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BISTABLE STATES OF TM POLARIZED NON-LINEAR WAVES GUIDED BY SYMMETRIC LAYERED STRUCTURES

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BISTABLE STATES OF TM POLARIZED NON-LINEAR WAVES GUIDED BY SYMMETRIC LAYERED STRUCTURES *

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ABSTRACT

Dispersion relations for TM polarized non-linear waves propagating in a symmetric single film optical waveguide are derived. The system consists of a layer of thickness d with dielectric constant ϵ_1 bounded at two sides by a non-linear medium characterized by the diagonal dielectric tensor $\epsilon_{11} = \epsilon_{22} = \epsilon_0$, $\epsilon_{33} = \epsilon_0 + \alpha|E_3|^2$, where E_3 is the normal electric field component. For sufficiently large d/λ (λ is the wavelength) we predict bistable states of both symmetric and antisymmetric modes provided that the power flow is the control parameter.

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I. INTRODUCTION

The field of optical bistability (OB) has grown very rapidly over the past few years (for a collection of papers on OB see [1]; also an excellent review is provided by [2]). As is well known, OB consists of the existence of two stable states for one set of optical input conditions. The interplay between the optical non-linearity and feed back forms the basis of OB.

A considerable interest has been devoted in the literature to OB because of possible applications to all optical logic and signal processing elements. The observations of OB in semiconductor materials (GaAs and InSb) at cryogenic temperatures [3],[4] and room temperature [5],[6] have been reported.

Recently, different guided-wave approaches to OB have been analyzed [7]. Several new features of the behaviour of non-linear three layer dielectric structures have recently been reported. [8]-[11].

Bistable states of TE polarized non-linear waves guided by a symmetric three layer dielectric structure consisting of a layer of thickness d with a dielectric constant ϵ_1 bounded at two sides by a non-linear medium characterized by the diagonal dielectric tensor

$$\epsilon_{11} = \epsilon_{22} = \epsilon_{33} = \epsilon_0 + \alpha(|E_1|^2 + |E_2|^2 + |E_3|^2) \qquad (1)$$

with $\alpha > 0$ (self-focusing medium) have been found provided that the power flow in the wave is the external parameter [12].

We remark that bistable states of TM polarized non-linear waves guided by the same symmetric layered structure as in [12] but when the non-linear material is described by the dielectric tensor

$$\begin{aligned} \epsilon_{11} &= \epsilon_{22} = \epsilon_\perp + \alpha\left(|E_1|^2 + |E_2|^2\right) \\ \epsilon_{33} &= \epsilon_\parallel \end{aligned} \qquad (2)$$

with $\alpha < 0$ (self-defocusing medium) have also been found provided that the total power flow is the control parameter [13].

In recent papers [14]-[18] exact dispersion relations for TE polarized non-linear waves propagating in an asymmetric single film waveguide were derived. The system under consideration consisted of a slab of thickness d with dielectric constant ϵ_2 bounded at the negative z-side by a linear medium with dielectric constant ε_1 and at the positive z-side by a non-linear substrate characterized by the diagonal dielectric tensor (1) with $\alpha > 0$ (self-focusing medium). It was shown in [15] that for sufficiently small d/λ (λ is the wavelength) the non-linear wave may exist only at power flows exceeding some certain minimal value. For sufficiently large d/λ bistable states of TE polarized non-linear surface and guided wave have been found provided that the total power flow is the control parameter [14]-[18]. For power flows exceeding some certain threshold value a single waveguide mode (with mode order $n = 0$) occurs [14],[15]. In a recent paper, the observation of intensity - dependent hysteresis in the transmission of guided wave through a thin film waveguide with a cladding characterized by an intensity-dependent refractive index (liquid crystal MBBA) has been reported [19].

It is the aim of the present paper to study the TM polarized non-linear waves travelling with propagation constant $n = k/k_0$ ($k_0 = \omega/c$) along a symmetric film waveguide which consists of a layer of thickness d with dielectric constant ε_1 (refractive index $n_1 = \varepsilon_1^{1/2}$) bounded at two sides by a non-linear medium characterized by the diagonal dielectric tensor [20]

$$\begin{aligned} \varepsilon_{11} &= \varepsilon_{22} = \varepsilon_0 \\ \varepsilon_{33} &= \varepsilon_0 + \alpha|E_3|^2 \end{aligned} \quad , \tag{3}$$

where E_3 is the normal electric field component.

