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MODEL OF HOMOGENEOUS NUCLEUS. TOTAL AND INELASTIC CROSS SECTIONS OF NUCLEON – NUCLEUS SCATTERING

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Abstract

It is shown that the nucleon-nucleus scattering amplitude at high energy can be easily calculated by generalization of the nucleon-nucleon scattering amplitude and satisfies a simple factorization relation.

As distinct from the Glauber model, the suggested approach makes no use of the nucleonic structure of the nucleus and the hadron-nucleus scattering amplitude is not expressed in terms of hadron-nucleon scattering amplitudes. The energy dependence of total and inelastic cross sections is successfully described for a number of nuclei.

1. In the present paper we study the hadron-nucleus scattering amplitude for sufficiently heavy nuclei (we shall mainly deal with $A > 50$ nuclei). These amplitudes are usually calculated by the Glauber method /1/, i.e. representing the hadron-nucleus amplitude as a sum of the amplitudes associated with n rescatterings of the incident hadron on the nucleons inside the nucleus. The result is presented in Fig.1: the nucleus is formed by A weakly correlated nucleons.

On the other hand, there are many arguments in favor of the idea that the nucleon is in its turn formed by three valence quarks (Fig. 2a). Then the simplest scheme of nuclear structure in the quark model is presented by the equilibrium distribution of quarks inside the nucleus (Fig. 2b). According to this scheme, the nucleus is a big nucleon (BN) which contains 3A quarks. The reality of the BN model is supported, in particular, by rather dense packing of nucleons inside the nucleus (the volume of the nucleus is practically equal to the volume occupied by A nucleons).

One of the simplest consequences of the quark structure of hadrons is factorization of the πN-, NN-, and $\pi\pi$ -scattering amplitudes and cross sections.

$$T_{\pi N}(0)/T_{NN}(0) \simeq \frac{2}{3}\ ;\ T_{\pi\pi}(0)/T_{NN}(0) \simeq \frac{4}{9}\ ; \tag{1a}$$

$$\sigma^{t}_{\pi N}/\sigma^{t}_{NN} \simeq \frac{2}{3}\ ;\ \sigma^{t}_{\pi\pi}/\sigma^{t}_{NN} \simeq \frac{4}{9}\ . \tag{1b}$$

Eqs. (1a), (1b) are well satisfied experimentally at high energies. They stem from the additivity hypothesis for quark-quark amplitudes first suggested in ref. /2/. In the BN model the additivity hypothesis naturally leads to the factorization relation for NN- and NA-scattering amplitudes, similar to those of Eq. (1a):

$$T^{P}_{NA}(0)\ /\ T^{P}_{NN}(0) \simeq \frac{3A}{3} = A \tag{2}$$

In this case, however, the factorization relation is satisfied not by total scattering amplitudes, but by "trial" one-pomeron-exchange amplitude. Due to large screening corrections to NA scattering there are not relations similar to Eq. (1b) for total cross sections. The main purpose of this paper is to confirm Eq. (2).

2. The BN model suggests a unified approach to the amplitude of hadron-nucleon and hadron-nucleus scattering. We begin with hadron-nucleon interactions. The best and most complete description of them is provided by the method of complex momenta. The elastic scattering amplitudes at high energy are calculated in the quasieikonal approximation /3/, e.g. the nucleon-nucleon scattering amplitude has the form /4/

$$T(S,q)=\frac{8\pi S\lambda_N}{C}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(Cg)^n}{n\,n!}e^{-\frac{\lambda_N}{n}q^2}$$

$$\lambda_N=r_N^2+\alpha_p'(0)\ln S;\quad g=\frac{\gamma_N}{\lambda_N}e^{(\alpha_p(0)-1)\ln S} \qquad (3)$$

with the following notation: S is the square of the invariant mass, q is the transferred transversal momentum, $\alpha_p(0)$ and $\alpha_p'(0)$ are parameters of the vacuum trajectory, $\gamma_N e^{-r_N^2 q^2}$ is a factor associated with one-pomeron contribution to the amplitude (γ_N is the multiple of the nucleon residues of the pomeron and r_N^2 is the corresponding effective radius squared), C is the shower enhancement constant which characterizes the cross section of quasielastic diffraction processes σ_{dif}:

$$C-1=\sigma_{dif}/\sigma_{el} \qquad (4)$$

Equation (3) is associated with the diagrams of Fig. 3. In this figure, each curly line means a one-pomeron exchange. The main nucleon-nucleon scattering characteristics are well described at the following values of parameters obtained by fitting procedure /5/

$$\alpha_p(0)=1.007,\quad \alpha_p'(0)=0.25\ \text{GeV}^{-2}$$
$$\gamma_N=3.64\ \text{GeV}^{-2},\quad r_N^2=3.56\ \text{GeV}^{-2},\ C=1.5 \qquad (5)$$

The energy dependence of the total proton-proton scattering cross section calculated by the formula (3) using Eq. (5) for the parameter values is presented in Fig. 4.

