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A.S.Gorsky

POWER CORRECTIONS TO THE ASYMPTOTICS OF THE PION ELECTROMAGNETIC FORMFACTOR

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A b s t r a c t

The first power correction to the pion electromagnetic formfactor is derived. A few asymptotic wave functions corresponding to the different series of operators and matrix elements of four-particle operators in pion have been found. The large scale of the first power correction $\frac{\Delta F}{F_0} \sim \frac{10^2 (GeV^2)}{Q^2}$ where Q^2 is the momentum transfer indicates that at low energies the whole series of power corrections seems to be taken into account.

I. Introduction

The pion electromagnetic formfactor (f.f.) attracts attention of theorists in the last few years because it allows one to obtain the rigorous QCD result which can be experimentally verified. After the pioneer works [1-3] it became clear that the asymptotic QCD prediction ($Q^2=-q^2\to\infty$ where $q=p'-p$, p and p' are initial and final state momenta, respectively, contradicts the low energy experiment and radiation corrections found in refs. [4,5] cannot remove this contradiction.

Some ways out of this situation were proposed. In a number of works (see, e.g. ref. [6]) basing on the analysis of the low-energy sum rules it was argued that the asymptotic regime starts at inaccessibly high energies. In these works the model description of wave functions was proposed and a reasonable agreement with experiment was found. It seems, however, that the model proposed is not completely grounded (see ref. [7]) and therefore other solutions of the problem cannot be excluded.

The situation when hard re-scattering mechanism is not principal at low energies was discussed in refs. [8,9]. It is assumed in such an approach that the main role belongs to diagrams with no hard gluon exchange whose contribution can be agreed with experimental data.

The third approach consists in assuming the important role of power corrections which, in authors opinion [10], are several times larger than the Q^2 ground term at $Q^2\sim 10$ GeV^2. The pion wave function in this approach is assumed to be asymptotic what is reasoned by meson consideration in relativistic quark model [8, 11] where wave function appears to be close to asymptotic at several GeV^2 energies.

The power correction connected with the two-quark component of the pion twist 3 wave function was calculated in ref. [10]. Basing on existence of the $\left(\frac{m_\pi}{m_q}\right)^2$ large factor it was concluded that just this correction is dominating.

However, the contribution of some corrections connected with the pion three-particle wave function which was estimated in ref. [12] appeared to be comparable in the value with correction from two-quark sector. So, of interest is the calculation of all power corrections and estimates of their scales and this is the purpose of the present paper. To logic of the paper is close to ref. [13].

To formulate the result let us introduce the definition of formfactor which we will follow hereafter:

$$\langle \pi^{b}(p')|J_\alpha|\pi^{a}(p)\rangle = F(Q^2)(p+p')_\alpha\, t_{3,ba} \quad , \tag{1}$$

where J_α is electromagnetic current, $t_{3ba}=-i\varepsilon_{3ba}$, $Q^2=-(p'-p)^2$; $p'=(0,p'_+,0,0)$, $p=(p_-,0,0,0)$ in the frame chosen. The asymptotic expression for $F(Q^2)$ at $Q^2\to\infty$ has the form

$$F_0(Q^2)=\frac{8\pi\alpha_s(Q^2)f_\pi^2}{Q^2}\;,\quad f_\pi=133\,\text{Mev}. \tag{2}$$

The calculations show that in each sectors (two-, three-, four-particle) the scale of corrections is $\Delta F\sim F_0\,\frac{\rho}{Q^2}$, where $\rho\sim 10^{2\mp3}(\text{GeV}^2)$ and it can be expected that their sum is of the same order. Unfortunately, we cannot present now the correct numerical result for the correction which is due to existence of divergences when integration with wave functions is performed.

The paper is arranged as follows. In Section 2 the operator expansion (OPE) appropriate for analysing possible power corrections is discussed. In Sec.3 the corrections in two-quark sector are discussed and mixing of different type operators is discussed. The corrections originated from $|q\bar{q}g\rangle$ and $|q\bar{q}gg\rangle$ configurations in the pion were found in Sec.4. The results of the work are briefly discussed in the last Section.

2. Operator Expansion

In what follows we will suppose that the main contribution to formfactor results from the diagram with hard scattering in Fig.1. The photon interaction is denoted by cross. The Heisenberg operators ψ , $\bar{\psi}$ correspond to external quark legs. All the lines in the diagram are assumed to be in external gluon field. We consider the situation when initial and final states are the pion ones.

The expression for the diagram can be written as

$$\int dy\, dz\; \bar{\psi}_\eta(z)\,\psi_\beta(0)\,T^{\alpha}_{\eta\beta\rho\xi}(z,y,Q^2)\,\bar{\psi}_\rho(y)\,\psi_\xi(0)$$

where T is the hard block which includes the propagators with large virtualities while small Fourier-components are essential in ψ .The next step consists in that after extracting from the hard block the external field stemming from the propagator expansion, we introduce vacuum intermediate state, fixing thereby the method of obtaining the hard block. The dominance of vacuum intermediate state which is valid on the tree level when the pion is built of free quarks, is a natural hypothesis for the real pion. Notice that in the free quark standard calculation when quark matrix elements of the operators are then replaced by the real pion matrix elements, the possibility of vacuum dominance is implied.

When taking into account radiation corrections the question arises on the validity of factorization of small and large distances and on introducing the normalization point. For the free quark case factorization has been checked at one-loop level for the leading power term (see, e.g. ref. [12]) but we will suppose that it is valid also for the real physical states in the power correction calculations. We will choose the normalization point to be μ^2 of order Q^2 in order that all the renormalization group correction would correspond to wave function.

