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THEORY OF FRACTIONAL QUANTUM HALL EFFECT *

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ABSTRACT

A theory of the Fractional Quantum Hall Effect is constructed based on magnetic flux fractionization, which lead to instability of the system against selfcompression. A theorem is proved stating that arbitrary potentials fail to lift a specific degeneracy of the Landau level. For the case of 1/3 fractional filling a model 3-particles interaction is constructed breaking the symmetry. The rigid 3-particles wave function plays the role of order parameter. In a BCS type of theory the gap in the single particles spectrum is produced by the 3-particles interaction. The mean field critical behaviour and critical parameters are determined as well as the Ginsburg-Landau equation coefficients. The Hall conductivity is calculated from the first principles and its temperature dependence is found. The simultaneous tunnelling of 3,5,7 etc. electrons and quantum interference effects are predicted.

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I. INTRODUCTION

The discovery of the fractional quantum Hall effect (FQHE) by Tsui, Stormer and Gossard [1] raised enormous interest even greater than that of the integral QHE by von Klitzing, Dorda and Pepper [2]. In these and the subsequent experiments the normal σ_{xx} and transverse σ_{xy} conductivities of silicon MOFETS and semiconducting heterostructures of the type GaAs-$Al_xGa_{1-x}As$ were measured in the quantum limit $\omega_c\tau \gg 1$, ω_c - being the cyclotron frequency and τ the impurity scattering time. Quantized values of σ_{xy} were observed equal to fractions of fundamental constants of the type e^2/h. The filling factor ν is defined as a ratio of the total number of electrons N_e.s to the total magnetic degeneracy of the Landay level N_ϕ. Up to now [3] the fractions $\nu = 1/3, 2/3, 4/3, 5/3, 1/5, 2/5, 3/5, 2/7, 3/7, 4/7$ were found and identified with fractional fillings of the Landau levels of the 2-D electronic liquid in strong magnetic field. The integral values of ν correspond to total filling of different Landau levels and represents the integral QHE, not discussed here. To some extent this is the first observation of fractional quantum numbers in physics. The extensive experimental work demonstrated that the observed plateaus of σ_{xy} (quantized values) depend critically on temperature and filling factor [3)-6)]. The application of an a.c. field with increasing frequency stimulates the fractional effect and suppresses the integral one [7),8)]. The disorder influence was found to be important by measurements of samples with low and high mobility.

A complete theory of this exciting phenomenon is absent. The most attractive explanation was suggested by Laughlin [9),10)], who discovered a set of new macroscopic quantum states of the 2-D electronic liquid. He related the fractional quantization with the fractional charge transport suggested by Su and Schrieffer [11)].

The fractional charge excitations were studied by Su [12)]. Fractional statistics and charge of the quasiparticles of the Laughlin ground states were found by Arovas, Schrieffer and Wilczek [13)]. The possible degeneracy of the ground state was discussed by Anderson [14)] and by Tao and Thouless [15)]. It could be stressed here that the theoretical work was mainly concentrated on explaining the equality of the Hall conductivity to fractions of fundamental constants with high precision [16)]. The critical behaviour connected with the onset of the new macroscopic quantum states were left without much attention in the hope that it will be easy to construct.

In a recent communication [17] a brief description was given of the results of a theory which also accounts for the critical behaviour and predicts a new quantum phenomena like 3, 5, 7, etc. electron tunnelling and Josephson type interference effects. In Ref.17 order parameters for fractional fillings were introduced and the coefficients of the corresponding Ginsburg-Landau equation were calculated. Here we give the whole scheme of this theory. It is restricted to one phase and it is essentially a mean field theory. The fluctuations can be taken into account as well as the frequency dependence reported by Pepper et al. [7),8)] The paper is organized as follows.

First in Sec.II we construct an orthogonal set which maps the states manifold to a circle. In Sec.III we show that there is a specific degeneracy persistent to the application of an arbitrary potential U(x,y) and the equivalent 2-D momentum representation is introduced in Sec.IV. Then in Sec.V the symmetry breaking is discussed and a model Hamiltonian of three-particles interaction is constructed. In Sec.VI the ordinary and anomalous propagator are introduced, the latter being the three-particles wave function and in Sec.VII the excitation spectrum is found to have a gap. In VIII we discuss the ground state and the gap equation. The temperature dependence of the gap is determined in Sec.IX and the thermodynamic potential is found in Sec.X, showing clearly that the coherent ground state has lower energy at T = 0. The critical filling number is found in Sec.XI, and the Ginsburg-Landau equation coefficients are determined in Sec.XII showing the fractional flux quantization. In the framework of the present theory a first principles FQHE derivation is given demonstrating the exponential temperature deviation from the exact quantization. In Sex.XIV the prediction of multi-electron tunnelling is discussed together with quantum interference effects and in the concluding section the results are discussed on the whole.

