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CALCULATION OF EXCITED VECTOR MESON ELECTRON WIDTHS USING QCD SUM RULES

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A b s t r a c t

The sum rules are suggested which allow one to calculate the electron widths of excited vector mesons of the ψ, Υ, ρ meson family assuming the values of their masses to be known. The calculated values of the electron widths agree with experiment.

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I. Introduction

The purpose of this work is to obtain in QCD the sum rules which allow one to calculate the electron widths of vector mesons from the ψ , Υ , ζ meson family assuming their masses to be known. The sum rules connected with the expansion in negative momenta[1,2] allow one, in principle, to calculate only the ground state widths. The sum rules connected with the expansion in positive momenta[3,4] contain excited vector mesons but if these sum rules are considered as equations for determining the electron widths, then because of insufficient exactness of these equations it was impossible to calculate the electron widths. In this paper I shall obtain the sum rules which make it possible to determine the electron widths of excited vector mesons of the ψ , Υ , ζ families assuming their masses to be known. The paper is arranged as follows. In the second Section I obtain in nonrelativistic quantum mechanics the sum rules which allow one to calculate the value $|\psi_k(0)|^2$ from the known values of the energy levels E_k . We shall consider the potential providing confinement ($V(r) \to \infty$ at $r \to \infty$) and asymptotic freedom ($V(0) = 0$). The consideration will be such that the going over to QCD will not encounter with difficulties but will be done by a simple re-denotion of the corresponding quantities. In the third Section the sum rules for QCD will be obtained. Using these sum rules we obtain the formulae which express the electronic widths of excited vector mesons versus their masses and calculate according to these formulae the electron widths of the mesons in view. In Conclusion the electron widths of the Υ family will be obtained with the help of a graph and the elect-

ron widths of the Υ meson ground state (9,46) will be also obtained.

2. Sum Rules in Nonrelativistic Quantum Mechanics

In this Section we obtain with a methodical purpose the sum rules in nonrelativistic quantum mechanics. These sum rules will allow us to calculate the value of the excited wave function at zero $|\psi_\kappa(0)|^2$ according to the known energy levels E_k. It will appear that the developed in this Section formalism can easily be transferred onto QCD. The sum rules in nonrelativistic quantum mechanics applying to the ground state was considered in ref.[5]. Bearing in mind the asymptotic freedom in QCD and quark confinement, we shall consider the motion in a spherically-symmetrical infinitely increasing at $r\to\infty$ potential. At the origin of coordinates the potential is zero, $V(0)=0$.

Consider the spectral representation for the time Green function at imaginary time and coinciding spatial coordinates

$$M(\tau)=\sum_{\kappa=0}^{\infty}|\psi_\kappa(0)|^2 e^{-E_\kappa\tau} \qquad (1)$$

In the considered potential at small τ function $M(\tau)$ must go over into the free Green function

$$M_0(\tau)=\left(\frac{m}{2\pi\tau}\right)^{3/2} \qquad (2)$$

At small τ the summation in formula (1) can be replaced by integration and it can be written

$$\int_0^\infty |\psi_\kappa(0)|^2 e^{-E_\kappa \tau} \frac{d\kappa}{dE_\kappa} dE_\kappa = \left(\frac{m}{2\pi\tau}\right)^{3/2} \qquad (3)$$

If eq.(3) is considered as an integral equation for the function $|\psi_\kappa(0)|^2 \frac{d\kappa}{dE_\kappa}$ then this equation can easily be solved and with the found solution $|\psi_\kappa(0)|^2$ can be expressed via the derivative

$$|\psi_\kappa(0)|^2 = \frac{m^{3/2}}{\sqrt{2}\,\pi^2} E_\kappa^{1/2} \frac{dE_\kappa}{d\kappa} \qquad (4)$$

Hereafter we assume that the energy E_k considered as a function of the number k is a smooth function of variable k, so that it has a sufficient number of derivatives with respect to k . Eq.(4) in quasiclassical approximation was obtained in refs. [6-8]. It is evident that at the above derivation the validity of eq.(4) has been proved only for large k . At the first sight it seems that formulae (3),(4) can be easily specified if the first correction of perturbation theory will be added to the r.h.s. of eq.(3). In fact, this is not so and it can be easily seen on an example of harmonic oscillator. If we take into account the first perturbation correction, then $M_0(\tau)$ will be replaced by $M_{p.t.} = M_0(\tau)\left(1 - \frac{1}{4}\omega^2\tau^2\right)$ [5] but eq.(3) with such a right-hand side has no solution at all. For defining the accuracy of eq.(4) applying it to small k we considered the Schrödinger equation with the following potentials:

(1) harmonic oscillator $V(r) = \frac{m\omega^2 r^2}{2}$

(2) a box with infinite walls $V(r) = \begin{cases} 0, & r < R \\ \infty, & r > R \end{cases}$

(3) linearly increasing potential.