Thus, after applying spinor index Fierz transformation we get the following OPE

$$\langle \pi | \int e^{iqz} J^{\alpha}(z) dz | \pi \rangle = \sum_{n,m} \int dy\, dz \langle \pi | O_n(z,\mu^2) | 0 \rangle T^{\alpha}_{nm}(z,y,Q^2) \langle 0 | O_m(y,\mu^2) | \pi \rangle \quad ($$

where O_n, O_m are some nonlocal operators containing quark and gluon fields and T_{nm} are coefficient functions. As normalization point of nonlocal operators we imply the normalization point of the operators resulting from expansion of these operators in the nonlocal ones. Nonlocal operators will be the corresponding wave functions in coordinate representation.

The following remark should be done here. In the framework of the diagram techniquethere may be the graphs depicted in Fig.: where dots denote the external classical gluon field. In such a diagram the classical gluon is radiated from small distances by a quark contained in initial state and faterwards this gluon enters the final state. In this case the way of deriving OPE (3) is erroneous. However, in the following we will use for external field the fixed point gauge which allows us to avoid the difficu ty. The diagram in Fig. 2 arises in the Heisenberg operator expansion (in g). In the fixed point gauge the covariant derivative in this expansion can be replaced by the usual one which allows one not to consider such a diagram.

When calculating the main asymptotics it is sufficient to use free propagators in Fig. 1 and retain only axial projector in the Fierz transformation. In this case the operatorsin (3) are axial currents whose matrix elements are of the form

$$\langle 0|\bar{\Psi}(y)\gamma_\mu\gamma_5\Psi(0)|\pi(p)\rangle_{\mu^2} = i f_\pi p_\mu \varphi_{A1}(yp,\mu^2)+\dots \qquad (4)$$

where dots stand for nonleading twists. The wave function φ_{A1} can be easily obtained at $\mu^2 \to \infty$. Hereafter we do not explictly write in matrix elements the isotopic factor $\frac{\tau^a}{\sqrt{2}}$ and the gauge exponent $exp\{i\int_0^y B_\mu(6)d6\}$ where B_μ is gluoinc field. It is worth noting that in the leading twist calculations the hard block is automatically gauge invariant with a desired accuracy.

Let us find now all operators in OPE for $\langle\pi(p')|J_\alpha|\pi(p)\rangle$ which lead to the first power correction for formfactor. In the case of local operators when only one vector p_μ may enter the matrix element of this operator, its contribution is determined by twist. If now the operator is nonlocal, then, as can be safely seen, its contribution is defined by the number $m=d+a-b$, where d is the operator dimension, a is the power of the coordinate in the expression for its matrix element, b is the momentum power in this expression. Of course, m characterizes the twist of local operators resulting from expansion of nonlocal ones, but m-classification of such operators seems to us more convenient.

The matrix element of two-particle operators with have the following form

$$\langle 0|\bar{\Psi}(x)\gamma_\mu\gamma_5\Psi(0)|\pi\rangle = if_\pi\left\{p_\mu\left[\varphi_{A1}(xp)+\frac{C_2x^2}{2}\varphi_{A2}(xp)\right]+x_\mu C_3\varphi_{A3}(xp)\right\}+\dots \quad (5a)$$
$$m=2,4.$$

$$\langle 0|\bar{\Psi}(x)\gamma^5\Psi(0)|\pi\rangle = C_p\varphi_p(xp)+\dots, \quad m=3 \quad (5b)$$

$$\langle 0|\bar{\Psi}(x)\sigma_{\mu\nu}\gamma_5\Psi(0)|\pi\rangle = C_T(p_\mu x_\nu - x_\mu p_\nu)\varphi_T(xp)+\dots, \quad m=3 \quad (5c)$$

Dots in formulae (5) and (7) stand for the terms with $m>4$. The origin of three functions φ_{Ai} $(i=1,2,3)$ can be easily understood if we consider the matrix element arising in expansion of the r.h.s. of eq.(5a) in series in x:

$$\langle 0|\bar{\Psi}\gamma_\mu\gamma_5\overleftrightarrow{D}_{\alpha_1}\overleftrightarrow{D}_{\alpha_2}\Psi|\pi\rangle = a_0p_\mu p_{\alpha_1}p_{\alpha_2}+a_1g_{\alpha_1\alpha_2}p_\mu + a_\mu(g_{\alpha_1\mu}p_{\alpha_2}+g_{\alpha_2\mu}p_{\alpha_1}) \quad ; \quad \overleftrightarrow{D}_\alpha = \overrightarrow{D}_\alpha - \overleftarrow{D}_\alpha, \; \overrightarrow{D}_\alpha = \overrightarrow{\partial}_\alpha - igB_\alpha, \; \overleftarrow{D}_\alpha = \overleftarrow{\partial}_\alpha + igB_\alpha . \quad (6)$$

Functions φ_{Ai} correspond to three tensor structures in (6) while at increasing the number of derivatives in the operators the rest derivatives $(\overleftrightarrow{D}_{\alpha_j}, j>2)$ transform into moments.

The operators mentioned above define the correction in the two-quark sector. Note, however, that besides the terms in OPE which correspond to the operators $m=2 \otimes m=4$, $m=3 \otimes m=3$ one should take into account two types of corrections. First, taking into account of the mass in the hard block results in the interference $m=2 \otimes m=3$ with no mass suppression, second, the kinematic correction to the hard block $\sim m_\pi^2$ give the power correction from the operators $m=2 \otimes m=2$.

