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RESISTIVE INSTABILITIES IN TOKAMAKS

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RESISTIVE INSTABILITIES IN TOKAMAKS*

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ABSTRACT

Low-m tearing modes constitute the dominant instability problem in present-day tokamaks. In this lecture, the stability criteria for representative current profiles with q(0)-values slightly less than unity are reviewed; "sawtooth" reconnection to q(0)-values just at, or slightly exceeding, unity is generally destabilizing to the $m = 2$, $n = 1$ and $m = 3$, $n = 2$ modes, and severely limits the range of stable profile shapes. Feedback stabilization of $m \geq 2$ modes by rf heating or current drive, applied locally at the magnetic islands, appears feasible; feedback by island current drive is much more efficient, in terms of the radio-frequency power required, than feedback by island heating. Feedback stabilization of the $m = 1$ mode -- although yielding *particularly beneficial effects for resistive-tearing and high-beta stability by allowing q(0)-values substantially below unity* -- is *more problematical*, unless the $m = 1$ ideal-MHD mode can be made positively stable by strong triangular shaping of the central flux surfaces. Feedback techniques require a detectable, rotating MHD-like signal; the slowing of mode rotation -- or the excitation of non-rotating modes -- by an imperfectly conducting wall is also discussed.

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I. INTRODUCTION

A small but finite amount of plasma resistivity has a remarkably strong destabilizing effect on the MHD stability of tokamaks. In particular, it can give rise to various modes -- especially resistive kink modes -- that grow on a time scale that lengthens only gradually with decreasing plasma resistivity. In present-day tokamak experiments, these resistive kinks (or "tearing modes") constitute, by far, the most dominant instability problem. Indeed, a tokamak with a highly non-optimal q(r)-profile may encounter gross instability and "disruptive" termination of the discharge.

While major disruptions can be avoided in normal tokamak operation, a low level of resistive-MHD activity generally remains present and often serves to restore the q(r)-profile periodically to a preferred form by reconnecting magnetic flux surfaces. For example, the "sawtooth" oscillations shown on the left in Fig. 1 (taken from PLT) serve to maintain q(0) slightly below unity, on average, by reconnecting the magnetic flux arising from $m = 1$ resistive-kink pertubations, thereby flattening the j(r)-profile (and T_e-profile) periodically within the region where $q(r) \leq 1$. Such discharges are generally free of significant $m = 2$ activity and exhibit relatively favorable confinement throughout the region $q(r) > 1$. On the other hand, discharges without sawteeth [i.e., with $q(0) > 1$], such as that shown on the right in Fig. 1, are generally characterized by a high level of $m = 2$ activity, often leading to a major disruption. [A "sawtoothing discharge" may undergo a transition to an "$m = 2$ discharge" just after the sawtooth reconnection phase, when q(0) attains its highest value ($\simeq 1$).] These observations can be understood in terms of the stability of tokamaks with $q(0) \sim 1$ against $m \geq 2$ tearing modes, which is the topic of Sec. II of this lecture.

Tearing modes are amenable to feedback stabilization by rf heating and current-drive techniques provided the rf power can be localized within the magnetic islands; this is the topic of Sec. III of this lecture. Feedback stabilization of the $m = 1$ mode would be especially benefical, since it would not only allow higher plasma current (thereby improving both confinement and beta-value) but would also indirectly stabilize the $m \geq 2$ modes by permitting q(0)-values significantly below unity; this is the topic of Sec. IV.

The experimental detection of MHD-like modes in tokamaks -- clearly necessary for the application of rf feedback -- requires that the mode be rotating. The possible role of an imperfectly conducting wall in stabilizing rotating tearing modes -- or provoking non-rotating instabilities -- is discussed briefly in Sec. V.

II. TEARING-MODE STABILITY

The stability of a tokamak to $m \geq 2$ tearing modes depends on the radial profile of the "safety-factor" $q(r)$, the m-value of the mode, and the position of the singular surface r_s with respect to the $q(r)$-profile. Stability criteria can be obtained by varying the perturbed magnetic energy

$$W = (\pi/m^2) \int \{r^3 (\frac{\partial\psi}{\partial r})^2 + [(m^2 - 1)r + \frac{1}{F}\frac{d}{dr} (r^3 \frac{dF}{dr})] \psi^2\}dr$$

and solving the resulting Euler-Lagrange equation

$$\frac{\partial}{\partial r} (r^3 \frac{\partial\psi}{\partial r}) - [(m^2-1)r + \frac{1}{F}\frac{d}{dr} (r^3 \frac{dF}{dr})]\psi = 0$$

on either side of the singular surface r_s. Here, $\psi(r)$ *is the radial component* B_r *of the perturbed magnetic field, and*

$$F(r) = \iota(r) - n/m,$$

where $\iota(r)$ is the rotational transform (divided by 2π), i.e., $\iota(r) = q^{-1}(r)$. If the Euler-Lagrange equations are satisfied, the perturbed magnetic energy becomes

$$W = - (\pi/m^2)\, r_s^3\, \psi_s^2\, \Delta_s',$$

$$\Delta_s' = [\partial\psi/\partial r]_s/\psi_s,$$

where $[\ ]_s$ denotes the jump across the singular surface r_s. Modes for which Δ_s' > 0 are unstable.

In an early paper on tearing modes in tokamaks /1/, the Euler-Lagrange equations were solved numerically for three representative tokamak current profiles, namely,

$$j_z(r) = j_z(0)/[1 + (r/r_0)^{2p}]^{1+1/p}$$

corresponding to

$$q(r) = q(0)\ [1 + (r/r_0)^{2p}]^{1/p}$$

for $p = 1$ ("peaked profile"), $p = 2$ ("rounded profile"), and $p = 4$ ("flattened profile"). These $j_z(r)$ and $q(r)$ profiles are shown in Fig. 2, arbitrarily normalized to unity at $r = 0$.

In the steady-state phase of the tokamak, the current profile invariably contracts until the central q-value is in the general neighborhood of unity. (The only exception to this pattern of behavior seems to be where high-Z impurity radiation from the central part of the discharge is unusually strong, depressing the central electron temperature and current density.) This contraction of the current profile is, no doubt, partly due to the cooling of the outer part of the discharge, but it may also represent a tendency for the discharge to evolve toward a configuration with relatively favorable stability properties against $m \geq 2$ tearing modes. Thus, it is of particular interest to consider the stability of profiles with $q(0) \sim 1$.

Noting that the quantity $r_s\Delta_s'$ provides a measure of the magnetic energy available to a tearing mode, we plot in Fig. 3 the calculated values of $r_s\Delta_s'$ as a function of $q(0)$ for the $m = 2$, $n = 1$ and $m = 3$, $n = 2$ modes for the three representative current profiles already discussed (assuming a conducting wall at $r_w/r_0 = 2.0$). It is evident from Fig. 3(a) that all three profiles are unstable to the $m = 2$ mode if $q(0) > 1$; the "flattened" and "rounded" profiles seem to be relatively unfavorable from an energetic viewpoint, and the effect of the $m = 2$ mode is presumably most severe in these cases. On the other hand, if $q(0)$-values below about 0.9 can be tolerated, the stability of the $m = 2$ mode is much improved, especially in the case of the "flattened" and "rounded" profiles. For $q(0)$-values above about 0.95, the "flattened" and "rounded" profiles are also strongly unstable to the $m = 3$, $n = 2$ modes; the simultaneous destabilization of the $m = 2$, $n = 1$ and $m = 3$, $n = 2$ modes and the resulting break-up of magnetic surfaces provides a persuasive explanation for the major disruption /2/.

Tokamak discharges with $q(0) < 1$ exhibit "sawtooth" behavior in which a strongly-growing $m = 1$ resistive kink reconnects the magnetic surfaces in the central region of the plasma in such a way that $q(0)$ is periodically restored to unity. The process of reconnection can be described quantitatively /3/ in terms of the ($m = 1$, $n = 1$) helical flux function

$$\chi(r) = \int^r B_\theta dr - (r^2 B_z/2R)$$

which undergoes the transformation illustrated in Fig. 4. Initial flux elements $d\chi$ at r_1 and r_2 combine into the final flux element at r in such a way that the toroidal flux (area) is conserved:

$$r dr = r_1 dr_1 + r_2 dr_2 \quad .$$

The final flux function $\chi_f(r)$ can be obtained from the initial flux function $\chi_i(r)$ by means of the relation

$$r \frac{dr}{d\chi_f} = r_2 \frac{dr_2}{d\chi_i}\bigg|_{r_2(\chi)} - r_1 \frac{dr_1}{d\chi_i}\bigg|_{r_1(\chi)} \quad .$$

The reconnected $\iota(r)$ and $j_z(r)$ profiles are shown in Fig. 5 ("Kadomtsev model") for the case of a parabolic current profile with $\iota(0) = 1.15$. A reversed surface current is induced at the outermost radius of reconnection; this surface current will survive only transiently, implying that a more physical model for the reconnected profiles might be the "flat-q model," also shown in Fig. 5, in which $\iota(r)$ is unity inside the initial q = 1 singular surface and unchanged outside it.

As we have already noted, small changes in the $\iota(r)$-profile in the central part of the plasma can have a remarkably strong effect on the stability of the m = 2, n = 1 and m = 3, n = 2 modes. The effect of such changes can be quantified by varying the F(r)-function, and consequently also the minimizing $\psi(r)$-function, in the perturbed magnetic energy W:

$$F \rightarrow F + \delta F,$$
$$\psi \rightarrow \psi + \delta\psi,$$
$$W \rightarrow W + \delta W.$$

If we limit ourselves to variations that leave ι and $d\iota/dr$ (i.e., q-value and current density) unchanged at the singular surface r_s of the mode (m = 2, n = 1 or m = 3, n = 2) under investigation, we obtain /1/:

$$\delta W = (\pi/m^2) \int \{r^3 \, [\tfrac{\partial}{\partial r} \, (\tfrac{\psi}{F})]^2 + (m^2-1) \, \frac{r\psi^2}{F^2}\} \; \delta \, (F^2) \; dr \; .$$

The expression { } is clearly positive definite: stability is improved to the extent that

$$\delta(F^2) = [\iota(r) - n/m]^2{}_{final} - [\iota(r) - n/m]^2{}_{initial}$$

can be made positive.

The formalism described above can be applied to the perturbations in $\iota(r)$ arising from sawtooth reconnection. It is immediately evident from Fig. 5(a) that reconnection according to the "flat-q model" is always destabilizing to the m = 2, n = 1 and m = 3, n = 2 modes: the magnitude of $\iota(r)$ - n/m is everywhere lowered (or unchanged) by reconnection. On the other hand, reconnection according to the "Kadomtsev model" has both stabilizing and destabilizing features: there are regions where $\iota(r)$ - n/m is lowered and regions where it is raised. Remarkably, a detailed calculation for the case of a parabolic current profile with an initial q(0)-value just below unity shows that the stability of the m = 2, n = 1 mode is unchanged by "Kadomtsev-model" reconnection -- the stabilizing and destabilizing contributions exactly cancel. The stability of the m = 3, n = 2 mode is actually improved slightly.