For these potentials the analytical expressions for E_k and $|\psi_k(0)|^2$ are known. Therefore the accuracy of eq.(4) can be verified on an example of these three potentials. It appeared that $|\psi_k(0)|^2$ determined with respect to eq.(4) differ from the exact values very little. So, for the ground states k=0 the accuracy of eq.(4) is $\lesssim 2\%$. For excited states $k > 0$ the accuracy of eq.(4) is portions of per cent. Notice that though eq.(4) is proved for large k, it is not a quasiclassical formula. So, if one takes for E_κ and $|\psi_\kappa(0)|^2$ their quasiclassical values, then at small k the r.h.and l.h. sides of formula (4) coincide only by the order of magnitude. Let us determine the aproximated Green function by

$$\tilde{M}(\tau)=\frac{\mu^{3/2}}{\sqrt{2}\,\pi^2}\sum_{\kappa=0}^{\infty} E_\kappa^{3/2} E_\kappa^{(2)} e^{-E_\kappa \tau} \qquad (5)$$

where we introduce the notation $E_\kappa^{(2)}=\frac{\partial^2 E_\kappa}{\partial \kappa^2}$.

It should be expected from the considered examples that the approximated Green function $\tilde{M}(\tau)$ coincides with the exact one up to a few per cent at any τ. Therefore everywhere hereafter we shall consider the function $\tilde{M}(\tau)$ and find $E_\kappa^{(2)}$ related to $|\psi_\kappa(0)|^2$ by formula (4) from the known E_k. The free Green function is obtained from $\tilde{M}(\tau)$ if summation in formula (5) is replaced by integration. The fact that at not too small τ the exact Green function differs from the free one is due to that when the sum in formula (1) is replaced by integral, a number of terms which should be taken into account, are neglected.

When replacing summation by integration we will use the

well-known Euler Macloren formula (E.M.F.) which will be written as

$$\sum_{k=p}^{m} F(k) = \int_{p}^{m} F(t)dt + \frac{1}{2}\left(F(m)+F(p)\right) + \sum_{k=1}^{n-1} \frac{B_{2k}}{(2k)!}\left(F^{(2k-1)}(m) - F^{(2k-1)}(p)\right) + R \qquad (6)$$

where

$$F^{(e)}(k) = \frac{\partial^e}{\partial k^e} F(k) \qquad (7)$$

B_{2K} are Bernulli numbers, R is the E.M.F. residual term which can be written as

$$R = \frac{B_{2n}}{(2n)!} \sum_{k=p}^{m} F^{(2n)}(k+\theta) \qquad (8)$$

$0 \le \theta \le 1$. The first five Bernulli numbers are equal to $B_0=1$, $B_1=-1/2$, $B_2=1/6$, $B_4=-1/30$, $B_6=1/42$. All the Bernulli numbers from the odd number are zero, $B_{2k+1} = 0$, except for B_1 . To derive the E.M.F. it is necessary for the function F(k) to have 2n continuous derivatives in k . The E.M.F. will effectively work if these derivatives are not too large. Let us emphasize that the E.M.F. contains finite number of terms and is exact.