There are also operators with gluon field giving a nontrivial contribution

$$\langle 0|\bar\Psi(x)\gamma_\mu\gamma_5 t^c G^c_{\alpha\beta}(y)\Psi(0)|\pi\rangle = if_\pi A p_\mu (x_\alpha p_\beta - x_\beta p_\alpha)\varphi_{3A}(xp,yp)+\cdots, m=4, \quad (7a)$$

$$\langle 0|\bar\Psi(x)\gamma_\mu t^c G^c_{\alpha\beta}(y)\Psi(0)|\pi\rangle = if_\pi C' \varepsilon_{\mu\alpha\beta\rho} p_\rho \varphi_{3v}(xp,yp)+\cdots, m=4, \quad (7b)$$

$$\langle 0|\bar\Psi(x)\sigma_{\mu\nu}\gamma_5 t^c G^c_{\rho\xi}(y)\Psi(0)|\pi\rangle = f_\pi D\{p_\mu(g_{\nu\rho}p_\xi - g_{\nu\xi}p_\rho)-(\nu\leftrightarrow\mu)\}\varphi_{3T}(xp,yp)+\cdots \quad m=3 \quad (7c)$$

$$\langle 0|\bar\Psi(x)\gamma_\mu\gamma_5 G^a_{\alpha\beta}(y)G^a_{\eta\rho}(z)\Psi(0)|\pi\rangle = iB^S p_\mu\{p_\alpha(p_\rho g_{\beta\eta} - p_\eta g_{\beta\rho})-(\alpha\leftrightarrow\beta)\}\cdot \varphi_{4A}(xp,yp,zp)+\cdots, m=4. \quad (7d)$$

$$\langle 0|\bar\Psi(x)\gamma_\mu\gamma_5 d^{abc} G^e_{\alpha\beta}(y)G^c_{\eta\rho}(z)t^a\Psi(0)|\pi\rangle = \frac{B^0}{B_S}\langle 0|\bar\Psi(x)\gamma_\mu\gamma_5 G^a_{\alpha\beta}(y)G^a_{\eta\rho}(z)\Psi(0)|\pi\rangle_S \quad m=4 \quad (7e)$$

Let us make some remarks on the operators (5) and (7). When calculating coefficient functions of importance are the components of the operators $\bar\Psi\gamma_-\gamma_5 G_{-\beta}G_{\beta-}\Psi$, therefore we consider only the symmetric structure constant $SU(3)-d^{abc}$. In operators (7) the definition includes the coupling constant g.
All wave functions (except for φ_{AS}) are normalized by the same condition $\varphi_k(0)=1$. Operators (5c) and (7a) vanish in the local limit by G-parity that is why they are proportional to coordinate. Some of the constants in operators (5), (7), namely: A, C'; C_P, C_T; C_2, C_3 are related to each other by equations of motion. It will be also shown that to provide the hard block invariance in the general case one should consider the corresponding wave functions in common.

Let us discuss now the renormalization problem. The operators in OPE are normalized on the scales $\sim Q^2$ and it is necessary to follow their evolution up to hadron scales where we can get some information about their matrix elements.

Besides the logarithmic dependence on the normalization point, the terms $\frac{\mu^2}{Q^2}$ may appear in loops. Following the logic of ref. [14] we suppose that such terms reduce in the sum. The power corrections of the type $\frac{\alpha_s m_\pi^2}{Q^2}$ arising in loops are neglected because of their numerical smallness.

Two types of mixing are possible at evolution. First, the mixing of operators from one series differing by the number of the derivative contained which arise in expansion of nonlocal operators in (5),(7). Second different series of operators with the same twist can also mix.

When taking into account the first type mixing, the operators which are conformal group representations (see ref. [15]) and which are featured by a definite value of conformal momentum n, are mutiplicatively renormalized. Anomalous dimensions of the operators with different n increase with n increasing, therefore at $Q^2 \to \infty$ the operator with minimum n is of the most importance. The wave function corresponding to this operator at $Q^2 \to \infty$ can be foundwhen exploiting the method proposed in ref. [16].

However, in the leading logarithmic approximation the mixing of different type operators with the same twist is possible, which was the case for singlet operators in the leading twist when quark and gluon operators mixed. But it is of importance that in such a mixing only evolution of matrix elements of the series considered changes while the structure of asymptotic wave functions remains untouched.

In numerical estimates, considering the contribution of a certain operator series, we will use the corresponding asymptotic wave functions, i.e. it is supposed that low energy wave functions are close to the asymptotic ones. For this reason, it is natural when considering power correction to restrict ourselves to the region of small Q^2 where the logarithmic factor is inessential and we neglect the evolution effects of matrix elements. In what follows we will make use of the values of matrix elements obtained with the sum rule method.

3. Corrections in Two-Quark Sector

When considering the power correction in two-quark sector we leave free the propagators in Fig.1 and in the Fierz expansion A,P,T terms. For example, for initial state we get

$$4\langle 0|\bar{\psi}_\alpha(0)\psi_\beta(z)|\pi(p)\rangle = -(\gamma_\mu\gamma_5)_{\beta\alpha}\, i f_\pi \Big\{ p_\mu \big[\varphi_{A1}(zp) + \frac{C_2 z^2}{2}\varphi_{A2}(zp)\big] + z_\mu C_3 \varphi_{A3}(zp)\Big\} + (\gamma_5)_{\beta\alpha} C_P \varphi_P(zp) - \big[(\hat{p}\hat{z} - \hat{z}\hat{p})\gamma_5\big]_{\beta\alpha} C_T \varphi_T(zp) + \dots \qquad (8)$$

Analogous expansion is also valid for final state.

The requirement of gauge invariance for the hard block imposes some relationships on the functions φ_T, φ_P ; $\varphi_{A2}, \varphi_{A3}$. For this requirement to be satisfied, it is necessary for the Dirac equation for field ψ to be true. For twist 3 operators this leads to the following equation :

$$C_P \varphi_P'(zp) = 3C_T \varphi_T(zp) + (pz)\varphi_T'(zp) C_T \qquad (9)$$

from which we can easily get the connection : $C_P = \frac{i}{6} C_T$.

The contribution of twist 3 operators into power correction was obtained in refs.[10] where the asymptotic wave function in the pseudoscalar channel was derived. Let us discuss the way of obtaining this result. In ref.[10] the effective projector which takes into account the current P, T contributions was built to provide gauge invariance of the hard block. This projector keeps its form in LLA which made it possible to examine the evolution of operators in the Bethe-Salpeter equation language in momentum representation. Solving this equation one can get the expression for $\varphi_P^{as}(zp) = \int_0^1 e^{ipzx} dx$.

