GHB

GORDON AND BREACH
SCIENCE PUBLISHERS S.A.

CONF-850507--7

DE85 011549

Title of Paper: Pairing Correction of Particle-Hole State Densities for Two Kinds of Fermions

Author(s): C. Y. Fu

Dear Contributor:

4132 000

We are pleased that you will be contributing a paper to the publication

"Radiation Effects (Nuclear Data Conference)"

We are confident that it will prove a prestigious addition to the literature.

We do need your assurance—which you can give us by endorsing and returning this letter—that the work you are preparing is original, and that you therefore own all publishing rights to it, and agree to transfer them to us (for government employees: to the extent transferrable) so that we may proceed to publish it in line with established practices.

Please countersign below and return this letter to us promptly, so that there will be no obstacle to production.

Cordially,

Gordon and Breach Science Publishers S.A.

Agreed:

(signature)

C. Y. Fu
(please print name)

Date:

Subject to the following:

P.S. If your paper includes material which someone else has already copyrighted, for our mutual protection we will require written permission from the copyright owner together with this form.

DISCLAIMER

CONF-850501--7

PAIRING CORRECTION OF PARTICLE-HOLE STATE DENSITIES FOR TWO KINDS OF FERMIONS*

C. Y. FU
Oak Ridge National Laboratory, Oak Ridge, Tennessee, 37831 USA

Abstract Pairing corrections in particle-hole (exciton) state-density formulas used in precompound nuclear reaction theories are, strictly speaking, dependent on the nuclear excitation energy U and the exciton number n. A general formula for (U,n)-dependent pairing corrections has been derived in an earlier paper for exciton state-density formulas for one kind of Fermion. In the present paper, a similar derivation is made for two kinds of Fermions. It is shown that the constant-pairing-energy correction used in standard level-density formulas, such as U_0 in Gilbert and Cameron, is a limiting case of the present general (U,n)-dependent results.

INTRODUCTION

If the two-Fermion state-density formula[1]

$$\omega_2(U) = \frac{\sqrt{\pi}}{12} \frac{\exp[2(aU^*)^{1/2}]}{a^{1/4}\ U^{*5/4}} \tag{1}$$

is used in a compound-reaction cross-section calculation, what particle-hole state-density formula should one use for a consistent precompound-reaction cross-section analysis?

In Eq. (1), the subscript 2 indicates the two-Fermion formulation in which neutrons and protons occupy different sets of single particle states; the parameter a is related to the single-particle state density g by $a = \pi^2 g/6$; and the effective excitation U^* is given by $U^* = U - U_0$, where U is the excitation energy and U_0 is a constant-pairing-energy correction, such as tabulated by Gilbert and Cameron.[1]

*Research sponsored by the U.S. Department of Energy under contract DE-AC05-840R21400 with Martin Marietta Energy Systems, Inc.

Equation (1) was derived under the assumption that $g_\pi = g_\nu = g/2$ and g_π, g_ν, and g are constants. The quantities g_π and g_ν are, respectively, single-proton and single-neutron state densities. Making the same assumption and introducing the pairing correction P_2 into the two-Fermion exciton state-density formula of Williams,[2] we have

$$\omega_2(p_\pi,h_\pi,p_\nu,h_\nu,U) = \left(\frac{g}{2}\right)^n \frac{(U - B - P_2)^{n-1}}{p_\pi!h_\pi!p_\nu!h_\nu!(n-1)!} , \tag{2}$$

where n is the exciton number equaling $p_\pi + h_\pi + p_\nu + h_\nu$, p_π the proton-particle number, h_π the proton-hole number, p_ν the neutron-particle number, and h_ν the neutron-hole number. The quantity B is the correction factor for the Pauli exclusion principle.[2] If $U_0 = 0$ and $P_2 = 0$, Eqs. (1) and (2) are based on the same assumptions and are consistent in principle. In fact, it can be shown numerically that if $U_0 = 0$ and $P_2 = 0$,

$$\omega_2(U) \sim \sum_{\substack{p_\pi=h_\pi \\ p_\nu=h_\nu}} \omega_2(p_\pi,h_\pi,p_\nu,h_\nu,U) , \tag{3}$$

within 10%. This small difference is probably due to approximations in deriving the Pauli correction in Eq. (2).

We now face the questions of (1) what the value of P_2 is and (2) whether the two pairing correction factors U_0 and P_2 can be related in a consistent manner. The goal of this paper is to answer these two questions.

PAIRING CORRECTIONS IN THE ONE-FERMION FORMULAS

The pairing correction $P_1(U,n)$ for the exciton state-density formula in the one-Fermion formulation derived earlier[3] can be utilized for deriving P_2 and is summarized below.

