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COMPARISON BETWEEN THEORETICAL PREDICTIONS AND TRACKING

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Abstract The beam-beam interaction in a proton-antiproton collider has been an outstanding issue for a long time. Several theoretical predictions have been made in the past which range from the appearance of single beam-beam driven resonances to the onset of stochasticity and Arnold diffusion and the presence of chaotic trajectories. All these effects would cause a limit on the maximum strength of the beam-beam interaction, the so called beam-beam tune-shift, and speculative values have been offered ranging from as low as 0.0005 to as large as a fraction of unit. The lower limit could be caused in a more complicated situation were the external focussing forces which keep the two beams in the same storage ring are also modulated in time. These theoretical predictions have been compared with extensive computer tracking where the motion of the particles are followed turn after turn over very long periods of time. Though it is indeed possible to observe the formation of several resonances, nevertheless the onset of connected stochasticity seems to occur at to large beam-beam tune-shift to be of any practical relevance. Moreover no Arnold diffusion has been observed to have any practical significance. Chaotic trajectories have been found to embed the phase space in disconnected regions of appreciable extension. They increase in numbers considerably when time modulation of external focussing forces is added.

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DEFINITION OF THE PROBLEM

A proton-antiproton collider is made of a single storage ring where the two beams move in opposite direction, thanks to their opposite charge, guided by the same magnetic field. The two beams are bunched and periodically they encounter with each other in one or more fixed locations where high energy physics experiments are done.

Where the two beam bunches cross with each other the particles receive a kick in both the horizontal and vertical directions perpendicular to the main direction of motion. The kick depends on each other intensity and particle distribution and on the location (x and y) of the particle in the transverse plane.

We will assume here that the bunches are so short that for any practical purpose the kick is also short and lumped and it can be approximated by a delta function. Moreover the usual case of interest is the so called weak-strong one where one beam (the proton) is very strong in intensity and the motion of its particles is not effected by crossing with the other beam (the antiproton) which is quite weaker in intensity. The particles in the weaker beam are nevertheless those that have their motion changed by the kicks when crossing with the other bunch.

We have been interested in the case of "round" beams where the dimensions of the two beams at the crossing point are about identical to each other and the same for both horizontal and vertical direction. We assume bigaussian distribution of particles and denote with σ the rms transverse width.

Between kicks the particles of the weak beam, which are those of which we want to investigate the motion, are transported by a sequence of magnets, typically made of dipoles for bending and quadrupoles for focussing and drifts. We will assume that in the transport between beam-beam kicks the motion of the particles is linear when expanded in terms of the deviations x and y from a reference closed orbit. We also assume that the particles have the same momentum which corresponds to the reference orbit.

One cycle (the n-th) is therefore made of two steps: (i) a transport between a crossing point and the next, (ii) a non-linear kick. The transport between two consecutive crossings is given by the transformation:

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{(n+1)} = \begin{bmatrix} \cos\mu_x & \beta^*\sin\mu_x & 0 & 0 \\ -(\sin\mu_x)/\beta^* & \cos\mu_x & 0 & 0 \\ 0 & 0 & \cos\mu_y & \beta^*\sin\mu_y \\ 0 & 0 & -(\sin\mu_y)/\beta^* & \cos\mu_y \end{bmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{n} \quad (1)$$

where x' and y' are angles, that is the local variation of the particle displacements with respect to the longitudinal coordinate (s), and are canonically conjugated to x and y.

$$\mu_x = 2\pi\nu_x \text{ and } \mu_y = 2\pi\nu_y \quad (2)$$

represent the phase advances of free (betatron) oscillations induced by the guiding and focussing magnet system with ν_x and ν_y the number of betatron oscillations per period, that is between two consecutive kicks. The other quantity β^* is the value of a lattice amplitude function at the crossing point. This configuration corresponds to the case where the crossing point is a location with a waist for what concerns the optical behavior of the beams. Finally we also assume that the same location is dispersionless that is the position of a particle does not depend on its momentum.

The nonlinear kick due to the beam-beam crossing is given by

$$\begin{aligned} \Delta x' &= -4\pi(\xi/\beta^*)\, F(x,y)x \\ \Delta y' &= -4\pi(\xi/\beta^*)\, F(x,y)y \end{aligned} \quad (3)$$

which gives the variations of x' and y' with $\Delta x = \Delta y = 0$. ξ is the strength parameter which describes the beam-beam interaction usually called as the beam-beam tune-shift, and for the assumed bigaussian distribution

$$F(x,y) = \frac{1 - \exp(-\frac{x^2 + y^2}{2\sigma^2})}{\frac{x^2 + y^2}{2\sigma^2}} \tag{4}$$

where σ, as defined before, is the rms dimension of the strong beam cross-section. The strength parameter ξ depends of course on the intensity of the strong beam. In a proton-antiproton collider typically $\xi = 0.005$, whereas in an electron-positron collider ξ is as large as 0.05. Usually these numbers are limitations on the charge densities that can be obtained for the strong beams (protons and electrons), rather than by a real beam-beam effect, though of course this, which is the subject of investigation of this paper, causes also observable deterioration of the beam behavior.

It is easily seen that the system described so far does not really depend on β^* and σ, because they can be easily eliminated by redefining for instance

$$u = x/\sigma \quad , \quad u' = \beta^* x'/\sigma \tag{5}$$

and similarly v and v' for y, y'. Therefore our problem has only three parameters, ν_x, ν_y and ξ. Another set of variables of course are the particle initial conditions

$$(x_0, x_0', y_0, y_0') \tag{6}$$

which can be given also in the scaled form (5).

THEORETICAL PREDICTIONS

The model we have outlined above is an example of nonlinear mapping. To be observed that all the nonlinearity is concentrated in the beam-beam kick. If the interaction parameter ξ goes to zero than the mapping is exactly linear as one can see from the relation (1).

Several theoretical predictions have been made for this model. First it has bee- pointed out that single isolated resonances will show up within the strong beam distribution. If these resonances get large enough they could overlap[1] causing the onset of stochasticity where the trajectory of a particle can take a chaotic behavior. Estimates have been provided in the past for the onset value of the strength parameter ξ which range from as low[2] as 0.05 to as large[3,4] as 0.25. Secondly, the model we have described is a time dependent system with two degrees of freedom and it has been suggested that such a system could be subject to Arnold diffusion[5], a slow process by which particles trajectories can move to large amplitudes. This effect does not have a threshold but the diffusion rate depends on the beam-beam tune-shift. It has been estimated [6] that for the model we have considered a tune-shift of 0.005 can cause significant growth in the beam dimension to warrant about its stability in actual existing colliders.

Finally a more significant effect is the one generated in the case the two betatron frequencies ν_x and ν_y are modulated in time by some external driving mechanism like power supply ripple or energy oscillations coupled to the chromaticity of the collider, that is the dependence of the betatron frequencies with the particle momentum. The addition of the frequency of modulation increases the number of the beam-beam induced resonances and could cause either a more chaotic behavior of the particle trajectories or a growth of their amplitude beyond an acceptable limit.

One can test these theoretical predictions with intensive simulation of the particle motion on a computer as we have done, by repetitively applying the mapping made of (1), (3) and (4). We have found that indeed major resonances appear within the beam cross section and the particle motion is affected by their presence. Eventually, stochastic behavior of the particle trajectories will show up but at a too large beam-beam tune-shift to be of any serious concern in actual and realistic colliders. Our search for the Arnold diffusion has been negative. Actually in some cases, we have also been capable to prove a theorem by which our system is integrable at least once.[7] This would reduce the degree of freedom of our system to only one and in this case it is known that Arnold diffusion does not exist. Finally we have indeed verified that as one deviates form the simplicity of our system, for instance by simply adding externally driven modulation of the betatron frequencies, the system can become quite unstable. This is the most worrisome aspect of our problem. In order to guarantee a good performance of the collider one has to pay attention to both the perturbations which are accidentally added to the system (noise) and the capability to tune away the working point (betatron tunes) to region which are less sensitive to the perturbations.

CHAOTIC TRAJECTORIES

I am always frustrated when I go to workshops or conferences on non-linear analysis. The pure theorists always show me a picture of trajectories like the one shown in Fig. 1. This is the result of the standard mapping as obtained by J. Greene.[8] The equations of the standard mapping are the following

$$r_{n+1} = r_n - \frac{K}{2\pi}\sin(2\pi\ \theta_n)$$
$$\theta_{n+1} = \theta_n + r_{n+1} \qquad (7)$$

The trajectories in Fig. 1 were obtained with K = 0.971635, and one of them has a very clear and peculiar behavior. It is a "chaotic trajectory" the presence of which is proof that one is past the onset of stochasticity.

Yet when I try to get similar pictures for the one-dimensional beam-beam mapping I fail. The beam-beam mapping we have outlined above can be easily reduced to one degree of freedom to give

$$x_{n+1} = Cx_n + Sy_n$$
$$y_{n+1} = -Sx_n + Cy_n - F(x_{n+1}) \tag{8}$$

where

$$C = \cos\mu,\ S = \sin\mu$$

with $\mu = 2\pi\nu$ and $F_n(x_{n+1})$ is the interaction form.

In Fig. 2 and 3 we show some of our results of the beam-beam mapping when

$$F_n(x_{n+1}) = g\,\frac{1 - \exp(-x_{n+1}^2/2)}{x_{n+1}} \tag{9}$$

and $g = 4\pi\xi$.

Clearly, in these examples, all the trajectories behave quite regularly. They show some typical "resonance islands", but none of them has a "chaotic" look like one in Fig. 1. Then, I ask, where is stochasticity for the beam-beam case?

Again the pure theorist would answer that I should be patient and keep tracking on my computer code a little longer.

