QUARKLEI: NUCLEAR PHYSICS FROM QCD

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ABSTRACT

The difficulties posed for nuclear physics by either recognizing or ignoring QCD, are discussed. A QCD model for nuclei is described. A crude approximation is shown to qualitatively reproduce saturation of nuclear binding energies and the EMC effect. The model is applied seriously to small nuclei, and to hypernuclei.

I. INTRODUCTION

Since 1973, nuclear physics, meaning the study of nuclei in terms of off-shell nucleons and possibly meson exchanges, has been intellectually untenable. Despite appearances, this may be a generous statement, since it could be argued that the critical time was 1963, when quarks were established, at least for some. I choose 1973 because the strong force, QCD, was theoretically determined at that time. Certainly 10 years later, the point should be understood.

Let me make the problem clear. For a moment, we will ignore the difficulty of understanding off-shell composite states of nucleons and mesons, despite the fact that no one knows how to do this. (Feynman is reputed to have asked: What the hell is an off-shell pion?) Start with conventional point nucleons in nuclear orbitals, and add form factors. But how can we add form factors without producing a physical extent in coordinate space? Having done that, we must address the question of how nucleons may pass through one another without affecting the internal constituents. It is not sufficient that the nucleons are in orthogonal eigenmodes, as their constituents are not, for the point nucleon Hamiltonian.

Aha! you may say. The nucleons do not pass through each other! The wounds in a Brueckner-Hartree-Fock (BHF) picture are similar to what appears in a hard sphere picture. But as they roil around, with their surfaces in close contact at nuclear densities, the question is still, how could these nucleons possibly avoid affecting each other's internal structure. When that structure was a cloud of off-shell pions, the effect was just what was involved to produce binding. But when that structure is a triplet of quarks, the situation looks different.

In fact, things are still very similar. Consider the analogy in chemistry. We do not describe molecular bonding in terms of photon exchanges between point atoms. Instead, despite the good approximations of electronegativity and electron affinities, we understand that the binding comes from a quantum mechanical sharing of electron wave functions. So it seems natural now to think of nuclear binding as due to a similar sharing of quark wavefunctions in a nucleus.

Thus at last, we come to the problem. It is not why we use point nucleons and mesons, plus form-factoring to calculate nuclear properties. The problem is why, in the light of the known composite structure of nucleons in terms of completely different degrees of freedom, does the conventional picture work so very well? I hope to convince you that the answer does lie in the known properties of QCD.

II. NUCLEAR PHYSICS AS AN ASPECT OF QCD

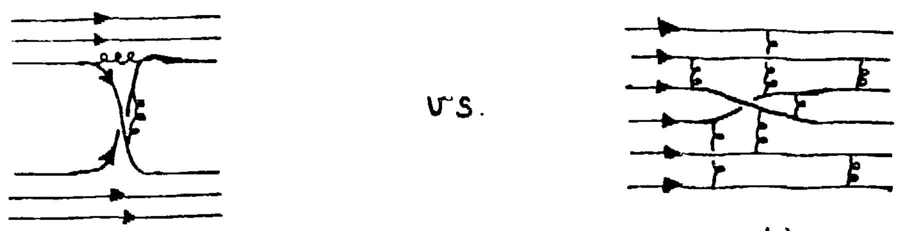
QCD is described by a path integral over quark and gluon fields. To obtain information, we introduce sources and take functional derivatives of the source-dependent path integral with respect to them, in order to obtain quark-quark correlation functions. If we take the 6A-th derivative with respect to 3A quark and 3A antiquark sources, we are studying the propagation of a system with baryon number A. This includes A nucleons propagating freely and independently, two nucleons scattering and A-2 moving freely, ... , and A nucleons scattering simultaneously. The resonance and bound state poles in each scattering subsystem describe nuclei and their excited states.

$$\mathbf{Z}[\eta,\bar{\eta}] = \int d[q]\,d[\bar{q}]\,d[G] \exp\left\{-iS[q,\bar{q},G] + \int d^4x\,(\bar{q}\eta + \bar{\eta}q)\right\}$$

When the nucleons are far apart, it is clear that their internal wavefunctions are those of isolated nucleons. When all 3A quarks are in a small region, on the order of the appropriate nuclear size, the gluons summed over in the path integral connect all of the quark lines. Further, no quark is required, even by confinement, to travel near a particular pair of quarks. Thus we cannot identify which quarks make up which nucleon. At this level, QCD offers no hope of describing a nucleus as any kind of a collection of nucleons. A clear molecular analog occurs in the benzene ring: Electrons are so delocalized that it makes no sense to think of even a fraction of an electron shifted between two atoms; they belong to the ring as a whole.

