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RANDOM ERRORS IN THE MAGNETIC FIELD COEFFICIENTS OF SUPERCONDUCTING MAGNETS*

J. Herrera, R. Hogue, A. Prodell, P. Wanderer, and E. Willen
Brookhaven National Laboratory
Upton, N.Y. 11973

Introduction

Random errors in the multipole magnetic coefficients of superconducting magnet have been of continuing interest in accelerator research.[1-5] The Superconducing Super Collider (SSC) with its small magnetic aperture only emphasizes this aspect of magnet design, construction, and measurement. With this in mind, we present a magnet model which mirrors the structure of a typical superconducting magnet. By taking advantage of the basic symmetries of a dipole magnet, we use this model to fit the measured multipole rms widths. The fit parameters allow us then to predict the values of the rms multipole errors expected for the SSC dipole reference design D, SSC-C5. With the aid of first-order perturbation theory, we then give an estimate of the effect of these random errors on the emittance growth of a proton beam stored in an SSC.

The Magnet Model

The fractional field coefficients,[6] normal (b_n') and skew (a_n'), that are produced in the central aperture by a specified current distribution, Fig. 1, are given in SI units by

$$b'_{n-1} - ia'_{n-1} = \frac{-R_o^{n-1}\mu_o}{B_o 2\pi} \int \frac{dI}{\rho^n}\left[1 + \frac{\mu-1}{\mu+1}\frac{\rho^{2n}}{R_f^{2n}}\right] e^{in\phi} \tag{1}$$

$$n = 1, 2, \ldots$$

where μ_o is the permeability of free space while μ is the relative permeability of the surrounding cylindrical iron boundary. B_o is the dipole reference field and R_o the reference radius. When we apply Eq. (1)

holds for the variation in the skew coefficients ($\delta a'_{n-1}$) with the appropriate partial derivatives Y. Assuming initially that the variations in $\delta\phi_o$, $\delta\rho_o$, $\delta\omega$, $\delta\Delta$ for the N_B current blocks making up the magnet are uncorrelated, we find that the total rms variation in the normal fractional field coefficients is

$$(\delta b'_{n-1})^2 = \sum_{\ell=1}^{N_B} \Big\{ \big(X_\ell(n,\phi_o)\big)^2(\delta\phi_o)_\ell^2 + \big(X_\ell(n,\rho_o)\big)^2(\delta\rho_o)_\ell^2 + \big(X_\ell(n,\omega)\big)^2(\delta\omega)_\ell^2 + \big(X_\ell(n,\Delta)\big)^2(\delta\Delta)_\ell^2 \Big\}. \tag{6}$$

A similar expression applies to the skew coefficients.

Random Coefficients and Magnet Symmetries

Superconducting dipole magnets are designed to have symmetry about the median plane (TB symmetry), as well as left-right symmetry (LR symmetry) about the normal plane passing through the center of the iron aperture. Such an arrangement allows the presence of only the even normal coefficients b_{2n}, and forbids the odd normal coefficients and all the skew coefficients. In the model we are considering, it is the unsymmetrical variation in the block variables which break these symmetries and permits the forbidden multipoles to appear. Consistent with this idea we arrive at a combination of partial derivatives ($\bar{X}$) which exhibits the basic four-fold symmetry of the blocks. Thus for the group of blocks numbered 1, 4, 5, 8 (see Fig. 3), we write

$$\bar{X}_1(n,\phi_o) = (1+\alpha)(1+\sigma)X_1(n,\phi_o) - (1+\alpha)(1-\sigma)X_4(n,\phi_o) \tag{7}$$