Of more practical relevance is the "flat-q model" of sawtooth reconnection, which might approximate the experimental situation after the reversed surface current has decayed. Using the perturbation formalism described above, we have calculated the effect of "flat-q" reconnection on the stability of the m = 2, n = 1 and m = 3, n = 2 modes for the "rounded" and "flattened" current profiles. [These profiles seem most representative of profile shapes inferred from measurements at low q_a-values in large tokamaks such as the Tokamak Fusion Test Reactor (TFTR) where the long skin time precludes too much peaking of j(r) during a sawtooth.] The results are given in Figs. 6 and 7, in which values of $r_s\Delta_s'$ are plotted against the initial value of q(0) (i.e., before reconnection) for two positions of a conducting wall (r_w/r_0 = 2.0 and $r_w/r_0 = \infty$). With a conducting wall at r_w/r_0 = 2.0, the "rounded" profile can remain stable to both modes after reconnection; the "flattened" profile can remain stable to the m = 2, n = 1 mode, but becomes unstable to the m = 3, n = 2 mode. The sensitivity of the m = 2, n = 1 mode to the position of the conducting wall is surprising, and it is further evidence of the "marginal" stability of all profiles with q(0) $\simeq$ 1.0. The requirement that "sawtoothing" profiles remain stable to the m = 2, n = 1 and m = 3, n = 2 tearing modes clearly imposes very severe constraints on the allowed current-profile shapes.

If the current profile becomes slightly hollow after sawtooth reconnection, as has been proposed in one theoretical model /4/, the m = 2, n = 1 and m = 3, n = 2 modes are even further destabilized.

III. RADIO-FREQUENCY FEEDBACK STABILIZATION OF TEARING MODES ($m \geq 2$)

The suppression of the $m = 2$ mode in a tokamak would have three important advantages: (i) it could provide disruption-free operation at relatively low $q(a)$-values; (ii) it would provide a modest improvement in the limiting beta-value for ballooning instabilities by shortening the connection length [i.e., lowering $q(a)$]; and (iii) it would provide a modest improvement in confinement, by allowing increased plasma current.

Feedback stabilization of $m \geq 2$ modes by rf heating and/or current drive has recently been proposed /5/. To produce a stabilizing effect, the feedback technique must increase the plasma current density at the O-point of the magnetic islands associated with the tearing mode and decrease the current density at the X-point (separatrix) of the island.

That the feedback-induced changes in plasma current density at the magnetic island must serve to reinforce the naturally occurring current-density perturbations (see Fig. 8) is apparent from a consideration of tearing-mode physics. In a tokamak geometry [where the shear $(B_\theta/r)'$ is opposite to the current density j_z], a lowering of the magnetic energy (corresponding to $\Delta' > 0$) requires localized increases in current density at the O-point of the magnetic island and localized decreases in current density on the separatrix. Since the helical flux function $\chi = \int^r [B_\theta(r) - nrB_z/mR]dr$ has a maximum at the singular surface, these perturbations in current density correspond to a flattening of the flux function across the magnetic island (i.e., a reconnection of negatively directed helical field $B_\theta - nrB_z/mR$ on the large-r side of the island with positively directed field on the small-r side). This reconnection can proceed only on the resistive time scale, which limits the growth rate of the mode. If the current-density perturbations required to produce magnetic islands of a certain size can be provided non-resistively (e.g., by rf feedback), then resistive reconnection will proceed even more slowly, and further growth of the islands will be inhibited.

There are two principal options for producing the desired current perturbations by rf-feedback techniques:

1. Heat the magnetic islands (O-points) by localized rf heating, thereby lowering the local resistivity;
2. Drive additional non-inductive currents within the magnetic islands (O-points).

(The converse of the first option -- island cooling by excess radiation -- has been proposed as a contributory factor in major disruptions /6/.) In both cases, the rf power must be phase modulated to match a perturbation signal from some suitable detector (for example, the electron temperature measured by electron cyclotron emission). Feedback techniques based on lower hybrid waves (for current drive) or electron cyclotron waves (for heating) are theoretically capable of providing the required localization of the rf power, and they are discussed in detail in a separate paper /7/.

The theory of feedback stabilization of tearing modes by island heating is based on the standard treatment of $m \geq 2$ modes in their slow-growing (nonlinear) phase /8/, except that the resistivity on flux surfaces interior to magnetic islands is allowed to be perturbed relative to that on exterior flux surfaces. The rf power density is modulated in phase with the rotating island:

$$P_{rf} = \tilde{P}_{rf} \cos\,(m\theta - n\phi - \omega t),$$

and the radial profile of power deposition is assumed to be quite narrow, but <u>not</u> as narrow as the island itself. The calculation proceeds most transparently in "slab" geometry, in which x replaces $r-r_s$ and ky replaces $m\theta-n\phi-\omega t$. For slow-growing modes, the perturbed current density δj_z and resistivity $\delta\eta$ must be constant on flux surfaces, i.e., surfaces of constant $\psi = \int^x B_y dx$. Faraday's and Ohm's laws, expressed in terms of the magnetic flux function ψ, can be written

$$\frac{\partial\psi}{\partial t} + \underline{v}\cdot\underline{\nabla}\,\psi = \eta\,\delta j_z(\psi) + j_z\delta\eta(\psi)\ ,$$

where

$$\psi = B_y' x^2/2 - \psi_1 \cos ky\ .$$

(Signs are chosen such that the O-point is at $x = y = 0$ for B_y', $\psi_1 > 0$.) For incompressible poloidal flow (strong toroidal field), a velocity stream function ϕ may be introduced, such that

$$\underline{v}\cdot\underline{\nabla}\,\psi = -\left(\frac{\partial\phi}{\partial y}\right)_\psi B_y'\,x\ .$$

Integrating over a full period in y (or fully around interior flux surfaces), we obtain

$$\delta j_z(\psi) = -\frac{1}{n}\frac{\partial\psi_1}{\partial t}\left\langle \frac{\cos ky}{(\psi+\psi_1\cos ky)^{1/2}}\right\rangle_y \Big/ \left\langle \frac{1}{(\psi+\psi_1\cos ky)^{1/2}}\right\rangle_y - \frac{j_z\delta n}{n}$$

where $\langle\ \rangle_y$ denotes an average over y. The solutions in the vicinity of the singular surface may be matched to the outer solutions in the usual manner: by integrating the relation

$$\nabla^2\psi = \delta j_z(\psi)$$

across the island region and including only the dominant Fourier component in y we obtain

$$\Delta'\psi_1 = -2\langle \cos ky \int \delta j_z\, dx\rangle_y$$

$$= \frac{4}{(2B_y')^{1/2}}\left[\frac{1}{n}\frac{d\psi_1}{dt}\int_{-\psi_1}^{\infty} d\psi \left\langle\frac{\cos ky}{(\psi+\psi_1\cos ky)^{1/2}}\right\rangle_y^2 \Big/ \left\langle\frac{1}{(\psi+\psi_1\cos ky)^{1/2}}\right\rangle_y \right.$$

$$\left. + \frac{j_z}{n}\int_{-\psi_1}^{\infty} d\psi \left\langle\frac{\cos ky}{(\psi+\psi_1\cos ky)^{1/2}}\right\rangle_y \delta n\right] .$$

The heat flux across a magnetic surface interior to the island must equal the rf power deposited within the surface:

$$-\int dy\, K_{\perp e}|B_y' x|\,\frac{\partial\delta T_e}{\partial\psi} \doteq \iint P_{rf}\, dx\, dy,$$

where $K_{\perp e}$ is the cross-field electron thermal conductivity, giving

$$\delta T_e(\psi) = -\frac{\tilde{P}_{rf}}{2K_{\perp e}B_y'}\int^{\psi} \frac{d\psi}{\langle(\psi+\psi_1\cos ky)^{1/2}\rangle_y}\int_{-\psi_1}^{\psi} d\psi' \left\langle\frac{\cos ky}{(\psi'+\psi_1\cos ky)^{1/2}}\right\rangle_y .$$

The resistivity perturbation may be obtained from the temperature perturbation by assuming $\delta n/n = -3\delta T_e/2T_e$. Changing the variable of integration from ψ to $X = \psi/\psi_1$, so as to reduce the various integrals to numerical constants, and integrating by parts where indicated, we obtain

$$\Delta' \psi_1 = \frac{4C_1 \psi_1^{1/2}}{\eta(2B_y')^{1/2}} \frac{\partial \psi_1}{\partial t} + \frac{6C_2 \tilde{P}_{rf} j_z \psi_1^{3/2}}{K_{\perp e} T_e (2B_y')^{3/2}} ,$$

where C_1 and C_2 are numerical constants:

$$C_1 = \int_{-1}^{\infty} dX \left\langle \frac{\cos ky}{(X+\cos ky)^{1/2}} \right\rangle_y^2 \Big/ \left\langle \frac{1}{(X+\cos ky)^{1/2}} \right\rangle_y ,$$

$$C_2 = \int_{-1}^{\infty} \frac{dX}{\langle (X+\cos ky)^{1/2} \rangle_y} \left(\int_{-1}^{X} dX' \left\langle \frac{\cos ky}{(X'+\cos ky)^{1/2}} \right\rangle_y \right)^2 .$$

Approximate numerical values are $C_1 = 0.7$ and $C_2 = 1.4$. Introducing the island width w, given by $w = 4(\psi_1/B_y')^{1/2}$, the result can be written

$$\frac{dw}{dt} = \eta \left(\Delta' - C_h \frac{\tilde{P}_{rf} w}{K_{\perp e} T_e}\right) ,$$

where

$$C_h = -0.75 \, (j_z/B_y') = 0.75 \, (rj_z/B_\theta) \, (q/rq') .$$

The latter expression for C_h represents the appropriate transformation from slab to cylindrical geometry. Typically -- for example, the m = 2 instability of a "rounded" or "peaked" profile with q(0) = 1 -- the value of C_h is about 0.5.

Unfortunately, the power requirements for this type of rf feedback turn out to be prohibitively large -- especially if the cross-field electron thermal conductivity within the magnetic island is as large as the observed global thermal conductivity. To estimate the rf power, we suppose that the feedback system can supply a fraction f of the total heating power and deposit it within a region of radial width d around the magnetic island. We also estimate the cross-field thermal conductivity from the overall electron power balance:

$$K_{\perp e}/n = a^2/4\tau_{Ee} \; ; \quad 3nT_e/2\tau_{Ee} = P_{tot} .$$

With these assumptions, suppression of islands of width w requires

$$\Delta' r_s < 2f(w/d).$$

Since typical values of $\Delta' r_s$ exceed unity (for example, $\Delta' r_s = 2$ for the $m = 2$ instability of a "rounded" current profile with $q(0) = 1$), a very large feedback power ($f \sim 1$) would be required to suppress islands of width d. If electron thermal conduction were a less important term in the power flow across the singular surface, or if the cross-field thermal conductivity within the island were to be much smaller than the global conductivity, then feedback stabilization by island heating would become a more attractive option.