D. Edwards,[9] at Fermilab, decided to look for an answer with his own means. He investigated both mappings (7) and (8) on the computer. He actually got the picture for the standard mapping very easily and consistent with Fig. 1. In particular, he shifted the origin to investigate the motion of a particle in proximity of the unstable fixed point and got one "chaotic trajectory" like the one shown in Fig. 4 very quickly, in only 670 iterations. He succeeded in observing "chaotic trajectories" also with the beam-beam mapping using the interaction form given by Eq. (9). Some of his results are shown in Fig. 5, but as one can see, he had to take extremely large values of the coupling parameter $g = 4\pi\xi$. For instance a $\xi = 0.08$ was not large enough to exhibit anything peculiar. The formation of a "chaotic trajectory" is very obvious for $\xi = 0.4$. Again, these pictures can be obtained very quickly with no more than one thousand iterations. The conclusion is that it is very difficult to find trajectories that look "chaotic" for the beam-beam case and small ξ parameters ($\lesssim .1$).

We like to stress a major difference between the standard mapping and the beam-beam mapping. In Eq. (7) the strength parameter is given by K, and if this constant is taken to be identically zero one reduces to an integrable system extremely non-linear. It is because of this basic non-linearity, I believe, that "chaotic trajectories" appear so easily as soon as a little amount of non-linear perturbation is added. On the contrary, the beam-beam model is governed by the strength parameter $g = 4\pi\xi$. If $g = 0$ then one reduces to a mapping that is extremely linear. At this purpose, observe that the variables (r,θ) can be thought of as the action-angle correspondents of the rectangular coordinates (x,y) in Eq. (8). It is basically more difficult to observe "chaotic trajectories" in a linear system to which some amount (small) of non-linearities has been added. We suggest that a modified version of the standard mapping is also investigated. This can be written as follows

$$r_{n+1} = r_n - \frac{K}{2\pi}\sin(2\pi\theta u)$$

$$\theta_{n+1} = \theta_n + K\, r_{n+1} \tag{10}$$

where the strength parameter K has also been added to the second equation. In this model when K goes to zero one reduces to a linear system as in the case of the beam-beam model.

Yet, there are many chaotic trajectories also in the pictures of Figs. 2 and 3. Only now these trajectories are more difficult to observe and eventually can be detected with other techniques than just tracking a mapping on a computer.

During our analysis we have discovered "chaotic trajectories" by pure chance during our search for the beam-beam effect in a way we are going to explain later. Before though, we should give a more specific definition of chaotic trajectories. This is done with reference to Fig. 6. We take two particles (A and B) very close to each other in the common phase space and close to a trajectory (ellipse) that for one reason or another we know to exist. After a long period of time we look at the new position of A and B. There are three possibilities: (i) A and B are still close to each other and to the reference orbit; in this case we say the trajectories are "regular". (ii) A and B are far away from each other and also moved to different amplitudes. In this case the two trajectories are "chaotic". (iii) It is also possible though, that the two particles are quite far apart from each other but they are still close to that initial reference orbit. Their trajectories, of course, are also "chaotic", though their divergence from each other is only in phase and not in amplitude. They are clearly confined to a very thin stochastic layer. It is the latter type of chaotic trajectories that exist in the beam-beam mapping (for moderate strength parameters) and are more difficult to be observed. It is this type of "chaotic trajectories" that eventually we had the chance to observe in our computer simulations.

SEARCH OF THE ARNOLD DIFFUSION

During our search for the Arnold Diffusion [10,11] with the beam-beam effect in two dimensions we have analyzed in quite some detail three cases which are shown in Fig. 7. The three cases are

A: $\nu_x = 0.245$ $\quad \nu_y = 0.245$
B: $\nu_x = 0.245$ $\quad \nu_y = 0.120$
C: $\nu_x = 0.3439$ $\quad \nu_y = 0.1772$

where we have disregarded the integer part of the betatron tunes since it does not enter our analysis. The beam-beam simulations were done with a tune shift parameter $\xi = 0.01$, one crossing per revolution, circular geometry and for weak-strong interaction. Tracking was done on the computer adopting the mapping given by (1), (3) and (4). We have followed the motion turn after turn typically of 100 particles at the time, the initial conditions (6) of which were selected randomly following gaussian distributions with rms widths that matched the strong beam distribution. Our goal was not only to observe the motion of individual particles but also to derive the dimension of the test (weak) beam in all directions by a straightforth measurement at the computer. The beam dimension was then plotted versus time to observe if any growth should have occurred. At this purpose we have developed a statistical method, very similar to those used by expeimentalists in the same conditions, to judge the significance of our results and to make positive conclusion well above statistical errors. We used the Fermilab CYBER system as computer and adopted double precision notation with 28 significant digits. We verified that the determinant of (1) is close to unit to 10^{-27} accuracy and we decided to calculate the non-linear kick (3) and (4) exactly for each particle and turn after turn with no approximation, truncation or interpolation. We have obviously some philosophical difficulties to justify the need for such accuracy when we know that even in the ideal case of no external perturbation added we have eventually to face the Heisenberg uncertainty principles... We do not believe particles would "know" their position and velocity so accurately as we were pretending during our computer simulations.

In case A and B we have simulated 40 minutes real time (120 million revolutions), for case C only 20 minutes. But with the statistical analysis mentioned above we have been capable to extrapolate the stability of the motion well beyond the actually simulated times, of which the reader should be capable to appreciate already the considerable length. No significant growth has been observed for case A and C with a double time for the beam emittance having a lower limit of approximately 100 days. In case B we have noticed some significant growth with a double time of approximately 10 days.

We have tried to explain our results. Case A corresponds to equal tunes $\nu_x = \nu_y$ and we have proven a theorem[7] by which indeed this case is integrable and the number of degrees of freedom can be reduced to only one. In this case Arnold diffusion does not exist and we believe this to be consistent with our results. Case C which has also shown a remarkable stability, sits in a region of the tune diagram which is free of any beam-beam induced resonance up to and including the 9-th order. To understand this we have to stress the significance of the betatron tunes ν_x and ν_y as they appear in the mapping notation of (1), (3) and (4). They would correspond precisely to those particles that have very large amplitude of oscillations, since in this case, to first order, there is a cancellation in average of the beam-beam kick (3) and (4) over long periods of time. But the particles with small amplitude oscillations have their actual tunes shifted both by ξ itself as one can see expanding the forms (3) and (4) and retaining only the linear terms. Thus ξ is also a measure of the spread of actual betatron tunes across the distribution of the test beam. We have verified that despite of this spread (0.01) there are no resonances of order lower than 10-th across the beam distribution for case C. This is not true for case B where actually resonances of order as low as fourth traverse the beam phase space. But even in this case diffusion rates are so small that we exclude they can have any significance to a realistic collider when more technical limitations are taken into account (vacuum, power supply noise, etc.).

REVERSIBILITY TEST AND CHAOTIC TRAJECTORIES

Because of the length of our simulations we decided, to subject some of our particles to a reversibility[12] (also said repeatability) test. After taking them forward for some 60 million turns recording their positions in the phase space every 200,000 turns or less, we reversed the tracking and compared the particle position before and after the reversal. A typical behavior of the error, i.e., of the accuracy lost, is shown in Fig. 8 for three particles. The error increases linearly at a rate as expected from cut-off error. Yet for case B we found one particle that behaved as shown in Fig. 9 with a loss of accuracy increasing exponentially. We call this a "chaotic trajectory" particle. The rate of accuracy loss is called the Lyapunov coefficient and it is a measure at which rate two particles, initially very close to each other, diverge from each other.

We decided to subject all the particles to the reversibility test. This test does not take long computer time because it is just enough to check whether the error grows linearly or exponentially. We decided to track particles 200,000 turn forward and 200,000 turns back. The results are given in Table 1. No chaotic trajectories have been found in cases A and C. "Chaotic trajectories" have been found only in case B. And this fact seems to be consistent with the rather marginal growth observed for this case. Our results say that about 25% of the phase space is

Table 1. Results of Reversibility Test

Case	No of particles tested	No of Particles that failed the reversibility test
A	100	none
B	500	127
C	100	none

embedded with chaotic trajectories. Indeed we believe that since particles have initial conditions taken randomly, the fraction of the particles that fail the reversibility test gives a measure of the fraction of the phase space that is "chaotic". In Fig. 10 we show the distribution of the initial conditions of all the chaotic trajectories for case B. In Fig. 11 we give the distribution of 100 particles chaotic or not for comparison. In the background of these figures we have shown typical trajectories indicating the presence of a 4-th order resonance. The chaotic trajectories are those that seem traverse the regions in the proximity of unstable fixed points. When we have repeated the tracking for case B but excluding those particles that have shown a chaotic behavior, we found that also this case gave a doubling time for the beam emittance in excess of 100 days similar to cases A and C.

Chirikov proposed a criterion[1] for estimating the density of stochastic phase space which is based on the stochastic parameter

$$s = \frac{\text{width of a non-linear resonance}}{\text{separation between resonances}}$$

To check this criterion we have made a systematic search[13] of "chaotic trajectories" in the tune diagram. Our results are shown in Figs. 12, 13, and 14. The numbers given at the intersections of resonances is the number of particles that failed the reversibility test out of 100. No chaotic trajectories have been found along the main diagonal $\nu_x = \nu_y$ and the upper left region behaves the same as the lower right one, obviously for symmetry reasons as one can see. Our result is independent of the beam-

beam strength parameter (the beam-beam tune shift) ξ. We have checked this for $\xi = 0.005$, 0.010 and 0.02. The explanation to this is that the picture of resonance islands in the phase space does not depend on ξ but only on the betatron tunes. We have also verified that the number of "chaotic trajectories" is larger at the intersection of major, low-order resonances. Somebody has suggested that the search for "chaotic trajectories" can also be done by observing how two particles initially close to each other move away from each other. Yet it is not clear how close the two particles ought to be at the start because this would require some knowledge of the thickness of the stochastic layers. The method we have established seems to be more effective. At the time the particle motion is reversed, its position is shifted by the smallest amount as possible compatible with the (double) precision of the computer. All the chaotic trajectories we have found so far did not show any considerable amplitude growth; their motion was always bounded but subject to a strong phase randomization.