So how can we recover the conventional picture of nuclear physics as an approximation? Let us look at one boson exchange in the quark picture. The leading contribution comes from exchanging a pair of valence quarks between two nucleons. If we look in the t-channel (exchange channel) it is clear that the amplitude can be dominated by resonances between the quark and antiquark. (If we move in the direction of one of the quarks, the quark going the other way appears as an antiquark because, with all of this occuring on a space-like hypersurface, we are effectively moving in a "time-reversed" direction for it.) At large separations between the two nucleons, the dominant resonance will be a pion. At shorter distances, other bosons can contribute, but so can non-resonant processes. The numerical question is whether this breakdown occurs at internucleon separations corresponding to densities greater or less than nuclear matter densities.

Fig. 1 Six quark interaction at a) large, b) small distances



(a) (b)

Still, even qualitatively, we can begin to see why one-boson exchange models have a chance to work, and yet cannot be exact. For from the extra gluonic effects (non-resonant corrections) it is clear that the free space couplings of bosons to nucleons should not be the correct values to use in nuclei, and that they should vary with A. So the question becomes: Can we find a better approximation which shows where, and by how much, this one fails?

III. ELEMENTS OF THE QUARK MODEL OF NUCLEI

We believe that we know that if a finite number of quarks are randomly introduced at low density into a compact region, then confinement will require them to form color singlet triples, locally. Any model we introduce should include this property. To make the model tractable, we may indirectly impose this by means of an ansatz, as in the BCS theory of superconductivity, or as a lattice is introduced in solid state theories. Our guide is to incorporate the critical features of BHF nuclei, reinterpreted in quark terms. We take these critical elements to be that the average quark density saturates at a constant volume per quark, and that the quark density is both large in a fractional subvolume, and small in other regions.

We go beyond BHF in allowing a strong phase correlation between quark amplitudes in different regions of the same nucleus. BHF assumes that there are no such phase correlations between quarks in different nucleons. We can see that this is unreasonable on general quantum mechanical grounds. Consider two localized wave packets that are widely separated, representing the same color and flavor of quark in two nucleons. The sum and difference of these wave functions are the correct symmetry preserving combinations to construct, but this will be irrelevant for large separations as the eigenenergies are degenerate. However, when the two packets are near each other, nonlinear interactions in the overlap region will in general split the degeneracy. The properly symmetrized wavefunctions, complete with phase correlations across the region between the maxima of the envelopes, will then form the proper, nondegenerate eigenmodes of the system.

G.J. Stephenson, Jr. and I have attempted to incorporate these features by assuming that the QCD path integral is dominated by a mean field of gluonic excitation which produces a multi-welled color electric potential within which the single-particle quark wavefunctions develop.[1] We take these wells to be the QCD potential measured between quarks and antiquarks in high energy (charmonium and upsilonium) experiments. For convenience, we will often take this potential to be linear in the radial distance from the origin of the well, which is a good approximation to the high energy results.[2] There is one such well for every three quarks (one "nucleon"). Later, I will describe how we determine the relative location of the centers of the wells in a self-consistent manner. For now, just imagine that they form some fixed array.

Where the values of these linearly rising potentials are equal, at the equidistant points between centers, we truncate the potentials. Thus, the potential seen by a quark continues to rise only at the nuclear surface. In this, we are assuming that, as in the work function for metals, the difficulty of separating a quark from a nucleus is not significantly greater than that required to separate it from an individual nucleon. The result is an average potential, similar in spirit to the conventional nuclear average potential, but more complicated in structure due to the confining effects of QCD.

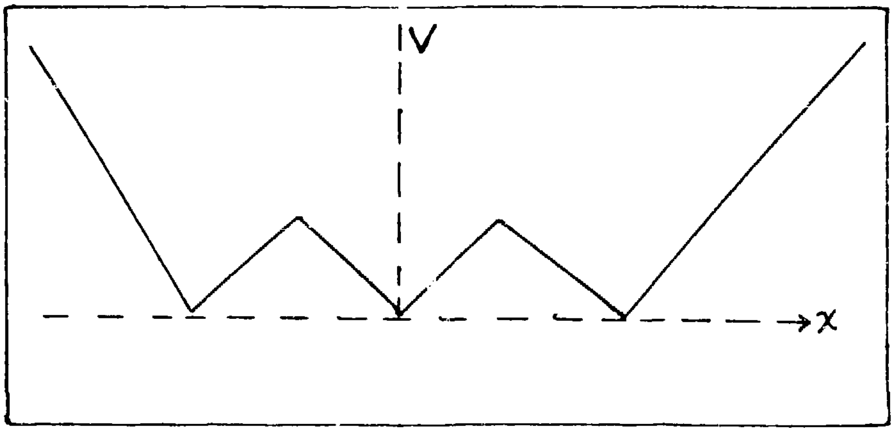


Fig. 2 Nuclear potential in coordinate space.

The question now arises: Does filling the quark eigenmodes of this potential, in a "quark shell model", produce anything that looks like real nuclei? A real calculation requires further that we project out states of good spin and parity. The full calculation is still in progress. In what follows, I shall first show you a crude calculation of the binding energy per nucleon in a kind of infinite nuclear matter which gives enough credence to the idea to warrant further effort. I will then indicate how this picture naturally produces the EMC effect[3], and indicate our progress on a realistic calculation of Helium-4. Finally, I will describe how hypernuclei can provide tests of the predictive power of our picture of nuclei.