The theory of feedback stabilization by island current drive proceeds along similar lines. The rf-driven current density is modulated in phase with the rotating island

$$j_{rf} = \tilde{j}_{rf} \cos(m\theta - n\phi - \omega t),$$

and the radial deposition profile is assumed to be quite narrow, but not as narrow as the island itself. The evolution of the magnetic flux function ψ is described by

$$\frac{\partial \psi}{\partial t} + \underline{v} \cdot \underline{\nabla}\, \psi = \eta[\delta j_z(\psi) - j_{rf}] \quad .$$

Proceeding (in slab geometry) as before, we obtain

$$\Delta' \psi_1 = \frac{4}{(2B_y')^{1/2}} \left(\frac{1}{\eta} \frac{d\psi_1}{dt} + \tilde{j}_{rf}\right) \int_{-\psi_1}^{\infty} d\psi < \frac{\cos ky}{(\psi + \psi_1 \cos ky)^{1/2}} >_y^2 / < \frac{1}{(\psi + \psi_1 \cos ky)^{1/2}} >_y \; .$$

Changing the variable of integration from ψ to $X = \psi/\psi_1$ and introducing the island width w, given by $w = 4(\psi_1/B_y')^{1/2}$, the result can be written

$$\frac{dw}{dt} = \eta \left(\Delta' - C_d \frac{\tilde{j}_{rf}}{j_{zo}} \frac{1}{w}\right) ,$$

where (including a transformation from slab to cylindrical geometry)

$$C_d = - 8 \,(j_z/B_y') = 8(r j_z/B_\theta)\,(q/rq') \quad .$$

Typically -- for example, the m = 2 instability with q(0) = 1 -- the value of C_d is about 5.

Unlike feedback by island heating, where only islands above a certain size can be stabilized, feedback by current drive will suppress all islands smaller than some critical size. To provide a concrete example, Fig. 9 shows the feedback current density $\tilde{j}_{rf}$ (expressed as a fraction of the initial local current density j_{zo}) required to stabilize the m = 2 instability of "rounded" and "peaked" current profiles with $q(a) \approx 3.0$, as a function of the q(0)-value and the maximum permitted island width w_{max}. We see that suppression of m = 2 islands with widths up to $w/a \sim 0.1$ requires values of j_{rf}/j_{zo} in the range 0.05 - 0.15. Thus, feedback by means of rf current drive represents a viable option for stabilizing $m \gtrsim 2$ magnetic islands.

IV. RADIO-FREQUENCY FEEDBACK STABILIZATION OF THE m = 1 MODE

The successful suppression of the m = 1 mode and the associated "sawteeth" would have more substantial benefits than the suppression of the m = 2 mode: (i) it could provide a significant improvement in the limiting beta-value for ballooning instabilities by reducing both q(0) and q(a); (ii) it would provide indirect stabilization of the m = 2 mode (and other "external" resistive kinks) by allowing q(0) to fall significantly below unity [corresponding to centrally peaked j(r) profiles that are known to be stable to $m \gtrsim 2$ modes]; (iii) it would provide a significant improvement in confinement by allowing increased plasma current; and (iv) it would enhance the maximum ohmic-heating power by increasing the central current density.

In a "cylindical" tokamak with circular cross section, the ideal-MHD m = 1 mode ("internal kink") is marginally stable in the limit of small $k_z a$. In this case, the resistive mode becomes strongly unstable, and it does not enter a slow-growing nonlinear phase. The rapid growth of the m = 1 mode is reproduced in computer simulations and makes rf-feedback stabilization highly problematical /9/.

However, the ideal-MHD mode can become positively stable in a tokamak with a strongly shaped (triangular) plasma cross section /10/. (Toroidicity will also stabilize the internal kink at low β_p-values, but this effect does not seem, of itself, strong enough for rf-feedback to be feasible.) If the ideal-MHD mode is positively stable, an effective Δ'-value can be calculated, and the rf-feedback theory developed in the previous section can be applied.

For m = 1, the perturbed magnetic energy may be written

$$W = \pi \int [r^3 (\frac{\partial\psi}{\partial r})^2 + \frac{1}{F} \frac{d}{dr} (r^3 \frac{dF}{dr}) \psi^2]dr \quad + \quad \delta W,$$

where corrections from toroidal effects and cross-sectional shaping, as well as higher order terms in $k_z r$, are all contained in the term δW, which can generally be reduced to the form

$$\delta W = \pi \int g\psi^2/F^2 \, dr.$$

Here, $F = \underline{k}.\underline{B} = (B/R)\,[\iota(r) - 1]$. Writing $\psi = F\xi$, the Euler-Lagrange equation becomes

$$\frac{\partial}{\partial r} (r^3 F^2 \; \frac{\partial \xi}{\partial r}) - g\xi = 0 \;,$$

which has lowest-order solutions

$$\xi = \xi_0 - \frac{\xi_0}{r_s^3 \; F_s'^2 \; x} \int_0^{r_s} g \, dr \;, \qquad (r < r_s) \;,$$

$$\xi = - \frac{\xi_0}{r_s^3 \; F_s'^2 \; x} \int_0^{r_s} g \, dr \;, \qquad (r > r_s),$$

where r_s is the $m = 1$ singular surface and $x = r - r_s$. Ideal-MHD stability requires

$$\delta w_{MHD} > 0,$$

where

$$\delta w_{MHD} = \pi\xi_0^2 \int_0^{r_s} g \; dr.$$

Assuming that this is satisfied, the resistive stability of the $m = 1$ mode is described by the quantity $\Delta_s' = [\partial\psi/\partial r]_s/\psi_s$, which can be written

$$\Delta_s' = r_s^3 \; F_s'^2 \; / \int_0^{r_s} g \; dr = \pi \; r_s^3 \; F_s'^2 \; \xi_0^2 / \delta w_{MHD} \quad .$$

Thus, the growth rate of the resistive mode varies inversely with δW_{MHD}: if the ideal-MHD mode can be given strong positive stability, the growth rate of the resistive mode can be correspondingly reduced.

The ideal-MHD $m = 1$ mode is positively stable in toroidal geometry at sufficiently low β_p-values. The stability of the mode can be enhanced by triangular shaping of the cross section /9/, and shaping becomes the dominant effect whenever the local distortion exceeds the local inverse aspect ratio; elliptical shaping is weakly destabilizing. For present purposes, we neglect toroidal effects and consider a D-shaped plasma boundary of the form

$$r/a = 1 - \xi_{2a}\cos 2\theta + \xi_{3a}\cos 3\theta \quad .$$

(The quantities ξ_{2a} and ξ_{3a} are related to the more familiar elongation κ and triangularity δ by $\kappa \approx 2\xi_{2a}$ and $\delta \approx 3\xi_{3a}$.) For this case, the perturbed magnetic energy is

$$\delta W_{MHD} = (\pi B^2 \xi_0^2 \, r_s^4 \, /R^2 a^2) \, (\delta W_3 \, \xi_{3a}^2 - \delta W_2 \, \xi_{2a}^2)$$

giving an effective Δ'-value

$$r_s \Delta_s' = (r_s q_s')^2 \, (a/r_s)^2 / (\, \delta W_3 \, \xi_{3a}^2 - \delta W_2 \, \xi_{2a}^2 \,) \, ,$$

where numerical quantities δW_2 and δW_3 (both positive) have been calculated previously for representative current profiles /9/ and are reproduced in Fig. 10(a). (The effect of reduced shaping of flux surfaces near the magnetic axis, relative to that of the plasma boundary, is included in the numerical values of δW_2 and δW_3, and it is calculated from an equilibrium that is consistent with the assumed current profiles.)

To provide a concrete example, we consider a strongly (bean) shaped cross section with $\xi_{2a} = 0.5$ and $\xi_{3a} = 0.5$ (corresponding to $\kappa \approx 2.0$ and $\delta \approx 1.5$). Figure 10(b) shows the values of $r_s\Delta_s'$ for this case, and Fig. 10(c) shows the feedback current density $\tilde{j}_{rf}$ (expressed as a fraction of the initial local current density j_{zo}) required to stabilize the mode for a parabolic current profile, as a function of the $q(0)$-value for various maximum permitted island widths w_{max}. We see that suppression of $m = 1$ islands with widths up to $w/a \sim 0.1$ requires values of $\tilde{j}_{rf}/j_{zo}$ in the range 0.3 - 0.4 -- a demanding requirement, but not entirely impossible. Thus, feedback stabilization of the $m = 1$ mode by means of rf current drive will require a strongly shaped plasma cross section and substantial feedback power, but it may be a feasible option for realizing and sustaining the $q(0) < 1$ regime in a tokamak.

If the regime $q(0) < 1$ can indeed be realized and sustained, the question naturally arises: What is then the ultimate lower limit on $q(0)$? The ideal-MHD $m = 1$, $n = 2$ mode certainly provides a firm lower bound at $q(0) = 0.5$. However, before such low $q(0)$-values are reached, the plasma will be vulnerable to higher order "fractional-m/n" tearing modes with singular surfaces falling in the region where $q(r) < 1$; the most relevant modes will be those with m,n-values given by $m/n = 4/5$, $3/4$, and $2/3$. The stability of such modes for the "peaked" current profile of Ref. 1 is given in Fig. 11, which shows Δ'-values plotted against $q(0)$ for the range $0.5 < q(0) < 1$. [In Fig. 11, the Δ'-values have been used to obtain a rough measure of the magnetic energy available to a tearing mode, namely, $\delta W = (r_s/r_0)^2 \Delta_s' r_s$, where r_s is the radius of the singular surface; a conducting wall is at $r_w/r_0 = 2.0$.] For this case, the $m/n = 4/5$ mode is not unstable ($m = 4$ is always stable for a peaked current profile) and the $m/n = 3/4$ mode is only mildly unstable. A small "window" of stability is evident for $q(0) \approx 0.75$. Careful tailoring of the current profile -- in the region where $q(r) < 1$, as well as in the vicinity of the $q = 2$ surface -- produces cases that are stable to all tearing modes without any conducting wall for $q(0)$-values as low as 0.7 /11/. It does not seem possible to stabilize the $m/n = 2/3$ mode. Thus, the optimum operating regime for a "sawtooth suppressed" tokamak would have $q(0)$-values of about 0.7. Relatively flat pressure profiles may be expected within the region $q(r) < 1$, because of the action of unstable resistive interchanges.