The total state density in the one-Fermion formulation is given by

$$\omega_1(U) = \frac{\exp[2(aU^*)^{1/2}]}{\sqrt{48}\ U^*} , \tag{4}$$

where U^* is $U - U_0$. The corresponding exciton state density is given by

$$\omega_1(p,h,U) = \frac{g^n(U - A' - P_1)^{n-1}}{p!h!(n-1)!} , \tag{5}$$

where p is the particle number, h is the hole number, and n is the exciton number equaling $p + h$. The quantity A' is a modified correction factor for the Pauli exclusion principle.[3] The pairing correction $P_1(U,n)$ is given by

$$P_1(U,n) = \frac{g}{4} [\Delta_0^2 - \Delta^2(U,n)] \quad , \tag{6}$$

where Δ_0 is the ground-state pairing gap. In Eq. (6) the quantity $\Delta(U,n)$ is the excited-state pairing gap calculated from the pairing theory[4] using Δ_0 and g as input parameters. The resulting values of $\Delta(U,n)$ were parameterized as simple functions of U, n, Δ_0 and g. Depending upon the values of U and n, the pairing correction $P_1(U,n)$ has values lying between 0 and U_0.

It has been shown[3] that the pairing correction U_0 for the total state density is equal to the value of $P_1(U,n)$ evaluated along the most probable exciton number, $\hat{n}$ for $U \geqslant 3.15\ U_0$. That is to say,

$$U_0 = P_1(U,\hat{n}) = \frac{1}{4} g \Delta_0^2 \quad . \tag{7}$$

This relation provides a means of calculating Δ_0 from a given U_0, which is well understood. It has further been shown that even for $U < 3.15\ U_0$, Eq. (7) is still sufficiently accurate. Therefore, the pairing correction $P_1(U,n)$ does not require any additional parameter and is consistent with U_0.

PAIRING CORRECTION IN THE TWO-FERMION FORMULAS

The major steps in the present development and the main results are summarized in this section.

1. It is assumed that there is no pairing interaction between the proton (π) and the neutron (ν) systems, so that the pairing correction P_2 can be written as a sum of independent contributions from two one-Fermion systems:

$$P_2(U,n_\pi,n_\nu) = P_1(U_\pi,n_\pi) + P_1(U_\nu,n_\nu) \quad . \tag{8}$$

2. The seven pairing equations contain three for the protons and three for the neutrons. Each set of the three equations is identical to that of the one-Fermion model.[2] The six equations are coupled by the same temperature and by the seventh equation: $U = U_\pi + U_\nu$. The seven equations are to be solved simultaneously with the seven input quantities: g_π, g_ν, $\Delta_{0\pi}$, $\Delta_{0\nu}$, n_π, n_ν, and U. The solutions of interest to the present problem are U_π, U_ν, $\Delta_\pi(U_\pi,n_\pi)$, and $\Delta_\nu(U_\nu,n_\nu)$, which are used in Eq. (8) for calculating P_2.

3. It is shown that under the approximation $\Delta_{0\pi} = \Delta_{0\nu}$ (the mean-gap approximation), the seven pairing equations can be solved approximately once for all nuclides (all combinations of g and U_0, see below) with an accuracy of a few percent for P_2. The energies U_π and U_ν are first determined by simple methods (essentially proportional to n_π and n_ν) satisfying the isothermal requirement; then the results given above for the one-fermion model can be utilized.

4. The constant-pairing-energy correction U_0 for the total state density can be equated to P_2 evaluated along the most probable exciton number, $\hat{n} = \hat{n}_\pi + \hat{n}_\nu$. This is to say that the exciton state at $\hat{n}$ is the most dominant component of the total-(compound) state. We then have

$$U_0 = P_2(U,\hat{n}_\pi,\hat{n}_\nu) \quad . \tag{9}$$

5. From Eqs. (6), (8), and (9), we obtain

$$\Delta_{0\pi}^2 = \Delta_{0\nu}^2 = 4U_0/g \quad . \tag{10}$$

Therefore, with the mean-gap approximation, the total state density, Eq. (1), the p-h state density, Eq. (2), and the generalized pairing correction $P_2(U,n_\pi,n_\nu)$ are all defined consistently by the same two parameters g and U_0.

CONCLUSION

It has been shown that the total state-density formula, Eq. (1), and the particle-hole state-density formula, Eq. (2), can be calculated consistently by the same two parameters, namely, the single-particle state density g and the constant-pairing-energy correction U_0. Although the pairing correction P_2 in the particle-hole state-density formula depends on the excitation energy U and the exciton numbers n_π and n_ν, the value of P_2 can be calculated from g and U_0 as well. Therefore, if Eq. (1) is used in a compound-reaction cross-section calculation, Eq. (2) can be used consistently in a simultaneous precompound-reaction analysis.

REFERENCES

1. A. Gilbert and A. W. G. Cameron, *Can. J. Phys.*, **43**, 1446 (1965).
2. F. C. Williams, Jr., *Nucl. Phys.*, **A166**, 231 (1971).
3. C. Y. Fu, *Nucl. Sci. Eng.*, **86**, 344 (1984).
4. L. G. Moretto, *Nucl. Phys.*, **A243**, 77 (1975).