IV. CUBICAL NUCLEONS AND CUBICAL NUCLEI

To make the problem simple, we start from the Dirac equation for massless quarks, with a scalar QCD potential. This eliminates Klein paradox problems. The next step is to assume that the Dirac wavefunction has the form of the conjugate Dirac operator acting on a spinor which has the lower pair of components identical, up to an overall sign possibly, to the upper pair of components. This was introduced some time ago by Feynman and Gell-Mann[4] for vector potentials. Here, it is not consistent unless the spin orbit term is negligible; i.e. - unless the gradient of the scalar potential dotted into the Pauli matrices annihilates the upper (or equivalently lower) two components. We chose to do this and ignore spin-orbit effects as the result is a Schroedinger-like equation with energy-squared eigenvalues. This is similar to what could be obtained by non-relativistic reduction, without the cost of making the incorrect assumption that the system is non-relativistic. I emphasize that this is done for convenience and simplicity, and that there is no problem in applying the exact Dirac equation to the actual problem.

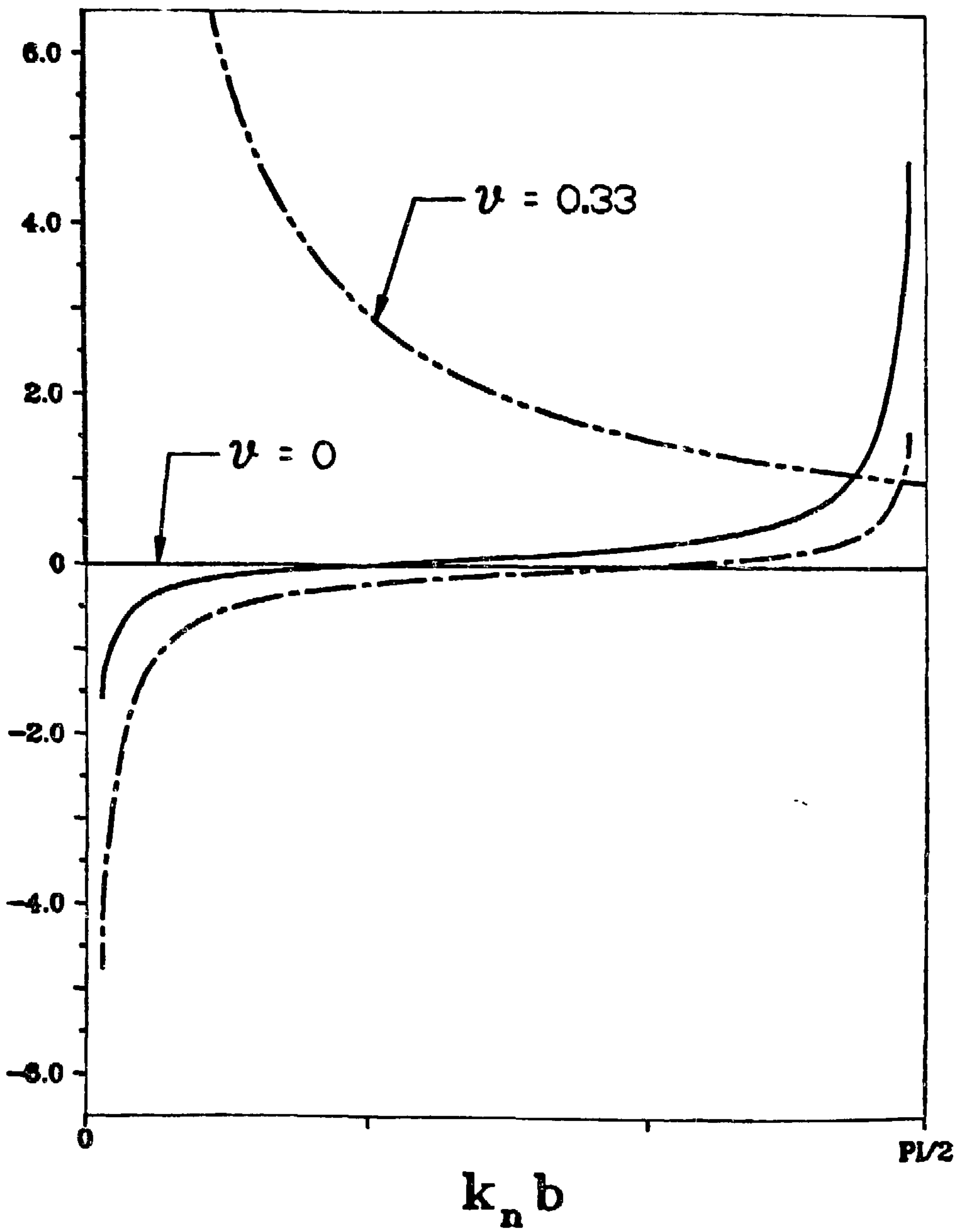
Since our point here is to simplify, we next replace the linear potential by a delta-function on the surfaces of an array of cubes. The strength of the delta-function is chosen to match the integrated strength of the linear potential across a cell. The strength is infinite on the outer surface of the cubical nucleus. The choice of cubical cells is of course made to produce a separation of variables in the Schroedinger-like wave equation. It is now a simple, one-dimensional problem in quantum mechanics to find the quark eigenmodes, by matching wavefunctions at the cell boundaries. We have checked that fluctuation of the potential strength over the face of a cube only lowers the resulting eigenenergies. Further, although our cube size (2b = 1.8 Fermi on a side) is fixed, it is clear that the eigenenergies will rise, due to increasing localization energy if we try to reduce the separation between them. This also occurs if we increase the separation, as the effective barrier strength increases; the linear potential must be integrated over a longer distance.