With the help of the E.M.F. formula (5) can be written as

$$\tilde{M}(\tau) = M_0(\tau) + \Delta M(\tau) \qquad (9)$$

where

$$\Delta M(\tau)=\frac{\mu^{3/2}}{\sqrt{2}\,\pi^2}\left(-\int_0^{E_0} e^{-\tau E}\sqrt{E}\,dE+\frac{1}{2}E_0^{(1)}\sqrt{E_0}\,e^{-\tau E_0}-\right.$$

$$\left.-\frac{1}{12}F'(0)+\frac{1}{720}F'''(0)-\frac{1}{30240}F^{(V)}(0)+\dots\right) \qquad (10)$$

$$M_0(\tau)=\frac{\mu^{3/2}}{\sqrt{2}\,\pi^2}\int_0^{\infty} e^{-\tau E}\sqrt{E}\,dE=\left(\frac{\mu}{2\pi\tau}\right)^{3/2} \qquad (11)$$

Here $F(\kappa)=E_\kappa^{(1)}\sqrt{E_\kappa}\,e^{-\tau E_\kappa}$

Note that the extra term $\Delta M(\tau)$ has no direct relationship with the power corrections of perturbation theory but is a correction arising when summation in formula (5) is replaced by integration. So, if the exact solution of the Schrödinger equation is known as for three considered potentials, the quantities $E_n^{(e)}$ entering the r.h.sides of the sum rules are known exactly, i.e. with taking account of all the orders of perturbation theory. If the exact solution of the Schrödinger equation is unknown, then $E_n^{(e)}$ should be considered as unknown parameters. Our aim is to write such sum rules with no unknown parameters $E_n^{(e)}$ at all.

Let us known obtain the sum rules from which $E_n^{(1)}$ can be calculated with respect to the known energy levels. Form the set of sums and apply the E.M.P. to them

$$\sum_{\kappa=n}^{\infty} F(\kappa)=\int_{E_n}^{\infty} F(E)\,dE+\frac{1}{2}F(n)-\frac{1}{12}\frac{d}{dn}F(n)+\frac{1}{720}\frac{d^3}{dn^3}F(n)- \qquad (12)$$

$$n=0,1,2,\dots.$$

where

$$F(\kappa) = \ell(E_\kappa) E_\kappa^{(1)}$$

$\ell(E_\kappa)$ is the weight function which will be chosen such that the terms with unknown numbers $E_n^{(e)}$ ($e > 1$) will drop out from the sum rules, or at least that they would enter with very small coefficients. The simplest such function is

$$\ell(E_\kappa) = e^{-\tau^2 (E_\kappa - E_n)^2} (E_\kappa - E_n)^2, \quad n = 0, 1, 2, \ldots \quad (13)$$

Substitute the weight function (13) into formula (12). After simple manipulations we get

$$\sum_{\kappa=n-1}^{\infty} e^{-\tau^2 (E_\kappa - E_{n-1})^2} (E_\kappa - E_{n-1})^2 E_\kappa^{(1)} = \frac{\sqrt{\pi}}{4\tau^3} + \frac{1}{60} E_{n-1}^{(1)^2} E_{n-1}^{(2)} \quad (14)$$

we neglect the rest terms in the r.h.s. of eq.(14) since there are quite small coefficients at them.

Introduce instead of variable τ the new dimensionless variable

$$x = \tau^2 (E_n - E_{n-1})^2 \quad (15)$$

and notations

$$a_{\kappa n} = \left(\frac{E_\kappa - E_{n-1}}{E_n - E_{n-1}} \right)^2 \quad (16)$$

$$b_{kn} = \frac{E_k^{(2)}}{E_n - E_{n-1}}\, a_{kn} \qquad (17)$$

Since $E_{n-1}^{(2)} \sim E_{n-1}^{(1)} \sim E_n - E_{n-1}$, the term $E_{n-1}^{(2)^2} E_{n-1}^{(2)}/60$ can be neglected. As a result, eq.(14) in new variables takes the form

$$\varphi_n(x) \equiv \sum_{k=n}^{\infty} e^{-a_{kn}x} b_{kn} = \frac{\sqrt{\pi}}{4x^{3/2}} \qquad (18)$$

At small x in the sum (18) many terms are essential, at large x the first term is essential but at very large x , the boundary terms should not be neglected and therefore it is invalid. Nevertheless, there are such x's at which the first term in the sum (18) can be remained while the r.h. and l.h.sides in eq.(18) will be equal with a reasonable accuracy ($\sim$ 10÷20%).

