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Abstract:

The pion multiplicity produced in nuclear collisions between 0.2 and 2 AGeV is calculated assuming shock formation. We also correct the procedure of extracting the nuclear equation of state as proposed by Stock et al. The nuclear equation of state would have to be extremely stiff for this model to reproduce the observed multiplicities. The assumptions of this model are critically analyzed.

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One of the prime objectives of high energy heavy ion research is to probe the properties of nuclear matter at high temperatures and densities. Recent data on collective flow phenomena [1] and the systematics of pion production [2] are particularly interesting for the program of extracting the nuclear equation of state. In this note we comment on the latter topic. First we correct the procedure used in Ref.[2] to deduce the energy functional, $W(\rho,T)$, from the difference between the observed and calculated pion multiplicity. Second we calculate the pion multiplicity assuming that the pion abundance is frozen out at the point when the highest density is reached, as suggested in Ref.[3]. We find that the equation of state would have to be extremely stiff if the latter assumption were valid.

The observed [2] negative pion multiplicity, $N_{\pi^-}^{obs}(E)$, produced in central nuclear collisions increases approximately linearly with incident kinetic energy E. While cascade calculations [3] using the Cugnon code [4] also lead to a linear dependence of $N_{\pi^-}^{cas}(E)$ on E, the calculated values systematically over estimate the observed multiplicity. In fact all models [5]-[8] except hydrodynamics [9] over estimate the pion yield. Stock et al. [2] suggested that this discrepancy could be exploited to extract information about the nuclear matter equation of state at high densities and temperatures. They noted that if thermal and chemical equilibrium is reached in such collisions, then the number of pions depends on the available thermal kinetic energy per nucleon, $K(E_{cm})$. For a given center of mass kinetic energy per nucleon, E_{cm}, $K(E_{cm})=E_{cm}-U(E_{cm})$, where $U(E_{cm})$ is potential energy per nucleon at the point where the pion abundance freezes out. That potential energy, on the other hand, clearly depends on the baryon density, $\rho(E_{cm})$, at the freeze out point. Since one expects ρ to be an increasing function of energy, the energy dependence of the pion multiplicity should then be sensitive to the form of $U(\rho)$.

To extract $U(\rho)$ from the data they noted further that the cascade calculation explicitly neglects potential energy. Therefore, N_π^{cas} is the pion multiplicity under the condition that $K = E_{cm}$. Hence, they proposed to extract $U(E_{cm})$ by equating

$$N_\pi^{cas}(E_{cm}-U(E_{cm})) = N_\pi^{obs}(E_{cm}) \quad . \tag{1}$$

Using (1) and the maximum compression $\rho(E_{cm})$ computed via cascade, they found a remarkable similarity between their $U(\rho)$ curve and the energy functional, $W(\rho,T=0)$, for compressibility K = 250 MeV. ($W(\rho,T=0)$ is also referred to as the zero temperature equation of state of symmetric nuclear matter.) They were therefore tempted to identified $U(\rho)$ with $W(\rho,0)$. The correct connection is, however,

$$W(\rho,T) = K(\rho,T)+U(\rho) \quad , \tag{2}$$

with

$$K(\rho,T) = \sum_\alpha K_\alpha(\rho,T) \quad , \tag{3}$$

where K_α is the kinetic energy per baryon for $\alpha = N,\Delta_{33},\pi,\cdots$. In the limit $T = 0$, K reduces to

$$K(\rho,0) = K_N(\rho,0) \approx (21\text{MeV})(\rho/\rho_0)^{\frac{2}{3}} \quad , \tag{4}$$

where $\rho_0 \approx 0.145\text{fm}^{-3}$ is the saturation density. Note that the conventional [9] compression energy functional, $W_c(\rho)$, is given by $W_c(\rho) = K_N(\rho,0)+U(\rho)$. The popular quadratic or linear forms of the equation of state refer to W_c rather than to U.

Therefore, to compare with nuclear models for $W(\rho,0)$ it is necessary to add the degeneracy kinetic energy (4) to the $U(E_{cm})$ deduced from (1). This procedure leads empirically to a much stiffer equation of state than deduced in [2].

The reliability of that empirical equation of state depends on the assumption made about the freeze out density, $\rho(E_{cm})$, at which the pion to baryon ratio ceases to change. However, as clearly shown in Ref.[10] the pion fraction not only depends on that freeze out density but also on the dynamical path leading toward that density. In thermal models [6][7] the evolution proceeds isoergically. The pion multiplicity therefore grows as the volume increases until freeze out. On the other hand, in hydrodynamical models [9] expansion proceeds isentropically as thermal energy is converted into collective flow kinetic energy. Therefore the pion multiplicity decreases until freeze out is reached. In the past [6][10] it was argued that freeze out should occur below normal nuclear density. This explains why thermal models tend to over predict and hydrodynamical models tend to under predict the pion multiplicity. However, the surprising result obtained from the cascade study of Ref.[3] was that the pion abundance froze out very early in the collision.