$$\frac{4\left[\cos^2\left(\frac{n\pi}{2N}\right)-\cos^2 k_n b\right]}{\sin 2k_n b} = \frac{v}{k_n}$$

[Eigenvalue equation for cubical ($A = N^3$) nuclei]

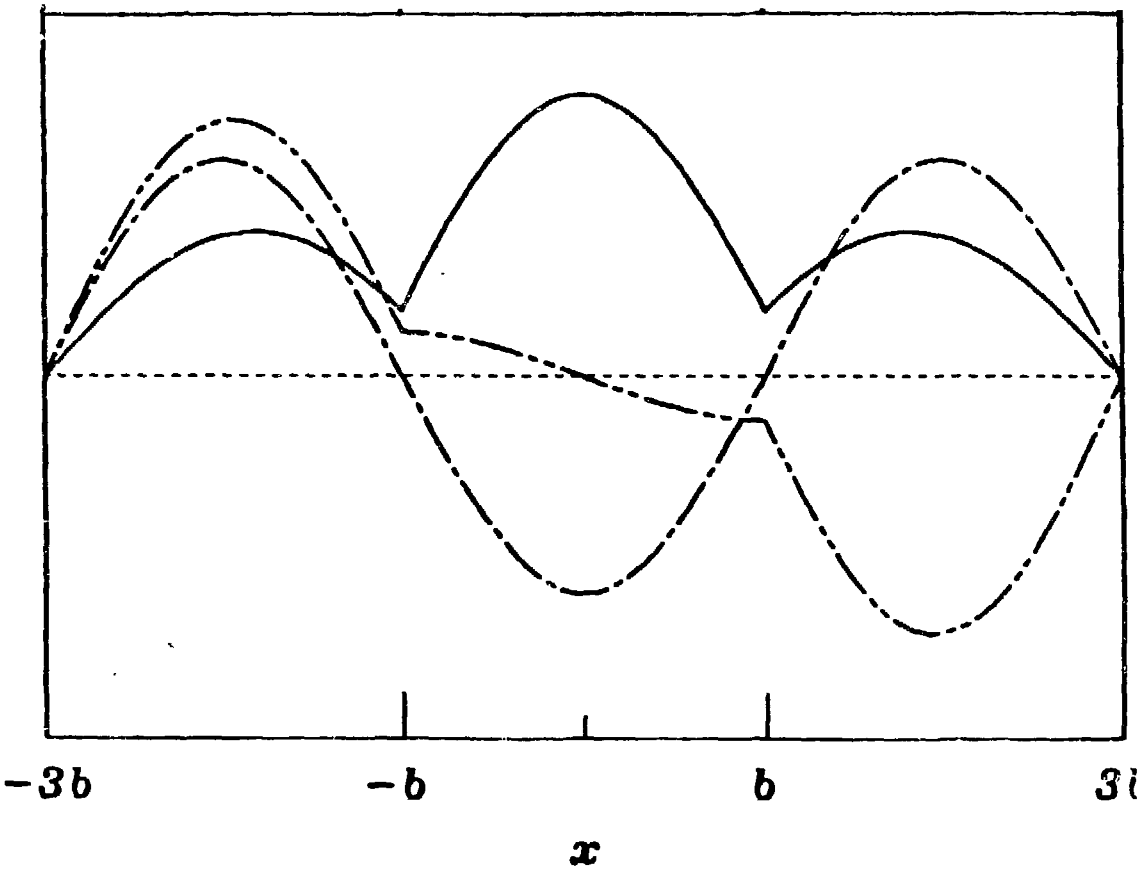
We find that[1], for realistic values of the potential strength, the wavenumbers of the eigenmodes are not enormously different from that for a single, isolated cube (nucleon). The first N modes for an $N \times N \times N$ cube cluster near, and of course

Fig. 3 Graphical solution of eigenvalue equation.



below, the value for a single cube. The absolute value of the wavefunction at the location of the delta function potential is about 10% of the maximum value. That is, the probability of finding a quark at an interstitial location, as far away as possible from the center of a cube (nucleon), is about 1% of the probability of finding it at the center. We have done little violence to the conventional view that quarks reside inside nucleons (beyond the horrible, cubical approximation itself). The next set of solutions occurs at close to twice the (momentum, and so for these completely relativistic quarks) energy of a quark in an individual nucleon. This large energy difference clearly suppresses the contributions of the higher Fock space states. Thus our first result is that valence quarks dominate the wave function.