This paper is structured as follows. Normal modes dispersion relations are derived in Sec.II. In Sec.III we calculated the power flow carried by TM polarized guided modes (i.e., $n < n_1$) and TM polarized non-linear surface modes (i.e., $n > n_1$) as a function of the propagation constant n. In the last section we briefly present our conclusions.

II. DISPERSION RELATIONS

We start by obtaining the symmetric modes of the symmetric layered structure consisting of a non-linear medium characterized by the dielectric tensor (3) in the region I ($-\infty < z < -d/2$), a dielectric film of thickness d with dielectric constant ε_1 (linear medium) in the region II ($-d/2 < z < d/2$) and the same non-linear medium in the region III ($d/2 < z < \infty$). We consider a TM polarized wave that propagates in the x-direction. It is therefore convenient

to work with the only non-vanishing components of $\vec{E}$ and $\vec{H}$ which can be written in the form

$$\begin{aligned} E_{1,3}(x,z,t) &= \mathcal{E}_{1,3}(z)\exp(-i\omega t + i\kappa x) \\ H_2(x,z,t) &= \mathcal{H}_2(z)\exp(-i\omega t + i\kappa x) \end{aligned} \tag{4}$$

Maxwell's equations are

$$\begin{aligned} &\frac{d\mathcal{E}_1}{dz} - i\kappa\mathcal{E}_3 = i\kappa_0\mathcal{H}_2 \\ &\frac{d\mathcal{H}_2}{dz} = i\kappa_0\mathcal{D}_1 \quad , \quad \kappa\mathcal{H}_2 = -\kappa_0\mathcal{D}_3 \end{aligned} \tag{5}$$

From Eqs.(5) it follows that

$$\begin{aligned} &\frac{d^2\mathcal{H}_2^{I}}{dz^2} - \frac{q^2}{\varepsilon_0}\left[\varepsilon_0 - \frac{\alpha}{\varepsilon_0}\frac{\kappa^4}{q^2k_0^2}\left(\mathcal{H}_2^{I}\right)^2\right]\mathcal{H}_2^{I} = 0 \quad , \quad z < -d/2 \\ &\frac{d^2\mathcal{H}_2^{II}}{dz^2} - (\kappa^2 - \varepsilon_1\kappa_0^2)\mathcal{H}_2^{II} = 0 \quad , \quad -d/2 < z < d/2 \\ &\frac{d^2\mathcal{H}_2^{III}}{dz^2} - \frac{q^2}{\varepsilon_0}\left[\varepsilon_0 - \frac{\alpha}{\varepsilon_0}\frac{\kappa^4}{q^2k_0^2}\left(\mathcal{H}_2^{III}\right)^2\right]\mathcal{H}_2^{III} = 0 \quad , \quad z > d/2 \end{aligned} \tag{6}$$

where $q^2 = k^2 - \varepsilon_0 k_0^2$.

We look for solutions localized near the surfaces of the thin film with fields that fall to zero as $|z| \to \infty$. The solutions of Eqs.(6) in the case $\alpha > 0$ (self-focusing medium) are

$$\mathcal{H}_2^{I} = \left(\frac{2}{\alpha}\right)^{1/2} f\left\{\cosh\left[q(z - z_1)\right]\right\}^{-1} \quad , \quad z < -d/2$$

$$\mathcal{H}_2^{II} = \begin{cases} A_1\cosh\left[k_1(z - z_0)\right] & , \ n > n_1 \\ A_2\cos\left[k_2(z - z_0)\right] & , \ n < n_1 \end{cases} \qquad -d/2 < z < d/2 \tag{7}$$

$$\mathcal{H}_z^{III} = \left(\frac{2}{\alpha}\right)^{1/2} f \left\{ \cosh[q(z - z_2)] \right\}^{-1}, \quad z > d/2$$

where $f = \frac{\varepsilon_0 q k_0}{k^2}$, $k_1 = (k^2 - \varepsilon_1 k_0^2)^{1/2}$, $k_2 = (k_0^2 \varepsilon_1 - k^2)^{1/2}$.