(b_n') and skew (a_n'), that are produced in the central aperture by a specified current distribution, Fig. 1, are given in SI units by

$$b_{n-1}' - ia_{n-1}' = \frac{-R_o^{n-1}\mu_o}{B_o 2\pi}\int \frac{dI}{\rho^n}\left[1 + \frac{\mu-1}{\mu+1}\frac{\rho^{2n}}{R_f^{2n}}\right] e^{in\phi} \tag{1}$$

$$n = 1, 2, \ldots$$

where μ_o is the permeability of free space while μ is the relative permeability of the surrounding cylindrical iron boundary. B_o is the dipole reference field and R_o the reference radius. When we apply Eq. (1) to a single keystone-shaped current block, Fig. 2, with N uniformly distributed conductors, we derive

$$b_{n-1}' = \frac{R_o^{n-1}\mu_o}{B_o\pi}\frac{NI}{4\rho_o\Delta}\cos n\phi_o \frac{\sin n\omega}{n\omega} M(n) \tag{2}$$

and

$$a_{n-1}' = \frac{-R_o^{n-1}\mu_o}{B_o\pi}\frac{NI}{4\rho_o\Delta}\sin n\phi_o \frac{\sin n\omega}{n\omega} M(n) . \tag{3}$$

Here M(n) is the integral

$$M(n) = \int_{\rho_o-\Delta}^{\rho_o+\Delta} \frac{d\rho}{\rho^{n-1}}\left[1 + \frac{\mu-1}{\mu+1}\frac{\rho^{2n}}{R_f^{2n}}\right]. \tag{4}$$

Since we are interested in the variations of the field coefficients with respect to small changes in the block variables ($\delta\phi_o$, $\delta\omega$, $\delta\rho_o$, $\delta\Delta$), we write

$$\delta b_{n-1}' = X(n,\phi_o)\delta\phi_o + X(n,\rho_o)\delta\rho_o + X(n,\omega)\delta\omega + X(n,\Delta)\delta\Delta \tag{5}$$

where the X's are the partial derivatives, such as, $\partial b_{n-1}'/\partial\phi_o = X(n,\phi_o)$. An expression similar to Eq. (5)

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$$\bar{X}_1(n,\phi_o) = (1+\alpha)(1+\sigma)X_1(n,\phi_o) - (1+\alpha)(1-\sigma)X_4(n,\phi_o) + (1-\alpha)(1-\sigma)X_5(n,\phi_o) - (1-\alpha)(1+\sigma)X_8(n,\phi_o) \tag{7}$$

and

$$\bar{X}_1(n,\omega) = (1+\alpha)(1+\sigma)X_1(n,\omega) + (1+\alpha)(1-\sigma)X_4(n,\omega) + (1-\alpha)(1-\sigma)X_5(n,\omega) + (1-\alpha)(1+\sigma)X_8(n,\omega) \tag{8}$$

The expressions for $\bar{X}_1(n,\rho_o)$ and $\bar{X}_1(n,\Delta)$ are like Eq. (8) with $\omega \to \rho_o$ and $\omega \to \Delta$ in turn. In these four equations, it is the parameter α, which, when different from zero, introduces a TB asymmetry. Similarly, the parameter σ introduces a LR asymmetry. Treating the block arrangement shown in Fig. 3 as consisting of four sets of four, we can write for the rms variation in the normal coefficients

$$(\delta b_{n-1}')^2 = \sum_{\ell=1,2,9,10}\left\{\left(\bar{X}_\ell(n,\phi_o)\right)^2(\delta\phi_o)_\ell^2 + \left(\bar{X}_\ell(n,\omega)\right)^2(\delta\omega)_\ell^2 + \left(\bar{X}_\ell(n,\rho_o)\right)^2(\delta\rho_o)_\ell^2 + \left(\bar{X}_\ell(n,\Delta)\right)^2(\delta\Delta)_\ell^2\right\}. \tag{9}$$

Fits to CBA and FNAL TEVATRON Data

We will employ our magnet model to fit the multipole rms widths derived from the magnetic measurement of the 10 CBA "field quality" dipoles.[7] As a first step, we assume a set of mechanical errors equivalent to 2 mils for the angular ($\delta\phi_o$) and radial ($\delta\rho_o$) positions of each block, and 1 mil for the two half-widths ($\delta\omega$) and ($\delta\Delta$). Since the azimuthal variations ($\delta\phi_o$) of the block positions have the most effect on the multipole widths, we vary them symmetrically in groups of four (maintaining the two asymmetry parameter α and σ equal to zero) according to

$$(\delta\phi_o)_\ell \rightarrow (\delta\phi_o)'_\ell = (1+\beta)(1+\eta)(\delta\phi_o)_\ell \qquad (10)$$

