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PARTICLE SPIN TUNE IN A PARTIALLY EXCITED SNAKE*

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INTRODUCTION

In this workshop, E.D. Courant[1] points out that a single snake can be turned on and off adiabatically without affecting the spin of the particles provided that the spin precessing frequency $\gamma G\pi$ satisfies $\cos(\gamma G\pi) = 0$. This is a novel idea. If this idea works, the conventional method of correcting spin depolarization resonances can be used at low energy and switched to the Siberian snake adiabatically for high energy. The switch between spin correction methods is essential for the 70 GeV SSC Booster I because the snake requires an uncomfortable large aperture at energy below 20 GeV and the conventional resonance tune jump method becomes impractical at energy higher than 25 GeV.

In this paper, we address the question on the effect of the particle spin when a snake is turned on adiabatically near a depolarization resonance while not accelerating. The spinor equation and its solution are reviewed briefly and the spin transfer matrix method in the presence of a snake are used to evaluate the spin tune and the precession axis.

Spinor Equation

The equation[2] for the spinor wave function ψ for a classical spin particle inside the circular accelerator at energy factor γ is given by

$$\frac{d\psi}{d\theta} = -\frac{i}{2}\begin{pmatrix}\gamma G & -\epsilon e^{-iK\theta} \\ -\epsilon^{*} e^{iK\theta} & -\gamma G\end{pmatrix}\psi$$

$$= -\frac{i}{2}(\vec{b}\cdot\vec{\sigma})\,\psi\ , \qquad (1)$$

where G is the Pauli anomalous magnet moment, $\epsilon = \epsilon_R + i\epsilon_I$ is the resonance strength at $\gamma G = K$ resonance. Making the transformation,

$$\phi = e^{+\frac{1}{2}iK\theta\sigma_3}\psi\ , \qquad (2)$$

we obtain

$$\frac{d\phi}{d\theta} = +\frac{1}{2}\left(\delta\sigma_3 + \epsilon_R\sigma_1 - \epsilon_I\sigma_2\right)\phi \qquad (3)$$

$$= \frac{i}{2}\lambda\hat{n}_\epsilon\cdot\vec{\sigma}\phi\ ,$$

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where $\delta = K-\gamma G$ and $\lambda^2 = \delta^2 + |\epsilon|^2$. For constant $\hat{n}_\epsilon$, Eq. (3) can be solved easily to give,

$$\phi(\theta) = e^{i \frac{\lambda(\theta-\theta_i)}{2} \hat{n}_\epsilon \cdot \vec{\sigma}} \phi(\theta_i) \tag{4}$$

or

$$\psi(\theta) = \left(e^{-\frac{i}{2} K\theta\sigma_3} e^{\frac{i}{2} \lambda\theta\hat{n}_\epsilon\vec{\sigma}}\right)\left(e^{-\frac{i}{2} \lambda\theta_i\hat{n}_\epsilon\vec{\sigma}} e^{-\frac{i}{2} K\theta_i\sigma_3}\right) \psi(\theta_i)$$

$$= e^{i\alpha(\theta)\hat{n}(\theta)\cdot\vec{\sigma}} e^{-i\alpha(\theta_i)\hat{n}(\theta_i)\vec{\sigma}} \psi(\theta_i)$$

$$= T_0(\theta) \quad T_0^{+}(\theta_i)\psi(\theta_i) \tag{5}$$

where the phase angle $\alpha(\theta)$ and the axis of precession $\hat{n}(\theta)$ in the rotating frame are respectively given by

$$\sin^2[\alpha(\theta)] = \cos^2\frac{\lambda\theta}{2}\sin^2\frac{K\theta}{2} - \frac{\delta}{4\lambda}\sin K\theta \sin\lambda\theta + \sin^2\frac{\lambda\theta}{2} - \frac{\delta^2}{\lambda^2}\sin^2\frac{\lambda\theta}{2}\sin^2\frac{K\theta}{2} \tag{6}$$

$$\hat{n}(\theta) = \frac{1}{\sin\alpha}\left\{\left(-\cos\frac{\lambda\theta}{2}\sin\frac{K\theta}{2} + \frac{\delta}{\lambda}\cos\frac{K\theta}{2}\sin\frac{\lambda\theta}{2}\right)\hat{k} + |\epsilon|\frac{\sin\frac{\lambda\theta}{2}}{\lambda}\left[\cos\left(\frac{K\theta}{2} - \theta\right)\hat{i} + \sin\left(\frac{K\theta}{2} - \theta\right)\hat{j}\right]\right\}, \tag{7}$$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors in the Frenet-Serret curvilinear coordinates. The axis of precession in the laboratory frame can be obtained by

$$\psi(2\pi) = e^{i\pi\nu_p \hat{n}_p\cdot\vec{\sigma}} \psi(0) \, . \tag{8}$$

We note here that Eq. (5) offers an interesting invariant,

$$T_0^{+}(\theta)\psi(\theta) = T_0^{+}(\theta_i)\psi(\theta_i) \, . \tag{9}$$

Spin Wave Function With a Snake

Let us consider a snake which rotates the spin around the $\hat{i}$ axis (i.e., type II). The snake can be expressed locally by a spin transfer matrix,

$$e^{i\frac{\phi_s}{2}\sigma_1}$$

where ϕ_s is the spin rotation angle around the $\hat{i}$ axis. A fully excited snake corresponds to $\phi_s = \pi$. We start with $\theta_i = 0$ with wave function $\psi(0)$. The spinor wave function in the presence of the snake after one turn, becomes

$$\psi(2\pi) = e^{i\frac{\phi_s}{2}\sigma_1} e^{i\alpha(\pi)\hat{n}_\pi\vec{\sigma}} \psi(0) . \tag{10}$$