V. EFFECTS OF ROTATION ON TEARING MODES

In the absence of a conducting shell, a tokamak with $q(0)$ approximately equal to unity is theoretically unstable to the $m = 2$, $n = 1$ tearing mode for virtually all typical current profiles $j_z(r)$. As we have seen in Section II, the addition of a fairly close-fitting conducting shell (for example, at a radius $r_W = 2r_0$, where r_0 is the effective radius of the current channel -- usually significantly less than the limiter radius a) has a substantial stabilizing effect, especially if $q(0)$ can also be made slightly less than unity. Experimentally, $m = 2$ instabilities are not as pervasive as the theory would predict for the case without a conducting shell, suggesting that the (resistive) vacuum vessel wall might be acting as a conducting shell, particularly for a rapidly rotating mode. Even if the mode is not completely stabilized, its growth may be sufficiently impeded so that a significant amplitude is not reached, especially in the case of a tokamak with a sawtooth cycle that extends to $q(0)$-values low enough to provide periodic intervals of stability against the $m = 2$ mode. Three questions naturally arise: (i) How small must the vessel resistance be for effective stabilization of rotating $m = 2$ modes? (ii) Do non-rotating instabilities remain, even in the presence of an effective conducting wall? (iii) On what time scale is the mode rotation slowed by the interaction of rotating magnetic perturbations with the resistive wall? These questions have been addressed previously for linear kink and tearing modes /12/, and they are considered here in the context of nonlinear tearing modes, i.e., slow-growing finite-amplitude magnetic islands. (Feedback techniques would, of course, be ineffective against non-rotating, experimentally "hidden" $m = 2$ tearing modes.)

The boundary condition to be applied on the inner side of a thin resistive wall at $r = r_W$ is:

$$(r B_r)'/B_r = -m + i\omega\tau_S , \qquad (r = r_W) ,$$

where $\tau_S = 4\pi r_W d/\eta_W c^2$ is the resistive time constant of the wall, d is the thickness of the wall, and the mode is assumed to vary like $\exp(im\theta - in\phi - i\omega t)$. In deriving this boundary condition, we have assumed that $d \ll \lambda$, where λ is the resistive skin depth of the wall $[\lambda = (\eta_W c^2/4\pi\omega)^{1/2}]$, and we have noted that the wall exerts a strong stabilizing effect on long wavelength modes even if its resistive skin depth greatly exceeds its thickness: a stabilizing effect requires only that $\lambda^2 \lesssim r_W d$.

The resistive wall modifies the solutions of the Euler-Lagrange equation for the perturbed magnetic fields only in the region outside the singular surface $r = r_s$. If the plasma current density is small in this region, the solutions for rB_r have the forms r^m and r^{-m}, which can be combined to satisfy the required boundary condition. The effective Δ'-value becomes

$$\Delta' = \Delta'_\infty - \frac{\Delta'_\infty - \Delta'_W}{1 + (2im/\omega\tau_S)} .$$

Here, Δ'_∞ is the value of Δ' with the wall absent (i.e., at infinity), and Δ'_W is the value of Δ' for the case of a perfectly conducting wall located at r_W. [An additional factor $(1 + r_s^{2m}/r_W^{2m})$ should multiply τ_S in the above expression, but is approximately unity for most cases of interest.] The real and imaginary parts of Δ' can be written:

$$\Delta'_R = \Delta'_\infty - (\Delta'_\infty - \Delta'_W)\frac{\omega^2\tau_S^2}{\omega^2\tau_S^2 + 4m^2} ,$$

$$\Delta_I' = (\Delta_\infty' - \Delta_W') \frac{2\, m\omega\tau_S}{\omega^2\tau_S^2 + 4m^2} .$$

The real part Δ_R' corresponds to current-density perturbations δj_z which are 90° out-of-phase with the field perturbation B_r (i.e., antinodes of δj_z at the O-points and X-points of the magnetic island); these are the current-density perturbations associated with tearing-mode growth, and they result in a finite-amplitude island of width w growing (in a rotating frame) according to the usual relation

$$\frac{dw}{dt} = \eta\Delta_R' .$$

The implications of our expression for Δ_R' are unsurprising. If $\omega\tau_S >> 1$ then $\Delta_R' \approx \Delta_W'$ and the wall is effectively perfectly conducting. If the mode is unstable even with a perfectly conducting wall at r_W (i.e., $\Delta_W' > 0$), the addition of wall resistivity simply produces a further positive contribution to Δ_R' (since $\Delta_\infty' > \Delta_W'$ always); the mode remains unstable at all rotation frequencies ω. The more interesting case is where the mode is stabilized by a perfectly conducting wall ($\Delta_W' < 0$) but is otherwise unstable ($\Delta_\infty' > 0$). Again, if $\omega\tau_S >> 1$, then $\Delta_R' \approx \Delta_W' < 0$, and the mode is effectively stabilized. However, for lower rotation frequencies (or more resistive walls), such that $\omega\tau_S << 1$, we obtain $\Delta_R' \approx \Delta_\infty' > 0$, and the mode remains unstable. In this case, there is a critical rotation frequency

$$\omega_{crit}\, \tau_S = 2m \left(-\Delta_\infty' / \Delta_W'\right)^{1/2}$$

below which the mode is unstable. (In this discussion, we have implicity assumed a slow-growing finite-amplitude island with a "growth rate" γ that is less than the rotation frequency ω.)

The imaginary part Δ_I' corresponds to current-density perturbations δj_z which are in phase with the field perturbation B_r; these current-density perturbations do not contribute to tearing-mode growth, but result in a torque on the plasma that tends to reduce its rotation and, thereby, also the frequency of the mode. The torque is exerted directly on an annular region of the plasma with radial width of order the magnetic island width. The rate at which mode rotation decreases will clearly depend sensitively on the degree of viscous coupling of this annular region to the rest of the plasma. In the present analysis, we neglect viscous effects entirely, and we assume that the plasma in the vicinity of the magnetic island can slip freely (along flux surfaces) through the surrounding plasma. In this case, the torque and resulting slowing of rotation can be calculated by means of a straightforward extension of nonlinear tearing-mode theory /8/. We obtain

$$\tau_H^2 \; \frac{d\omega}{dt} = - C \, m^2 \, (\Delta_I' r_s) \, (w/r_s)^3 \; ,$$

where $C = 0.5 \times 10^{-2}$ is a numerical constant, and $\tau_H = \rho^{1/2}/[r(B_\theta/r)'] = (Rq/v_A)\,(q/rq')$ is a characteristic hydromagnetic time. The damping of plasma rotation is given by

$$\frac{dv_\theta}{dt} = \frac{r_s}{m} \frac{d\omega}{dt} \; ; \quad \frac{dv_\phi}{dt} = \frac{Rq}{m} \frac{d\omega}{dt}$$

for poloidal and toroidal rotation, respectively.

The value of Δ_I', and hence the rate of rotation damping, peaks at $\omega\tau_S \sim 2m$; for $\omega\tau_S \lesssim 2m$, the characteristic time scale for rotation damping is given by

$$\tau_D = - \frac{\omega}{d\omega/dt} \sim 2 \times 10^2 \; \frac{\tau_H^2}{\tau_S} \; \left(\frac{r_s}{w}\right)^3$$

for an $m = 2$ mode with $(\Delta_\infty' - \Delta_W')r_s \approx 1$. For a TFTR-size tokamak with a close-fitting high-resistance vacuum vessel, we have $\tau_H \sim 10^{-6}$ sec and $\tau_S \sim 2 \times 10^{-3}$ sec (corresponding to a toroidally continuous vessel with toroidal resistance 2 mΩ). [In general, $\tau_S(\text{sec}) = 1.3\, r_W(\text{m})\, d(\text{cm})/\eta(\mu\Omega\text{-cm})$, with $\eta = 80$ $\mu\Omega$-cm for stainless steel.] For an island width $w \sim r_s/10$, the shortest characteristic time scale for rotation damping is $\tau_D \sim 10^{-4}$ sec, i.e., very short. For the usual case where $\omega\tau_S \gg 2m$ (frequencies $\sim$ 10 kHz, $\omega\tau_S \sim 10^2$), the characteristic time scale for rotation damping is much longer ($\tau_D \sim 0.1$ sec). Thus, rotation decelerates rather slowly at first, but then much more rapidly as the frequency drops below the kHz range.

In the special case where the plasma is non-rotating, the mode is always purely growing, and its growth rate is described by the relations

$$dw/dt = \eta \, \Delta'(\gamma\tau_S) \quad ,$$

$$\Delta'(\gamma\tau_S) = \Delta_R'(\gamma\tau_S) = \Delta_\infty' - \frac{\Delta_\infty' - \Delta_W'}{1 + (2\,m/\gamma\tau_S)} \; ,$$

where γ is a "growth rate" to be interpreted as $\gamma = (2/w)dw/dt$. It is convenient to define a characteristic time scale of the slow-growing nonlinear mode $\tau_I \sim w/\eta\Delta'$. For small islands or low wall resistivity, i.e., $\tau_I \ll \tau_S$, growth on the faster time scale τ_I occurs only if $\Delta'_W > 0$; in the more interesting case $\Delta'_W < 0 < \Delta'_\infty$, growth is exponential but on the slower time scale τ_S:

$$\gamma\tau_S = 2m\,(-\Delta'_\infty/\Delta'_W) .$$

For large islands or high wall resistivity, i.e., $\tau_S \ll \tau_I$, growth proceeds on the slower time scale τ_I and is algebraic as in the usual nonlinear case; the resistive wall plays no role.

Even when the plasma is rotating, a non-rotating mode is theoretically possible, with the same growth rate as in the non-rotating plasma case discussed in the previous paragraph. In particular, if $\tau_I \ll \tau_S$, exponential growth occurs, given by $\gamma\tau_S = 2m(-\Delta'_\infty/\Delta_W)$. Such solutions, while theoretically possible in an ideally inviscid plasma, require the plasma to rotate with finite velocity past the magnetic island, with streamlines lying on magnetic surfaces, and with null or vortex flow within the island; a strongly sheared flow occurs near the magnetic separatrix. Although the damping of rotation by the interaction of a rotating mode with the resistive wall (discussed above) might lead ultimately to such non-rotating modes with highly sheared flows, we have seen that the mode becomes quite strongly unstable below a small but finite critical rotation frequency ω_{crit}, implying that the non-rotating state might not actually be reached. In any case, when plasma viscosity is introduced, the physical significance of the non-rotating mode in a rotating plasma is questionable.

The introduction of plasma viscosity will clearly reduce the rate at which rotation is damped by increasing the inertial forces opposing the magnetic torque. Moreover, in a toroidal plasma of low collisonality, the so-called "bulk viscosity" will play a role, since the plasma cannot rotate in the toroidal direction past helical magnetic islands without introducing a component of rotation in the poloidal direction -- a component that is strongly damped by bulk viscosity. Nonetheless, the simplified analysis of the effects of rotation presented above has some appealing features. In particular, it suggests that m = 2 modes may form two classes: (i) effectively stabilized rotating modes, where a resistive vessel wall plays the role of a conducting shell; and (ii) truly unstable modes, occurring either when the rotation frequency drops below a critical value, or when the wall is not close enough to provide stability even if perfectly conducting.

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FIGURE CAPTIONS

Fig. 1 Illustration of "sawtooth" and "m = 2" tokamak discharges (from PLT); the "sawtooth" discharge has q(0) just below unity, leading to m = 1 activity at the plasma center, but it is quiescent near the plasma edge; if q(0) rises above unity, strong "m = 2" activity near the edge leads to a major disruption.

Fig. 2 Representative tokamak j(r) and q(r) profiles normalized to unity at r = 0.

Fig. 3 Values of $r_s\Delta_s'$ for representative current profiles plotted against q(0), for (a) the m = 2 mode and (b) the m = 3, n = 2 mode; a conducting wall is at $r_w/r_o = 2$.