II. ORTHOGONAL SET

The electrons in the inversion layer with typical concentration n_0 of the order of 5.10^{11} are at a distance of the order of 10^{-6} cm. apart from each other and are occasionally scattered by impurities or imperfections of the interface. Thus, it is acceptable to consider first the case of free electrons in strong magnetic field and then to include the crystal lattice periodic potential and/or the impurity scattering. In order to display the specific degeneracy of the electronic states and to discuss its consequences it is useful to have a full orthogonal set of free-electron eigenstates of the inversion layer in strong magnetic field. Here we consider a rectangular

sample with area $L_0 \times L$, which is rolled onto a cylinder of radius $R = L_0/2\pi$ and simultaneously is continuated periodically in z direction. The magnetic field is realized by a magnetic monopole wire along the cylinder axis. When the cylinder radius R tends to infinity the flat geometry is recovered. The vector potential is used in the Landau type of gauge $A = A_r = 0$, $A_z = q$ with radial field $B = q/R$. Note here that the Laughlin theorem can be applied by the inclusion of a thin solenoid coaxial with the cylinder producing pure vector potential $A = \phi_{ext}/2\pi R$, where ϕ_{ext} is the magnetic flux of this source through the cylinder. This theorem, however will not be used, since a more general first principle's calculation of the Hall current will be given. In a large part of this paper magnetic units defined by $r_H = (\hbar c/eH)^{1/2} = \hbar\omega_c = 1$ will be used, but some results are displayed with actual dimensions. For this geometry shown in Fig.1 the Schrödinger equation has the form

$$2E\Psi(z,\varphi) = \left[(i\partial_z + R\varphi)^2 - \frac{1}{R^2}\partial^2_{\varphi\varphi} \right]\Psi \quad . \tag{1}$$

The Landau type of solution is

$$|p\rangle = \Psi_p(z,\varphi) = A_p \frac{e}{\sqrt{L}}^{ipz - \frac{1}{2}(R\varphi - p)^2} \cdot \mathcal{H}_n(R\varphi - p) \tag{2}$$

A_p being the normalization constant.

This solution is defined as a periodical function of φ and a Bloch condition is applied in z direction. The momentum is then of the type $p = 2\pi m/L$, $m = 1,2,\ldots,L$. A rather trivial remark is that this set if not complete. The problem for construction of Bloch functions in magnetic field was discussed recently by Thouless [18)], who pointed out also the impossibility to construct Wannier functions with zero Hall current. The solutions with n = 0 will be considered here for simplicity. When R increases these functions tend to δ functions centered at angles $\varphi_p = p/R$. When $p = 2\pi$, $\varphi_p = 2\pi/R$ and thus only a small fraction of the cylinder is accessible. It is clear then that p should vary in much larger regions $p = m2\pi/L$, $m = 1,2,,,.L,\ldots,2L,\ldots,RL$. In this sense the relevant quantum number is the location angle φ_p, which covers the circle only once. The set obtained is a discrete complete subset of the coherent states, the latter forming an overcomplete set by just one state [18),19)]. In agreement with Thouless' discussion it is seen that it is impossible to construct a set localized in both directions. For instance, with another gauge the following set is obtained (E = 1/2):

$$|m\rangle = \frac{const}{\sqrt{2\pi}} \cdot e^{im\varphi - \frac{1}{2}(z-z_m)^2}, \quad z_m = \frac{m}{R}, \quad m = 1,2,\ldots L.R. \tag{2'}$$

These wave functions are localized in z direction but delocalized in φ direction. This is a property of the von Neumann (p,q) lattice, i.e. the lattice in the phase space of elementary cell area $p\,q = h$. The s-type functions (2), (2') are well defined by periodical continuation only far from the boundaries $(0,2\pi)$. In the vicinity of the boundaries the energy levels bend up, a situation clarified by Halperin [20].

III. DEGENERACY THEOREM

The orthogonal set determined in the previous section is of the Landau type. It was discussed here because with this particular type of states a very important degeneracy theorem will be proved. Let the total number of states $M = L.R$ be of the type

$$M = L.R = (2k+1)NR \quad , \tag{3}$$

where k is a small integer. Further, the case of $M = 3.NR$ will be considered as the simplest one. In this case the whole set of functions $|p\rangle$ can be divided into three subsets shown in Fig.2 as the three sectors of the circle. Specifically, an equivalent symbol will be used $|p\rangle = |m,l,\alpha\rangle$, where $m = 1,2,\ldots,N$; $l = 1,2,\ldots R$ and $\alpha = 1,2,3$. The degeneracy theorem already mentioned states: For arbitrary potential $U(z,\varphi)$ the intersector matrix elements are vanishingly small $U_{\alpha,\alpha'} = \langle ml\alpha|U|m'l'\alpha'\rangle = 0$. Conversely this theorem provides that whatever the potential is (random or periodic) the three huge subsets $\alpha = 1,2,3$ remain degenerate. The prove is straightforward. The matrix elements $U_{p,p'}$ are of the type

$$U_{p,p'} = \int_0^L \frac{dz}{L} \cdot e^{-i(p-p')z} \cdot U_{p,p'}(z) \quad . \tag{4}$$

Here $U_{p,p'}(z)$ is the matrix element of $U(z,\varphi)$ between two distant $\alpha \neq \alpha'$ localized wave functions

$$U_{pp'}(z) = A_p A_{p'} \int_0^{2\pi} d\varphi \cdot e^{-\frac{R^2}{2}\left[(\varphi-\varphi_p)^2 + (\varphi-\varphi_{p'})^2\right]} \quad . \tag{5}$$

The overlap integral of the two exponents is extremely small for $\alpha \neq \alpha'$ and $R \gg 1$

$$U_{\alpha,\alpha'} \sim e^{-\frac{R^2}{2}(\varphi_\alpha - \varphi_{\alpha'})^2} \sim e^{-\frac{R^2}{2}\left(\frac{2\pi}{3}\right)^2(\alpha-\alpha')^2} \simeq 0 \quad . \tag{6}$$

This result is a property of the von Neumann lattice. Consequently, each Landau level will remain 3-fold degenerate in the presence of disorder and periodic potential. In this sense the disorder is irrelevant for the splitting of the Landau subbands at fractional fillings $\nu = 1/3, 1/5$, etc.