Fig. 4 Wave functions for 3^3 nucleus.



To determine a binding energy for our "nucleus", we need to compare the energy of 3A filled states to the mass of A cubic nucleons. The mass of even a cubical nucleon is somewhat less than three times the energy of a quark wave restricted to a single cube, due to the color-magnetic spin-splitting interaction between pairs of quarks, which raises the delta and lowers the nucleon. We applied a crude correction to account for the difference between the empirical value in spherical nucleons and our cubes. The result is a set of average binding energies per nucleon between 40 and 90 MeV (depending on the potential strength), and which show evidence of saturation as A increases. These are not expected to be realistic values, but they are in the right ballpark. Since the systematics are as we had expected, we have been encouraged to attempt a more realistic case.

Table I

Binding Energy per Cubical Nucleon for Cubical Nuclei of various Atomic Weights, A.

v \ A	8	27	64	125	216	343
1.0	36	76	96	108	116	122
k_{min}	279	258	250	246	244	242
E_{min}	483	447	432	426	422	420
1.25	16	50	67	78	85	89
k	287	269	261	258	256	255
E	497	465	453	447	444	442
1.5	4	31	46	55	61	65
k	293	277	270	267	266	265
E	508	480	468	463	460	458
1.75	-11	16	29	37	43	46
k	298	283	277	275	273	272
E	516	491	480	476	473	471
2.0	-21	4	16	23	28	31
k	302	288	283	280	279	278
E	523	499	490	486	483	482

k_{max} = 333 MeV/c, E_{max} = 576 MeV

V. THE EMC EFFECT

Before turning to the more realistic problem, we briefly apply the results above to explain the EMC effect. This is the discovery by the European Muon Collaboration[2], since confirmed by electron scattering data from SLAC[5], that the scaling structure function of a nucleus, measured in deep inelastic muon scattering, is not just A times that of an average (isoscalar) nucleon. This startling observation shows that nuclear binding effects are apparent at even very high energies, where they had been naively expected to be negligible.

Many proposals have been put forth to explain the observation of a (relative to the nucleon) excess of structure function strength at low quark fractional momentum, including expanding nucleons, six-quark bags, and excesses of pions. None of these has been conclusively confirmed, and indeed the last runs into some trouble with polarization transfer measurements in proton scattering at much lower energies.[6] All we really have to say here is: Me too!

Geoffrey West has developed a formalism which shows that these structure functions may be determined from a three dimensional integral over the non-relativistic constituent wavefunction in any system. Thus the EMC effect can be seen to correspond to an increase in nuclei, over the amount found in individual (isolated) nucleons, of long wavelength, low momentum components of the quark wavefunction. It is clear from this why our picture produces the EMC effect, at least qualitatively. Even in the cubical nucleus, the boundary condition between adjacent cubes allows the quark wavefunction to flatten, relative to that in a nucleon, increasing the amplitude of low momentum Fourier components. In addition, the coherence of the quark waves over a space of many nucleon diameters also affects the strength of the Fourier components.

We have seen these effects in both the Fourier transforms of the quark wavefunctions, and in the West integral over them. In both cases, the EMC effect shows up as a low momemtum (or momentum fraction) enhancement, followed by a suppresion at higher values relative to a nucleon. Finally, at large momentum fraction (x>1 in nucleon terms), we have seen again an excess of strength, that in conventional models would be ascribed to Fermi momentum effects. In performing these calculations, we have averaged over orientations of the cube in order to restore rotational symmetry, and so as to smooth out Bragg reflection-like effects. We have not attempted to compare so crude a model to actual data.

$$f_{q/A}(x) \cong (k_0 - k_3) \int \frac{d(k_0+k_3)\, d^2k_\perp}{(2\pi)^4} \left|\psi_q(\vec{k})\right|^2 2\pi\, \delta\left((k-p_A)^2 - \omega_0^2\right)$$

[West integral]

Fig. 5 Comparison of quark wavefunctions in nucleon (—) and 3^3 cubical nucleus (---).