In OPE language we get the corrections of the type considered in (9) as O_n the operators from series $\bar{\Psi}\gamma_5 \overleftrightarrow{D}^{2n}\Psi$, $\bar{\Psi}G_{\alpha\beta}\gamma_5 \overleftrightarrow{D}^{2n+1}\Psi$ $(n=0,1\ldots)$. When operators evolve from scales Q^2 up to $(500\,\mathrm{Mev})^2$ the operators from the first series mix with those from the second one (but not vice versa), so it is convenient to express the operators with T current via the operators with P currentand to consider the evolution of the latter. For example, we have for the simplest operators

$$\langle 0|\bar{\Psi}G_{\alpha\beta}\gamma_5 \overleftrightarrow{D}_\nu \Psi|\pi\rangle = \frac{i}{3}\langle 0|\bar{\Psi}\gamma_5\Psi|\pi\rangle (p_\alpha g_{\nu\beta} - p_\beta g_{\nu\alpha}) \qquad (10)$$

Note that at re-expression of the operators with T current the operators $G_{\mu\nu}\bar{\Psi}\sigma_{\nu\rho}\gamma_5 \overleftrightarrow{D}^{2n+1}\Psi$ also erise but they do not give a contribution into pion because of G-parity.

Re-expanding now the effective series of the operators with ρ current in the Legendre polynomials system which is multiplicatively renormalized in LLA we can get the coefficient in front of the zero Legendre polynomial $\bar{\Psi}\gamma_5\Psi$ which is the main at $Q^2 \to \infty$. Thus obtained coefficient in front of this operator coincides with the result of refs.[10] Note also that no one of the twist 3 operators with gluon field mix with the operator $\bar{\Psi}\gamma_5\Psi$. This is because of the fact that the operators with gluon field having a definite conformal moment mix with the operators from the ρ series with the same moment. The operator $\bar{\Psi}\epsilon_{\alpha\beta}G_{\beta\rho}\gamma_5\Psi$ with minimal conformal moment mixes only with the operator $\bar{\Psi}\gamma_5 P_2(\frac{D}{\partial})\Psi$ where $P_2(x)$ is the second Legendre polynomial. Finally, we get the following result for the correction in view [10]:

$$
\rho_{PP} \approx \frac{2}{3} \ln^2 \frac{Q^2}{\Lambda^2} (GeV^2) \ , \ \Lambda = 100 MeV. \qquad (11)
$$

Let us calculate now the contribution of the other two-quark operators corresponding to nonleading operators in (5a). This contribution originates from effective account of transverse degrees of freedom in the pion. In general case we must consider (in common) the series of the operators corresponding to wave functions φ_{A2} , φ_{A3} to provide the gauge invariance. For example, we can re-express the operators from the first series through the operators from φ_{A3}. But the direct calculation of the diagrams shows that the contribution of the function $\varphi_{A2}(xp)$ which corresponds to the series $\bar{\Psi}\gamma_\mu\gamma_5 \overleftrightarrow{D}_1^2 \overleftrightarrow{D}_-^{2n}\Psi$ (see ref.[13]) vanishes and we can consider the second series operators separately.

To find the contribution of function $\varphi_{A3}(xp)$

one should recognize to which operators it corresponds. To this end, differentiate (5a) over $X_{\perp\mu}$ at $X_\perp = 0$. The resultant expression is

$$< 0 | \bar{\Psi}(0) \hat{\vec{D}}_\perp \gamma_5 \Psi(x_+) | \pi > \sim \varphi_{A3}(xp) \qquad (12)$$

The asymptotoc wave function can be found using the method proposed in ref. [16]. The calculations lead to the following result

$$\varphi^{as}_{A3}(xp) = - \frac{35 i}{2} \int_{-1}^{1} e^{ipx\frac{(1+\xi)}{2}} \xi^3 (1-\xi^2) d\xi , \qquad (13)$$

$$\varphi'_{A3}(0) = 1$$

After integrating over dimensional variables the expression for formfactor can be represented in the form

$$F(Q^2) = \int_0^1 dx \int_0^1 dy \; \varphi(x) \varphi(y) T(x, y, Q^2) , \qquad (14)$$

$X(y)$ are the initial (final) parton variables, $\varphi(x) [\varphi(y)]$ is the wave function of initial(final) pion state and $\int_0^1 dx$ stands for $\int_0^1 \prod_{i=1}^{N} dx_i \, \delta(1 - \sum_{i=1}^{N} x_i)$ $(N = 2, 3, 4)$. When integrating over variables x, y in a number of cases logarithmic integrals $J \sim \int_0^1 \frac{dx}{x}$ arise. Such divergences were analysed in ref. [18] where it was shown that the integral cut-off stemed from account of the double-logarithmic corrections to the hard block. These corrections were explicitly calculated by the authors of these works for the case of $\rho \times \rho$ interference. In this work we did not calculate the double-logarithmic corrections and in the cases when such integrals arise we use for estimate $J \approx 10$. The result for the correction originating from $\varphi_{A3}(zp)$, is of the form

$$\rho_{AA3} = -4C_3 \int dx\,dy\, \varphi_{A1}(x)\varphi_{A3}(y) \frac{x(1-x)-1}{y^2 x^2} \qquad (15)$$

The constant C_3 which can be connected with C_2 by equations of motion appears to be equal to $\approx 10^{-2}(GeV^2)$ (see ref.[12]). Substituting the values C_3, φ_{A3}^{as} into (15) we have

$$\rho_{AA3} \approx -3\cdot 10^2 (GeV^2) \qquad (16)$$

The hard block corrections for operators $m=2 \otimes m=2$ are proportional to m_π^2 and, as can be easily seen, lead only to logarithmic integrals. For these reasons we will neglect them in the following. When calculating the interference of operators $m=3 \otimes m=2$ the logarithmic divergence arises but comparing with $\rho \times \rho$ interference this term has a relative smallness and we will not take it into account.