We have searched for chaotic trajectories also for the one-dimensional mapping (8). We scanned resonances up to tenth order included. We failed to observe any chaotic trajectories for $\xi < 0.15$. Since we were taking 100 particles at the time in our simulation, this implies that less than 1% of the phase space is chaotic. For $\xi \sim 0.15$ chaotic trajectories suddenly appear at 10% level to increase to 30% for $\xi \sim 0.3$. Chaotic trajectories were observed mostly when major low-order resonances were included within the tune-spread.

MODULATION OF BETATRON TUNES

The number of chaotic trajectories increases considerably when tune modulations are added. Moreover in this case a new class of chaotic trajectories appears: these are chaotic trajectories with amplitude growth.

To study the effect of the betatron tune modulation we have considered case C as defined in the previous section and let the tunes vary[14] according to

$$\nu_x = 0.3439 + a \sin\Theta$$
$$\nu_y = 0.4772 + b \sin\Theta$$

where

$$\Theta = 2\pi \frac{n-1}{N}, \; n = 1,2,3.....$$

N an integer, describing the periodicity of the tune modulation, and

$a = 0.001$ to 0.01 in step of 0.001

$b = +a$ or $-a$

The beam-beam strength parameter was taken again to be $\xi = 0.01$. We followed 100 particles for 6 million turns (about two minutes of real time). We remind that no chaotic trajectories were observed for case C in absence of tune modulation. In Fig. 15 we show the tune diagram and three major, low-order resonances surrounding a square of 0.01 side where the 100 particles are located initially.

Summary of the results: $N = 1000$

i) In the $b = +a$ case we saw no significant changes in the rms beam emittance for $a < 0.009$. However, for $a = 0.01$ some statistically significant changes appear. There is a strong anti-correlation between horizontal and vertical emittances. There is a statistically significant rms emittance change of about 1% over 6 million turns, corresponding to a double time of 0.1 days.

ii) In the $b = -a$ simulations more dramatic changes occur. For $a < 0.003$ nothing significant happens, but for $a > 0.004$ there is a fast blow-up of the beam emittances, with doubling times of minutes rather than days. The blow-up is evident within 200,000 turns of particle motion and continues at slower rate throughout 6 million turns. The result of the doubling times are shown in Fig. 16 versus $\delta\nu = a$, the amplitude of the modulation.

We can explain these results by pointing out that only one major resonance of the 8-th order is within reach of the tune modulation for the b = +a case, whereas beam particles can cross two resonances of even lower order (3rd and 6th) for b = -a. Clearly this result is strongly tune dependent and it is reasonable to expect a better performance with a careful choice of the working point.

REVERSIBILITY TESTS WITH TUNE MODULATION

We have subjected all our particles to the reversibility test including the tune oscillations. The results are as follows.

i) In the case b = +a we have found chaotic trajectories only for a = 0.01 and there were only few of them (3-6%).

ii) For b = -a we did not observe chaotic trajectories for a < 0.003. The results for a > 0.003 are given in Table 2.

Table 2. Results of Reversibility Test with Tune Modulation

a b = -a	Number of Chaotic Trajectories
0.004	10 out of 100
0.005	39
0.01	36

We have found three groups of particles:

1. "Non-chaotic" (repeatable) trajectories which do not change their amplitude substantially in long-time simulations.
2. "Chaotic" trajectories which may undergo some change in mean amplitude but do not diverge to large amplitudes.
3. "Chaotic" trajectories which do diverge to large amplitudes.

We clearly have noticed a strong correlation between the appearance of the last kind of particles and beam emittance growth. Regular particles are those that do not cross the low-order resonances as shown in Fig. 17; they correspond to small betatron amplitude. Chaotic trajectories appear when particles cross at least one the major low-order resonances. Chaotic trajectories that grow to larger amplitudes already start with large values, that is, those located at the tail of the beam distribution. In Fig. 17 we show the distribution of the 100 particles averaged over the first tune oscillation in our simulation. The two straight lines are major resonances that the beam has a chance to cross. The small amplitude particles are located in the upper right corner, and the betatron tunes increase moving along the diagonal downward. The same distribution is shown in Fig. 18 but 3 million turns later. As one can see the large amplitude, "chaotic" particles have been pushed to even larger amplitude in a region where they do not cross any further the two major resonances shown.

We have also varied N, the number of revolutions per tune oscillation period, the results are given in the following Table 3.

Table 3. Results of Repeatability Test with Betatron Tune Modulation ($\pm$ case, $\xi = 0.01$, $a = 0.005$)

N	No. of Chaotic Trajectories out of 100	No. of Trajectories that diverge to large amplitude out of 100
8	0	0
16	0	0
32	10	0
64	7	0
100	5	0
200	13	3
400	28	18
600	29	-
800	40	27
1000	41	-
2000	50	24
4000	46	22
10000	42	18
100000	39	7

These results are also shown in Fig. 19 where also the growth rates of the beam emittance are plotted versus N. Chaotic trajectories appear for N > 16 and their number increases until they saturate around the 50% level. At the same time the onset for fast beam growth is around N = 100. This growth seems to peak at N = 10,000 and decreases after that. We believe that the maximum growth rate corresponds to a frequency modulation which equals the precession frequency of the particle trajectories captured by the major resonance in range. This is supported by the fact that also the trajectories that increase to large amplitude, and therefore cause beam emittance growth, appear for N > 100 and their number peaks around N = 1000 to decrease again for longer periods of modulation. Nevertheless we should also point out that all our simulation runs for this case included only and always 200,000 turns and therefore the number of modulation cycles varied accordingly. In Fig. 20 we plot the Lyapunov coefficient, that is the rate at which two chaotic trajectories, initially close to each other, diverge from each other. As one can see the coefficient is actually constant up to N = 4000 and then drops rather considerably.

It seems to us that the phenomenon we have discovered during our tracking is consistent with the observations in the $Sp\bar{p}S$ collider where indeed a beam loss in the weak beam (the antiprotons) has been observed as due to crossing of major resonances due to magnet imperfections and to a tune spread in the beam induced by the beam-beam interactions.[15]

CONCLUSION

The beam-beam interaction in a proton-antiproton collider is a very important issue for accelerator physicists considering the difficulties involved in collecting precious particles like antiprotons. In the past we have been involved with tow different kinds of problems that, because of the nonlinear nature of the beam-beam interaction, can lead to a reduction of the collider performance.

In the first group we have very basic effects like the presence of resonances, the onset of stochasticity and Arnold diffusion, that mathematicians have discovered for other nonlinear systems. They are very intrinsic instabilities associated to our system defined by (1), (3) and (4) where no other ingredients need to be added. We believe now that, though these effects can indeed exist, they are not of any serious concern to the performance of the collider. The time and the strength involved before these effects are noticeable are of too large size and other technical limitations are to be considered first. The picture of single resonances is certainly correct but we like to dismiss the importance of stochasticity and Arnold diffusion.

The second group includes those effects to which an accelerator physicist should really pay more attention. These effects cover the realistic range of performance of a collider when engineering details like power supply noise, residual gas in the vacuum chamber, phase oscillations coupled to lattice behavior could cause a more reasonably expected limitations. We have covered one of such details where the betatron tunes are modulated at some frequency by external causes. The hope is that once one of these effects has been found with simple computer experiments like tracking we have shown in this paper, one can turn to an analytical explanation of the effect to replace the need for long computer searches. For instance a complete theory of a single resonance crossing is, in our opinion, at this date still outstanding.

We agree with the mathematicians on the existence of chaotic trajectories. We hope the reader appreciates the methods we have invented to discover them like the reversibility test. We are planning to include these methods in other tracking codes (PATRIS) which deal with nonlinearities from magnet imperfections. But we do not share the concern of the mathematicians that the presence of chaotic trajectories leads necessarily to instabilities. We have learned that we deal often with different concepts of instability. Ours, for a system to be unstable, has to cause amplitude growth. For the system we are interested in, for most of the time, chaotic trajectories have only a phase randomization. But it is important to scan the phase space and the tune diagram to measure the local density of their presence; in fact, as we have seen, the addition of external perturbations can cause an increase of their number and can make them grow to large amplitude, that is unstable.