for the inner blocks, and according to

$$(\delta\phi_o)_\ell \rightarrow (\delta\phi_o)'_\ell = (1+\beta)(1-\eta)(\delta\phi_o)_\ell \qquad (11)$$

for the outer blocks. In this way we obtain the values of the symmetrical parameters, β and η, which give a least-square fit to the allowed normal multipole widths (δb_2, δb_4, and δb_6). We note that in contrast to the parameter β which treats all the blocks the same, the parameter η changes the inner block azimuthal variation relative to the outer block azimuthal variations. With the symmetric parameters fixed, we now vary the top-bottom asymmetry parameter (α) and obtain a least square fit to the odd skew multipoles (δa_1, δa_3, δa_5). The final step, increasing the left-right asymmetry parameter (σ) from zero, allows us to fit the remaining odd b's and even a's (δb_1, δb_3, δb_5, δa_2, δa_4, and δa_6).

Figures (4) and (5) show the overall fit to the CBA data,[7] and the corresponding symmetry parameters. In the interest of getting a better fit to the FNAL data,[8] we have omitted the quadrupole widths. The resultant fits are shown in Figs. (6) and (7).

Multipole Widths for the SSC Dipole

A design for a 4 cm collared dipole for the SSC has been developed by Fernow and Morgan.[9] Starting with the same linear errors that we previously assumed (2 mils for $\delta\phi_o$ and $\delta\rho_o$, 1 mil for $\delta\omega$ and $\delta\Delta$) and adopting the symmetry parameter of the CBA fit ($\beta = -0.4$, $\eta = -1.2$, $\alpha = 0.67$, and $\sigma = 0.39$), we can calculate the expected rms random multipole errors for the dipole magnet SSC-C5. These are presented in Table 1.

Random Errors and Emittance Growth

The effect of random multipole errors in the dipole magnets is to cause the betatron emittance to grow with time. The exact manner in which this growth takes place depends critically on the betatron [illegible] from the resonance values. However, for the lower order multipoles, it may be desirable to provide harmonic correction coils around the ring to cancel the random multipoles and thereby control the rate of growth of the beam emittance.

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10. Report of the Reference Designs Study Group on the Superconducting Super Collider, May 8, 1984.

Table I. Multipole Widths ($R_o = 1$ cm) for SSC-C5 Dipole (SSC Ref Design D)

Multipole Number	Normal Multipole Width	Skew Multipole Width
n	$\delta b_n'(10^{-4})$	$\delta a_n'(10^{-4})$
0	6.5	2.1
1	1.8	3.3

In the interest of getting a better fit to the FNAL data,[8] we have omitted the quadrupole widths. The resultant fits are shown in Figs. (6) and (7).

Multipole Widths for the SSC Dipole

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Random Errors and Emittance Growth

The effect of random multipole errors in the dipole magnets is to cause the betatron emittance to grow with time. The exact manner in which this growth takes place depends critically on the betatron tune of the machine, and the combined effect due to all the random errors must be studied by analysis and beam tracking studies. However, in order to put the random errors that we have predicted in proper perspective, we would like to give an estimate of the beam growth that occurs when the tune of the beam is exactly on a resonant value. Using first-order perturbation theory, one obtains, for the number of turns (T) required to double an initial betatron emittance of the beam, ϵ, the relation

$$\frac{1}{T} = \frac{\pi}{2^{n-2}} \frac{\beta_{AV}}{\sqrt{M_D}} \left(\frac{\epsilon\beta_{AV}}{\pi\gamma}\right)^{\frac{n-1}{2}} \frac{(\delta b_n')_{rms}}{R_o^n} \qquad (12)$$

where β_{AV} is the average beta function value, M_D the total number of dipoles having rms errors $\delta b_n'$ (at a reference radius R_o), and γ the relativistic gamma of the circulating protons. We have applied Eq. (12) to the SSC Reference Design A[10], that is, with $\epsilon = 6\pi 10^{-6}$ rad m (95% phase space), $\beta_{AV} = 146.5$ m, and $M_D = 3780$. In Table II we present these results for a beam stored at high energy (20 TeV) and low energy (1 TeV). We emphasize that the numbers of turns (rms) necessary to double the emittance represent lower bounds, since the SSC machine tune will be kept away

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9. R.C. Fernow and G.H. Morgan, BNL, SSC Tech Note 19, Oct. 8, 1984.

