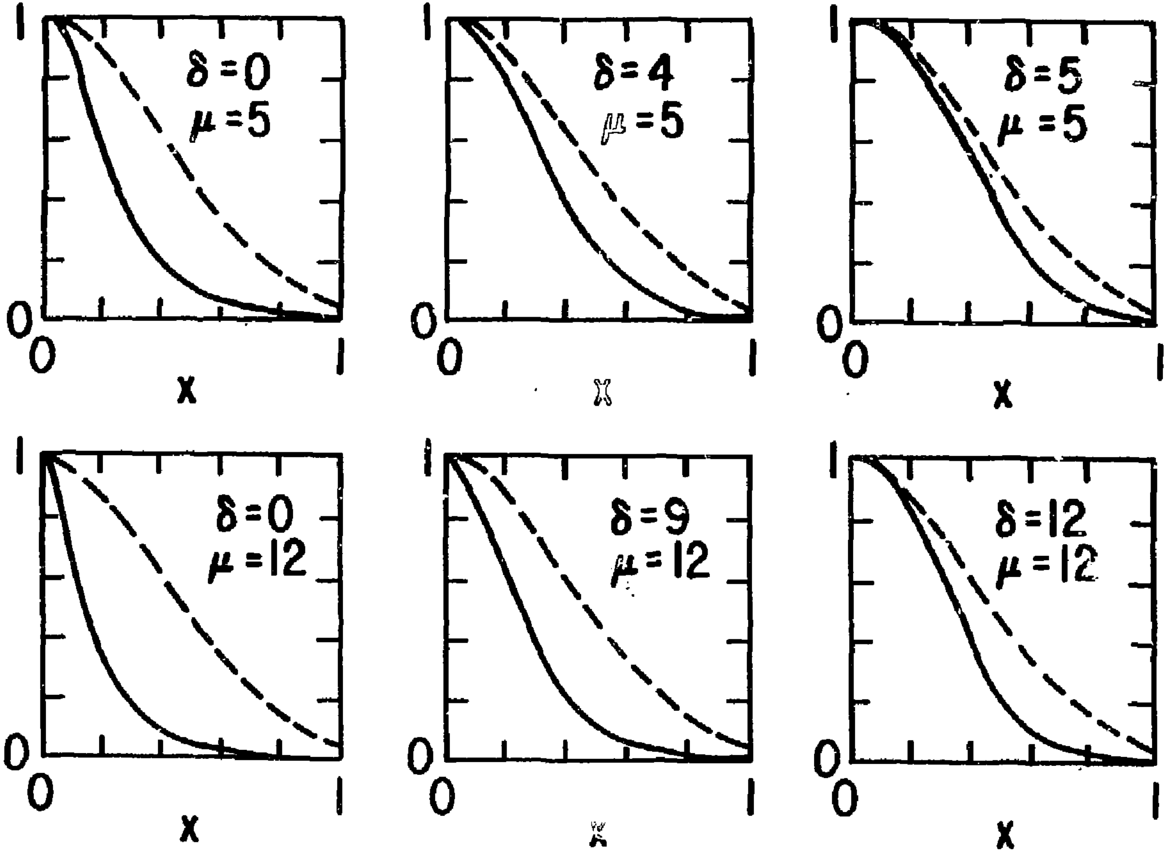

Fig. 4

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