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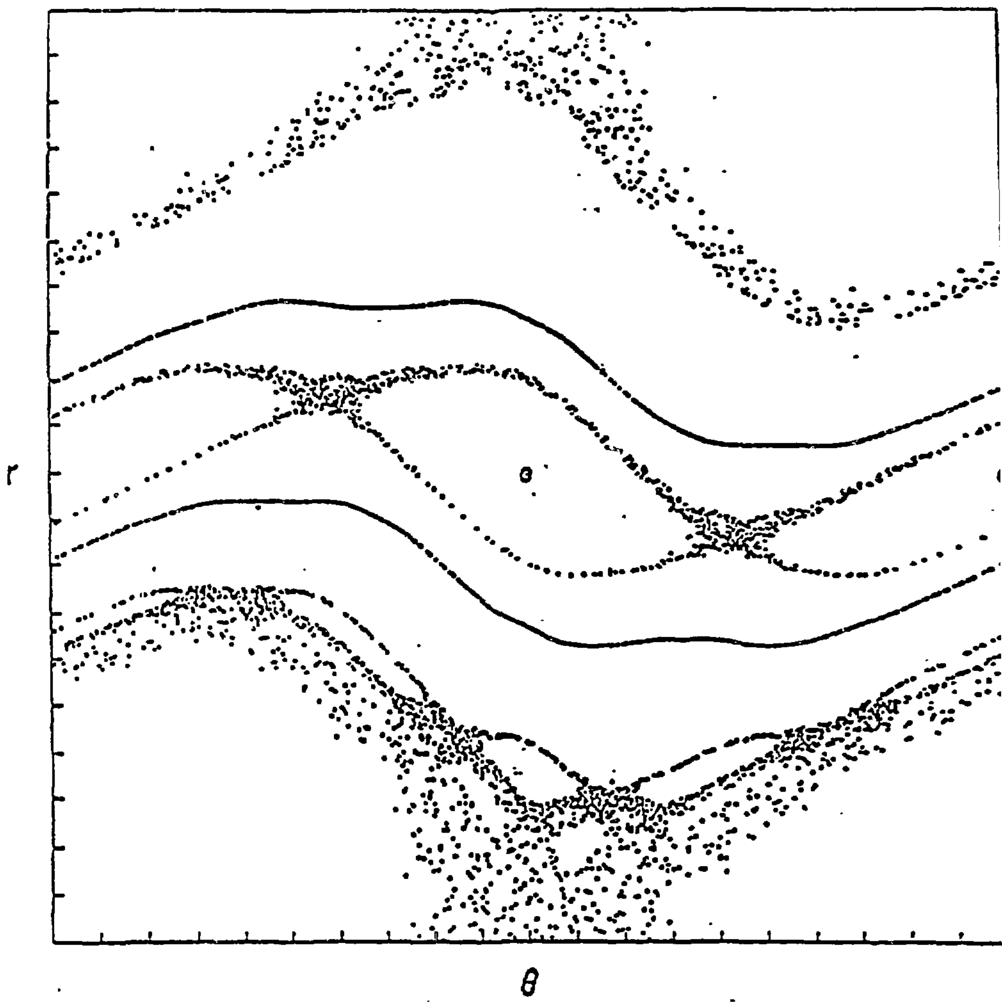


FIGURE 1 Standard Mapping for K = 0.971635.
(J. Greene)

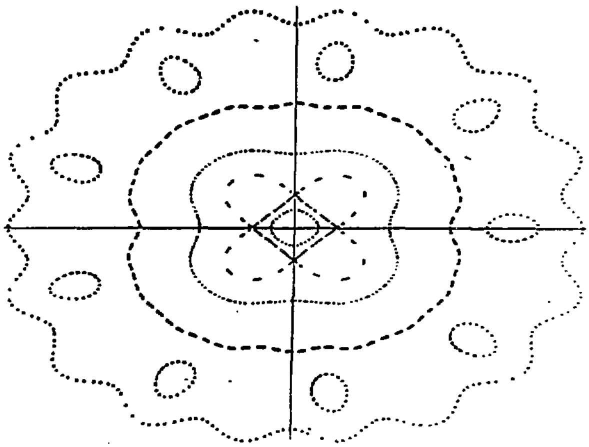


FIGURE 2 Beam-Beam Mapping: $\nu = 0.22$, $\xi = 0.08$.

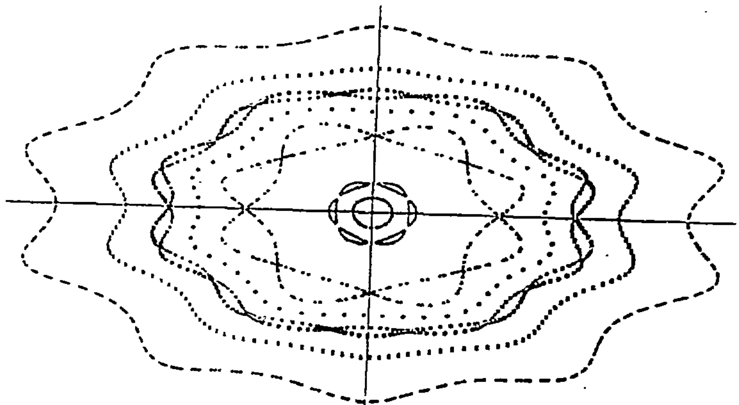


FIGURE 3 Beam-Beam Mapping: $\nu = 0.10$, $\xi = 0.10$.

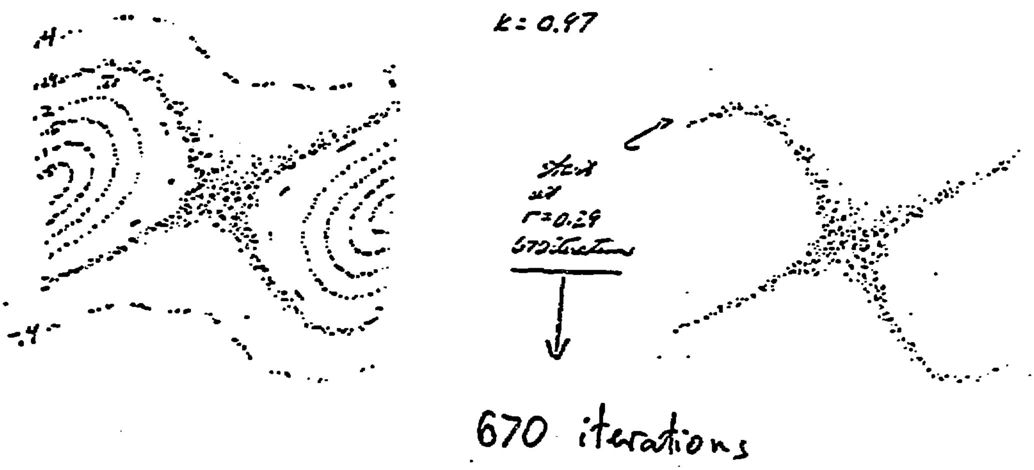


FIGURE 4 Standard Mapping for K = 0.97. (D. Edwards)

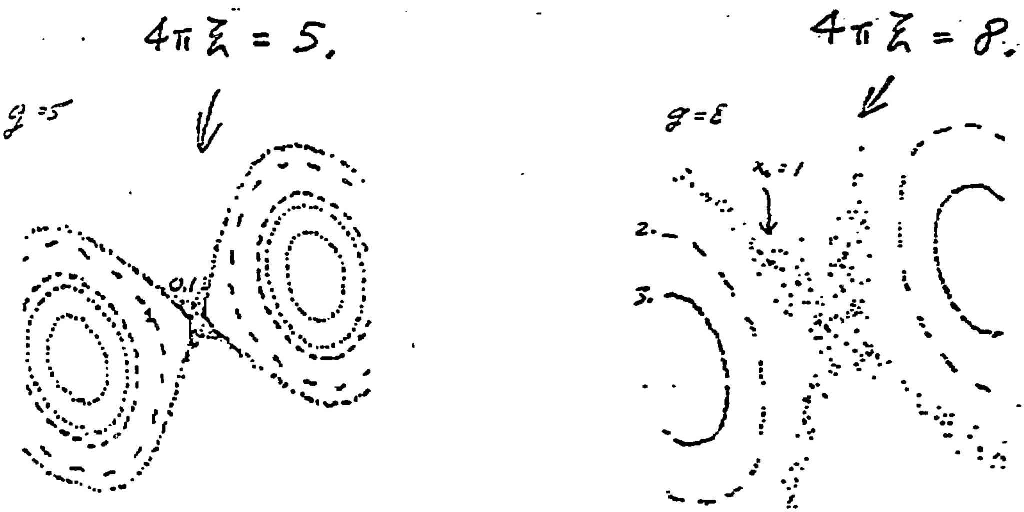


FIGURE 5 Beam-Beam Mapping: g = 5. and 8. (D. Edwards)