4. Corrections in the Quark-Gluon Sector

Let us now calculate the corrections due to operators with the valence gluon field. Following the method elaborated in ref. 14 we divide a gluon field into classical and quantum parts $A_\mu = a_\mu + A_\mu^{cl}$. Re-writing QCD lagrangian in terms of fields we obtain the following quantum field quadratic terms:

$$L_2 = \frac{1}{2} a_\mu^a (D^{2ab} g_{\mu\nu} + 2g G_{\mu\nu}^{ab}) a_\nu^b + i\bar{\Psi}\hat{D}\Psi \qquad (17)$$

where $D_\mu^{ab} = g_{ab}\partial_\mu - ig\, A_\mu^{\alpha ab}$, $G_{\mu\nu}^{ab} = f^{acb} G_{\mu\nu}^{c}$ in the first factor and $A_\mu^{cl} = A_\mu^{acl} t^a$ in the second.

Discuss now the gauge conditions on the fields A_μ^{cl}, a_μ. To have a simple connection A_ν^{cl} with $G_{\mu\nu}^{cl}$ it is convenient to introduce the fixed point gauge for external field. Making use of the remained gauge freedom for the field a_μ we choose the background gauge $D_\mu a_\mu = 0$. Hereafter we shall perform all calculations in these gauges but the final results is, of course, independent of them.

Formulae for gluon and quark propagators in external field can be easily found from (17). In these formulae gluon fields are expressed through A_μ^{cl} according to the fixed point gauge relationship $A_\mu(x) = \int_0^1 \alpha\, d\alpha\, G_{\mu\nu}(\alpha x)$. In a somewhat conditional form the gluon propagator takes the form:

$$D_{\mu\nu}^{ab}(q,q',q'') = \frac{g_{\mu\nu}\delta^{ab}}{q^2} + \frac{2g\, G_{\mu\nu}^{ab}}{q^2 q'^2} + \frac{4g^2}{q^2 q'^2 q''^2} G_{\mu\rho}^{ac}(x) G_{\rho\nu}^{cb}(y) +$$
$$+ \frac{2g^2 g_{\mu\nu}}{q^6} \int \frac{\alpha\, d\alpha\, \alpha' d\alpha'}{q'^2} \left[G_{\xi\rho}^{ac}(\alpha x) G_{\xi\rho}^{cb}(\alpha' x) q^2 - 4 q_\rho q_\gamma G_{\xi\beta}^{ac}(\alpha x) G_{\gamma\beta}^{cb}(\alpha' x) \right] + \cdots \tag{18}$$

Four terms of expression correspond to the diagrams depicted in Fig.3. The expression should be understood as follows. The strength tensor of external gluon field which interacts with the quantum field at the point $x(y)$ enters afterwards the wave function and the term e^{ikx} where k is the momentum of the gluon, corresponds to it. Momenta in q', q'' are fixed after coordinate integration in the diagrams taking into account the mentioned coordinate dependence of the field strength.

Analogously we obtain the expression for the quark propagator:

$$\hat{S}(q,q',q'') = \frac{1}{\hat{q}} + ig \int \frac{\hat{q}'\gamma_\mu \hat{q}\gamma_\rho \hat{q}}{q^4 q'^2} G_{\rho\mu}(x\alpha)\, \alpha\, d\alpha +$$
$$+ g^2 \int \frac{\hat{q}''\gamma_\rho \hat{q}''\gamma_\nu \hat{q}'\gamma_\mu \hat{q}\gamma_\eta \hat{q}}{q^4 q'^2 q''^4} G_{\rho\nu}(\alpha x) G_{\eta\mu}(\alpha' x)\, \alpha\, d\alpha\, \alpha' d\alpha' + \cdots \tag{19}$$

Three terms (19) correspond to the diagrams shown in Fig.4. Emphasize that the external field is not considered to be constant unlike the usual situation when only a few first terms from (18), (19) are needed, we must sum all the terms of the same twist of the type $(qD)^n G$ in propagators. Note that momenta q', q'' are parameter α -dependent and thus they are to be integrated over. The terms not written in (18), (19) lead to operators not giving a contribution to the first power correction.

Prior to direct calculations of the diagrams, let us make the following remark. In ref. [12] the wave functions for the operators with gluon field were calculated over states with free quarks and gluons and the relationship A_μ with $G_{\mu\nu}$ for free gluons was supposed. This was possible when components $G_{-\perp}$ worked but in general case the free gluon calculation of wave functions, as calculation shows, leads to erroneous results.

In this connection note that free quark calculation also appears to be possible because of that the nor~~ical matrix element of the Heisenberg quark fields in the asymptotics can be modelled by free quarks:

$$\langle 0|\bar{\psi}(-y)\,\Gamma\,\psi(y)|p\rangle = \int dx\, \varphi_\Gamma^{as}(x)\, e^{ipy(2x-1)} + \dots \qquad (20)$$

where Γ is some Dirac structure.

The fixed point gauge calculation demands fixation of the origin by virtue translation invariance violation by the gauge condition. We will follow the choice of the origin mentioned in Fig.1. Below we calculate only the diagrams which give a nontrivial contribution into correction while a great number (~ 20)

of diagrams vanishing for some reason are not considered.

The nonvanishing contribution into the first correction is given by the operator interference $(5a)\otimes(7b)[A_2\otimes V_4^g]$; $(7c)\otimes(7c)[T_3^g\otimes T_3^g]$; $(7c)\otimes(5b,c)[T_3^g\otimes P]$; $(5a)\otimes(7d,e)$. Let us start with the interference $T_3^g\otimes T_3^g$. The diagrams contributing to this interference are shown in Fig.5. They give the following result for ρ_{TT}

$$\rho_{TT}=D^2\int dx\,dy\,\varphi_{3T}(y)\varphi_{3T}(x)\left\{\frac{9}{(1-y_1)^2(1-x_1)x_2^2y_2}+\frac{1}{2}\int_0^1\frac{\alpha\,d\alpha}{(1-\alpha x_3-x_1)^2(1-\alpha x_3)(1-y_1)^2}\right\} \quad (21)$$