IV. UNITARY TRANSFORMATION

Before discussing the consequences of the degeneracy established already, a brief description of a model Hamiltonian reflecting the presence of a periodical potential will be given. In the same way the disorder can also be taken into account. What comes out as a result is a 2-D momentum representation equivalent to the set in Sec.II.

For a periodic potential the matrix elements

$$H^0_{m,m+k} = \langle m,l,\alpha|\, U \,|m+k,l,\alpha\rangle \simeq H(k) \tag{7}$$

are all non-zero and of the same order of magnitude, but since realistic calculation of the density of states is not the goal here only the diagonal and the nearest non-diagonal elements will later be taken into account. The other type of matrix elements is

$$m\,e,m',l+k = \langle m,l,\alpha|\, U \,|m',l+k,\alpha\rangle \cong e^{-\frac{R^2}{2}\left(\frac{2\pi}{3}\right)^2} \cdot U(m-m') \quad . \tag{8}$$

Since this expression rapidly decreases with $k = 0,1$ the quasi-diagonal approximation is quite acceptable. Collecting these terms the following matrix has to be diagonalized:

$$H_{tot} = \begin{pmatrix} H & 0 & 0 \\ 0 & H & 0 \\ 0 & 0 & H \end{pmatrix}, \quad H = \begin{pmatrix} H^0 & V & 0 & \ldots & 0 \\ V & H^0 & V & \ldots & 0 \\ 0 & V & H^0 & \ldots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \ldots & H^0 \end{pmatrix} , \tag{9}$$

where the matrices H_0 and V are of the type

$$H^0 = \begin{pmatrix} H(0) & H(1) & H(2) & \dots & H(N-1) \\ H(1) & H(0) & H(1) & \dots & H(N-2) \\ H(2) & H(1) & H(0) & \dots & H(N-3) \\ \dots & \dots & \dots & \dots & \dots \\ H(N-1) & \dots & \dots & \dots & H(0) \end{pmatrix} \qquad V = \begin{pmatrix} V(0) & V(1) & V(2) & \dots & V(N-1) \\ V(1) & V(0) & V(1) & \dots & V(N-2) \\ V(2) & V(1) & V(0) & \dots & V(N-3) \\ \dots & \dots & \dots & \dots & \dots \\ V(N-1) & \dots & \dots & \dots & V(0) \end{pmatrix} . \quad (10)$$

The symbols H(0) and V(0) were used for the diagonal elements and H(1), V(1) for the nearest non-diagonal ones. The three-fold degeneracy is stressed by the displayed form of H_{tot}. It is clear also that using random variables as matrix elements it is also possible to include the disorder at this stage. This Hamiltonian is diagonalized with the following two unitary transformations:

$$U^{I}_{\ell m,\ell' m'} = \delta_{m,m'} \frac{e^{iq_x^{\ell'}\cdot\ell}}{\sqrt{R}} , \quad q_x^{\ell'} = \frac{2\pi\ell'}{R} , \quad \ell' = 1,2,\dots R$$

$$U^{II}_{\ell m,\ell m'} = \delta_{\ell\ell'} \frac{e^{iq_y^{m'}\cdot m}}{\sqrt{N}} , \quad q_y^{m'} = \frac{2\pi m'}{N} , \quad m' = 1,2,\dots N . \quad (11)$$

The eigenvalues of the diagonalized matrix H are related to the parameters by the relations

$$E(q_x,q_y) = H(0) + 2V(0)\cos q_y + 2\left[H(1) + 2V(1)\cos q_y\right]\cos q_x + \dots \quad (12)$$

The one-dimensional type of states are converted here into an equivalent 2-D representation. The tight binding form of $E(q^x,q^y)$ is a result of the truncation of the matrix elements in the last equation (12) and the approximation $H_{p,p'} \simeq H_{p-p'}$, (Eq.(7)). For instance, in this way the complicated Cantor-type level structure discussed by Thouless et al.[21] and Aers and MacDonald[22] is left out. However, the two-dimensional form is useful. It will be emphased by using 2-D vector $\vec{q} = (q_x,q_y)$ or $\vec{p}$. The difference $\xi(\vec{p}) = E(\vec{p}) - E_F$ will be written as usual in the form $\xi_p = v_F(p-p_F)$. It could be mentioned that the two-dimensional behaviour was clearly seen experimentally.

The 3-fold degeneracy remains intact under the unitary transformation. Thus the single particle potentials fail to split the magnetic degeneracy of the Landau subband and next the electron-electron interaction is to be examined in respect to intersector elements.

V. THREE-PARTICLES INTERACTION MODEL

The electron-electron interaction in the already specified $|m,l,\alpha\rangle = |p\rangle$ representation has the usual form

$$H_i = \sum_{pp',qq'} V_{pp'}^{qq'} \; a_p^+ a_{p'}^+ a_q a_{q'} \; , \quad (13)$$

where a_p, a_p^+ are the anticommuting fermion operators.