Fig. 4 Illustration of sawtooth reconnection according to the Kadomtsev model. Initial flux elements at r_1 and r_2 combine into the final flux element at r in such a way that the toroidal flux (area) of an element $d\chi$ of helical flux is conserved.

Fig. 5 Profiles (a) of $\iota(r)$ and (b) of $j_z(r)$, before and after sawtooth reconnection according to the "Kadomtsev" and "flat-q models," for the case of a parabolic initial current profile with $\iota(0) = 1.15$.

Fig. 6 Values of $r_s\Delta_s'$ for (a) the m = 2, n = 1 mode and (b) the m = 3, n = 2 mode before and after sawtooth reconnection according to the "flat q model" for a "rounded" current profile. In the case of the m = 2 mode, two positions of the conducting wall r_w/r_o are considered; the m = 3, n = 2 mode is insensitive to the position of the wall.

Fig. 7 Same as Fig. 6 for the "flattened" current profile.

Fig. 8 Current-density perturbations for a tearing mode in a tokamak; the current density is increased at the O-point and decreased on the separatrix. Feedback must <u>reinforce</u> these naturally occurring perturbations.

Fig. 9 Feedback current density $\tilde{j}_{rf}$ required to stabilize the m = 2 mode for (a) "rounded" and (b) "peaked" current profiles with q(a) = 3.0; $\tilde{j}_{rf}$ is expressed as a fraction of the initial local current density j_{zo} and is plotted as a function of the q(0)-value for various maximum island widths w_{max}.

Fig. 10 (a) Values of δW_2 and δW_3 as a function of the position of the singular surface $[q(r_s) = 1]$ (from Ref. 9); (b) Values of the effective $\Delta_s' r_s$ for the m = 1 mode in the case of a strongly shaped cross section, triangularity $\xi_{3a} = 0.5$ ($\delta = 1.5$) and elongation $\xi_{2a} = 0.5$ ($\kappa = 2.0$); (c) Feedback current density $\bar{j}_{rf}$ required to stabilize the m = 1 mode for a parabolic current profile; $\bar{j}_{rf}$ is expressed as a fraction of the initial local current density j_{z0}, and is plotted as a function of the q(0)-value, for various maximum permitted island widths w_{max}.

Fig. 11 Stability of higher order "fractional-m/n" tearing modes with singular surfaces falling in the region where $q(r) < 1$ for the "peaked" current profiles (with a conducting wall at $r_w/r_0 = 2$ to contribute to the stability of the m = 2 mode). Rather than Δ_s' itself, we plot the quantity $\delta W = (r_s/r_0)^2 \Delta_s' r_s$, which is a measure of the magnetic energy available to an unstable tearing mode localized to $r \lesssim r_s$.

EFFECTS OF M=1 AND M=2 MHD ACTIVITY

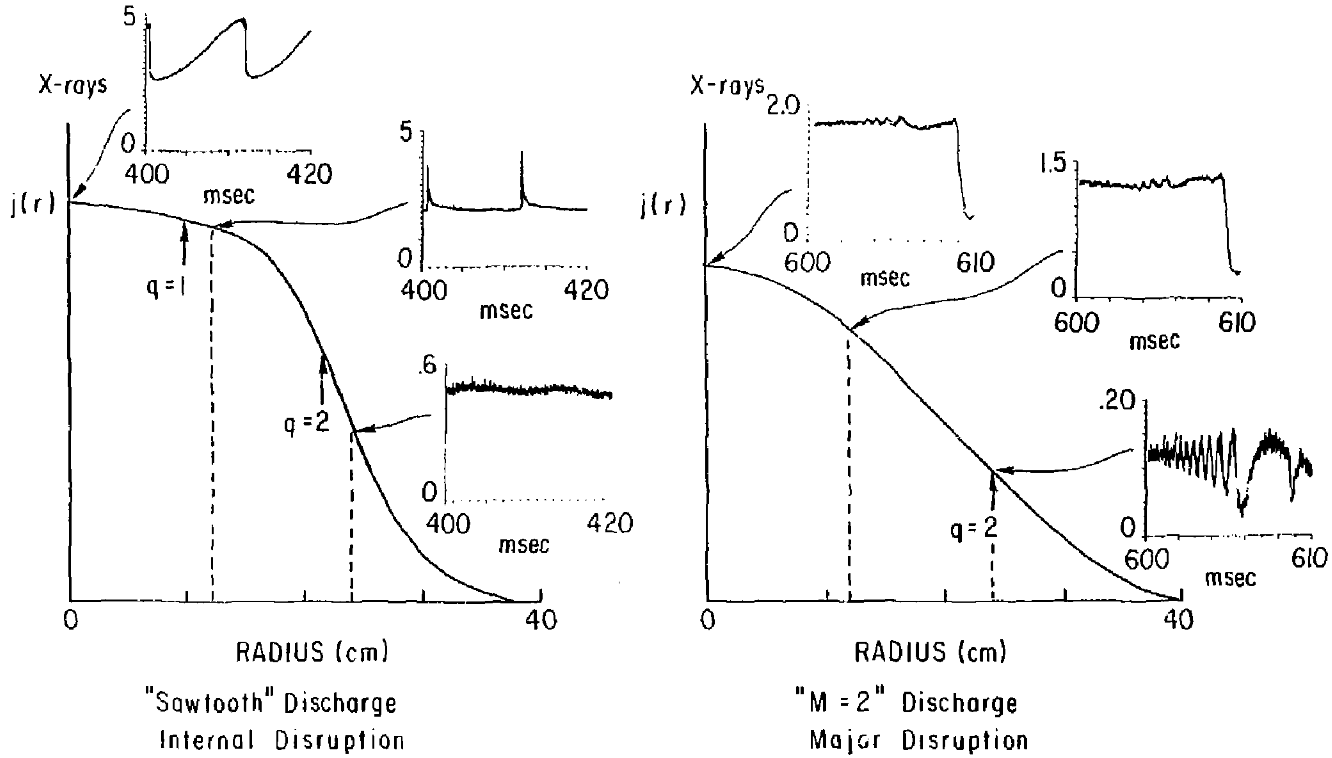
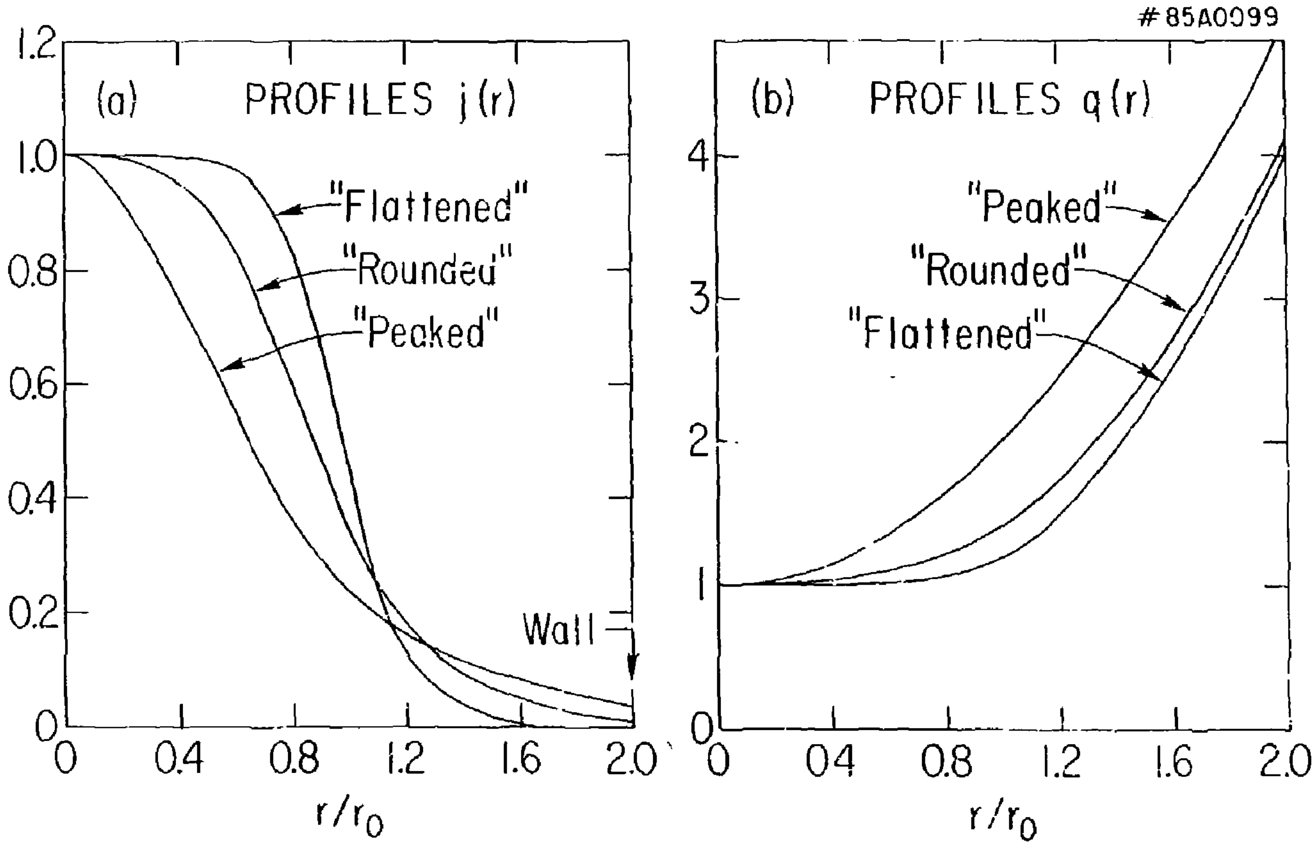


Fig. 1



#85A0099
(a) PROFILES j(r)
"Flattened"
"Rounded"
"Peaked"
Wall
1.2
1.0
0.8
0.6
0.4
0.2
0
0
0.4
0.8
1.2
1.6
2.0
r/r0
(b) PROFILES q(r)
"Peaked"
"Rounded"
"Flattened"
4
3
2
1
0
0
0.4
0.8
1.2
1.6
2.0
r/r0