In fact, to 15% accuracy the pion plus delta multiplicity was fixed at the time when the compression reached its highest value $\rho_{max}(\dot{E}_{cm})$. If this result is more general than the specific cascade model used, then it would eliminate the problem of which precise dynamical path was followed on expansion. We will comment further on this result, but for now we follow Ref.[3] and assume that it holds generally. The value of ρ_{max} depends on dynamical details of the compression phase unless the Rankine-Hugoniot shock conditions are attained. If the following conditions are satisfied

1. Shock densities are attained
2. pion chemical equilibrium is reached in the shock zone
3. the pion abundance freezes out before the shocked matter expands appreciably

then the pion to baryon ratio would be fixed by kinematics and the sought after equation of state.

In Ref.[3] the above assumptions were used to derive the equation of state independent of the empirical formula (1). The result obtained was that the two methods led to the same equation of state within errors. However, the Rankine-Hugoniot equation was not solved consistently because the thermal pressure was assumed to of the ideal gas form $p_T = \rho T$ while the compression pressure, $p_c = \rho^2 \partial W(\rho,0)/\partial\rho$, again included the zero temperature degeneracy pressure, cf. (2).

Because of the importance of this independent determination of the equation of state we recalculated the Rankine-Hugoniot conditions using eq.(2) and the following ansatz for $U(\rho)$:

$$U(\rho) = -B - K_N(\rho,0) + \frac{K}{18\rho_0^2}(\rho-\rho_0)^2 , \qquad (5)$$

where $B \gtrsim 8$MeV is the binding energy and K is the compressibility. Note that the zero temperature energy $K_N(\rho,0)$ must be removed from the quadratic term in order for nuclear matter to saturate at ρ_0. With eqs.(2,5) the pressure is given by

$$p(\rho,T) = \rho^2\frac{\partial U(\rho)}{\partial\rho} + \sum_\alpha p_\alpha(\rho,T) , \qquad (6)$$

where $p_\alpha = \langle p^2/3\varepsilon\rangle_\alpha$ is the usual thermal pressure expectation value over Bose or Fermi distributions which we evaluate numerically using relativistic kinematics.

The Rankine-Hugoniot equation [11] relates the shock compression to the energy density, ε, and pressure, p, in the shock as

$$\left(\frac{\rho}{\rho_0}\right)^2 = \frac{(\varepsilon+p_0)(\varepsilon+p)}{(\varepsilon_0+p)(\varepsilon_0+p_0)} , \qquad (7)$$

where for a given $E_{cm} = (\gamma_{cm}-1)m_N$ the energy density and pressure are further constrained by

$$\gamma_{cm}^2 = \frac{(\varepsilon+p_0)(\varepsilon_0+p)}{(\varepsilon+p)(\varepsilon_0+p_0)} . \qquad (8)$$

Given the conditions, ε_0, p_0, of the incoming nuclear matter and its fluid velocity, eqs.(7,8) determine the temperature, T, and chemical potential, μ, in the shock. The equation of state enters through the dependence of ε and p on T and μ.

Once T and μ are known the pion to baryon ratio in chemical equilibrium is given by

$$\frac{N_\pi}{A} = \frac{1}{\rho} \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{3}{e^{\omega_\pi/T}-1} + \frac{16}{e^{(\omega_\Delta-\mu)/T}+1} \right\} . \qquad (9)$$

In (9) $\omega_\alpha = (p^2+m_\alpha^2)^{\frac{1}{2}}$ for $\alpha = \pi, \Delta$ is the single particle spectrum of pions and Δ_{33} resonances respectively.

The dependence of ρ, T, and N_π/A on cm kinetic energy are shown in Fig.(1,2). First consider the curves labeled 1 and 2 corresponding to the case with U = 0. Curve 1 corresponds to the pure ideal gas case for a mixture of nucleons, pions and Δ_{33} resonances. Indicated by the + symbols are the results of cascade calculations [3][4]. We note the large difference between the expected compression in the ideal gas case and the cascade results. This difference is due to finite range of hadronic forces as implemented by the cascade prescription of scattering two particles at the distance of closest approach. Indeed as past studies [12] have shown the compression reached is sensitive to the details of the scattering prescription. Physically the finite range induces an effective excluded volume effect that enhances the pressure in the system while leaving the energy functional invariant. For a hard sphere gas the enhancement factor to lowest order is $(1+b\rho)$, where $b = 2\pi d^3/3$ in terms of the hard sphere diameter d. In curve 2 we have implemented this Enskog correction [13] by enhancing the thermal part of the pressure (6) as