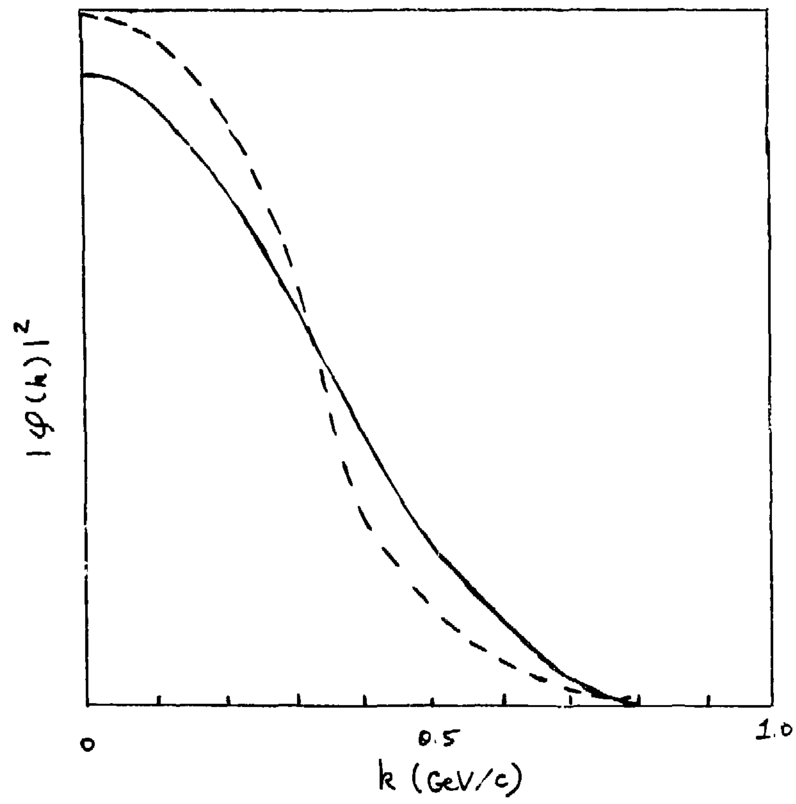
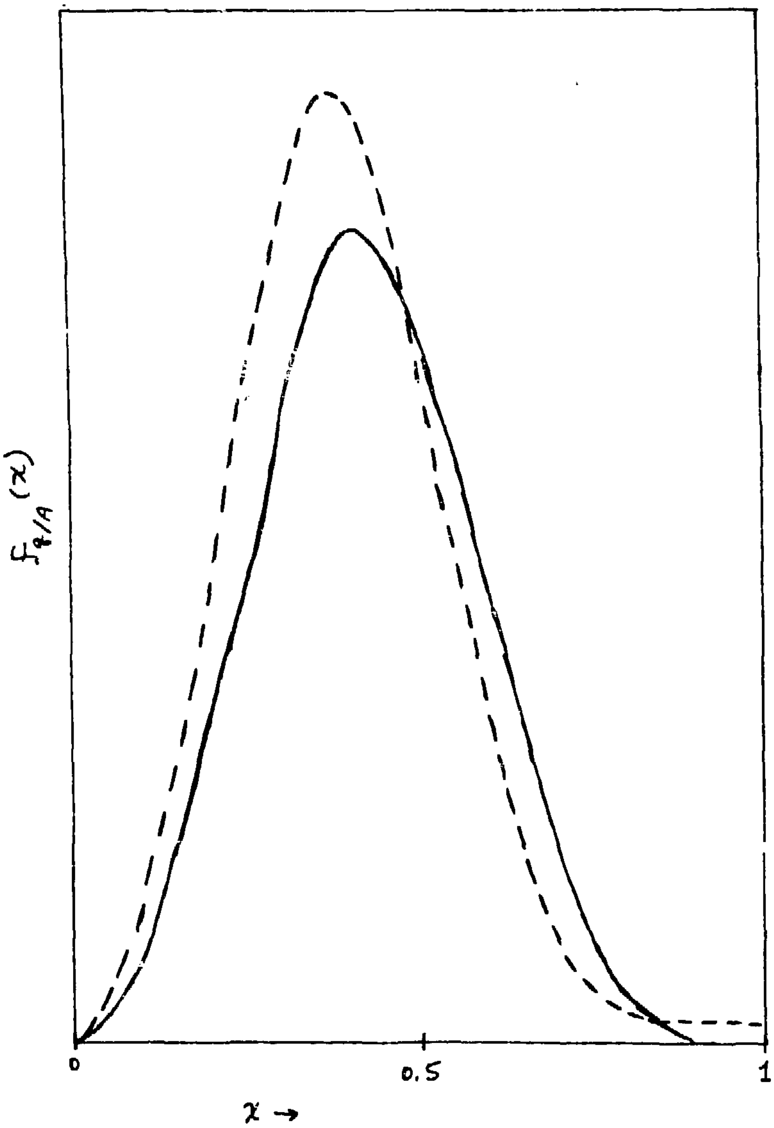


Fig. 6 Probability f of finding a quark at momentum fraction x in a nucleon (—) or 3^3 cubical nucleus (---)



VI. SMALL QUARKLEI: A=2,3,4

I would like to note, just in passing, that the linear potential fitted to heavy quark systems crosses zero at just under 0.6 Fermis from the origin. With the Feynman-Gell-Mann reduction technique, this produces a squared potential (for massless quarks) which is very flat from the origin, to about 1 Fermi out. This potential in the Schroedinger-like equation bears a strong resemblance to the structure envisaged in bag models of confinement. Thus, despite the difficulty of justifying a potential approximation for relativistic quarks, it is clear that our approach will do very well, as does the MIT bag model[7], in reproducing the hadronic spectrum.