Fig. 2

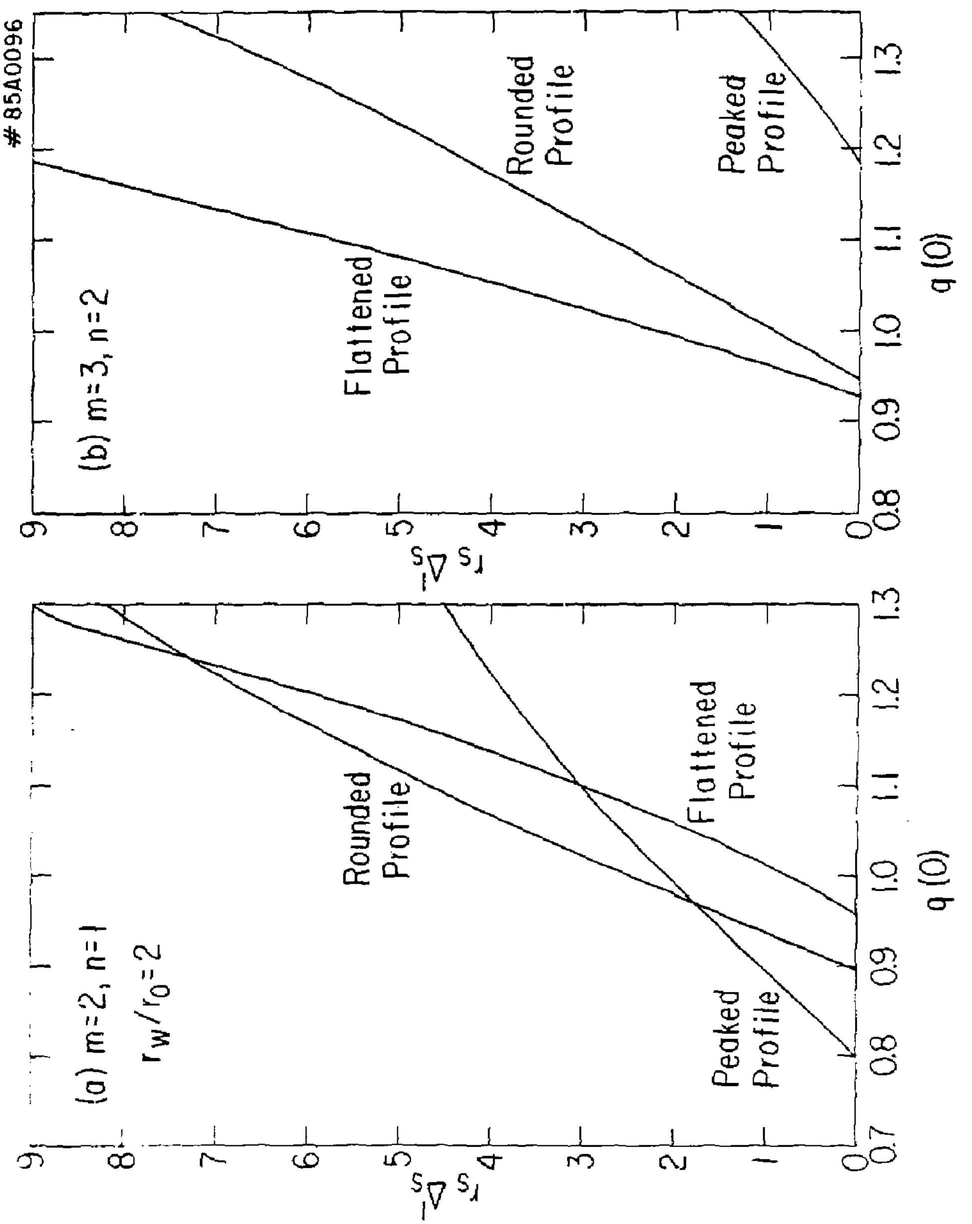

#85A0096
(a) m=2, n=1
$r_w/r_0=2$
Rounded Profile
Flattened Profile
Peaked Profile
(b) m=3, n=2
Flattened Profile
Rounded Profile
Peaked Profile
$r_s \Delta'_s$
q (0)
0
1
2
3
4
5
6
7
8
9
0.7
0.8
0.9
1.0
1.1
1.2
1.3

Fig. 3

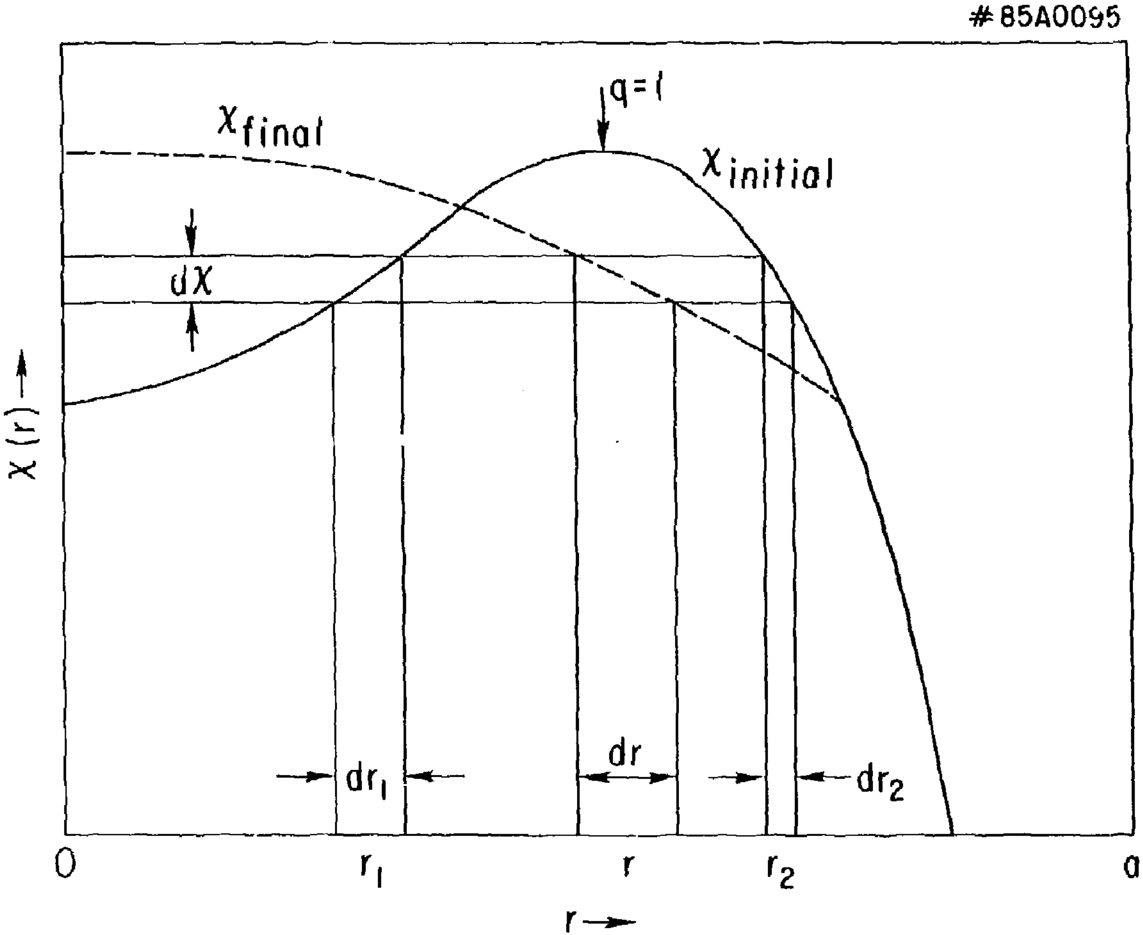
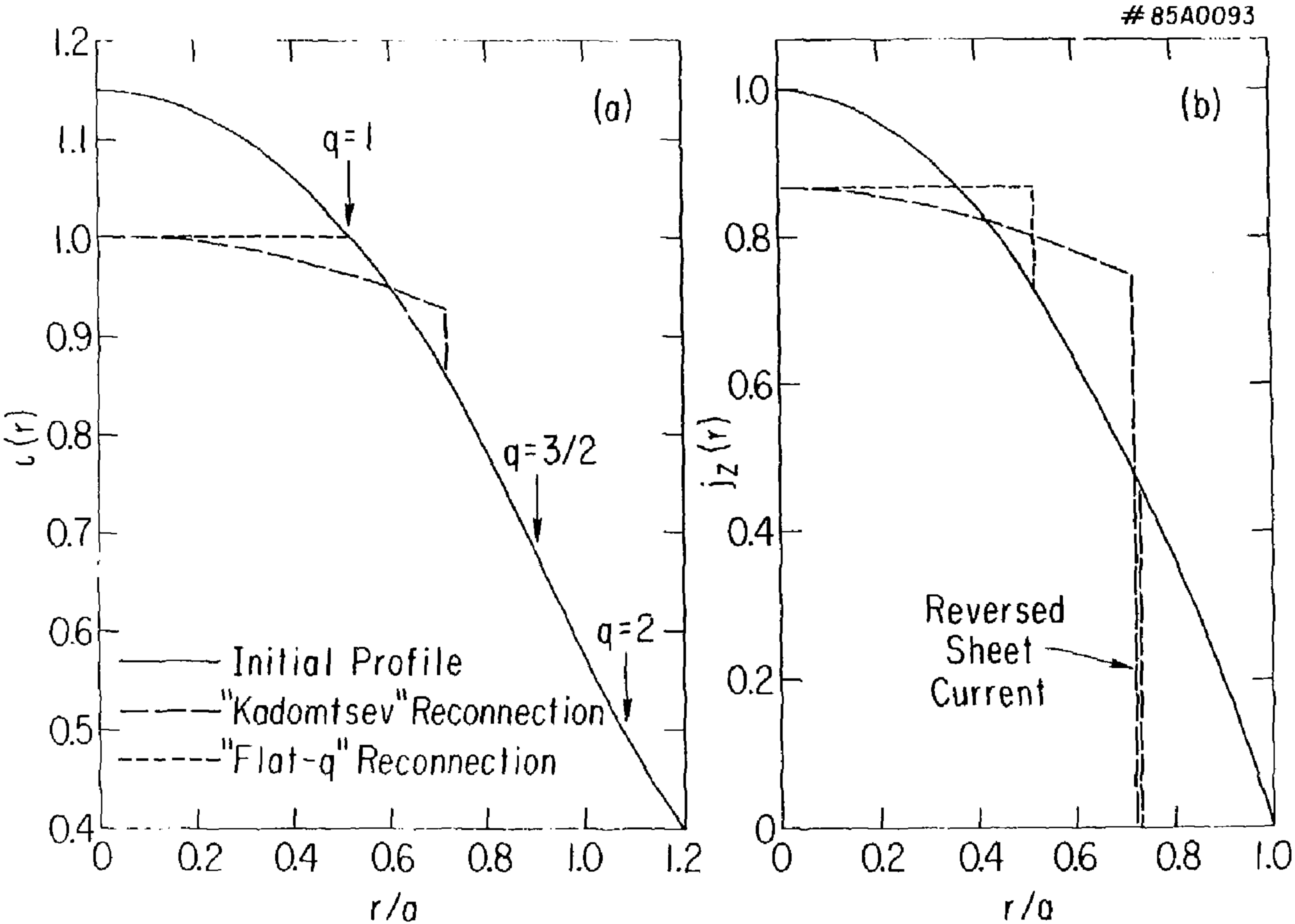
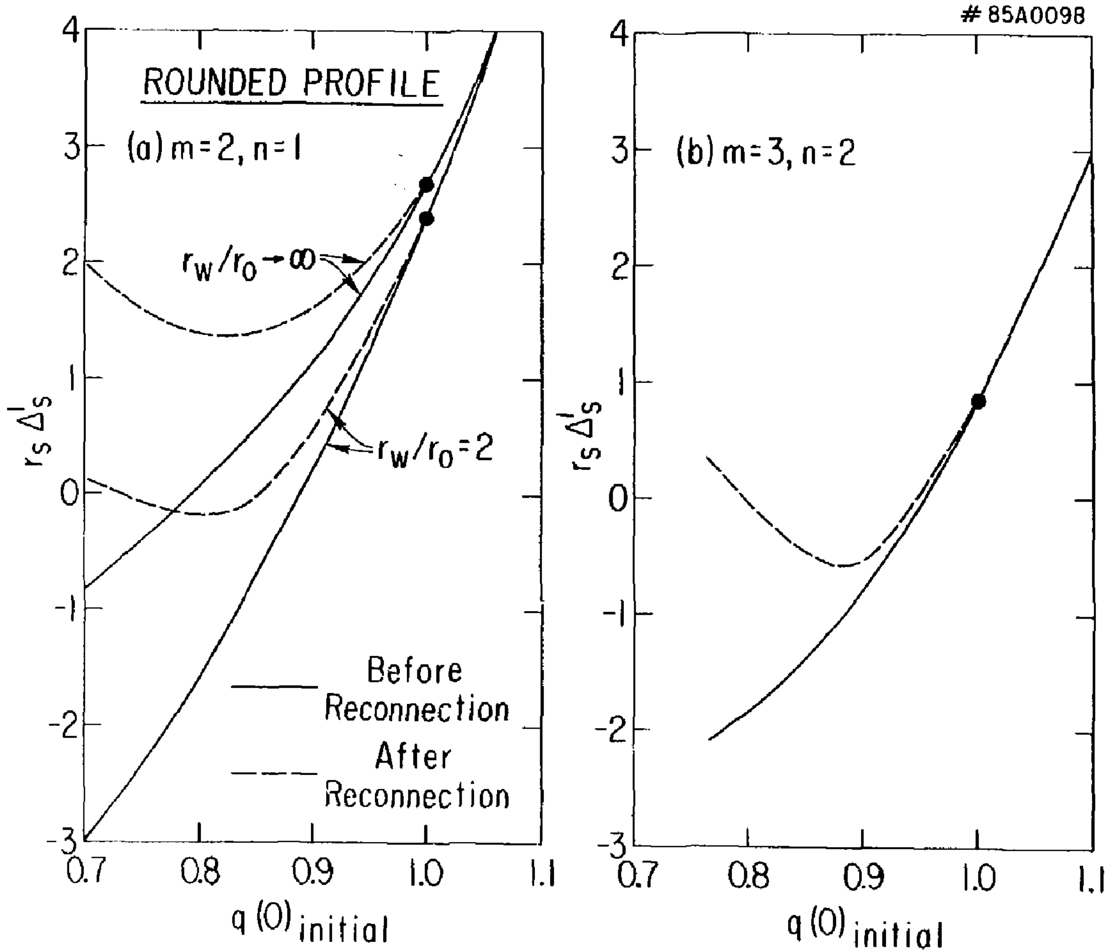