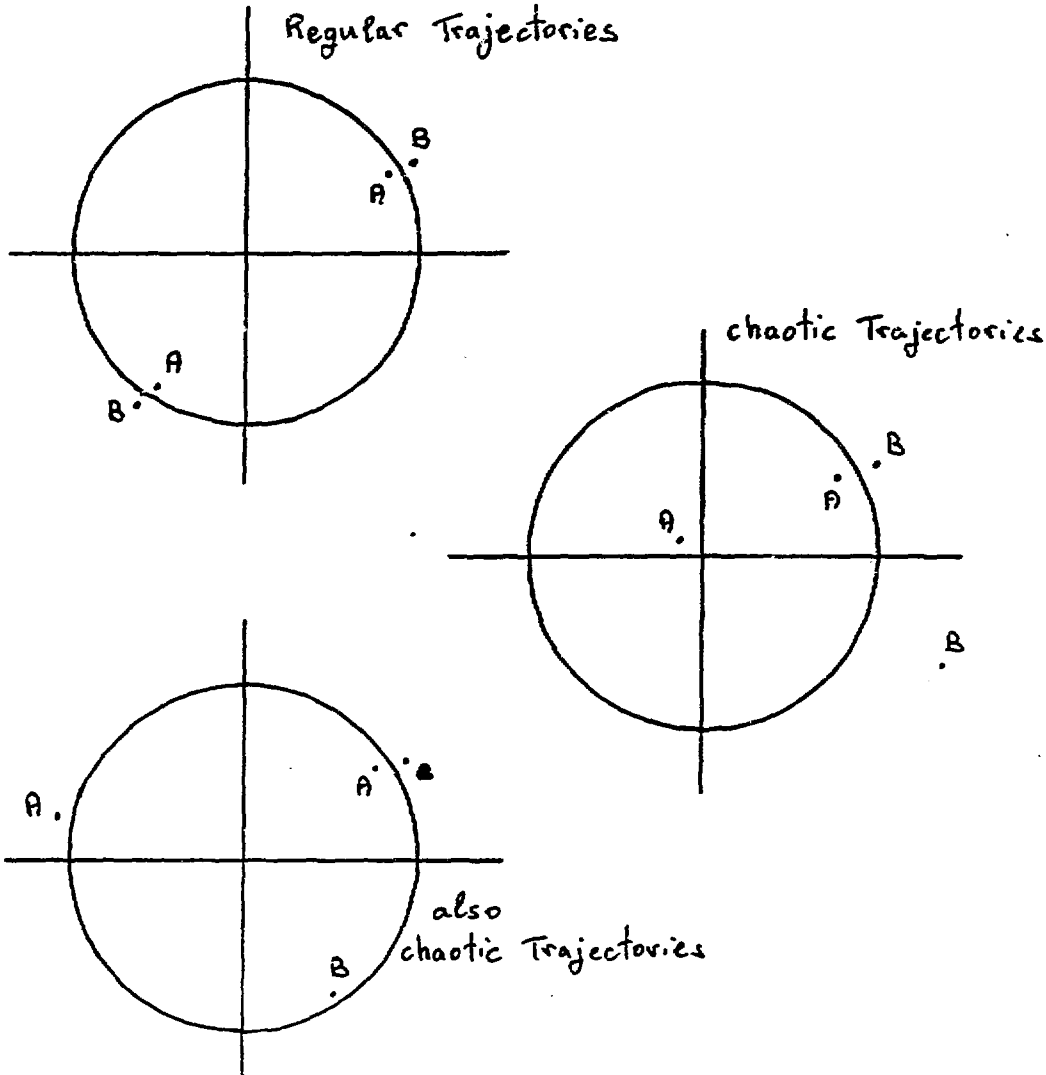


FIGURE 6 Behavior of regular and chaotic trajectories.

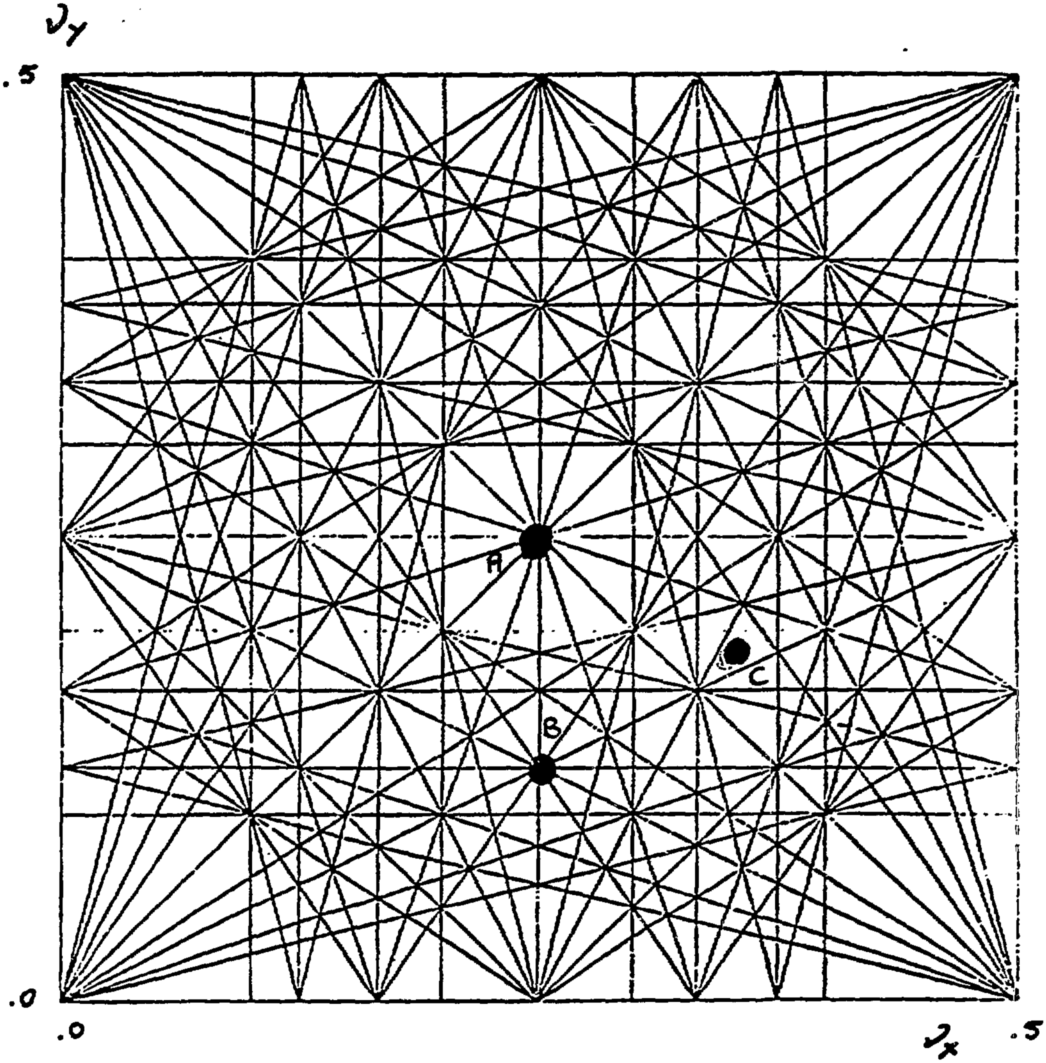


FIGURE 7 The three cases explored for the search of Arnold Diffusion.

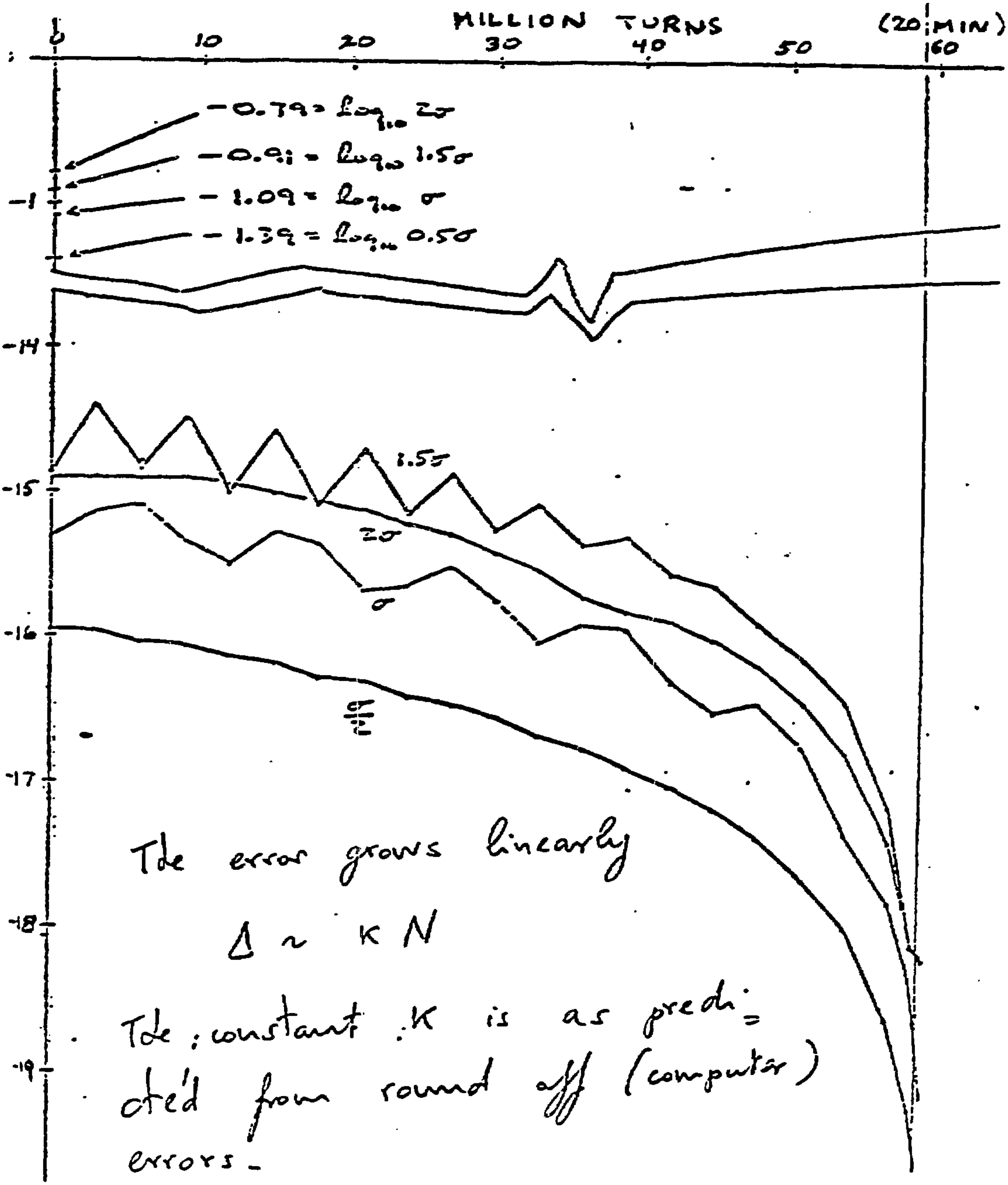


FIGURE 8 Error propagation during reversibility test. The error grows linearly.