Let us remain the first term (k=n) in the sum (18) and write (18) as

$$x^{3/2} e^{-x} b_{nn} = \sqrt{\pi}/4 \qquad (19)$$

The r.h.s. of (19) is x-independent, let us find at which x the l.h.s. of eq.(19) will be also x-independent. To this end, differentiate eq.(19) over x . At x=3/2 the derivative from the l.h.s. of eq.(19) is zero. At x=3/2, if it is assumed that $E_n^{(2)} \sim E_n^{(2)} \sim E_n - E_{n-1}$ the term $E_{n-1}^{(2)^2} E_{n-1}^{(2)}/60$ neglected in eq.(14) is $\sim$ 7% from the ground term $\sqrt{\pi}/4x^{3/2}$. Substituting x=3/2 into (19) we find in the zero approximation the value $E_n^{(2)}$

$$\left(E_n^{(1)}\right)_0 = \frac{\sqrt{\pi}}{3\sqrt{6}}\, e^{3/2}\cdot (E_n - E_{n-1}) \qquad (20)$$

Eq.(20) for $E_n^{(1)}$ can be easily specified if one takes into account the next level. For this, determine $E_{n+1}^{(1)}$ in the zero approximation using eq.(20) and remain two first terms in eq.(18). As a result we get

$$E_n^{(1)} = \left(E_n^{(1)}\right)_0 (1 - \delta_n) \qquad (21)$$

$$\delta_n = \left(\frac{E_{n+1} - E_{n-1}}{E_n - E_{n-1}}\right)^2 \left(\frac{E_{n+1} - E_n}{E_n - E_{n-1}}\right) e^{-\frac{3}{2}\left[\left(\frac{E_{n+1} - E_{n-1}}{E_n - E_{n-1}}\right)^2 - 1\right]} \qquad (22)$$

Let us test formulae (21),(22) on potentials for which the solution of the Schrödinger equation is known:

(1) harmonic oscillator $V(z) = \frac{\mu\omega^2 z^2}{2}$

The eqs.(21),(22) calculation gives $\delta_n = 0.044$, $(E_n^{(1)})_0 = 1{,}08 \cdot 2\omega$, $(E_n^{(1)})_1 = 1{,}03 \cdot 2\omega$. The exact value $E_n^{(1)} = 2\omega$.

(2) The linearly rising potential $V(z) = f \cdot z$

$\delta_1 = 0.085$, $(E_1^{(1)})_0 = 1.89 \left(\frac{f^2}{2\mu}\right)^{2/3}$, $(E_1^{(1)})_1 = 1.73 \left(\frac{f^2}{2\mu}\right)^{2/3}$

The exact value $E_1^{(1)} = 1.56 \left(f^2/2\mu\right)^{2/3}$. For higher levels the accuracy is better.

(3) a box with infinite walls $V(z) = \begin{cases} 0, & z < R \\ \infty, & z > R \end{cases}$

$\delta_1 = 0.0012$, $(E_1^{(1)})_1 \cong (E_1^{(0)}) = 3.24 \frac{\pi}{2\mu R^2}$

The exact value $E_1^{(1)} = 4 \frac{\pi^2}{2\mu R^2}$. For higher levels the accuracy is better.

A rather low accuracy of eqs.(21),(22) apllying to the

potential "a box with infinite walls" is because of the fact that in this potential the contribution of higher levels is very small, and therefore the higher derivatives are rather large. Other weight functions, for example $f(E_\kappa) = e^{-\tau^2(E_\kappa - E_n)^2} \cdot (E_\kappa - E_n)^3$ $f(E_\kappa) = e^{-\tau(E_\kappa - E_n)} \cdot (E_\kappa - E_n)^3$ were also considered. The results are weakly dependent of the weight function. Nevertheless, it should be mentioned that the weight function $f(E_\kappa) = e^{-\tau(E_\kappa - E_n)} \cdot (E_\kappa - E_n)^3$ is better for the box with infinite walls ($\delta_1 = 0.04$, $E_n^{(1)} = 3{,}68 \frac{\pi^2}{2\mu R^2}$). It seesm that the best weight function is that at which the contribution of the next level has the order of 10÷20 from the lowest level.

3. Sum Rules in QCD.

In this Section we obtain the sum rules in QCD which allow us to calculate the electron widths of excited vector mesons (ψ , γ , φ meson family). For definiteness, we will consider the ψ meson family. Consider the polarization operator connected with the charmed quark current $\Pi_c(s)$ and write the dispersion relation for $\Pi_c(s)$ as [4]

$$\Pi_c(s) = \frac{s^2}{\pi} \int_{4m_c^2}^{\infty} \frac{\mathrm{Im}\,\Pi_c(s')\,ds'}{(s'-s)\,s'^2} \qquad (23)$$

where

$$\mathrm{Im}\,\Pi_c(s) = \frac{1}{3}\alpha \cdot s \cdot R_c(s) \qquad (24)$$

We consider the approximation of infinite number of narrow resonances with the masses M_n , and the electron widths Γ_n^{ee}.