To calculate the splitting between deltas and nucleons requires, that in addition to the electric interaction given by the potential, the spin-spin color-magnetic correlation energy between pairs of quarks must be included. Using standard SU(6) spin and flavor wavefunctions, and our own spatial wavefunctions, we can reproduce the experimental value of the delta-nucleon splitting if the value of the dimensionless strong coupling constant is about 1. For comparison, the MIT bag calculation required this value to be about 2.2, and a value of about 0.75 has been suggested by T. Barnes[8] on the basis of short distance corrections to the wavefunction due to the strength of the (otherwise universally ignored) color-Coulomb interaction near the origin. My point is not that any of these values is any better than another, but that they are all in the same ballpark. Thus, our model is not wildly different from others that have been found quite acceptable for the hadron spectrum.

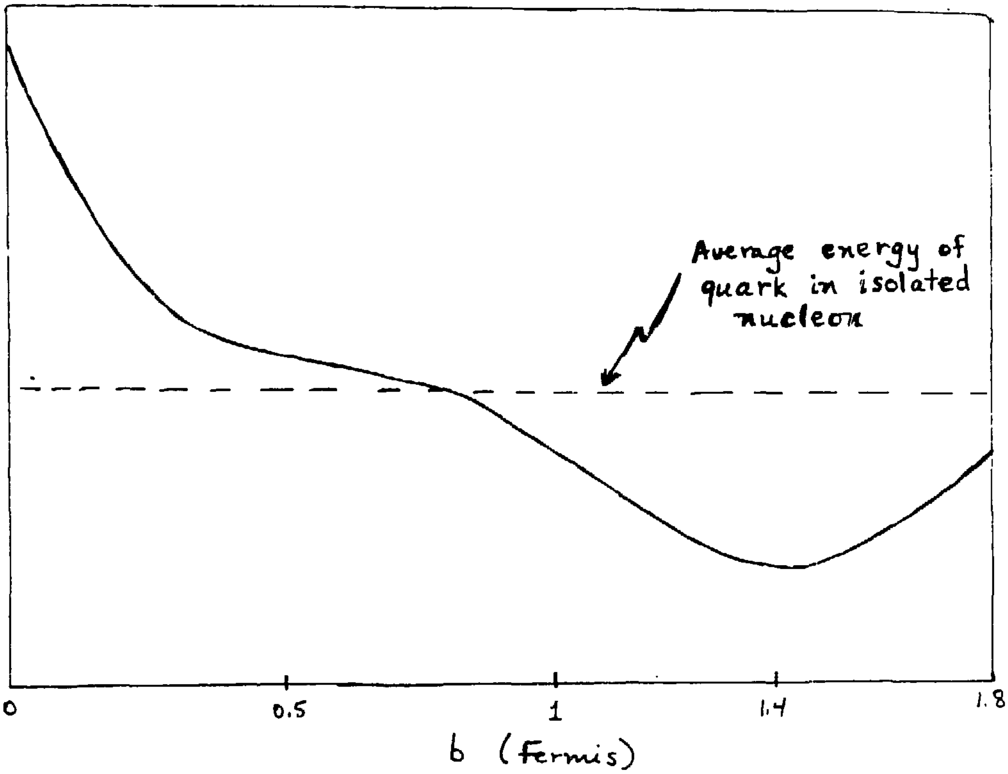
We (that is, Kevin Schmidt, our post-doc at the time) have constructed a program to calculate the full Dirac quark eigenenergies in arbitrary geometry systems of up to four potential wells (A=4). Some fortunate arithmetic errors quickly taught us that the symmetric arrays, equilateral triangle for A=3 and tetrad for A=4, are indeed the ones that provide the lowest eigenenergies for the quarks. The A=2 system is not bound, and we would certainly not expect to reproduce a dueteron with our poor description of long distance interactions. For the A=3 and 4 systems, the minima occur at separations of 1.3 and 1.4 Fermis, respectively, between the well centers. It is these energy minima which determine the nuclear structure. The binding energies are 12 and 24 MeV per quark, before including color-magnetic spin-splitting effects. These calculations do not yet include any variational adjustments to the quark wavefunctions; we have only used the symmetric sum of wavefunctions for the individual wells.

The color-magnetic spin-splitting effect overwhelms this binding energy. We concentrate on Helium-4 as its density and compactness offer a "best case" for minimizing omitted long distance (single pion) effects. It is also a convenient "closed shell" system as 12 quarks in 3 colors, 2 flavors (up and down) and 2 spin states are the maximum that can be placed in the 1S spatial wavefunction, consistent with Pauli exclusion. We find 43 MeV additional energy, on average, per quark in this case, despite a roughly factor of three decline in the matrix element of the color-magnetic operator, which arises because of the increased volume occupied by the wavefunction. (The minimum overall energy now occurs at a center-to-center distance of 1.6 Fermis between the wells in the tetrad.) The large value of the additional energy is due to a combination of the effects of the occurrence of quark pairings similar to those which appear in the delta (S=1, T=1), and of new pairings which occur in the 6-dimensional color representation. We do not know if symmetric variational adjustments to the quark wavefunctions will improve this, but adding in antisymmetric (P-wave) combinations certainly will. Note, however, that the later additions make the system look even more like one of separate nucleons than the fluctuations in the symmetric quark amplitudes did.