$$p(\rho,T) = p_c(\rho)+\{p_T(\rho,T)-p_N(\rho,0)\}(1+b\rho) , \qquad (10)$$

where $p_T = \sum_\alpha p_\alpha(\rho,T)$. Note that the degeneracy contribution $p_N(\rho,0) \approx 2\rho K_N(\rho,0)/3$ to p_c is not enhanced in (10). We chose d = 1fm for illustration for curve 2. We observe that the Enskog corrected nucleon, pion, Δ_{33} gas reproduces the cascade results much better than curve 1. Therefore, the cascade results can be interpreted as supporting the contention [3] that shock densities are reached in finite nuclear systems. However, the evidence for shock conditions depends crucially on the non idealness of the cascade dynamics at high densities. Unfortunately, the correction to the ideal gas pressure in (10) is only a crude approximation to the cascade situation. Herein lies one of the main weaknesses of the present approach. It would be useful to carry out cascade studies that map out the particular equation of state corresponding to the scattering prescriptions adopted.

While the compression in Fig.(1a) is sensitive to both the Enskog correction and the nuclear potential, the temperature in the shock is sensitive only to the potential. In general, the temperature is smaller with increasing value of K in eq.(5). However, T is sensitive to the functional form of $U(\rho)$ as well. For curve 5 we modified the term proportional to K by replacing $(\rho-\rho_0)^2/\rho_0^2$ by a "linearized" form $(\rho-\rho_0)^2/(\rho\rho_0)$. Note that for higher energies the temperatures in curves 3 and 5 are nearly the same despite the fact that K=250 MeV for curve 3 and K=800 MeV for curve 5. Because the pion to baryon ratio is mostly sensitive to T and not ρ, we find in Fig.(2) that curves 3 and 5 are also similar there. We therefore conclude that the form of $W(\rho,0)$ near ρ_0 is not important for the pion to baryon ratio. What counts is the magnitude of the potential energy at high densities. Thus the stiff K≈800 MeV quadratic potential (5) is better able to reproduce the data. Note also that we can recover the results of [3], see the curve labeled S in Fig.(2) by setting K=250 MeV and neglecting the K_N in (5). Therefore, we have verified the consistency of the empirical potential obtained via eq.(1) and that obtained by fitting N_π/A using shock dynamics. Our main conclusion is, however, that this empirical equation of state is rather unphysical from the point of view of conventional nuclear physics.

Given the pathological form of that equation of state it is necessary to analyze critically the assumptions made. The most questionable assumption is that the pions drop out of chemical *equilibrium before the shocked matter has a chance of expand.* That shock densities are reached is plausible from our analysis. That the pions freeze out so early is far from clear. Indeed, reasonable assumptions [10] about the time scales of the expansion and pion production and absorption rates have led to the expectation that pions freeze out rather late, when the density falls below ρ_0. The resolution of this problem could lie in the nonisentropic nature of the expansion. As shown by Stöcker [9] only a 20% enhancement of the entropy is enough to fit N_π/A with a *hydrodynamic model* that *freezes out below* ρ_0. That amount of entropy production is consistent with cascade estimates [14]. Therefore, an alternate interpretation of the approximate

constancy of N_π/A is that pions remain in chemical equilibrium until the density falls considerably below the shock density but that due to entropy production that ratio increases relative to the isentropic case by just the right amount to leave the ratio constant in time. With this interpretation the constancy of that ratio in cascade calculations [3] after maximum compression is only accidental. In the absence of further compelling reasons to assume early freeze out, we interpret the unphysical equation of state obtained as evidence against it.

The resolution of this pion puzzle must ultimately come from detailed cascade calculations including mean field effects. Recently, there has been progress [8] [15] toward constructing such models. In Ref.([15]) the absolute pion yield was successfully reproduced for the first time assuming a conventional equation of state. However, they also found very little sensitivity to *variations of K between 200 - 400 MeV.* The *reason* for the agreement and lack of sensitivity to K remain unclear at this time. Dynamical details such as the treatment of Pauli blocking may play an important role [5].

While we have shown that the corrected methodology of Refs. [2][3] leads to an implausibly stiff equation of state, the idea of using pion production to probe dense nuclear matter is still provocative. The pion data [2] together with the data on collective flow [1] may eventually lead to useful constraints on that *elusive* equation *of* state. *However, entropy* production and the freeze out process must first be better understood.