In this approximation

$$R_c(s) = \frac{9\pi}{\alpha^2} \sum_{n=0}^{\infty} \Gamma_n^{ee} M_n \delta(s - M_n^2) \qquad (25)$$

In the following the parameters in the sum rules will be chosen such that they would be sensitive only to the low lying resonances which are narrow. One should not be embarassed that the resonant structure at $s \gtrsim (4.5)^2$ GeV^2 is not seen because resonances at these s expand and fuse into a fluent curve. The polarization operator $\Pi_c(s)$ in this approximation will be written as

$$\Pi_c(s) = \frac{s^2}{\pi} \cdot \frac{1}{3} \alpha \sum_{\kappa=0}^{\infty} F(\kappa) \qquad (26)$$

Here

$$F(\kappa) = \frac{9\pi}{\alpha^2} \Gamma_\kappa^{ee} \frac{1}{(M_\kappa^2 - s) M_\kappa} \qquad (27)$$

Transform the sum (26) into integral with E.M.F.:

$$\sum_{\kappa=0}^{\infty} F(\kappa) = \int_0^{\infty} F(\kappa) d\kappa + \frac{1}{2} F(0) - \frac{1}{12} F'(0) + \frac{1}{720} F'''(0) - \cdots \qquad (28)$$

where the derivatives in the r.h.s. of eq.(28) are taken with respect to k at k=0. We assume that the mass of the k-th resonance considered as a function of k is continuous and has a sufficient number of derivatives with respect to k.

Write the integral term in eq.(28) as

$$\int_0^\infty F(\kappa)\,d\kappa = \int_{M_0}^\infty \frac{9\pi}{\alpha^2}\,\Gamma_\kappa^{ee}\,\frac{1}{(M_\kappa^2 - s)M_\kappa}\,\frac{d\kappa}{dM_\kappa}\,dM_\kappa \tag{29}$$

M_0 is the lowest resonance mass.

At $s \to -\infty$, $\Pi_c(s) \to \Pi_c^{(0)}(s)$ where

$$\Pi_c^{(0)}(s) = \frac{s^2}{\pi}\,\frac{1}{3}\,\alpha \int_{M_0^2}^\infty \frac{R_c^{(0)}(s')\,ds'}{(s'-s)\,s'} \tag{30}$$

and

$$R_c^{(0)}(s) = \frac{3}{2}\,Q_c^2 \sqrt{1 - \frac{4m_c^2}{s}}\left(2 + \frac{4m_c^2}{s}\right). \tag{31}$$

$Q_c = 2/3$ is the c-quark charge. The asymptotic behaviour of the polarization operator $\Pi_c(s)$ at large negative s is determined by the integral term (29) and therefore if one introduces the notation

$$\chi(M_\kappa) = \frac{9\pi}{\alpha^2}\,\frac{\Gamma_\kappa^{ee}}{M_\kappa}\,\frac{d\kappa}{dM_\kappa} \tag{32}$$

then at large s the function $\chi(M_\kappa)$ must satisfy the integral equation

$$\int_{M_0^2}^\infty \frac{\chi(M_\kappa)\,dM_\kappa}{M_\kappa^2 - s} = \int_{4m_c^2}^\infty \frac{R_c^{(0)}(s')\,ds'}{(s'-s)\,s'} \tag{33}$$

The solution of eq.(33) allows one to express the electron width Γ_κ^{ee} via the derivative $\mathcal{M}_\kappa^{(1)} \equiv \frac{d\mathcal{M}_\kappa}{d\kappa}$

$$\Gamma_\kappa^{ee} = \frac{2\alpha^2}{9\pi} R_c^{(0)}(\mathcal{M}_\kappa^2)\mathcal{M}_\kappa^{(1)} \qquad (34)$$