Integrating in (21) with asymptotic wave functions $\varphi^{as}(x)=360x_3^2x_1x_2$ and substituting the found with sum rule method the value $D=-3\cdot10^{-2}Gev$ (see ref. [12]) we get

$$\rho_{TT}=0{,}9\left\{108\left(\frac{5}{3}-\frac{J}{2}\right)+\frac{13}{2}\right\}(Gev^2)\;,\quad J=\int_{x_0}^1\frac{dx}{x} \quad (22)$$

Substituting for estimate $J\approx10$ we have

$$\rho_{TT}\approx-3\cdot10^2\,(Gev^2) \quad (23)$$

The results of the calculation of the diagrams contributing to the interference $A_2\otimes V_4^g$ and $T_3^g\otimes P$ which are shown in Figs.6 and 7, respectively, are:

$$\rho_{AV}=C'\int dx\,dy\,\varphi_{SV}(x)\varphi_A(y)\left\{\frac{2(1+x_1)}{(1-x_1)^2(1-y_1)^2x_2}+\frac{6}{y_2^2(1-x_1)x_2}-\right.$$
$$\left.-\frac{4}{3y_2}\int\alpha\,d\alpha\left[\frac{2}{y_2(1-\alpha x_3)(1-\alpha x_3-x_1)}+\frac{(x_1+\alpha x_3)}{(1-x_1)(1-x_1-\alpha x_3)^3}\right]\right\} \quad (24)$$

$$\rho_{TP}=\frac{16m_\pi^2D}{9m_q}\int\varphi_P(x)\varphi_T(y)\,dx\,dy\,\alpha\,d\alpha\,\frac{1}{x^2(1-\alpha y_3-y_1)^3} \quad (25)$$

After integrating with asymptotic wave functions and substituting numerical values of the constants ($\varphi^{as}_{3v}(x) = 120 x_1 x_2 x_3$ [12] , $C' = -0{,}2\,Gev^2$ (see ref. [17])) we have

$$\rho_{TP} \approx 5\ Gev^2 \tag{26}$$

$$\rho_{AV} \simeq 3\cdot 10^3\ Gev^2 \tag{27}$$

The following remark should be done. In general case to satisfy the gauge invariance we are to consider the series of operators originating from (7a) and (7b) in common. The operators (7a) should be afterwards expressed via the operators of the second series, i.e. the situation is analogous to ρ – ω mixing. But in our case the direct calculations show that the coefficient functions corresponding to the series of three-particle-operators with the axial current vanish and thus these operators do not contribute to the correction.

Let us consider in more detail the axial current interference with four-particle operators. The diagrams corresponding to such interference are shown in Fig.8. Denote the values of ρ for singlet and octet operators as ρ^{s}_{AA} , ρ^{o}_{AA} , respectively. The values of these parameters obtained in the diagram calculations are

$$\rho^{o}_{AA} = B^{o}\int \varphi_{4A}(x)\,\varphi_{A1}(y)\,dx\,dy\Big\{\int \alpha\, d\alpha\, \alpha'\, d\alpha' \Big[-\frac{12}{(1-x_1-\alpha x_3-\alpha' x_4)(1-x_1)y_2^2} + \frac{8}{9(1-x_1)(1-x_1-\alpha x_3)(1-x_1-\alpha x_3-\alpha' x_4)^2 y_2} + \frac{1}{9y_2^2(1-\alpha x_3-\alpha' x_4)^2(1-\alpha x_3-\alpha' x_4-x_1)} - \int \frac{\alpha\, d\alpha}{y_2^2(1-x_1-\alpha x_3-x_4)(1-x_1-\alpha x_3)^3}\Big\} \tag{28}$$

$$\rho^{S}_{AA} = B^{S}\int \varphi_{4A}(x)\,\varphi_{A}(y)\,dx\,dy\Big\{\int d\alpha\,d\alpha'\Big[-\frac{2}{3(1-x_1-\alpha x_3-\alpha' x_4)\,y_2^2(1-x_1)} + \frac{8}{27(1-x_1)(1-x_1-\alpha x_3)(1-x_1-\alpha x_3-\alpha' x_4)^2 y_2} - \frac{8}{27 y_2^2(1-\alpha x_3-\alpha' x_4)^2(1-\alpha x_3-\alpha' x_4-x_1)}\Big] - \frac{1}{3}\int\frac{d\alpha}{y_2^2(1-x_1-\alpha x_3-x_4)(1-x_1-\alpha x_3)^3}\Big\} \qquad (29)$$

For numerical estimates we should know the function $\varphi^{as}_{4A}(x)$ and the values of constants B. The function φ^{as}_{4A} can be easily found using the method of ref. [16]. For this we should consider the polarization operator of the form

$$\int d^4x\, e^{iqx}\langle 0|T\,A^{0000}(x)\,A^{n_1n_2n_3n_4}(0)|0\rangle = I^{n_1n_2n_3n_4}(q^2)(\Delta q)^{6+n_1+n_2+n_3+n_4} \qquad (30)$$

where $A^{n_1n_2n_3n_4}(x) = \bar\psi(\overleftarrow{D}_-^{\,n_1})(G^a_{\nu_2\beta}\overleftarrow{D}_-^{\,n_3})(G^a_{\nu_3\beta}\overleftarrow{D}_-^{\,n_4})\gamma_{\nu_1}\gamma_5(\overrightarrow{D}_-^{\,n_2})\psi\cdot\Delta_{\nu_1}\Delta_{\nu_2}\cdot\Delta_{\nu_3}$,
$\Delta^2 = 0$.