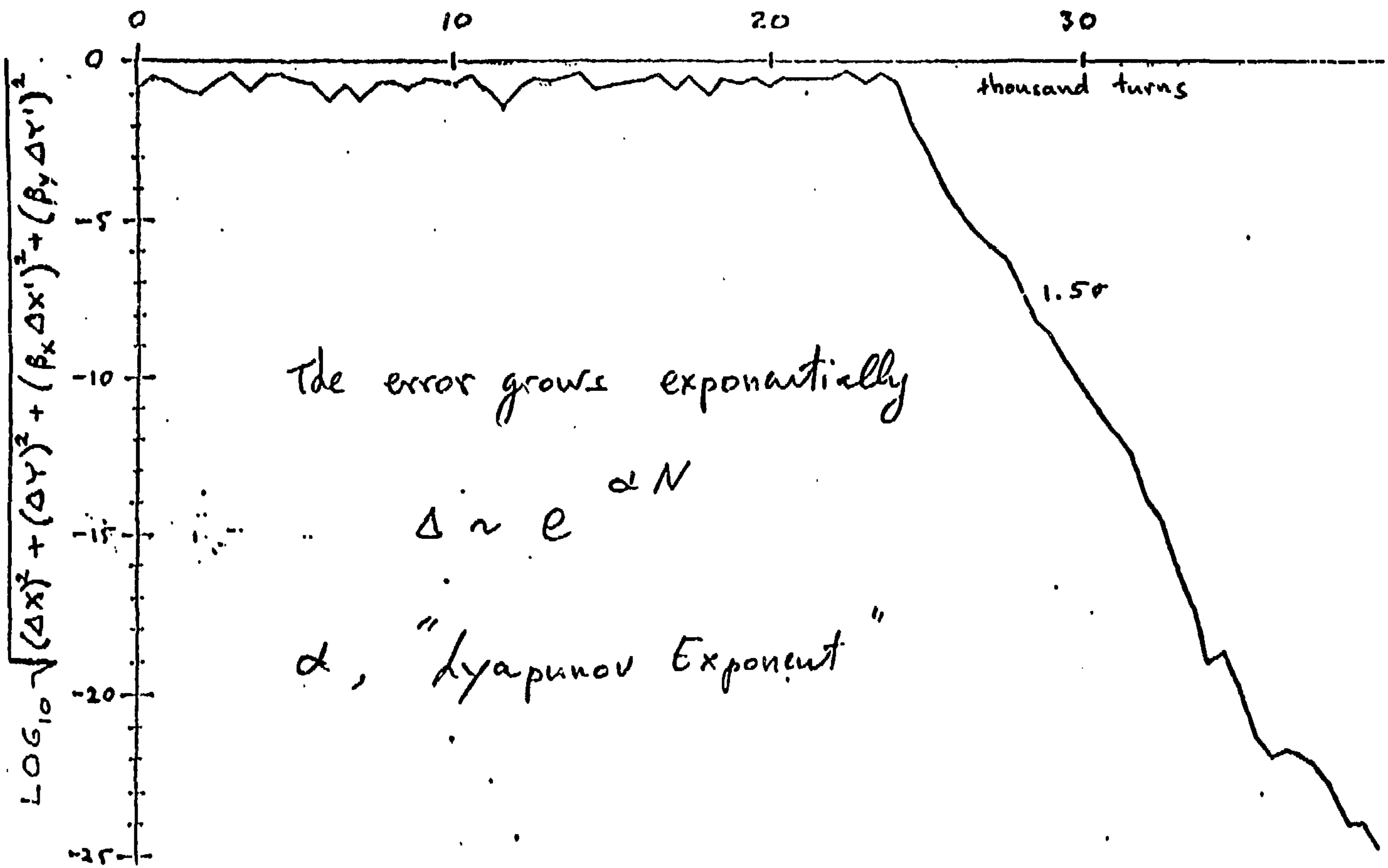


FIGURE 9 Error Propagation during reversibility test. The error grows exponentially.

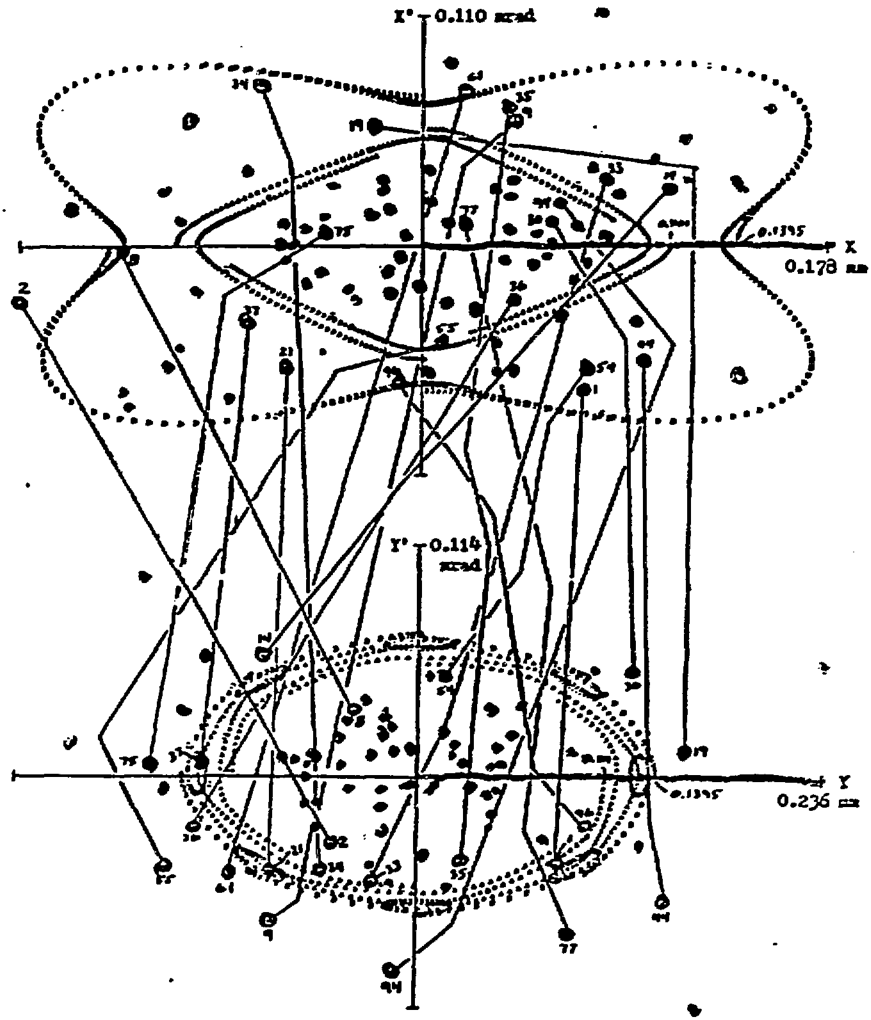


FIGURE 10 Initial coordinates of the 100 particles used in Case B. Twenty-one particles lose all accuracy in a few hundred thousand turns.

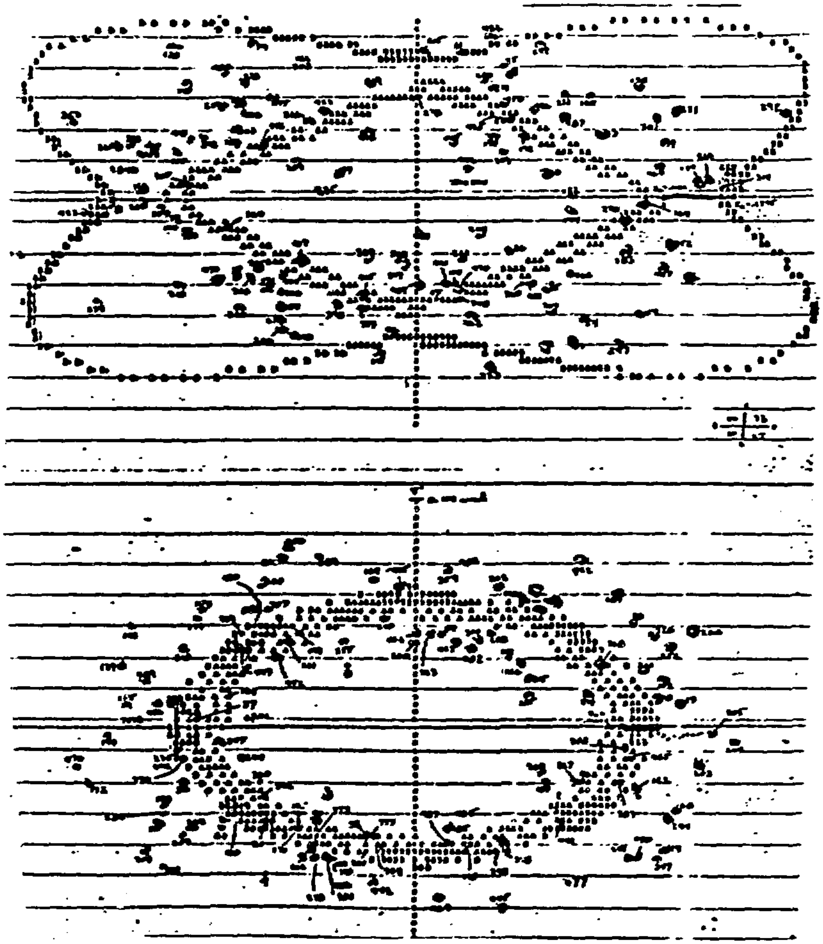


FIGURE 11 Initial locations of the 127 out of 500 particles that failed the reversibility test for Case B.