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Figure captions:

1. The baryon density (A) and the temperature (B) as a function of center of mass kinetic energy per nucleon. Curves labeled 1 and 2 correspond to the case $W(\rho,T) = K(\rho,T)$ including pions, nucleons, and Δ_{33} with Enskog diameter d = 0, 1fm respectively, cf. (10). Curves 3 and 4 includes the potential (6) with K=250,800 MeV respectively. Curve 5 corresponds to the "linearized" potential with K=800 MeV. The + symbols correspond to cascade results [3].
2. The pion to baryon ratio as a function of E_{cm}. Curves labeled as in Fig.(1). Solid dots are data points extrapolated to zero impact parameter [2] for Ar+Ca, while open square is for La+La. Curve S corresponds to the calculations in Ref.([3]) where the ground state kinetic energy, $K_N(\rho,0)$, was doubly counted.

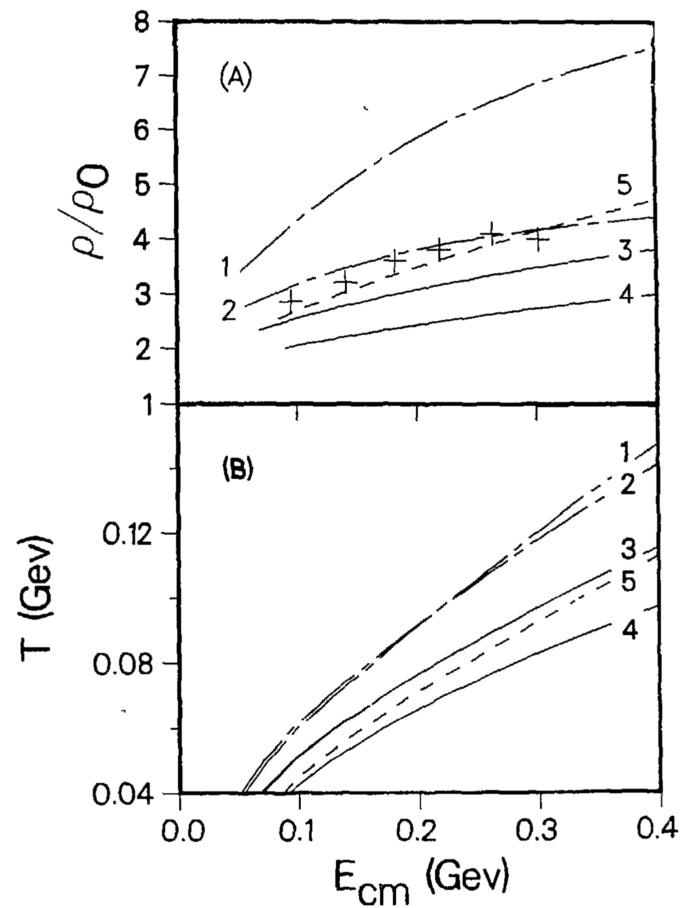

Fig.1

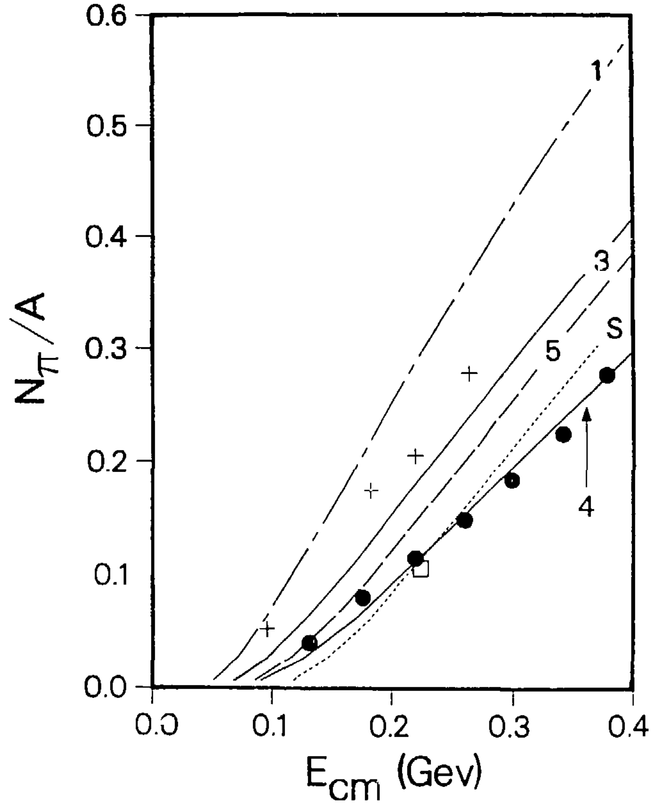

Fig.2