For further calculations the amplitude may be conveniently written in a standard manner in the impact parameter b representation:

$$T(s,q) = 4\pi s M(s,q)$$
$$M(s,q) = \frac{1}{2\pi}\int e^{i\vec{q}\vec{b}} M(s,b)\, d^2b \qquad (6)$$
$$M(s,b) = \frac{1}{c}\left(1 - e^{-M_1(b)c}\right)$$

The quantity M (b) is the Fourier image of the one-pomeron exchange amplitude

$$M_1(b) \cong \frac{1}{2\pi}\int e^{-i\vec{q}\vec{b}}\left(\gamma_N e^{\xi\Delta} e^{-\lambda_N q^2}\right) d^2q = \frac{\gamma_N}{\lambda_N} e^{\xi\Delta} e^{-\frac{b^2}{4\lambda_N}} \qquad (7)$$
$$\xi = \ln s, \quad \Delta = \alpha_P(0) - 1$$

The total (σ_t), elastic (σ_{el}), inelastic (σ_{inel}) and diffraction dissociation (σ_{dif}) cross section may be calculated according to

$$\sigma_i = \int \sigma_i(b)\, d^2b$$

where

i = t, el, inel, dif

$$\sigma_t(b) = \frac{2}{c}\left(1 - e^{-M_1(b)c}\right)$$
$$\sigma_{el}(b) = \frac{1}{c^2}\left(1 - e^{-M_1(b)c}\right)^2 \left(\frac{1}{2}\sigma_t(b)\right) \qquad (8)$$
$$\sigma_{inel}(b) = \frac{1}{c}\left(1 - e^{-2M_1(b)c}\right)$$
$$\sigma_{dif}(b) = (c-1)\,\sigma_{el}(b)$$

Now we proceed to the nucleon-nucleus scattering amplitude in the BN model. Let us formulate the main point of the paper. If a nucleus interacting with high energy nucleons really behaves as a big nucleon, then the nucleon-nucleon scattering amplitude must transform into the quasieikonal nucleon-nucleus scattering amplitude by means of two procedures:

a) The pomeron-nucleus coupling vertex γ_A is by a factor of A greater than the nucleon vertex (the additivity hypothesis)

$$\gamma_A = A \gamma_N \tag{9}$$

b) The pomeron form factor of the nucleon must be substituted by the nuclear form factor. To do it we shall write the nucleon-nucleus scattering amplitude $M_{1A}(q)$ as a product

$$M_{1A}(q) = \gamma_A F_N(q^2) F_p(q^2) F_A(q^2) \tag{10}$$

Functions F_N, F_p, and F_A describe the q dependence of the nucleon vertex, of the pomeron propagator and of the nuclear vertex, correspondingly. All the functions are normalized by F (0) = 1. Since we shall deal with only medium-weight and heavy nuclei we suggest that all the q dependence is defined by the nuclear form factor $F_A(q^2)$, whereas $F_N = F = 1$ (optical approximation)

$$F_A(q^2) = \int e^{i\vec{q}\vec{r}} \rho(r)\, d\vec{r} \tag{11}$$

For $\rho(r)$ we used the Fermi density of the nuclear matter distribution

$$\rho(r) = C_\rho \left[1 + \exp\left(\frac{r-R}{a}\right)\right]^{-1} \tag{12}$$

The parameters R and a are best fits to the electron-nucleus scattering data /6/. The constant C_ρ was defined by the normalization condition $\int \rho(r)\, d\vec{r} = 1$.

Less essential additional remark is associated with modification of the shower enhancement factor C . For both NN and NA interactions C takes account of production of two showers corresponding to the beam and target particles (see Eq. (4)) [1].