Thus, at present, our model is even further from describing nuclei than is a collection of non-interacting nucleons! Our point is, however, that these represent opposite extremes of zero and maximal phase correlations of the nucleon constituent quark waves. Real nuclei undoubtedly lie somehwere in between these two extremes. Introducing spatial-color correlations into our many-body wavefunctions will undoubtedly improve our results. We therefore expect that it will be possible to describe nuclei entirely in terms of quark and gluon constituents, without the need to define off-shell composite objects.

The most exciting prospect of our program is the absence of free parameters. What we need about QCD can be derived from the hadronic particle spectrum, if not (at present) ab initio. Thus, for nuclear calculations, there are no free parameters. The shape and scale of well arrays is determined by the need to minimize the quark energies. Nuclear binding energies, excitation spectra, and structures become as calculable (and independent of experimental input) as are the corresponding quantities for molecules. However, to prove that we are on the right track, it is necessary to find critical tests of features unique to our model, or to predict new phenomena based on it. The most striking new element is the occurence of color-6 representations of pairs of quarks. Unfortunately, we do not yet have any definite

Fig. 7 Quark eigenergies in tetradal nuclei as a function of the length of a side (b) of the tetrad



physical predictions based on this. Since the EMC effect has already been observed, we turn next to applying our model to new phenomena in a new nuclear sector.

VII. HYPERNUCLEI

We have calculated the eigenenergies for strange quarks of mass 130 or 170 MeV in the A=4 system, using the code referred to previously. These yield about 16 MeV more binding energy than for light quarks (it costs less energy to localize a massive quark) but this minimum occurs at a well separation of 0.9 Fermis. That is, the strange quark would prefer to have the system a little smaller than that fixed by the light quarks. Since the size of so-called Lambda-Helium-4 is more likely determined by the 11 light quarks than the one strange quark, we expand the system to its light quark size. This reduces the strange quark binding energy to about the same as for the light quarks. With just this information, we can make qualitative predictions for the systematics of hypernuclei.

The first is that, contrary to the expectation based on an entire Lambda being able to fall to the lowest (nucleon) shell model state, only the strange quark can fall into the lowest quark shell model state. The two light quarks are Pauli blocked, although they may still provide some binding energy. The separation of the strange quark from its light partners may cost further in energy. Thus, the binding energy for adding a Lambda should be about 1/3, and certainly less than 1/2, that expected from the binding energy of the lowest state nucleon in a nucleus of the same total baryon number (A). The light quarks hold the Lambda up at a higher average level in the nuclear well than that for a point particle.

Secondly, when the nucleus in which we will trade a nucleon for a Lambda is a closed shell nucleus, we predict that the hypernucleus will always show evidence of, in conventional terms, strong Lambda-Sigma mixing. This is because the strange quark which is trying to sink down to an unblocked lower energy state, leaving its light partners behind, will lose track of which light pair it "belongs" too. Since the light pair it is "near" at any time may be either T=1 or T=0, the strange-light system has roughly equal probability to appear as either hyperon. Coversely, when the A-1 nucleus is a closed shell, there is a light quark pair which is distinguishable, namely the one that is trying to act like a nucleon in the first orbital outside the closed shell. This distinguishable pair helps the Lambda to retain its identity and mixing is suppressed.

Finally, as our calculations directly show, hypernuclei are smaller than ordinary nuclei. This is opposite to the expectation in the conventional (nucleon) picture. There, the short range of the Lambda-nucleon interaction leads to the Lambda residing on the surface on the nucleus, if it is to be lightly bound, consistent with experiment. Alternatively, the Lambda may be viewed as sinking to the center of the nucleus, but there its relatively weak interaction with nucleons weakens the average nuclear potential, and allows the nucleus to expand, overall. Either way, the hypernucleus is larger. Unfortunately, the prospect is remote, to say the least, that electron scattering experiments may determine whether hypernuclei are larger or smaller than the same A nucleus. We must think of a better way.

VIII. CONCLUSION

I have tried to convince you that we know nuclei are made of quarks, and that direct QCD calculations offer hope of making nuclear physics as well defined, and as powerfully predictive, as modern quantum chemistry. We are a long way from proving this, but we are up against several thousand more man-years of effort applied to the conventional picture, including efforts by some of the "giants" of physics. More importantly, the problem is really two-fold: Both to do better, and to understand deeply why the old ways have worked so well. An improved ability to make accurate extrapolations beyond experimentally known regimes will be the hallmark of success for our approach. But for me, and I hope for you, the most stirring rallying cry for this effort has been and will continue to be: No parameters!

It is a pleasure to acknowledge many useful conversations with my colleagues in T-, P-, and MP-Divisions regarding this subject.

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