The spin transfer operator is therefore

$$\begin{aligned} T(2\pi) &= e^{i\frac{\phi_s}{2}\sigma_1} e^{i\alpha(\pi)\hat{n}_\pi\vec{\sigma}} \\ &= \cos\frac{\phi_s}{2}\{\cos K\pi\ \cos\lambda\pi + \frac{\delta}{\lambda}\sin K\ \sin\lambda\pi\} - \\ &\quad \frac{|\varepsilon|}{\lambda}\sin\lambda\pi\ \cos(K\pi-\beta)\ \sin(\frac{\phi_s}{2}) \\ &\quad + i\sin\pi\nu_p\hat{n}_p\cdot\vec{\sigma} \ , \end{aligned} \tag{11}$$

where $\tan\beta = \varepsilon_I/\varepsilon_R$ and ν_p is the spin tune with

$$\begin{aligned} \cos\pi\nu_p &= \cos\frac{\phi_s}{2}\{\cos K\pi\ \cos\lambda\pi + \frac{\delta}{\lambda}\sin K\pi\ \sin\lambda\pi\} \\ &\quad - \frac{|\varepsilon|}{\lambda}\sin\lambda\pi\ \cos(K\pi-\beta)\ \sin\frac{\phi_s}{2} . \end{aligned} \tag{12}$$

When the resonance strength is zero, $|\varepsilon| = 0$, $\delta = \lambda$, a partially excited snake, $0 \le \phi < \pi$, does not change the spin tune[1], provided that $\cos(\gamma G\pi) = 0$. That is spin tune ν_p is transparent to the excitation of a snake when there is no spin detuning resonances.

In the presence of a spin resonance with strength $|\varepsilon|$, the spin tune is not constant when the snake is turned on adiabatically.

Equation (12) gives a measure on the effectiveness of the snake in curing the resonance. At the fully excited snake, $\phi_s = \pi$, the spin tune ν_p deviates from ideal 1/2 linearly with respect to $|\varepsilon|\sin\lambda\pi/\lambda$.

Figure 1 shows the spin tune ν_p vs the snake excitation angle ϕ_s for various resonance strengths $|\varepsilon|$. We choose $\delta = K - \gamma G = 1/2$. Two curves for each $|\varepsilon|$ correspond to maximum and minimum ν_p depending on the phase β of the resonance strength ε (the uncontrollable phase of ε may vary as the particle travels through the accelerator). The width of ν_p is proportional to $|\varepsilon|$ for small $|\varepsilon|$.

When $|\varepsilon| > 0.1$, e.g., $|\varepsilon| = 0.5$, the width of ν_p becomes uncomfortably large. The partially excited snake becomes impractical. A single fully excited snake cannot cure the resonance of this strength.

CONCLUSION

We study the effect of a partially excited snake on the spin tune in the region of resonance. We found that when the resonance strength $|\varepsilon| < .1$, the adiabatic excitation of the snake processing angle ϕ_s does not affect the spin tune. When $|\varepsilon| > .1$, the width of the spin tune is uncomfortably large for a single fully excited snake. It is concluded that a partially excited snake will work provided that $|\varepsilon| < .1$.

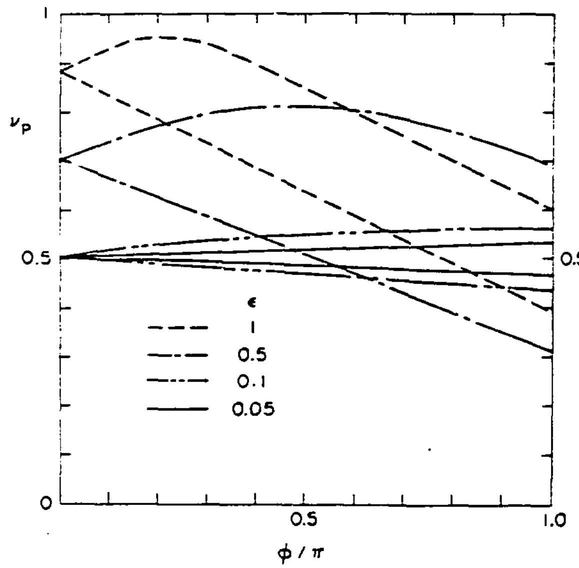

Fig. 1. The maximum and minimum spin tunes ν_p vs the precession angle ϕ of the snake for various resonance strength. We observe that when $\varepsilon > 1$, the spin tune modulation becomes too large for the snake to be useful.

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