For NA scattering the role of one of the showers is played by coherent diffraction excitation of nuclei. The experimental total cross sections of these processes are almost unknown. If we suggest that these cross sections are less than the beam shower production cross section, then the factor C for NA scattering (we shall call it C_A) can be calculated making use of its earlier value /5/ for nucleon-nucleon scattering

$$C_A = \sqrt{C} = 1.23 \qquad (13)$$

This value characterizes the asymptotic cross section of shower production, since at fixed energy a beam with the mass M_d can be produced only at transferred momentum $q^2 > q_{min}$ with

$$q^2_{min}(S) \simeq m^2 \left(\frac{M_d^2 - m_N^2}{S} \right)^2 \qquad (14)$$

Therefore in the BN model the energy dependence of the factor C_A is

$$C_A(S) = 1 + \Delta_c \exp\left[-2 z_0^2 A^{2/3} q^2_{min}(\bar{M}_d) \right], \qquad (15)$$

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[1]) Note, that in the model under consideration the account of shower production processes leads to decrease of the total cross section (inelastic screening /7/), similarly to the Glauber case.

where $\Delta_c = 0.23$ (see Eq. (13)), M_d is the mean mass of the produced beam shower.

To obtain the analytic expression (15) for C_A we used the exponential form of the nuclear form factor, with the radius $r_A = r_0 A^{1/3}$ ($r_0 = 1.12$ fm).

It is found experimentally that the mass spectra of diffraction beams decrease fast with increase of the mass, the maximum being at $M_d \simeq (1.5 - 2)$ GeV. Therefore M_d^2 was taken to be $M_d^2 = 3\ GeV^2$. Now we see that making a somewhat arbitrary choice of M_d we have built a model which involves no free parameters.

Weak correlation of quarks inside the nucleus is an essential condition in the BN model. It is expected to be satisfied only for sufficiently heavy nuclei (say, with A 50), and the best fit to the data will be for nuclei with big atomic number. However, we present for comparison our results for two light nuclei ^{12}C and ^{27}Al. It has turned out that a satisfactory fit to the ^{12}C and ^{27}Al data can be obtained by introducing A dependent C_A . The found $\Delta_c(A)$ dependence is presented in Fig. 5. Besides, the total nucleon-nucleus scattering cross sections calculated in the BN model must be in a poorer agreement with the data then inelastic cross sections because the formulated BN model does not include hadron-nucleus quasielastic scattering followed by the target nucleus desintegration.

3. The calculations have been carried out for the nuclei ^{12}C, ^{27}Al, ^{64}Cu, and ^{208}Pb. The choice of nuclei was governed by the existence of the experimental data.

All the available data have been used from Batavia /8,9/, Serpukhov /10/, CERN /11/, and Argonn National Laboratory /8/ at energies from 10 to 273 GeV. In Figs. 6-9 the theoretical curves are compared with the experimental values of the total /8-12/ and inelastic /12-14/ cross sections of nucleon-nucleus scattering.

We have made use of the energy dependence (15) of the factor $C_A(S)$. For A = 64 and A = 208 Δ_c has been fixed (Δ_c = 0.23). For A = 12 and A = 27 the dependence presented in Fig. 5 has been used. The following nuclear radii were involved in the calculations: r_A = 2.3, 3.07, 4.16, and 6.5 fm for A = 12, 27, 64, and 208, correspondingly. The theoretical results are in obvious agreement with the data: the mean deviation is of about 5%.

To test quantitatively the additivity hypothesis for the nucleon-nucleus scattering amplitude (Eq. (2)) the value of the pomeron residue γ_N (3) has been varied. In Fig. 8 the dashed and dashed-dotted curves correspond to 10% variation of γ_N : γ_N = 4 (dashes) and γ_N = 3.28 (dots). It can be seen that variation of γ_N by 10% induces the cross section variation within the experimental errors. Hence, the present experiment indicates that the additivity hypothesis is valid within a 10% accuracy.

4. We have demonstrated that the model for the nucleon-nucleus scattering amplitude constructed by simple extrapolation of the nucleon-nucleon scattering amplitude leads without free parameters to satisfactory description of the experimental data on the total and inelastic cross sections for not too light nuclei. In the quark model this means that the formulas for the cross

sections can be obtained taking no account of quark correlations inside the nucleus. These correlations should be included when more complicated characteristics are being calculated. A concrete example is the Glauber model.

The sucsess of the model is not unexpected. Actually, the total and elastic cross sections of nucleon-nucleon scattering are commonly calculated making use of eikonal formulas which do not include the intrinsic structure of the nucleon likewise our formulas do not include the intrinsic structure of the nucleus.