The matrix elements $V_{pp'}^{qq'}$ can be written as

$$V_{pp'}^{qq'} = \frac{e^2}{2\varepsilon_0}\int dz \int dz' \; \psi_p^*(z)\,\psi_{p'}^*(z')\,\frac{1}{|z-z'|}\,\psi_q(z)\,\psi_{q'}(z') = \delta_{p+p'-q-q'}\,V_{p-q}^{p'} \; . \quad (14)$$

In accordance with the theorem already proved the matrix elements are diagonal with respect to the sector indices p,q and p'q' , respectively. The intersector elements are different from zero only for the two groups p,q and p'q' . These elements are easily seen to be of the order of $e^2/\varepsilon_0 R$, where R is the cylinder radius. In the limit of large R the states $|p\rangle$ are well localized at a fixed angle φ_p and consequently the distance between the two electrons is of the order R - the distance among different sectors. Thus, the symmetry breaking term in the interaction is of the type

$$\tilde{H}_i = \sum_{p,p',q,\alpha\neq\beta} U_{pq}^{p'}(\alpha,\beta)\; a_{p\alpha}^+ a_{p'-p,\beta}^+ a_{q\alpha} a_{p-q,\beta} \; , \quad (15)$$

where the sector indices α,β are written explicitly.

This symmetry breaking term is very small, however, and vanishes in the limit $R \to \infty$. It fails to produce a sufficient gap in the single-particle spectrum, and the three-fold degeneracy still remains. The term actually breaking the C_3 symmetry is the three-particle bound state wave function. It is an anomalous average of the type

$$\langle a_{\vec{p}_1,\alpha}^+ \; a_{\vec{p}_2,\beta}^+ \; a_{\vec{p}_3,\gamma}^+ \rangle \; , \quad \vec{p}_1+\vec{p}_2+\vec{p}_3 = 0 \; , \quad \alpha+\beta\neq\gamma \; . \quad (16)$$

Here the two dimensional momentum representation introcuded in the previous section is used. This anomalous three-particle's wave function appears in the second order of the perturbation theory in the chain of equations for the Green

functions. Due to the small denominator this term is not small, reflecting the magnetic degeneracy. Instead of diaplaying the more complicated form of the theory constructed in this way, here is presented a simplified version of the theory restricted to the rigid wave function regime. In this way the internal excitations of the three-particles aggregates are left out being of higher energy. It should be mentioned also that this internal excitation may have fractional charge as in the theory of Laughlin [10] and Arovas, Schriffer and Wilczek [13]. In this sense, the present theory does not contradict the existent theoretical explanations but is centered on lowest energy states and transport. Later when discussing the ground state, however it will be pointed out also that in fact it is rather different from that considered by Laughlin [10] and Avoras, Schrieffer and Wilczek [13].

The rigid wave function regime can easily be described by a model Hamiltonian which is essentially a contact three particle's interaction of the type

$$H_{int} = \frac{g}{3!} \int d\vec{r}\ \psi_\alpha^+(\vec{r})\psi_\beta^+(\vec{r})\psi_\gamma^+(\vec{r})\psi_\gamma(\vec{r})\psi_\beta(\vec{r})\psi_\alpha(\vec{r}) \quad . \qquad (17)$$

Here the interaction strength g is proportional to third power of the electronic charge

$$g \sim \left(\frac{e}{\sqrt{\varepsilon_0}}\right)^3 .$$

Instability of the ground state occurs only for $g < 0$. In general the (2k+1) fold degeneracy of the Landau level persistent to random potentials can be lifted by a (2k+1) particle interaction of the type

$$H_{2k+1} = \frac{g_{2k+1}}{(2k+1)!} \int d\vec{r}\ \psi_{\alpha_1}^+(\vec{r}) \ldots \psi_{\alpha_{2k+1}}^+(\vec{r})\ \psi_{\alpha_{2k+1}}(\vec{r}) \ldots \psi_{\alpha_1}(\vec{r}) , \qquad (18)$$

where

$$g_{2k+1} \sim \left(\frac{e}{\sqrt{\varepsilon_0}}\right)^{2k+1} .$$

In this case the anomalous propagator is

$$\langle \psi_{\alpha_1}^+(\vec{r})\ \psi_{\alpha_2}^+(\vec{r}) \ldots \psi_{\alpha_{2k+1}}^+(\vec{r}) \rangle \quad .$$

It is in fact the order parameter describing a new phase transition from a normal Fermi liquid ro a new macroscopic quantum atate - Laughlin liquid.

When writing the model Hamiltonian an energy cut-off ω_0 is also supposed. It will enter the theory later.

The model Hamiltonian serves as effective intersector scattering providing the momentum conservation at the Fermi level in the same way as the BCS Hamiltonian for superconducting pairs.

VI. ANOMALOUS PROPAGATOR

In this section only the excitations of lowest energy will be discussed. The internal excitations described by the multiparticle correlator are not considered here. For this reason we focus our attention on the rigid wave function regime described by the propagators defined here

$$G_{\alpha\beta}(x,x') = -i \langle T\ \psi_\alpha(x)\ \psi_\beta^+(x') \rangle$$

$$\Phi_{\alpha\beta\gamma}^+(x,x') = \langle T\ \psi_\beta^+(x)\ \psi_\gamma^+(x)\ \psi_\alpha(x') \rangle \quad . \qquad (19)$$

Here the symbol x is used for $(\vec{r},t)$ variables and the angular brackets denote quantum-mechanical average at $T = 0$. Only the anomalous particle correlator Φ is important in the Green functions chain equations. These equations are written here in order to introduce Gor'kov type equations for G and Φ^+. Following the common scheme the terms contributing to the self energy are included in the chemical potential μ. The antisymmetry of the anomalous wave function is ensured by the form $\Phi_{\alpha\beta\gamma}(\vec{r}) = \varepsilon_{\alpha\beta\gamma}\Phi(r)$ assumed here.