For TM waves E_1 and D_3 are continuous across the interfaces $z = \pm d/2$. From the boundary conditions we are left with an equation for the unknown z_0

$$\left\{1 - b_1^2 \tanh^2[k_1(\tfrac{d}{2} - z_0)]\right\} \cdot \cosh^2[k_1(\tfrac{d}{2} + z_0)] = \\ = \left\{1 - b_1^2 \tanh^2[k_1(\tfrac{d}{2} + z_0)]\right\} \cdot \cosh^2[k_1(\tfrac{d}{2} - z_0)] \tag{8}$$

where $b_1 = \frac{\varepsilon_0}{\varepsilon_1} \frac{k_1}{q}$.

Eq.(8) has the unique solution $z_0 = 0$ for all $n > n_1$. The solution (7) for which $z_0 = 0$ corresponds to the symmetric mode (S) of our symmetric layered structure. In this case $z_1 = -z_2$ and the dispersion relations for the symmetric mode are

$$\tanh[q(\tfrac{d}{2} + z_1)] = -b_1 \tanh(k_1 \tfrac{d}{2}), \quad n > n_1 \tag{9a}$$

$$\tanh[q(\tfrac{d}{2} + z_1)] = b_2 \tan(k_2 \tfrac{d}{2}), \quad n < n_1 \tag{9b}$$

where $b_2 = \frac{\varepsilon_0}{\varepsilon_1} \frac{k_2}{q}$.

We remark that if $\alpha \to 0$, then $z_1 \to +\infty$ and Eq.(9b) reduces to the dispersion relation for the symmetric modes of the symmetric dielectric waveguide. For the non-linear symmetric mode the amplitude of the magnetic field inside the film is given by

$$A_1^2 = \frac{2}{\alpha} f^2 \cosh^{-2}(k_1 \tfrac{d}{2})[1 - b_1^2 \tanh^2(k_1 \tfrac{d}{2})], \quad n > n_1$$
$$A_2^2 = \frac{2}{\alpha} f^2 \cos^{-2}(k_2 \tfrac{d}{2})[1 - b_2^2 \tan^2(k_2 \tfrac{d}{2})], \quad n < n_1 \tag{10}$$

Next we obtain the dispersion relations for the antisymmetric mode (AS). For this purpose we write down the second solution of Maxwell's equations (6) inside the film

$$\mathcal{H}_z^{II}(z) = \begin{cases} B_1 \sinh[k_1(z - z_0)], & n > n_1 \\ B_2 \sin[k_2(z - z_0)], & n < n_1 \end{cases} \tag{11}$$

In this case the equation for the unknown z_0 has the form

$$\left\{1 - b_1^2 \coth^2[k_1(\tfrac{d}{2} - z_0)]\right\} \cdot \sinh^2[k_1(\tfrac{d}{2} + z_0)] = \\ = \left\{1 - b_1^2 \coth^2[k_1(\tfrac{d}{2} + z_0)]\right\} \cdot \sinh^2[k_1(\tfrac{d}{2} - z_0)] \tag{12a}$$

for $n > n_1$ and

$$\left\{1 - b_2^2 \cot^2[k_2(\tfrac{d}{2} - z_0)]\right\} \cdot \sin^2[k_2(\tfrac{d}{2} + z_0)] = \\ = \left\{1 - b_2^2 \cot^2[k_2(\tfrac{d}{2} + z_0)]\right\} \cdot \sin^2[k_2(\tfrac{d}{2} - z_0)] \tag{12b}$$

for $n < n_1$.

It is easily verified that Eqs.(12a) and (12b) have the solution $z_0 = 0$. This solution corresponds to the antisymmetric mode (AS) of the symmetric layered structure. We remark that Eqs.(12a) and (12b) also have solutions $z_0 \neq 0$ (asymmetric mode).

From the boundary conditions we get the following dispersion relations for the antisymmetric mode

$$\tanh[q(\tfrac{d}{2} + z_1)] = -b_1 \coth(k_1 \tfrac{d}{2}), \quad n > n_1 \tag{13a}$$

$$\tanh\left[q\left(\frac{d}{2}+z_1\right)\right] = -b_2 \cot\left(k_2 \frac{d}{2}\right), \quad n < n_1 \tag{13b}$$

If $\alpha \to 0$, then $z_1 \to +\infty$ and from Eq.(13b) we obtain the dispersion relation for the antisymmetric modes of the linear dielectric waveguide.