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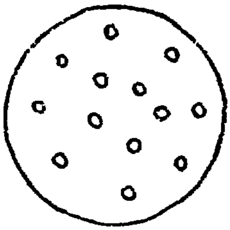


Fig.1

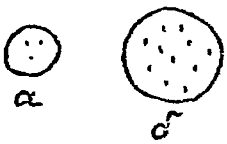


Fig.2

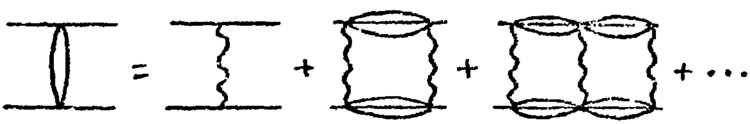


Fig.3

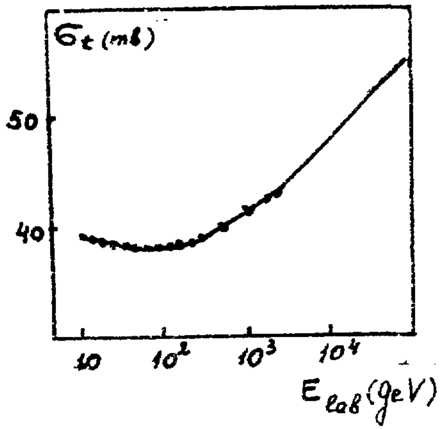


Fig.4

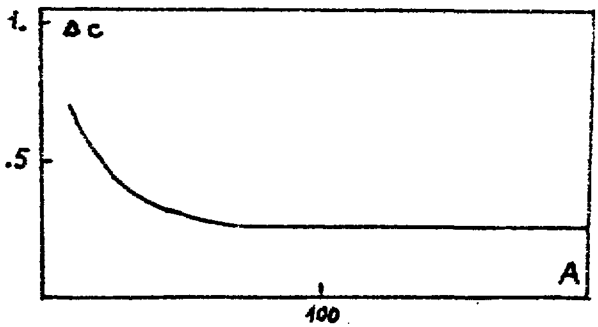


Fig.5