Fig. 4



85A0093
(a)
q=1
q=3/2
q=2
ι(r)
r/a
Initial Profile
"Kadomtsev" Reconnection
"Flat-q" Reconnection
(b)
$j_z(r)$
Reversed Sheet Current
r/a

Fig. 5



85A0098
ROUNDED PROFILE
(a) m=2, n=1
$r_w/r_0 \to \infty$
$r_w/r_0 = 2$
Before Reconnection
After Reconnection
$r_s \Delta'_s$
q (0) initial
(b) m=3, n=2
$r_s \Delta'_s$
q (0) initial

Fig. 6

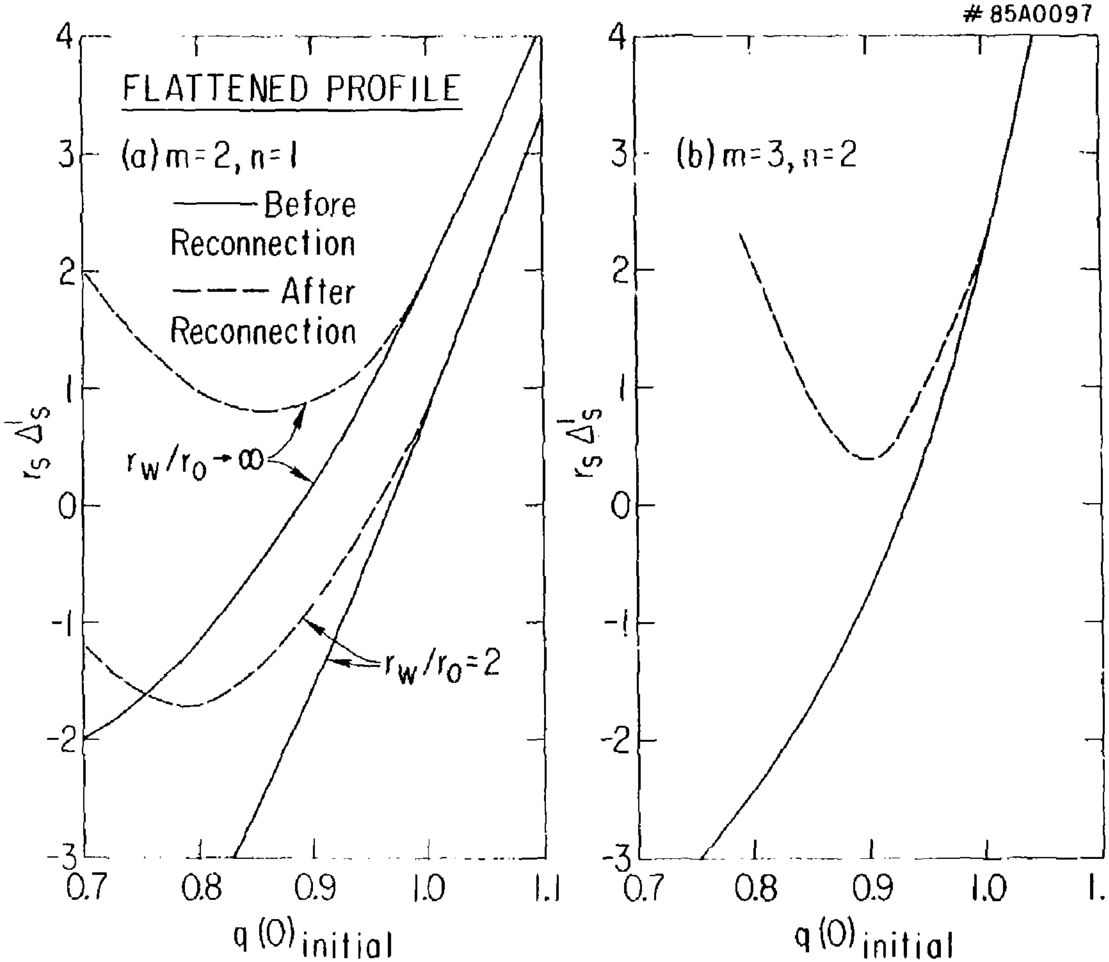

#85A0097
FLATTENED PROFILE
(a) m=2, n=1
Before Reconnection
After Reconnection
$r_w/r_0 \to \infty$
$r_w/r_0 = 2$
$r_s \Delta_s'$
q (0) initial
(b) m=3, n=2
$r_s \Delta_s'$
q (0) initial