In the first order with respect to g the equations for $G_{\alpha\beta} = \delta_{\alpha\beta}G$ and Φ take the form

$$[i\partial_t - H_0(\vec{r})]\,G(x,x') = \delta(x-x') + ig\,\Phi_0(x)\,\Phi^+(x,x')$$

$$[i\partial_t + 2H_0(\vec{r})]\,\Phi(x,x') = -ig\,\Phi_0^+(x).N.\,G(x,x') \quad , \qquad (20)$$

where the particle's concentration is $N(\vec{r}) = \langle \psi^+(x)\psi(x) \rangle$. In the case of homogeneous 2-D layer $N = \text{const} = N_0$. The three particle's wave fuhction $\Phi_0(\vec{r}) = \langle \psi_1(\vec{r})\psi_2(\vec{r})\psi_3(\vec{r}) \rangle$ is independent on $\vec{r}$ in

homogeneous external magnetic field and describe essentially the gap in the excitations spectrum. Performing Fourier transformation with respect to t-t' and applying the unitary transformation already discussed one reaches an algebraic set of two equations for $G(p,\omega)$ and $\Phi^+(p,\omega)$

$$\left[\omega - \xi_{\vec{p}}\right] G(\vec{p},\omega) = 1 + ig\,\Phi_0\,\Phi^+(\vec{p},\omega)$$

$$\left[\omega + 2\xi_{\vec{p}}\right] \Phi^+(\vec{p},\omega) = -ig\,\Phi_0^+ . N_0 . G(\vec{p},\omega) \quad . \tag{21}$$

Here the chemical potential was introduced by shifting ω to $\omega + \mu$ and $\Phi^+(t)$ to $e^{i3\mu t}\,\Phi^+(t)$. The energy difference $\xi_{\vec{p}} = E(p) - \mu$ can be written as usual in the form $\xi_{\vec{p}} = V_F(p-p_F)$ and $\vec{p}$ is a 2-D wave vector.

The excitations spectrum and density of states will be discussed next.

VII. EXCITATIONS SPECTRUM. DENSITY OF STATES

The poles of the two functions $G(\vec{p},\omega)$ and $\Phi^+(\vec{p},\omega)$ determine the excitation's spectrum of the type

$$\omega = \omega^{\pm}(\vec{p}) = \tfrac{1}{2}\left[-\xi_p \pm \sqrt{g\xi_p^2 + 4\Delta_0^2}\,\right] \quad , \tag{22}$$

where the gap parameter Δ_0 is

$$\Delta_0^2 = g^2 N_0 |\Phi_0|^2 \quad . \tag{23}$$

The two branches of the spectrum $\omega^{\pm}$ are plotted in Fig.3. It is seen that the extrema of both curves $\omega^{\pm}(\xi_p)$ are displaced with respect to ξ by $\pm\xi_0 = \pm\,\Delta_0/3\sqrt{2}$. The energy gap $2E_g$ is then $2E_g = 4\sqrt{2}\Delta_0/3$.

The density of states $N(\omega)$ is determined essentially by the derivative

$$N(\omega) = N(0)\,\frac{d\xi}{d\omega} = \frac{N(0)}{4}\left(1 \pm \frac{3\omega}{\sqrt{\omega^2 - \xi_g^2}}\right) \quad .$$

The well-known one-dimensional singularities are clearly seen from this form and from Fig.4. The account of the disorder due to impurity centers lead to replacement of the singularities by finite maxima. It can be stressed here that this type of behaviour is clearly seen in the observed behaviour of σ_{xx} as a function of the filling number ν .

The solutions of the equations for the two functions $G(p)$, $\Phi^+(p)$ have a standard form

$$G(\vec{p},\omega) = \frac{U^2(\xi)}{\omega - \omega^+(\xi) + i\delta} + \frac{V^2(\xi)}{\omega - \omega^-(\xi) - i\delta}$$

$$\Phi^+(\vec{p},\omega) = -\frac{ig\,\Phi_0^+ N_0}{(\omega - \omega^+(\xi) + i\delta)(\omega - \omega^-(\xi) - i\delta)} \quad . \tag{24}$$

Here the $U(\xi)$, $V(\xi)$ pair is

$$U^2(\xi) = \tfrac{1}{2}\left[1 + \frac{3\xi}{\sqrt{g\xi^2 + 4\Delta_0^2}}\right]$$

$$V^2(\xi) = \tfrac{1}{2}\left[1 - \frac{3\xi}{\sqrt{g\xi^2 + 4\Delta_0^2}}\right] \quad . \tag{25}$$

VIII. GAP EQUATION. GROUND STATE

It may be stressed here that the present theory is a single phase theory in the sense that the other phases with 5, 7, etc. particle order parameters are not considered simultaneously with the three particles correlations. The set of such transitions can form a "devil staircase" and will be discussed elsewhere. For the order parameter $\Phi^+(0)$ a BCS type equation is valid .

$$\Phi_{(0)}^+ = \int \frac{d\vec{p}}{(2\pi)^2} \int \frac{d\omega}{2\pi}\, \frac{(-iN_0 g\,\Phi_{(0)}^+)}{(\omega - \omega^+(\xi) + i\delta)(\omega - \omega^-(\xi) - i\delta)} \quad . \tag{26}$$