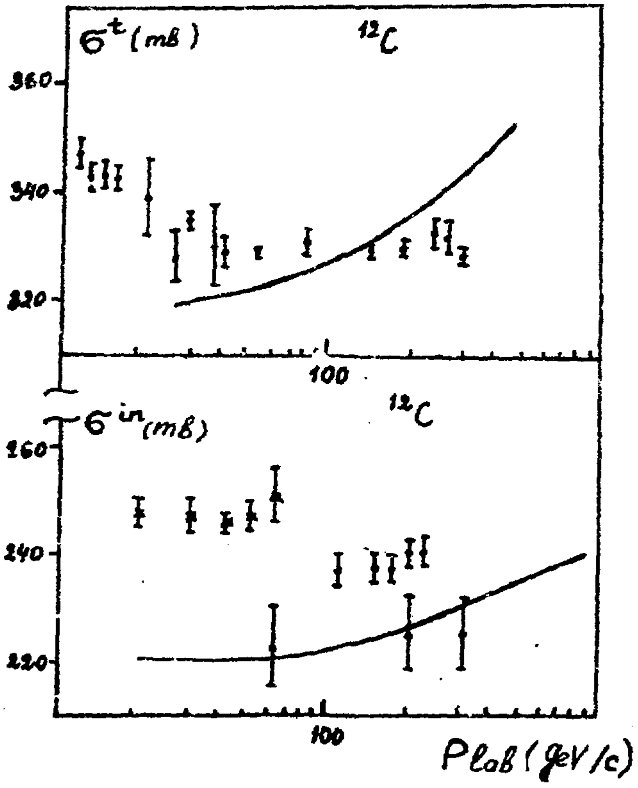


Fig.6

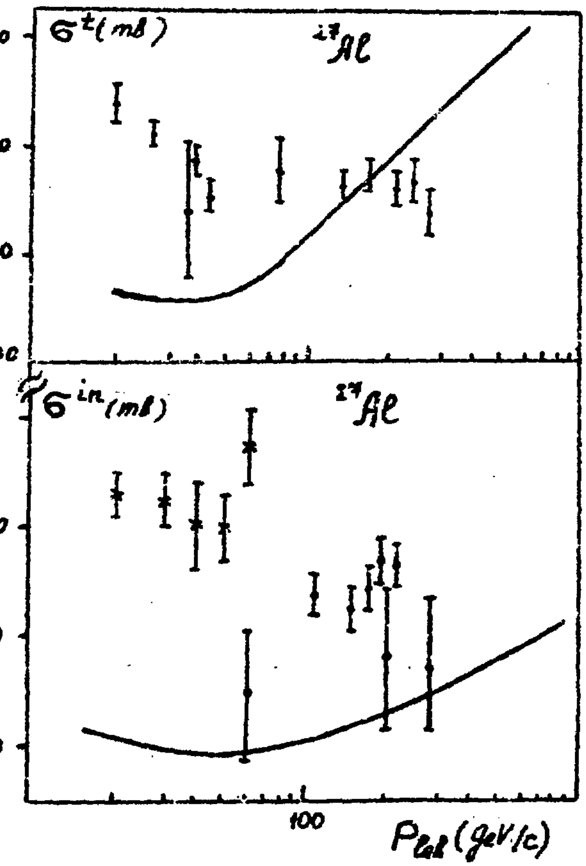
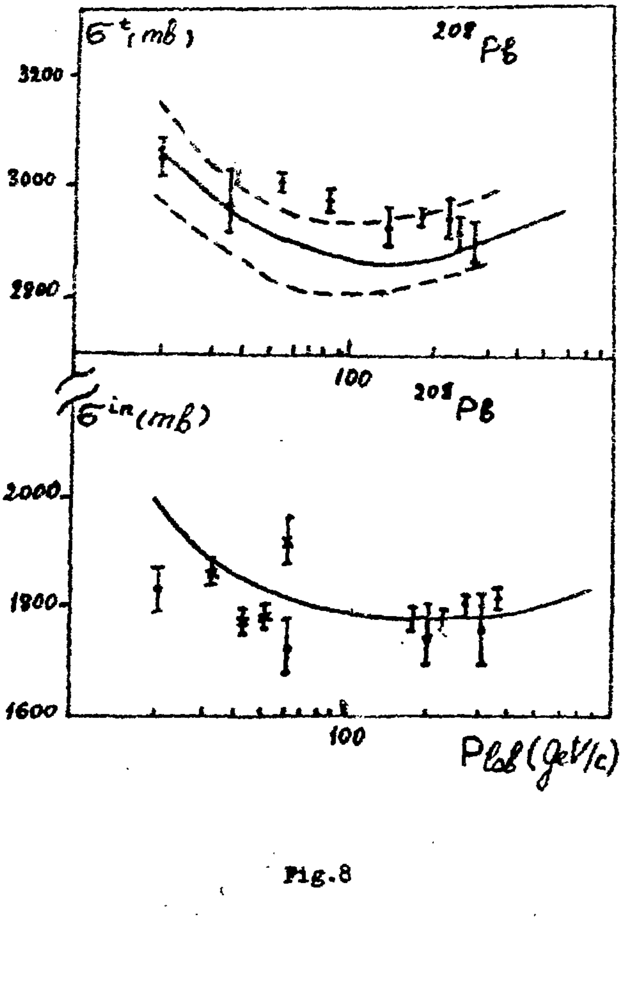


Fig.7



σ^t (mb)
208 Pb
3200
3000
2800
100
σ^in (mb)
208 Pb
2000
1800
1600
100
Plab (GeV/c)

Fig.8

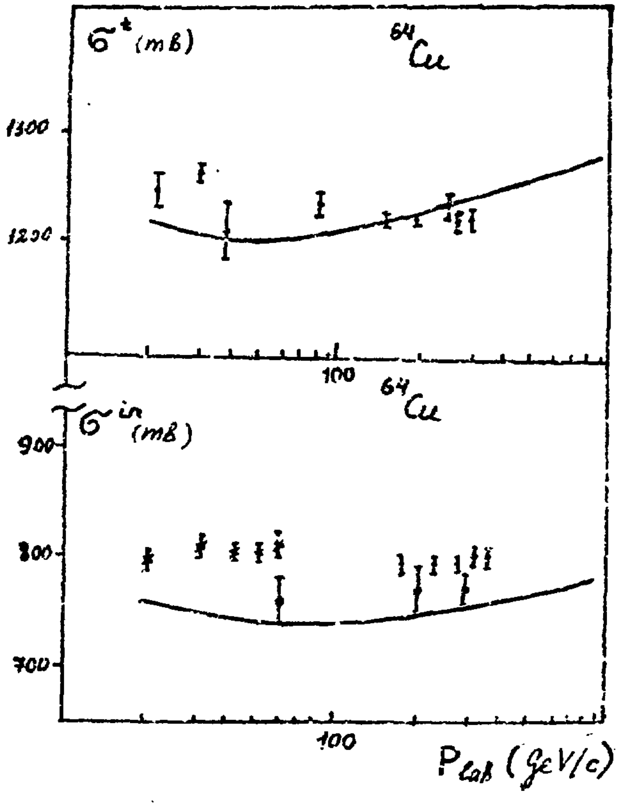


Fig.9

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