Eq.(34) was obtained in quasiclassical approximation in ref.[4]. All the following calculations are performed analogously to quantum mechanics, one should only replace E_κ by $\mathcal{M}_\kappa$, $|\psi_\kappa(0)|^2$ by Γ_κ^{ee}, $E_\kappa^{(e)}$ by $\mathcal{M}_\kappa^{(e)} \equiv \frac{d^e\mathcal{M}_\kappa}{d\kappa^e}$. The resultant sum rules are

$$\phi_n(x) = \frac{\sqrt{\pi}}{4x^{3/2}}, \qquad n = 1, 2, \ldots \qquad (35)$$

where

$$\phi_n(x) = \sum_{\kappa=n}^{\infty} e^{-a_{\kappa n} x} b_{\kappa n}, \qquad n = 1, 2, \ldots \qquad (36)$$

$$a_{\kappa n} = \left(\frac{\mathcal{M}_\kappa - \mathcal{M}_{n-1}}{\mathcal{M}_n - \mathcal{M}_{n-1}}\right)^2 \qquad (37)$$

$$b_{\kappa n} = \frac{\mathcal{M}_\kappa^{(1)}}{\mathcal{M}_n - \mathcal{M}_{n-1}} a_{\kappa n} \qquad (38)$$

Replacing $E_n \to \mathcal{M}_n$ by $E_n^{(1)} \to \mathcal{M}_n^{(1)}$ in formulae (22),(24) we obtain for $\left(\mathcal{M}_\kappa^{(1)}\right)_{th.}$ the following formulae

$$\left(\mathcal{M}_n^{(1)}\right)_{th.} = \frac{\sqrt{\pi}}{3\sqrt{6}} e^{3/2}(1-\delta_n)(\mathcal{M}_n - \mathcal{M}_{n-1}) \qquad (39)$$

$$\delta_n = \left(\frac{M_{n+1} - M_{n-1}}{M_n - M_{n-1}}\right)^2 \left(\frac{M_{n+1} - M_n}{M_n - M_{n-1}}\right) e^{-\frac{3}{2}\left[\left(\frac{M_{n+1} - M_{n-1}}{M_n - M_{n-1}}\right)^2 - 1\right]} \qquad (40)$$

The experimental data taken from refs. [10,12] for ψ and Υ families are given in Table I. Making use of the mass values we get:

for the ψ -family

$$\delta_1 = 0.12, \quad \delta_2 = 2{,}8 \cdot 10^{-9}, \quad \delta_3 = 0{,}17, \quad \delta_4 = 1{,}2 \cdot 10^{-4} \qquad (41)$$

for the Υ family

$$\delta_1 = 0{,}15, \quad \delta_2 = 0{,}13, \quad \delta_3 = 8{,}6 \cdot 10^{-3}, \quad \delta_4 = 0{,}17 \qquad (42)$$

The values $(M_n^{(1)})_{th.}$ for both families are given in Table I. Note that if a higher resonance is unknown, the corresponding δ_n is equalled to zero. In this case $(M_n^{(1)})_{th.}$ and the electron width $(\Gamma_{ee}^n)_{th.}$ seems to be overestimated. Perhaps, this is due to the overestimated value $(\Gamma_{ee}^n)_{th.}$ for ψ and Υ mesons comparing to experimental values $(\Gamma_{ee})_{exp.}$ for these mesons.

The going over to the Υ -family in theoretical formulae is obtained by replacing $R_c^{(0)} \to R_b^{(0)}$, $Q_c \to Q_b = 1/3$, $m_c \to m_b$. To calculate the electron widths of excited ψ and Υ families one should know the masses of c- and b-quarks. The quantities $R_c^{(0)}$ and $R_b^{(0)}$ contain c- and b-quark masses with no radiation correction. This is connected with the fact that the sum rules contain physical masses of mesons and therefore we assume that all the radiation corrections enter the values of these masses and the values $M_0^{(2)}$ in the r.h.s. of the sum rules. For the c-quark mass such a mass is calculated in ref. [1] and for it $m_c = 1.25$ GeV is obtained. This value was exploited in all formu

lae. Consequently, in the calculation of the electron widths of excited ψ family free parameters were absent.