In the framework of the model three particles interaction a symmetrical energy cut-off ω_0 is introduced such that the solution of this equation is

$$\Delta_0 = 3\omega_0\, exp\left(-1/\left(\tfrac{2}{3}|g|N_0 P_F\right)\right) \quad . \tag{27}$$

This expression determines the gap at $T = 0$ for $\nu = 1/3$, where exact three-fold symmetry holds. In general Δ is a function of ν and T of the mean field type. Experimentally the filling number ν can be varied in

two regimes: gate voltage V_g = const or magnetic field B = const. The discussion of these two situations will be given in Sec.XI. Here is constructed the ground state $|\psi_0>$ in the case of three-fold degeneracy. The normal 2-D Fermi liquid ground state $|0>$ is acted upon by three-particles operators in the same way as in the BCS theory giving rise to

$$|\Psi_0> = \prod_{\vec{p}} \left(u(\xi) + \frac{\varepsilon_{\alpha\beta\gamma}}{6!} u(\xi)\, a^+_{\vec{p}_1,\alpha} a^+_{\vec{p}_2,\beta} a^+_{\vec{p}_3,\gamma} \right) |0> . \quad (28)$$

The product is taken over all 2-D vectors $|\vec{p}_1| = p_F$ and $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$. This ground state differs from the charge density wave construction intensively discussed recently.

Being a coherent state $|\psi_0>$ can be presented as a superposition of states with fixed number of particles $|\psi_N>$

$$|\Psi_0> = \sum_N \lambda_N |\Psi_N> \quad . \quad (29)$$

Calculating the mean values one has

$$<N> = 3 \sum_{|p|<p_F} v_p^2 \quad (30)$$

and

$$<(N-<N>)^2> = 9 \sum_{|p|<p_F} u^2(\xi_p) v^2(\xi_p) \quad .$$

The states with fixed number of particles are found using Anderson type projection

$$|\Psi_N> = \int_0^{2\pi} d\Phi \; e^{-iN\Phi} |\Psi_0>$$

the uncertainty relation $\Delta N \, \Delta \Phi \geq 1$ being valid.

This coherent state $|\psi_0>$ is current carrying one, when longitidunal voltage is applied. The Hall current is non-dissipative type and the quantized values of σ_{xy} at fractional fillings can be obtained applying the Laughlin theorem, once the gap in the spectrum is present.

In the preaent theory, however, the Hall current can be reduced by increasing the temperature T or other parameters, such as the a.c. field frequency ω_{ac}, which were observed lately. In order to calculate the current in these more complicated conditions another first principles derivation of the Hall current will be given in Sec.XIII. Next the temperature dependence of the gap will be calculated.

IX. TEMPERATURE DEPENDENCE OF THE GAP

In the framework of the temperature diagram technique, the Fourier serial expansion is applied to the Matsubara Green functions. Practically these are obtained by the substitution of ω by $i\omega_n$ in the Gor'kov equations with the condition $\omega_n = (2n+1)\pi T$. Also $i\Phi_0$ is replaced by Φ_0 . In this way one has the equations

$$\left[i\omega_n - \xi_{\vec{p}} \right] G(\vec{p}, i\omega_n) + g \Phi_0 \Phi^+(p, i\omega_n) = 1$$

$$\left[i\omega_n + 2\xi_{\vec{p}} \right] \Phi^+(\vec{p}, i\omega_n) + g \Phi_0^+ N_0 G(\vec{p}, i\omega_n) = 0 \quad . \quad (31)$$

The gap is now defined as $\Delta^2 = g^2 N_0 |\Phi_0|^2$, the following equation being fulfilled:

$$1 = g N_0 T \sum_{\omega_n} \int \frac{d\vec{p}}{(2\pi)^2} \frac{1}{\omega_n^2 + \Delta^2 + 2\xi_{\vec{p}}^2 - i\omega_n \xi_{\vec{p}}} \quad . \quad (32)$$

Working it out as usual, one reaches the temperature expansion

$$\Delta(T) = \Delta_0 - 2\sqrt{\frac{\pi T \Delta_0}{3\sqrt{2}}} \; e^{-\frac{3\Delta\sqrt{2}}{3T}} \left(1 - \frac{3T}{16\Delta_0\sqrt{2}} \right) \quad . \quad (33)$$

The critical temperature T_c is defined by the condition $\Delta(T_c) = 0$. It is found to be

$$T_c = \frac{2\Delta_0}{3\pi} \cdot \gamma \cdot \sqrt{2} \quad , \quad (34)$$

where $C = \ln \gamma$ is the Euler constant (0,577). In the critical region close to T_c, Δ is small and the mean field type behaviour holds.

It could be mentioned here that the experimental temperature dependence of the plateau's length found by Chang et al. is qualitatively of this type as seen from Fig.5. Precise temperature measurements can verify this prediction. In the same time the account of impurity scattering alters this result to some extent.