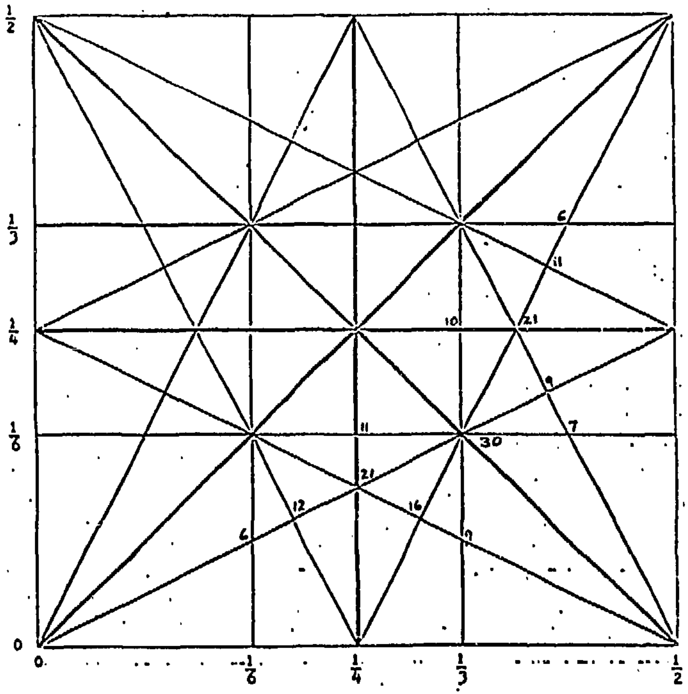


FIGURE 12 Number of chaotic trajectories out of 100 at 13 intersections between resonances of order 4 and 6.

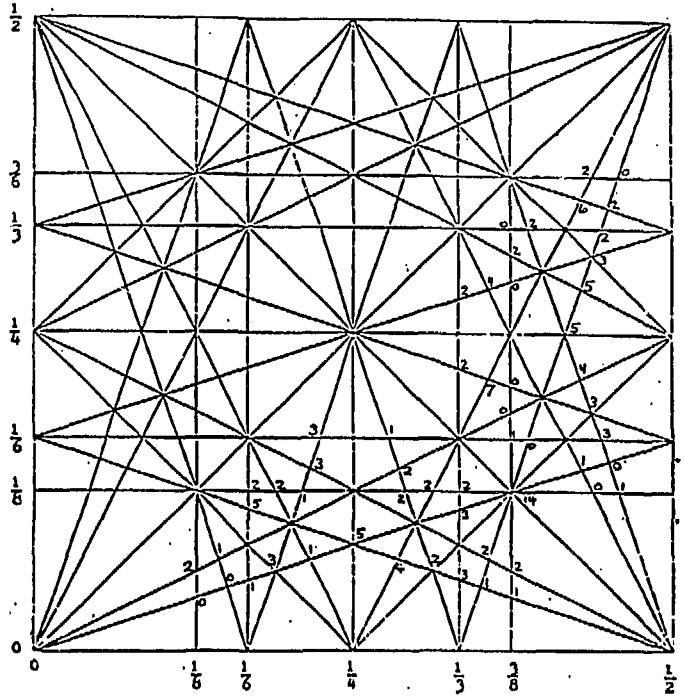


FIGURE 13 Number of chaotic trajectories out of 100 at 55 intersections between resonances of order 4, 6 and 8.

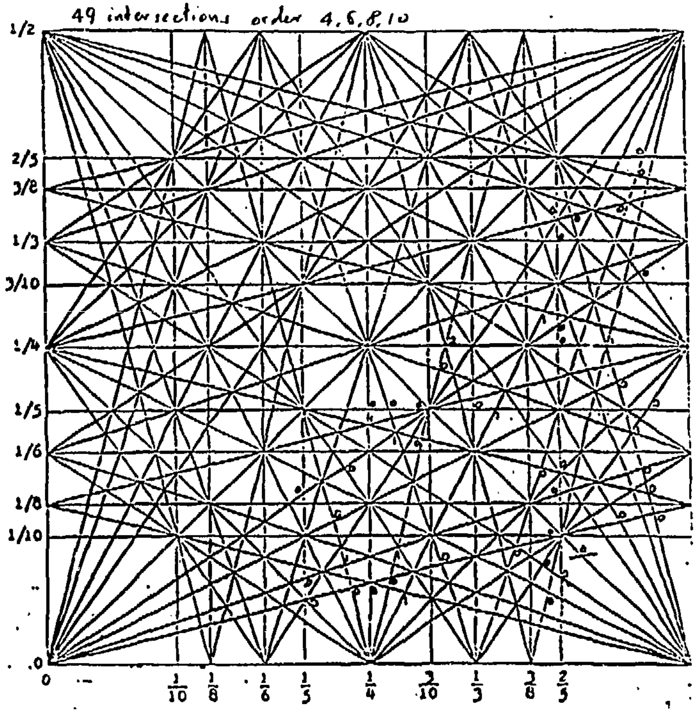


FIGURE 14 Number of chaotic trajectories out of 100 at 49 intersections between resonances of order 4, 6, 8 and 10.

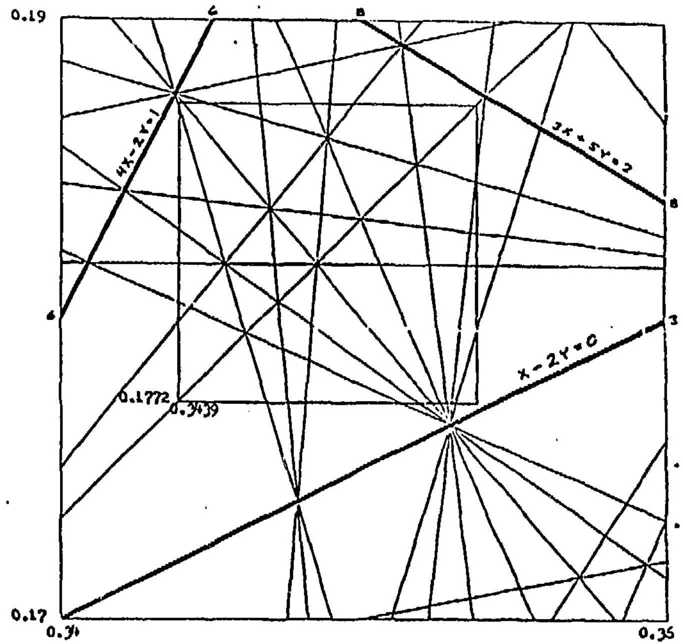


FIGURE 15 Tune space showing the limits of Case C and how close the corners come to the lives of order 3, 6 and 8.

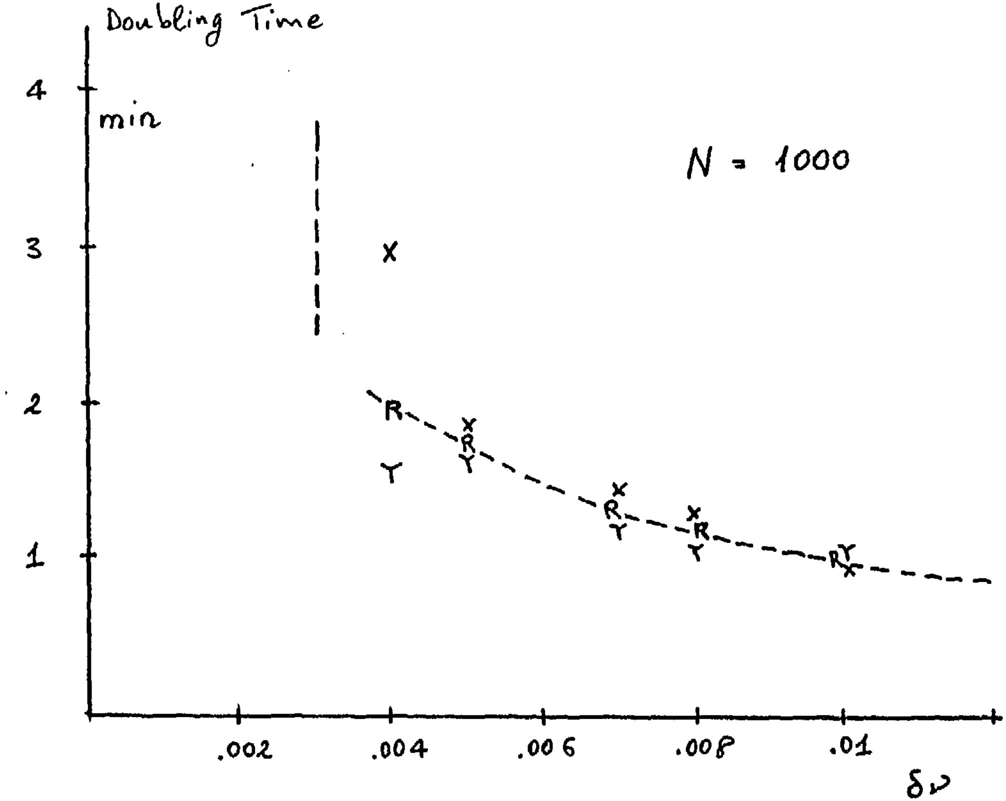


FIGURE 16   Emittance doubling time versus amplitude of tune modulation for Case C.

Figure 17 Tunes averaged over turns 0 to 800. [illegible] = 800.

$\mu_x/2\pi = 0.3539 + 0.005 \sin\beta$ $\beta = 2\pi N/800$, N = Turn number.

$\mu_y/2\pi = 0.1872 - 0.005 \sin\beta$ $\omega = CY = 2\pi\Delta\nu$, $\Delta\nu = 0.01$

The lower-left corner corresponds with $\mu_x/2\pi - \Delta\nu$, $\mu_y/2\pi - \Delta\nu$.

The upper-right corner corresponds with $\mu_x/2\pi$, $\mu_y/2\pi$.

Labels: A Reversible and small emittance after 3 million turns.

C Chaotic and small emittance after 3 million turns.

F Chaotic and large emittance after 3 million turns.

N Reversible and large emittance after 3 million turns.