Saturating the spectral density of this correlator by the states with the pion quantum number, we can get the n dependence of the wave function moments. The simple calculation of the diagram Fig.9 leads to

$$\langle x_1^n\rangle \sim \langle x_2^n\rangle \sim \frac{(n+1)!}{(n+9)!}\;,\quad \langle x_3^n\rangle \sim \langle x_4^n\rangle \sim \frac{(n+2)!}{(n+9)!} \qquad (31)$$

It corresponds to the normalized asymptotic wave function

$$\varphi^{as}_{4A}(x) = 90720\, x_1 x_2 x_3^2 x_4^2 \qquad (32)$$

Let us find now the constant B_S which defines the scale of the correction from the four-particle operator. The simplest way is to investigate the nondiagonal correlator

$$\Pi(q^2)=i\int d^4x\, e^{iqx}\langle 0|T\,\bar\Psi\hat\Delta\gamma_5\Psi(x),\bar\Psi\hat\Delta\gamma_5 G^a_{\mu\beta}G^a_{\beta\nu}\Psi(0)|0\rangle\times$$
$$\times\Delta_\mu\Delta_\nu=(\Delta q)^4 L(q^2)$$
$$q^2=-Q^2 \tag{33}$$

The coefficient in front of the unit operator in OPE for the current product in (33) is determined by the three-loop diagram whose contribution is suppressed by phase factors. The corrections from operators of dimension 4 - $\langle G^2\rangle$, as can be easily understood, are also determined by three-loop graphs, therefore in sum rule in the chiral limit of importance will be operator $\langle\bar q q\rangle^2$. The one-loop diagram givihg a nonvanished contribution into coefficient function of this operator is shown in Fig.10. As shown by the calculations, other possible diagrams in Fig.11 vanish.

The sum rule for constant B^s after borelization is of the form

$$-\frac{8}{27}\;\frac{\langle\bar q q\rangle^2\alpha_s^2(M)}{M^2}=f_\pi^2 B^s \tag{34}$$

where M^2 is the Borel parameter. We take for estimate $M^2\simeq 1\,Gev^2$ and "standard" value of condensate $\langle\alpha_s(\bar q q)^2\rangle\approx 1.8\cdot 10^{-4}(Gev^2)$ [18] . Substituting these values into (34) we obtain the result for a constant B^s , normalized on the scales $\sim 1\,Gev^2$

$$B^s\simeq -10^{-3}Gev^2 \tag{35}$$

Taking into account the change of the colour factor in the diagram Fig.10 we have

$$B^{\circ} \approx -1{,}2 \cdot 10^{-3} Gev^2 \qquad (35)$$

Another way to derive these constants is to consider the diagonal sum rules. The contribution of the unit operator for this sum rule is determined by the diagram in Fig.9. The power corrections in this case are also determined by the three-loop diagrams, so the free-loop estimate is justified. We consider the accuracy of formulae (34),(35) to be not worse than 50%. Substituting (34),(35) into (28) and (29) we have

$$\rho^{s}_{AA} + \rho^{\circ}_{AA} \approx -3 \cdot 10^{3} (Gev^2) \qquad (36)$$

Summarizing the consideration of many-body operator contribution it may be concluded that they lead to a correction $\sim \frac{10^2 \div 10^3 (Gev^2)}{Q^2}$ and at small Q^2 significantly exceed the asymptotic term.

5. Conclusion

Thus, the analysis of the power corrections shows that most probably their common effect is of order $\frac{\Delta F}{F_0} \sim \frac{10^2 (Gev^2)}{Q^2}$ which considerably exceeds the experimental data at small energies . There is un uncertainty in this estimate due to the presence of divergent integrals but the possibility of reducing different contributions which change the common effect by an order, seems to be rather unnatural.

The absence of numerical suppression of the first power

power correction is an argument in the favour of that even higher power corrections can be important at small energies. Such corrections may arise when accounting for the next terms of expansion in x in the r.h.s. of (5), (7) a possible interaction with vacuum condensates $\langle \bar{q} q \rangle$, $\langle G^2 \rangle$ and also due to new types of the operators, e.g. five-particle. Let us emphasize that the smallness of many-particle matrix elements is compensated by a large normalization factor in wave function. Thus, there are some doubts whether it is possible to obtain an exact QCD result comparable with low energy experiment.

It is also of a great interest to find all the low energy wave functions considered in the problem. Such wave function can be tried to find using the sum rule method.

I am indebted to M.V.Terentyev, who proposed this problem, for useful discussions and to V.M.Belyaev and Ya.I.Kogan for remarks as to sum rule method.