X. THERMODYNAMIC POTENTIAL. GROUND STATE ENERGY

The thermodynamic potential deviation $\Delta\Omega$ from the normal state has the form

$$\Delta\Omega = \int_0^{\Delta} \frac{d\left(\frac{1}{|g|}\right)}{d\Delta} \cdot \Delta^2 d\Delta \qquad (35)$$

In the low temperatures limit one reaches the expression

$$\Delta\Omega = \rho_F \left[\frac{\pi^2 T^2}{4} - \frac{\Delta_0^2}{3} - 2\Delta_0 \sqrt{\frac{\pi \Delta_0 T}{3\sqrt{2}}} \cdot e^{-\frac{2\sqrt{2}\Delta_0}{3T}} \left(1 + \frac{45T}{16\Delta_0\sqrt{2}} \right) \right] . \qquad (36)$$

Here the term proportional to T^2 is analogous to the 2-D Fermi liquid energy. At $T = 0$ the system is in the coherent fractional quantum state $|\psi_0\rangle$ with

$$\Delta\Omega = -\rho_F \frac{\Delta_0^2}{3} \qquad (37)$$

providing lower energy of this state in respect to the normal one. The second term in $\Delta\Omega$ proportional to $\Delta_0^{3/2}$ is the entropy. The calculated heat capacity in the low temperature limit is

$$C_e = \frac{20}{9} \sqrt{\frac{\pi\sqrt{2}}{3}} \rho_F \Delta \left(\frac{\Delta}{T}\right)^{3/2} \cdot e^{-\frac{2\sqrt{2}\Delta}{3T}} . \qquad (38)$$

Close to T_c the thermodynamic potential is still negative

$$\Delta\Omega = -\frac{2\gamma\pi^2}{28\zeta(3)} \rho_F T_c^2 \left(1 - \frac{T}{T_c}\right)^2 . \qquad (39)$$

The jump of the heat capacity between the normal and coherent state has the form

$$\Delta C = \rho_F T_c \cdot \frac{2\gamma\pi^2}{28\zeta(3)} \qquad (40)$$

and the numerical relation is of the type

$$\frac{\Delta C}{C_n} = \frac{2\gamma}{7\zeta(3)} . \qquad (41)$$

These relations were displayed here in order to make comparison with the BCS theory. Measurements of heat capacity are not reported.

XI. CRITICAL FILLING NUMBER

The filling number was already defined as the ratio of the total number of electrons $S.N_a$ to the total number of states N_B, the latter being equal to the flux quanta content of the magnetic flux ($N_B = \phi/\phi_0$).

$$\nu = \frac{N_0 S}{N_B} = \frac{N_0 h c}{e B} . \qquad (42)$$

The gap parameter $\eta = \frac{2}{3}(g)\, \mu_0 \rho_F$ depends on ν. This dependence is different in the two experimentally important situations, namely: constant gate voltage (V_g = const) and constant magnetic field (B = const).

In the first case (V_g = const) the total number of electrons is constant but the density of states at the Fermi level ρ_F varies. In this

case a rough estimate of ρ_F can be a rectangular one of width E_0 such that $\rho_F E_0 = N_B$. The gap parameter η is written in this case as

$$\eta = \frac{\nu_g}{\nu} \quad , \tag{43}$$

where

$$\nu_g = \frac{2}{3}|g| \frac{N_0^2 S}{E_0} \quad . \tag{44}$$

For the gate voltage control (B = const) the same form of ρ_F leads to

$$\eta = \frac{\nu}{\nu_B} \quad , \tag{45}$$

where

$$\nu_B = \frac{3hc}{2eB\rho_F|g|} \quad . \tag{46}$$

The critical value of the filling number ν_c is defined as one for which the gap vanishes

$$\Delta(\nu_c) = 0 \quad . \tag{47}$$

It is determined precisely in the same way as T_c is found. In the V_g = const case and $T \simeq T_c$ it is

$$\nu_c = \nu_g \ln \frac{2\omega_0 \gamma \sqrt{2}}{\pi T_c} \quad . \tag{48}$$

For the B = const one has

$$\nu_c = \nu_B / \ln \frac{2\omega_0 \gamma \sqrt{2}}{\pi T_c} \quad . \tag{49}$$

In the critical region a mean field gap dependence on ν is predicted

$$\Delta = 3\pi T_c \sqrt{\frac{3|\nu_c - \nu|}{14\zeta(3)\nu_g}} \tag{50}$$

V_g = const

$$\Delta = \frac{3\pi T_c}{\nu_c} \sqrt{\frac{3\nu_B|\nu - \nu_c|}{14\zeta(3)}} \tag{51}$$

B = const

The experimental data of Cheng et al. for $\Delta(V_g)$ are in accordance with these results. In the vicinity of ν_c however the mean field theory is no longer valid, and the fluctuations have to be accounted for. It may be mentioned here that the Ginsburg number Gi in this case is of the order of $\mathrm{Gi} = \left(\frac{T_c}{\varepsilon_F}\right)^2$. Experimental values of T_c are of order 1K and ε_F is of the order of 10 mev. The number Gi appears to be small of the order of Gi $10^{-4} \ll 1$.