The b-quark mass determined analogously is uknown and we determined it in a way for the calculated electron width of Υ meson (Γ_{ee})$_{th}$ to coincide with its experimental value $(\Gamma_{ee})_{exp}$. For the b-quark mass it was obtained m_b = (4.1±0.3) GeV. This value of the b-quark mass entered all formulae. The usually given mass values m_c=1.4 GeV and m_b=4.8 GeV[11] differ from those of this work because they were calculated taking into account radiation corrections. The nonrelativistic quantum mechanics experience of the sum rule treating (Sec.2) shows that perturbation corrections should not be taken into account to avoid double counting. At a chosen mass m_b=4.1 GeV the quantity $R_b^{(0)}(\mathcal{M}_n^2)$ weakly depends on n , varying from $R_b^{(0)}(\mathcal{M}_0^2) = 0.23$ to $R_b^{(0)}(\mathcal{M}_5^2) = 0.28$. Table I presents also the values $(\mathcal{M}_n^{(1)})_{exp}$ calculated according to experimental data (Γ_{ee}^n) using formula (34) at given mass values m_c=1.25 GeV and m_b=4.1 GeV. Note that quite a simple formula $\mathcal{M}_n^{(1)} = \mathcal{M}_n - \mathcal{M}_{n-1}$ reasonably describes the experiment. The values calculated according to this formula for the $\mathcal{M}_n^{(1)}$/GeV ψ family are the following:

$\mathcal{M}_1^{(1)} = 0.59$, $\mathcal{M}_2^{(1)} = 0.084$, $\mathcal{M}_3^{(1)} = 0.26$, $\mathcal{M}_4^{(1)} = 0.13$, $\mathcal{M}_5^{(1)} = 0.26$.

For the Υ -family

$\mathcal{M}_1^{(1)} = 0.56$, $\mathcal{M}_2^{(1)} = 0.33$, $\mathcal{M}_3^{(1)} = 0.22$, $\mathcal{M}_4^{(1)} = 0.29$, $\mathcal{M}_5^{(1)} = 0.15$.

The obtained formulae can be used for calculating the ζ_2 meson electron width. For this $R_c^{(0)} \to R_\zeta^{(0)} = 3/2$ should be replaced. The light quark mass can be neglected. The experimental

values of the Υ_4 and Υ_5 masses and electron widths are taken from transparencies of the Silvermann's rapporteur talk[12] at the Leipzig Conference on high energy physics. The experimental values of masses and electron widths of all the rest mesons (except for φ_1 -meson) are taken from the recent review[10] on the elementary particle properties . The φ_1 meson electron widths given in Table I are taken from refs.[12,13]. Note the following interesting experimental fact. The values $(M_0^{(1)})_{exp}$ for all three families ψ , Υ , φ coincide up to experimental errors.

4. Conclusion

Let us plot for the Υ -family the meson number n on the abscissa axis and its mass M_n on the ordinate axis. Let us draw a fluent curve through these points (graph 1). The tangents to this curve at the points n= 0.1,... 5 determine the values $M_n^{(1)}$ which should be compared with $(M_n^{(1)})_{exp}$. For derivatives $M_n^{(1)}$ Get the following values are obtained:

$M_0^{(1)} = 1.16$, $M_1^{(1)} = 0.42$, $M_2^{(1)} = 0.28$, $M_3^{(1)} = 0.21$, $M_4^{(1)} = 0.21$, $M_5^{(1)} = 0.13$.

which agree well with the values $(M_n^{(1)})_{exp}$ given in Table I. The n-dependence of M_n through the first four mesons is monotonous therefore it is natural to trust the values of the derivative $M_n^{(1)}$ for these n . It is possible that the curve irregularity in the region of Υ_4 and Υ_5 mesons disappears in the course of improving the experimental data. The analogous curve for the ψ -family exhibits at least two bends and for this reason one cannot reliably determine $M_n^{(1)}$ from the diagram. Perhaps, these bends are because of the fact that ψ_2 -meson is, with

respect to the potential model $^1D_2$ state and it should not be included into analysis. Note that the potential model prediction for the ψ_2 meson electron width agrees rather similarly with experiment as predictions of this work. For ψ_4 -meson, the potential model predictions give for the ψ_4 meson electron width a value by an order of magnitude smaller than the experimental one. For these reasons we did not take into account the conclusions of the potential model at all.