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Multipole Number n	Normal Multipole Width $\delta b_n'(10^{-4})$	Skew Multipole Width $\delta a_n'(10^{-4})$
0	6.5	2.1
1	1.8	3.3
2	2.6	0.65
3	0.57	0.73
4	0.68	0.15
5	0.06	0.26
6	0.08	0.05
7	0.02	0.04
8	0.02	0.008
9	0.004	0.009
10	0.003	0.002

Table II. Turns to Double Emittance

Multipole No.	Turns for 20 TeV*	Turns for 1 TeV*
1 (QUAD)	3.7	3.7
2 (SEX)	253	56.6
3 (OCT)	114000	5700
4	9.4×10^6	105000
5	10×10^9	2.6×10^7
6	8×10^{11}	4.3×10^8
7	3×10^{14}	3.8×10^{10}

*Turns/Day = 3×10^8

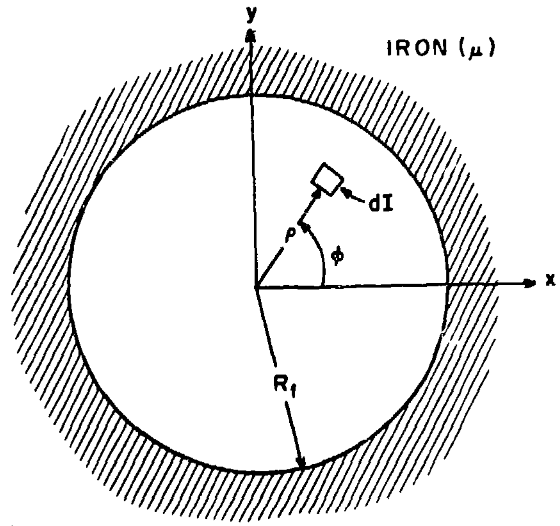
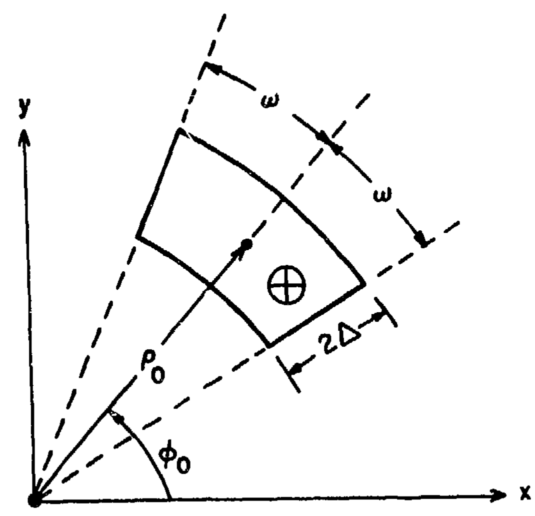
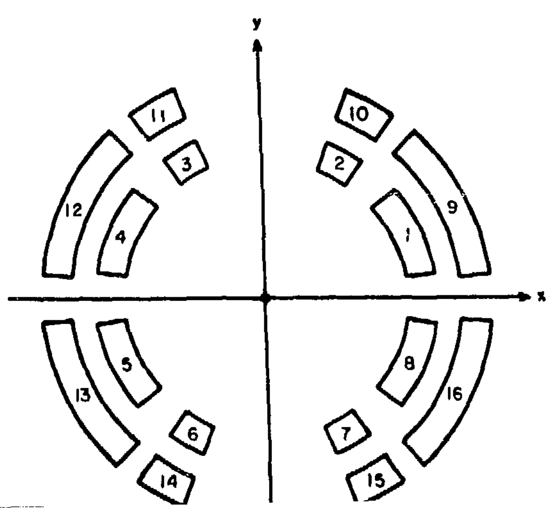


Figure 1. Coordinate system for a current element within a circular iron boundary.