The amplitudes B_1 and B_2 of the magnetic field inside the linear medium in the case of antisymmetric mode are given by

$$B_1^2 = \frac{2}{\alpha} f^2 \sinh^{-2}\left(\kappa_1 \frac{d}{2}\right)\left[1 - b_1^2 \coth^2\left(\kappa_1 \frac{d}{2}\right)\right], \quad n > n_1 \tag{14}$$

$$B_2^2 = \frac{2}{\alpha} f^2 \sin^{-2}\left(k_2 \frac{d}{2}\right)\left[1 - b_2^2 \cot^2\left(k_2 \frac{d}{2}\right)\right], \quad n < n_1$$

III. THE POWER FLOW IN THE NON-LINEAR WAVE

The time averaged power flow in the x-direction per unit width in the y direction can be expressed as

$$\mathcal{P} = -\frac{c}{8\pi} \int_{-\infty}^{\infty} Re\,(E_3 H_2^*)\,dz \tag{15}$$

or

$$\mathcal{P} = \frac{c}{8\pi}\frac{\kappa}{k_0}\left[\frac{1}{\varepsilon_0}\int_{-\infty}^{\infty} \mathcal{H}_2^2(z)\,dz - \frac{\alpha}{\varepsilon_0^3}\left(\frac{\kappa}{k_0}\right)^2 \int_{-\infty}^{\infty} \mathcal{H}_2^4(z)\,dz\right] \tag{16}$$

Making use of Eqs.(10) we find the power flow carried by non-linear symmetric mode (S)

$$\mathcal{P} = \mathcal{P}_0 \frac{\kappa}{q}\frac{4f^2}{\varepsilon_0}\left\{1 - \frac{2}{\varepsilon_0^2}\left(\frac{\kappa}{k_0}\right)^2 f^2\left[1 - \frac{1}{3}\left(1 + r_1 + r_1^2\right)\right]\right\}(1 - r_1) +$$
$$+ \mathcal{P}_0 \frac{f^2}{\varepsilon_0}\left(1 - r_1^2\right)\cosh^{-2}\left(\kappa_1 \frac{d}{2}\right) \times \tag{17}$$
$$\times\left[\left(\kappa d + \frac{\kappa}{\kappa_1}\sinh \kappa_1 d\right) - \frac{1}{\varepsilon_0^2}\left(\frac{\kappa}{k_0}\right) f^2 \cosh^{-2}\left(\kappa_1 \frac{d}{2}\right)\left(1 - r_1^2\right)\left(\frac{1}{4}\kappa d + \frac{\kappa}{2\kappa_1}\sinh \kappa_1 d + \frac{\kappa}{16\kappa_1}\sinh 2\kappa_1 d\right)\right]$$

for $n > n_1$, where $P_0 = \frac{c}{8\pi\ \alpha\ k_0}$ and $r_1 = -b_1 \tan h(k_1 \frac{d}{2})$.

$$\mathcal{P} = \mathcal{P}_0 \frac{\kappa}{q}\frac{4f^2}{\varepsilon_0}\left\{1 - \frac{2}{\varepsilon_0^2}\left(\frac{\kappa}{k_0}\right)^2 f^2\left[1 - \frac{1}{3}\left(1 + r_2 + r_2^2\right)\right]\right\}(1 - r_2) +$$
$$+ \mathcal{P}_0 \frac{f^2}{\varepsilon_0}\left(1 - r_2^2\right)\cos^{-2}\left(k_2 \frac{d}{2}\right) \times \tag{18}$$
$$\times\left[\left(\kappa d + \frac{\kappa}{k_2}\sin k_2 d\right) - \frac{1}{\varepsilon_0^2}\left(\frac{\kappa}{k_0}\right) f^2 \cos^{-2}\left(k_2 \frac{d}{2}\right)\left(1 - r_2^2\right)\left(\frac{1}{4}\kappa d + \frac{\kappa}{2k_2}\sin k_2 d + \frac{\kappa}{16k_2}\sin 2k_2 d\right)\right]$$

for $n < n_1$, where $r_2 = b_2 \tan(k_2 \frac{d}{2})$.