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Table I

Name	Mass (MeV)	Γ_{tot} (MeV)	$M_\chi^{-1/4}$ / GeV	$(M_\chi)_{exp}$ / GeV	$(\Gamma_{ee})_{th}$ / keV	$(\Gamma_{ee})_{exp}$ / keV
ψ_0	3096.9±0.1	0.063±0.009		1.17±0.10		4.60±0.39
ψ_1	3686±0.1	0.215±0.40	0.56	0.45±0.05	2.54	2.05±0.21
ψ_2	3770±3	25±3	0.09	0.056±0.01	0.42	0.257±0.046
ψ_3	4030±5	52±10	0.23	0.16±0.03	1.08	0.75±0.15
ψ_4	4159±20	78±20	0.14	0.16±0.05	0.66	0.77±0.23
ψ_5	4415±6	43±20	0.28	0.10±0.03	1.34	0.49±0.13
Υ_0	9460.0±0.3	0.044±0.007		1.27±0.13		1.10±0.12
Υ_1	10023.4±0.3	0.030±0.005	0.52	0.52±0.05	0.50	0.507±0.051
Υ_2	10355.5±0.5	0.018±0.005	0.31	0.35±0.05	0.31	0.362±0.05
Υ_3	10573 ±4	14±5	0.23	0.23±0.05	0.24	0.240±0.053
Υ_4	10866 ±9		0.26	0.20±0.05	0.28	0.21 ±0.05
Υ_5	11020 ±8		0.17	0.075±0.02	0.17	0.08 ±0.02
ζ	769 ±3	154±5		1.25±0.07		7.08 ±0.38
ζ_1	1590 ±20	260±10	0.89	0.50±0.04 1.33±0.27	5.03	2.8±0.2 [12] 7.5±1.5 [13]

The calculated values of the electron widths $(\Gamma_{ee})_{th}$ compared with the experimental values $(\Gamma_{ee})_{exp}$ for the ψ, Υ ζ families.

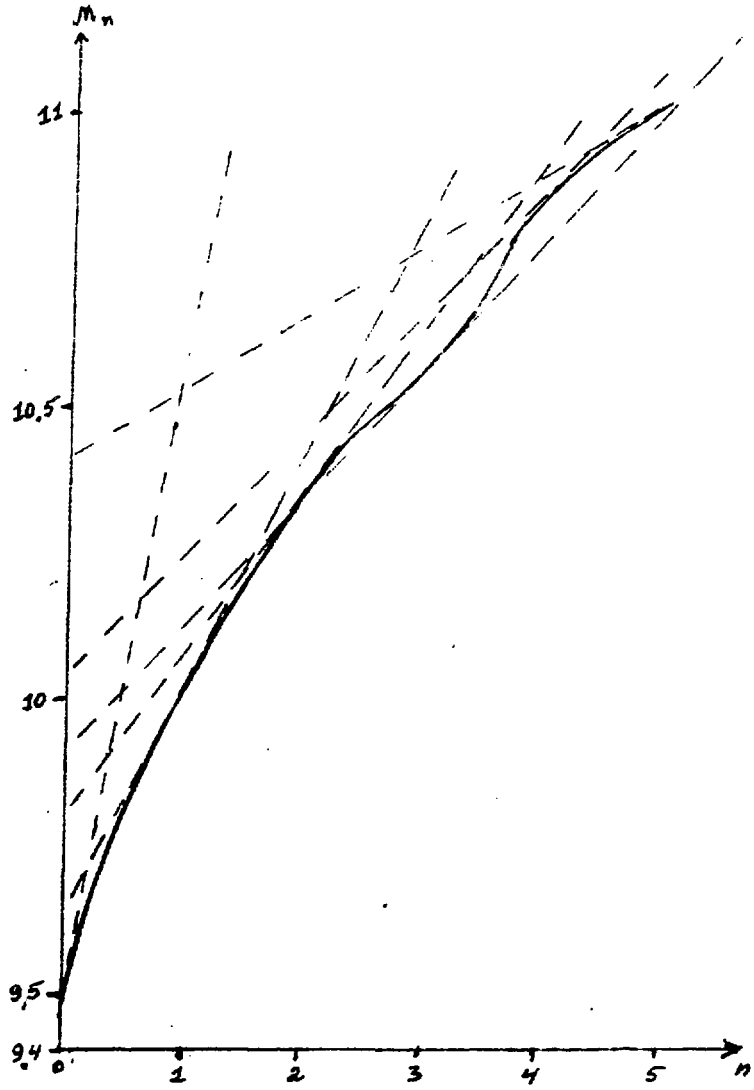

Fig.1

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