Notation defining location and size of a current block.

Figure 2



Locations and numbering arrangement for the current blocks.

Figure 3

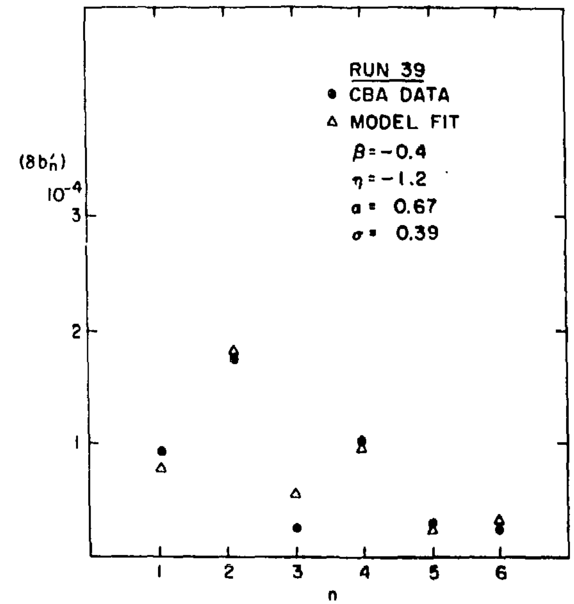


Figure 4. Comparison of the normal multipole widths ($\delta b_n'$) of the CBA data (solid dots) with the model fit (triangles).

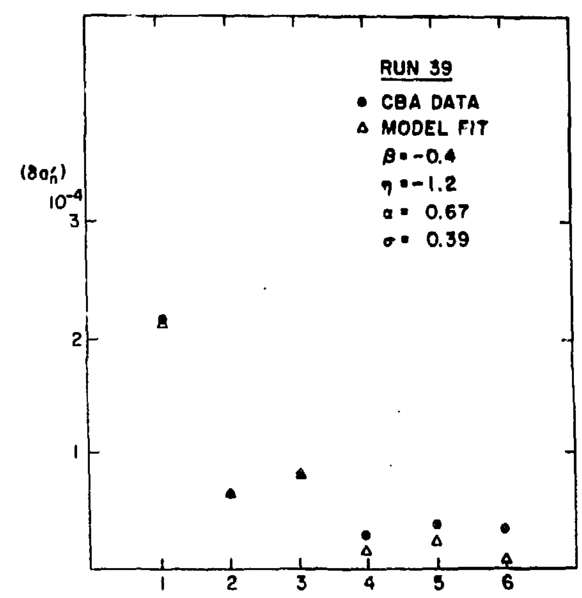


Figure 5. Comparison of the skew multipole width ($\delta a_n'$) of the CBA data (solid dots) with the model fit (triangles).

a current block.