Fig. 7

#85A0043

CURRENT PERTURBATIONS IN UNSTABLE MAGNETIC ISLANDS

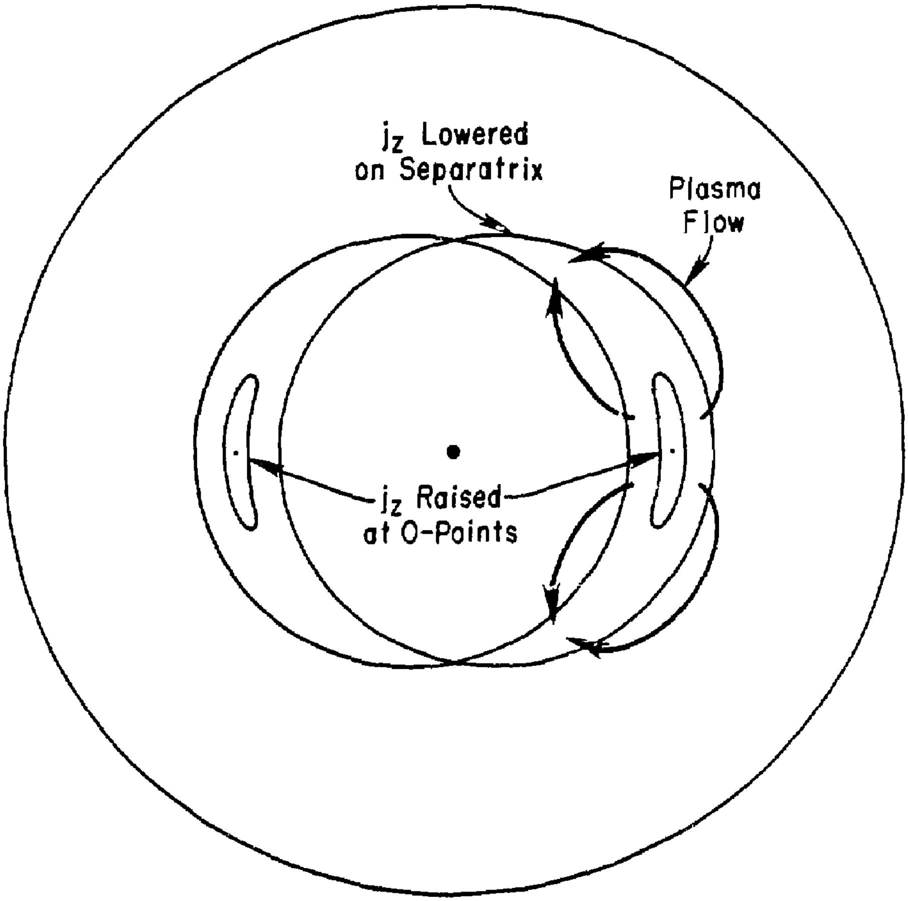


Fig. 8

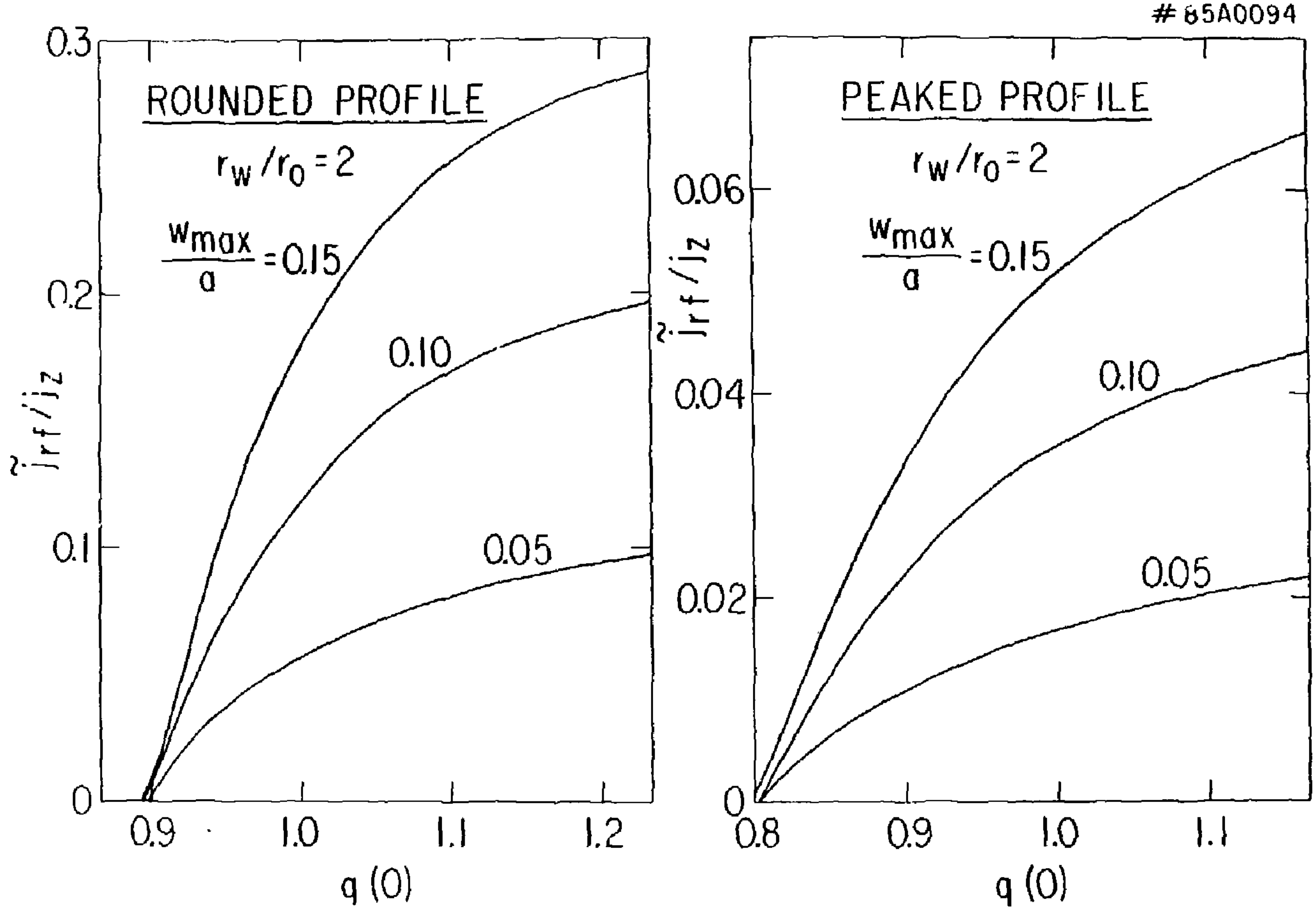

#85A0094
ROUNDED PROFILE
$r_w/r_0 = 2$
$\frac{w_{max}}{a} = 0.15$
0.10
0.05
$\tilde{j}_{rf}/j_z$
0.3
0.2
0.1
0
0.9
1.0
1.1
1.2
q (0)
PEAKED PROFILE
$r_w/r_0 = 2$
$\frac{w_{max}}{a} = 0.15$
0.10
0.05
$\tilde{j}_{rf}/j_z$
0.06
0.04
0.02
0
0.8
0.9
1.0
1.1
q (0)

Fig. 9

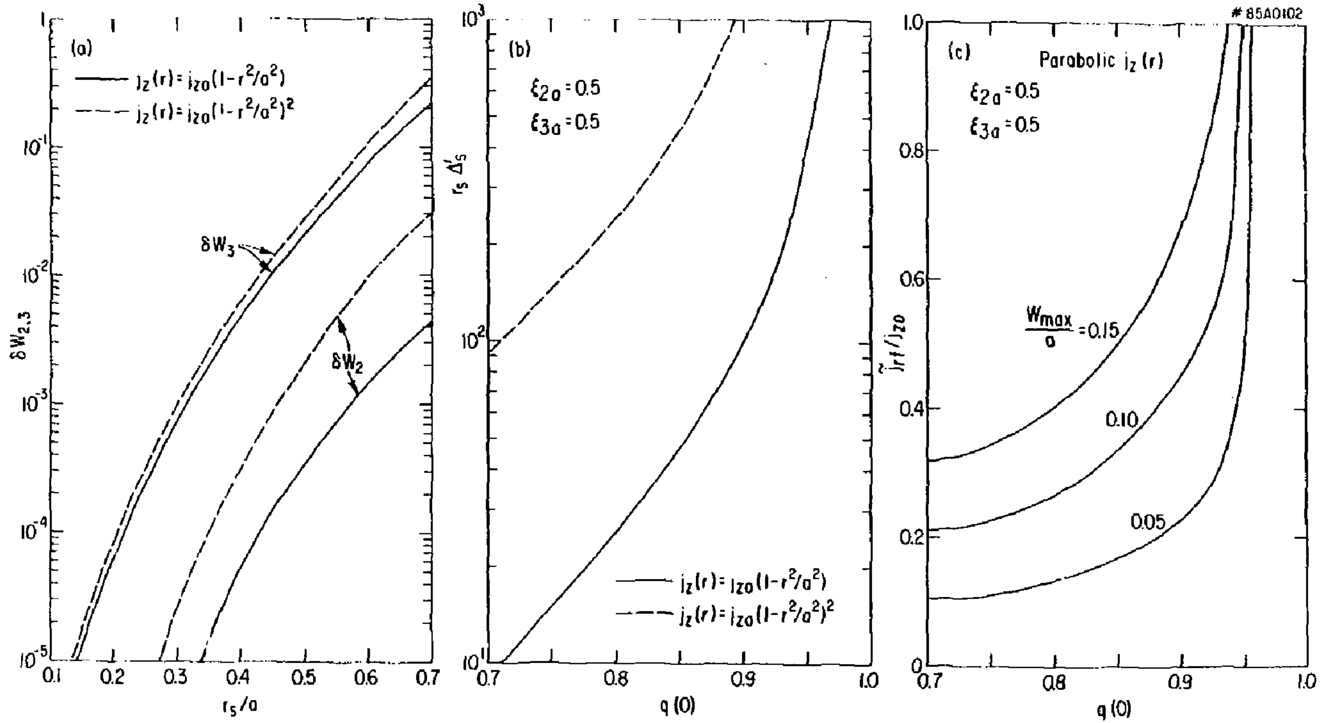

(a)
$j_z(r) = j_{z0}(1-r^2/a^2)$
$j_z(r) = j_{z0}(1-r^2/a^2)^2$
δW_3
δW_2
$\delta W_{2,3}$
r_s/a
(b)
$\xi_{2a} = 0.5$
$\xi_{3a} = 0.5$
$r_s \Delta'_s$
$j_z(r) = j_{z0}(1-r^2/a^2)$
$j_z(r) = j_{z0}(1-r^2/a^2)^2$
q (0)
(c)
Parabolic $j_z(r)$
$\xi_{2a} = 0.5$
$\xi_{3a} = 0.5$
$\frac{W_{max}}{a} = 0.15$
0.10
0.05
$\tilde{j}_{rt}/j_{z0}$
q (0)
85A0102

Fig. 10

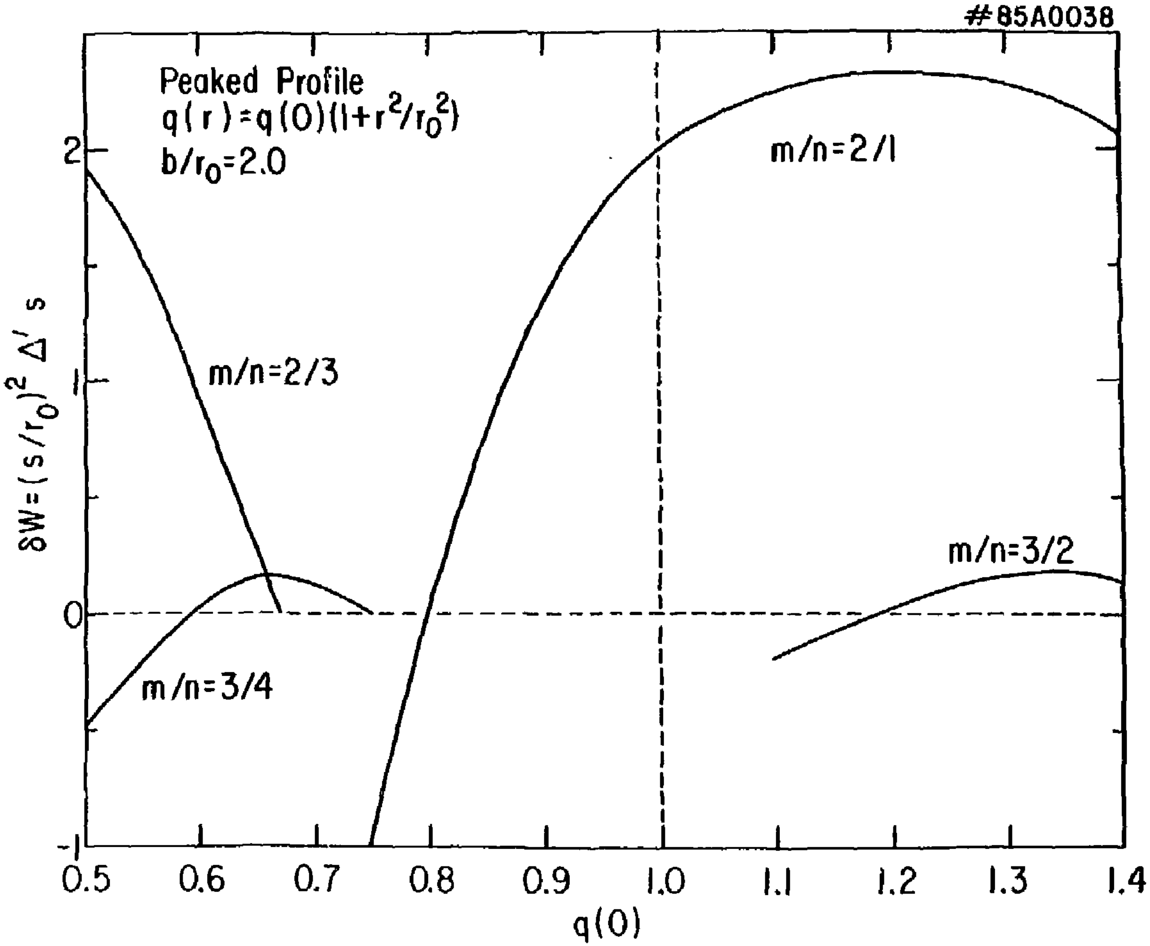

#85A0038
Peaked Profile
q(r)=q(0)(1+r²/r₀²)
b/r₀=2.0
m/n=2/1
m/n=2/3
m/n=3/4
m/n=3/2
δW=(s/r₀)² Δ' s
q(0)
2
1
0
-1
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4

Fig. 11

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