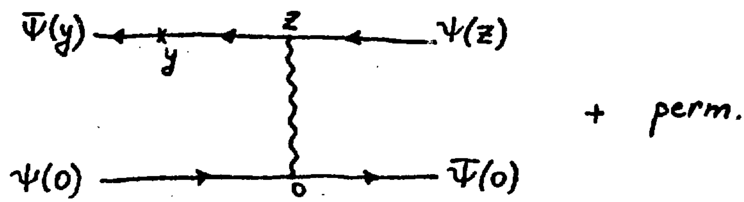

Fig.1

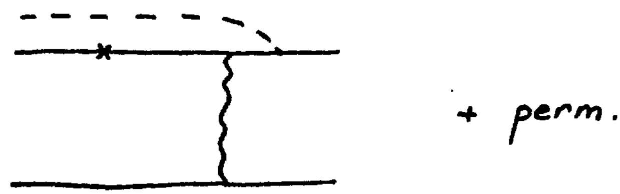

Fig.2

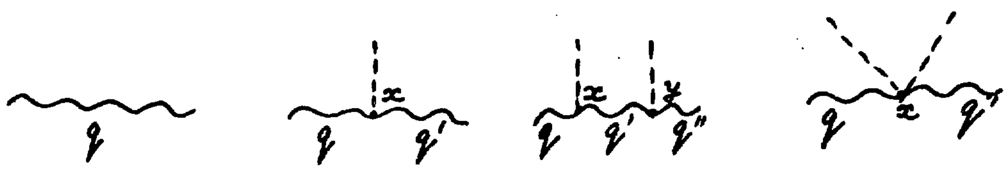

Fig.3

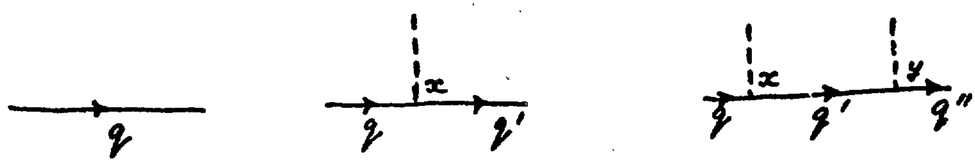

Fig.4

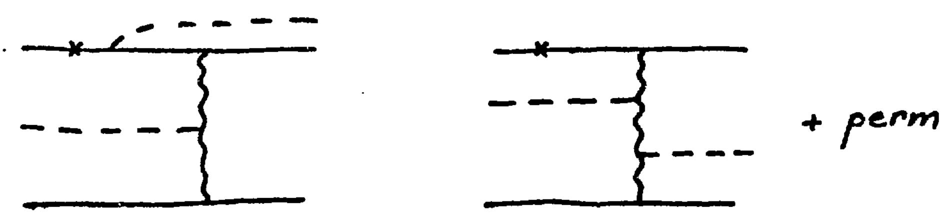

Fig.5

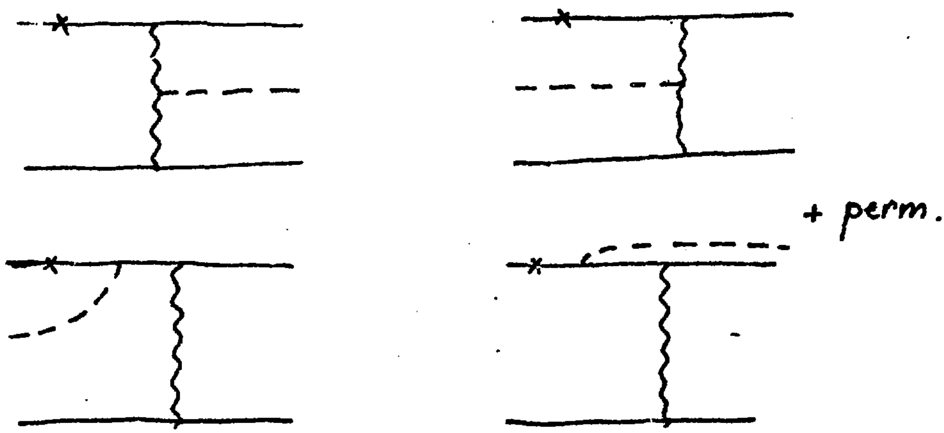

Fig.6

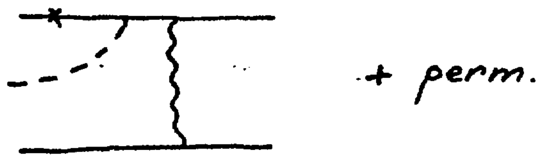

Fig.7

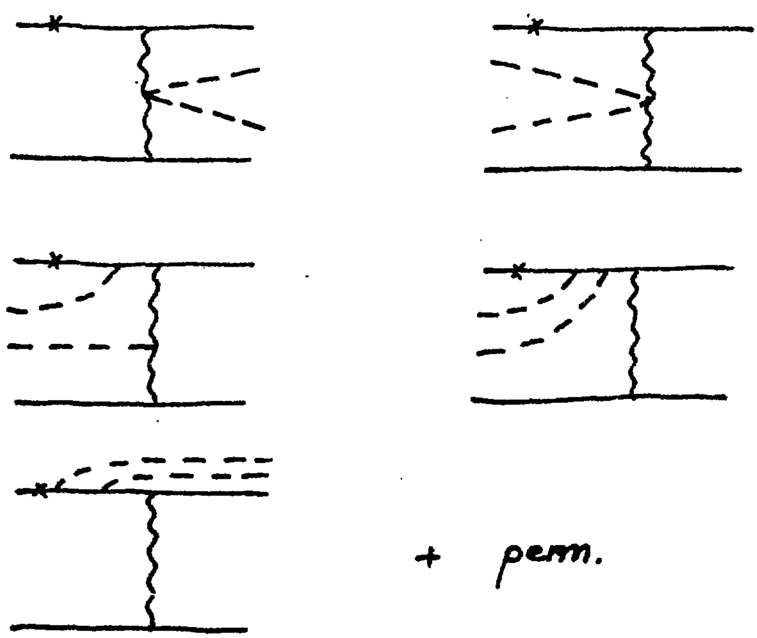

Fig.8

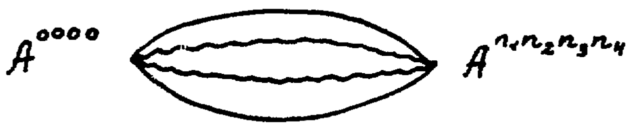

Fig.9

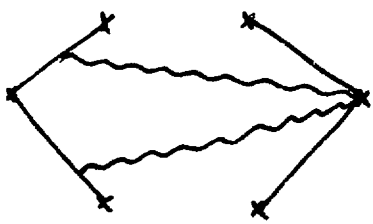

Fig.10

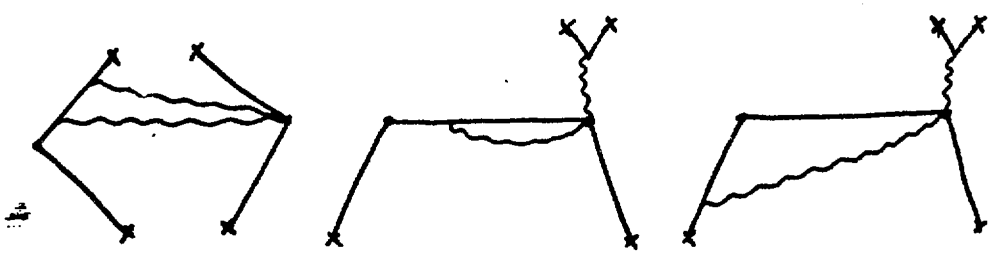

Fig.11

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А.С.Горский

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