Eqs.(17) and (18) give us the dependence $\omega = \omega(k,P)$, i.e., the dispersion relation for the non-linear symmetric mode (S). (for $P=0$, Eq.(18) gives $1-r_2=0$, i.e., We see that the dispersion relation of TM-polarized symmetric modes of the linear waveguide.

In a similar manner, making use of Eqs.(14) we obtain the power flow in the non-linear mode (AS)

$$\mathcal{P} = \mathcal{P}_0 \frac{\kappa}{q}\frac{4f^2}{\varepsilon_0}\left\{1 - \frac{2}{\varepsilon_0^2}\left(\frac{\kappa}{k_0}\right)^2 f^2\left[1 - \frac{1}{3}\left(1 + t_1 + t_1^2\right)\right]\right\}(1 - t_1) +$$
$$+ \mathcal{P}_0 \frac{f^2}{\varepsilon_0}\left(1 - t_1^2\right)\sinh^{-2}\left(\kappa_1 \frac{d}{2}\right) \times \tag{19}$$
$$\times\left[\left(-\kappa d + \frac{\kappa}{\kappa_1}\sinh \kappa_1 d\right) - \frac{1}{\varepsilon_0^2}\left(\frac{\kappa}{k_0}\right) f^2 \sinh^{-2}\left(\kappa_1 \frac{d}{2}\right)\left(1 - t_1^2\right)\left(\frac{1}{4}\kappa d - \frac{\kappa}{2\kappa_1}\sinh \kappa_1 d + \frac{\kappa}{16\kappa_1}\sinh 2\kappa_1 d\right)\right]$$

for $n > n_1$, where $t_1 = b_1 \cot h(k_1 \frac{d}{2})$.

$$
\begin{aligned}
P = {} & P_0 \frac{\kappa}{q} \frac{4f^2}{\varepsilon_0}\left\{1 - \frac{2}{\varepsilon_0^2}\left(\frac{\kappa}{k_0}\right)^2 f^2 \left[1 - \frac{1}{3}\left(1+t_2+t_2^2\right)\right]\right\}(1-t_2) + \\
& + P_0 \frac{f}{\varepsilon_0}\left(1-t_2^2\right)\sin^{-2}\left(k_2 \frac{d}{2}\right) \times \\
& \times \left[\left(-\kappa d + \frac{\kappa}{k_2}\sin k_2 d\right) - \frac{1}{\varepsilon_0^2}\left(\frac{\kappa}{k_0}\right) f^2 \sin^{-2}\left(k_2 \frac{d}{2}\right)\left(1-t_2^2\right)\left(\frac{1}{4}\kappa d - \frac{\kappa}{2k_2}\sin k_2 d + \frac{\kappa}{16k_2}\sin 2k_2 d\right)\right]
\end{aligned}
\tag{20}
$$

for $n < n_1$, where $t_2 = -b_2 \cot(k_2 \frac{d}{2})$.

We remark that for $P = 0$, Eq.(20) gives $1 - t_2 = 0$, i.e., the dispersion relation of TM polarized antisymmetric modes of a linear symmetric waveguide.

IV. CONCLUSIONS

We conclude with a few comments about the results we have obtained in this paper.

We showed that the more realistic case of a dielectric tensor whose normal component is proportional to the square of the normal electric field component [20] and not to the parallel electric field component as assumed previously [21] is solvable to a good approximation in terms of magnetic field components. The physics derived from these solutions is similar to that obtained from the exact solvable TE case [12].

Thus for $\alpha > 0$ (self-focusing medium) there exist three different modes in the non-linear symmetric three-component layered structure: the symmetric, antisymmetric and asymmetric modes. For d/λ sufficiently large (λ is the wavelength) we expect bistable behaviour of both symmetric and antisymmetric modes, i.e., to a fixed value of the power flow there correspond two stable values of the propagation constant $n = k/k_0$. The state with smaller value of n is related to the non-linear wave whose power flow is confined inside the thin film and the state with larger value of n corresponds to the non-linear wave whose power flows mainly outside the waveguiding layer.

We note that the asymmetric mode which may exist only at power flow exceeding some threshold value has no counterpart in the corresponding linear optics of surface and guided waves.

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