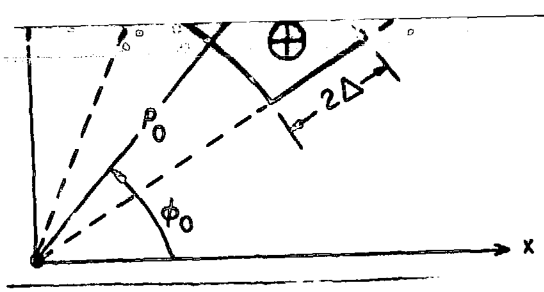
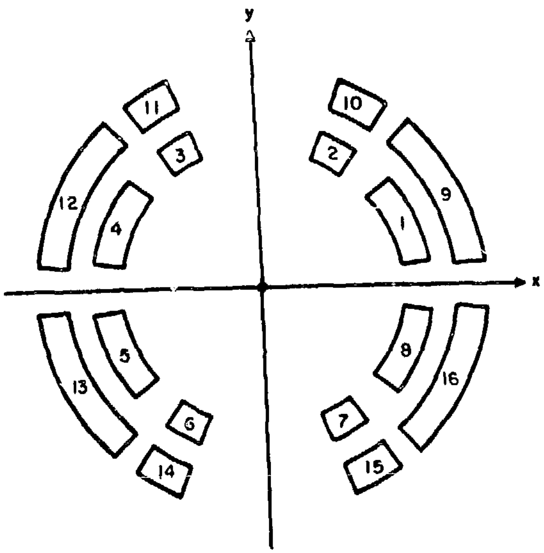
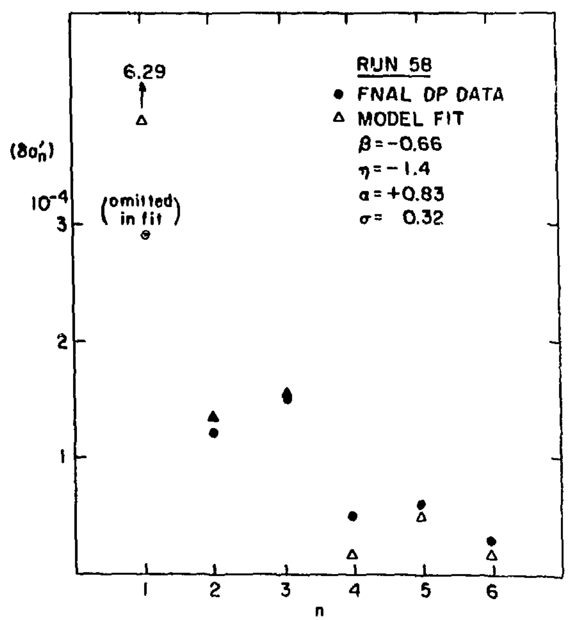


Figure 2



Locations and numbering arrangement for the current blocks.

Figure 3



Comparison of the skew multipoles ($\delta a_n'$) widths of the FNAL data (solid dots) with the model fit (triangles).

Figure 7

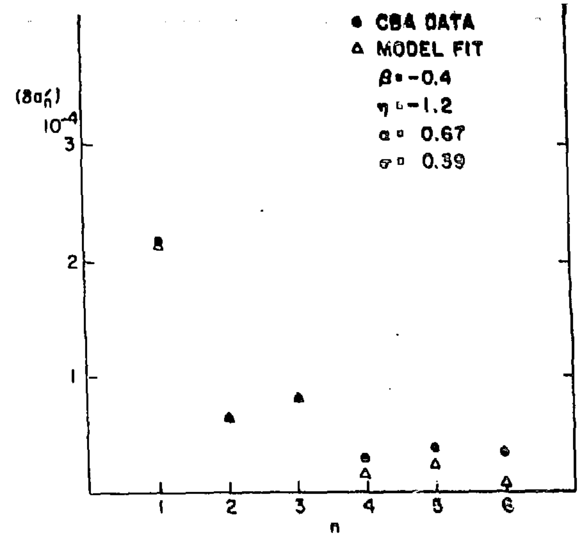


Figure 5. Comparison of the skew multipole width ($\delta a_n'$) of the CBA data (solid dots) with the model fit (triangles).

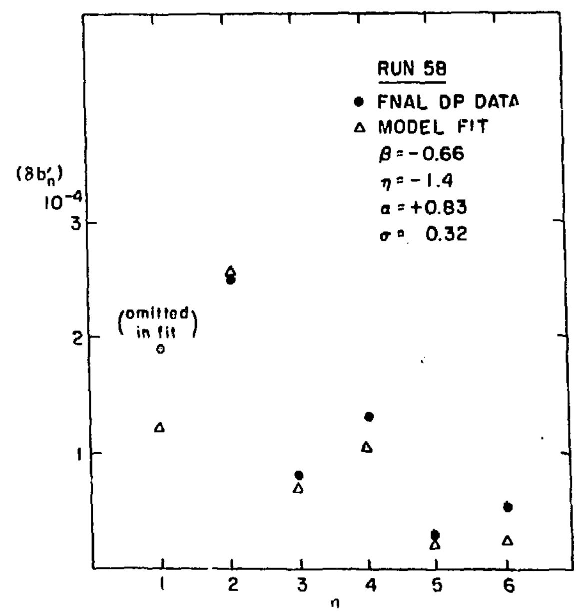


Figure 6. Comparison of the normal multipole widths ($\delta b_n'$) of the FNAL data (solid dots) with the model fit (triangles).

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