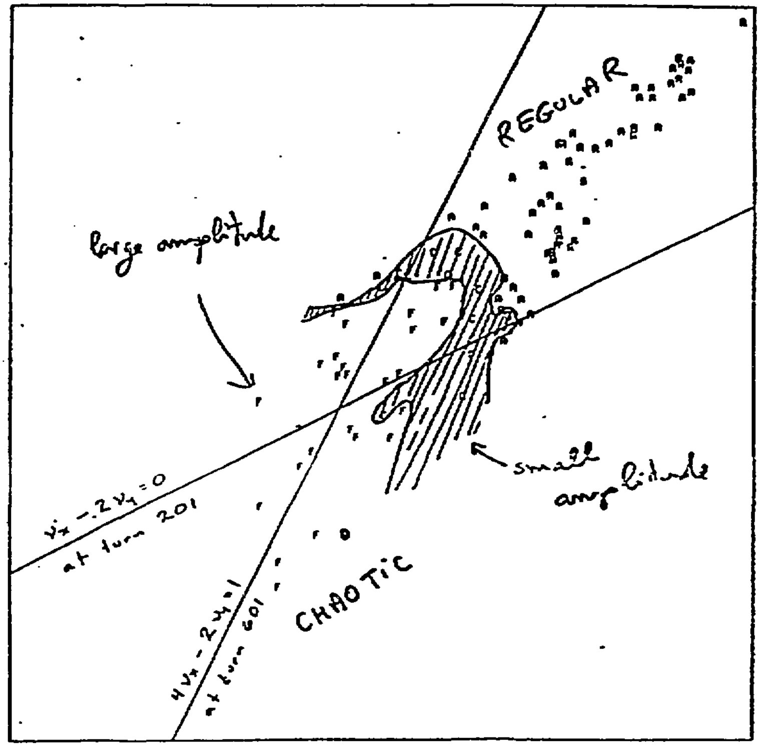


Figure 18 Tunes averaged over turns 3,000,000 to 3,000,800. NCYCLE = 800.

$\mu_x/2\pi = 0.3539 + 0.005 \sin\theta$ $\theta = 2\pi N/200$, N = Turn number.

$\mu_y/2\pi = 0.1872 - 0.005 \sin\theta$ CX = CY = $2\pi\Delta\nu$, $\Delta\nu = 0.01$

The lower-left corner corresponds with $\mu_x/2\pi - \Delta\nu$, $\mu_y/2\pi - \Delta\nu$.

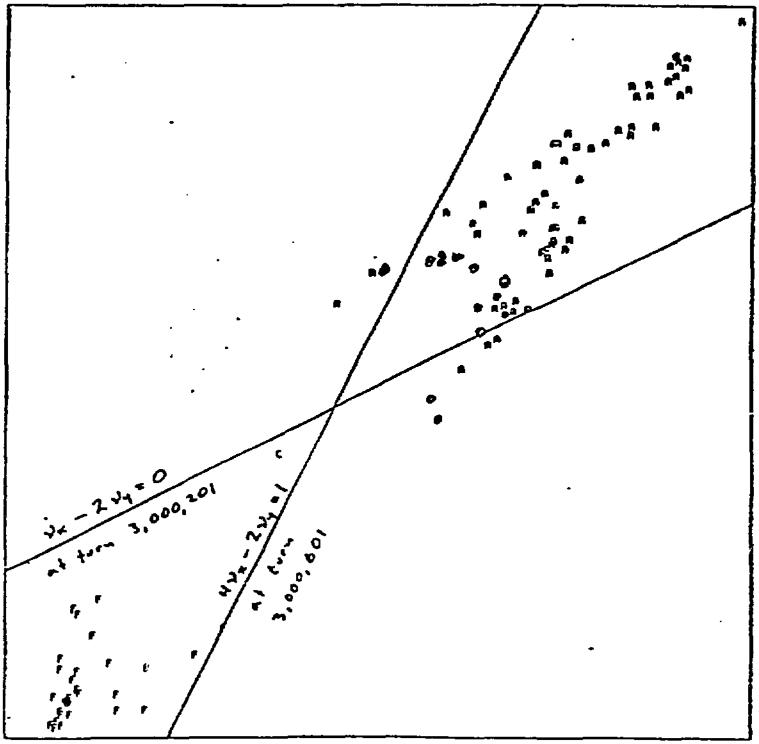
The upper-right corner corresponds with $\mu_x/2\pi$, $\mu_y/2\pi$.

Labels: A Reversible and small emittance after 3 million turns.

C Chaotic and small emittance after 3 million turns.

F Chaotic and large emittance after 3 million turns.

N Reversible and large emittance after 3 million turns.



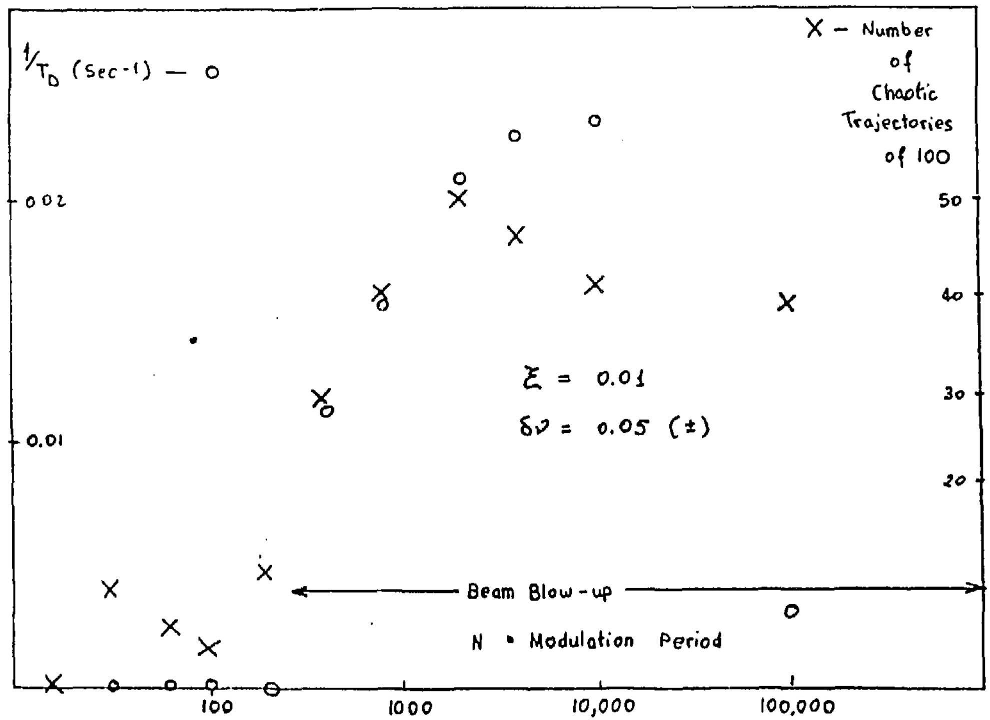


FIGURE 19 Number of chaotic trajectories and doubling rate versus modulation period.

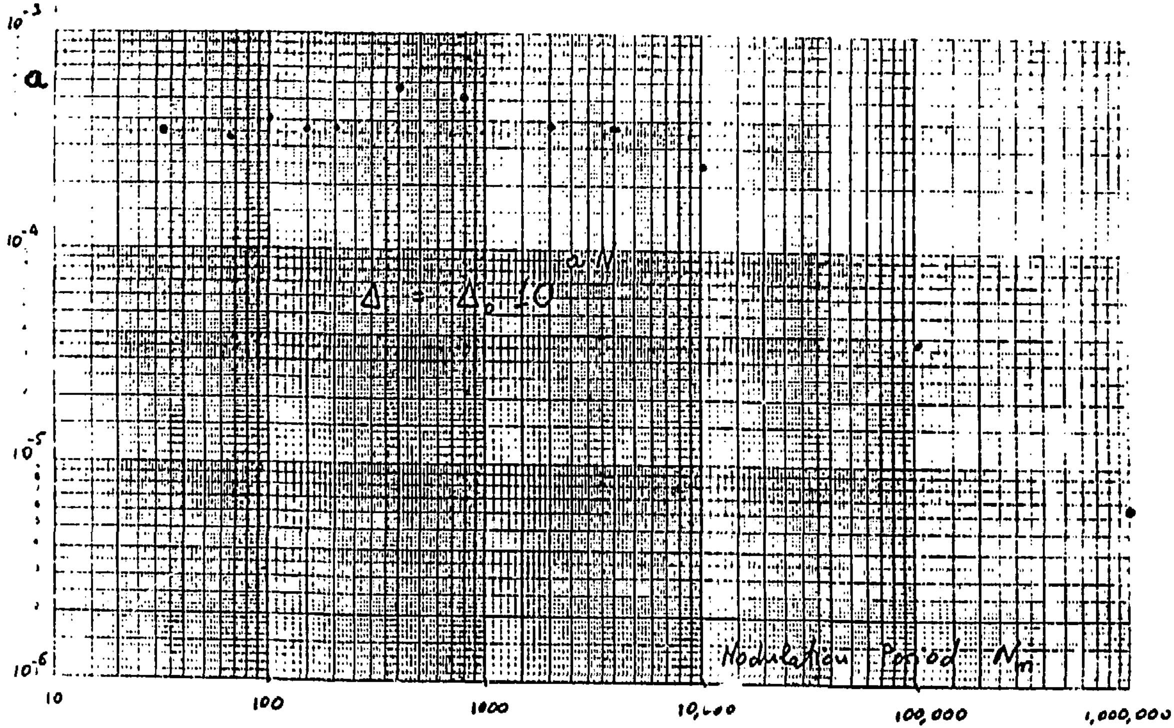


FIGURE 20 Lyapunov exponent as function of the modulation period.

DISCLAIMER