XII. GINSBURG-LANDAU EQUATION. FRACTIONAL FLUX QUANTIZATION

As was already mentioned, the macroscopic wave function is discussed here only in the rigid wave case. The internal excitations being of higher energy are left out. The calculation of the Ginsburg-Landau equation coefficients is done according to the standard scheme. The Gor'kov equations are written in integral form and the gap Δ defined as

$$\Delta^*(\vec{r}) = g T \sum_{\omega_n} \Psi^+(r, r; i\omega_n)$$

is assumed small in the critical region. In this way the following equation is found for the three particles order parameter $\Psi(r)$

$$\left[\frac{1}{2m^*} \left(-i\hbar\vec{\nabla} + \frac{e^*}{c}\vec{A} \right)^2 + \alpha_0 \left(1 - \frac{T}{T_c} - |\Psi|^2 \right) \right] \cdot \Psi(\vec{r}) = 0 \quad . \tag{52}$$

The effective mass here is $3m$ and the effective charge $e^* = 3e$ as is expected. The coefficient α_0 determines the correlation length ξ as usually

$$\xi^2 = \xi_0^2 \frac{1}{1 - \frac{T}{T_c}} \quad , \tag{53}$$

where

$$\xi_0^2 = \frac{\hbar^2}{2m^*\alpha_0} \quad , \quad \alpha_0 = \frac{3(\pi T_c)^2}{14\zeta(3)\varepsilon_F} \quad . \tag{54}$$

The coherent state Ψ determines the current density

$$\vec{j} = \frac{e^*}{2m^*} \left\{ \Psi^*(\vec{r}) \cdot \left(-i\hbar\vec{\nabla} + \frac{e^*}{c}\vec{A} \right) \Psi(r) + c.c. \right\} = e^* |\Psi|^2 \vec{V}_c \quad . \tag{55}$$

Here the velocity $\vec{V}_c$ is defined as usually by the phase ϕ of $\Psi(r)$

$$V_c = \frac{\hbar}{m^*} \left(\vec{\nabla}\phi - \frac{e^*}{\hbar c}\vec{A} \right) \quad . \tag{56}$$

By integration over a closed contour in the interface plane the exact fractional flux quantization is found to be

$$\Phi = n\,\Phi^* \quad , \quad \Phi^* = \frac{hc}{3e} \quad . \tag{57}$$

In the present scheme in the plateau's region the 2DEG undergoes a phase transition into new macroscopic quantum states-Laughlin states. In these states the coherent fermionic liquid has gaps in the single particle spectrum.

The gaps are of multiparticle origin. At $T = 0$ the dissipative conductivity σ_{xx} vanishes in the region of the gaps. Thus the phase transition is of metal-insulator type.

It could be mentioned that due to the fractional flux quantum Φ^* the total number of states $N_B^* = \frac{\Phi}{\Phi^*} = 3\,N_B$ increases 3 times. This makes the fractional filling $\nu = \frac{N_e}{N_e}$ equivalent to the integer one.

XIII. FIRST PRINCIPLES FQHE DERIVATION

It is remarkable that the quantum mechanic current density definition allows direct calculation of the Hall conductivity in plateau's conditions. In an infinite system in the absence of external electric field the wave function can be chosen real and the current is zero. In the presence of external electric field at $T = 0$ the gap in the spectrum provides $\sigma_{xx} = 0$ and the only existing current is the nondissipative Hall current. Since the electric and magnetic fields are perpendicular, a unique reference frame exists, where the electric field is zero. In this frame the Hall current also vanishes. Hence the velocity $\vec{V}_c = 0$ and in this way the phase of the rigid wave function is fixed in the moving frame. Returning to the reference frame with actual electric field according to the Galilean transformations of the wave function gives

$$\Psi_{\mathcal{E}} = \Psi_{\mathcal{E}=0} \cdot e^{i\frac{m^*\vec{V}_c(r+\vec{V}t)}{\hbar}} \quad , \quad |\vec{V}| = c\frac{\mathcal{E}}{B} \tag{58}$$

The current calculated from the definition has the form

$$\vec{j} = e^* \cdot |\Psi_{\mathcal{E}=0}|^2 \cdot e\frac{\mathcal{E}}{B} = \frac{e^2}{h} V_c(T) \cdot \mathcal{E} \tag{59}$$

The result is highly unexpected. For $T = 0$ because of the energy gap the concentration of the coherent electrons N_c coincides with the total density of the electrons N_o. For $T = 0$ N_c is different from N_o being temperature dependent according to (compare with (33))

$$N_r = |\Psi|^2 = N_o - \tilde{N}(T) = N_o\left[1 - 2\sqrt{\frac{\pi T}{3\Delta_o\sqrt{2}}}\left(1 - \frac{3T}{16\Delta_o\sqrt{2}}\right)e^{-\frac{2\sqrt{2}\Delta_o}{3T}}\right] \qquad (60)$$

where a fraction of electrons $\tilde{N}(T)$ is normal, activated in states with higher energy than the gap E_g. The Hall conductivity becomes

$$\sigma_{xy} = \frac{e^2}{h}\frac{1}{3}\left[1 - 2\sqrt{\frac{\pi T}{3\Delta_o\sqrt{2}}}\left(1 - \frac{3T}{16\Delta_o\sqrt{2}}\right)\cdot e^{-\frac{2\sqrt{2}\Delta_o}{3T}}\right] . \qquad (61)$$

Here $v_c(T) = N_c(T)/N_o$.

Therefore (61) also contains as well the observed[4,6] thermally activated behaviour. The data of Chang, Paalanen, Tsui, Stormer and Hwang[4] demonstrates the same activation energy for σ_{xx} and σ_{xy}. The result (61) relates the plateau's only to the coherent electrons. It gives a remarkable explanation of the observed deviations from the exact quantization. The two fluids model with activated normal electrons contributing to the dissipative transport can account for the effect of an a.c. field applied to the 2-D inversion layer[7,8].

XIV. MULTIELECTRON TUNNELLING

Here we give only the prediction of simultaneous tunnelling of 3,5,7 etc. electrons through barriers and quantum interference effects. Following Josephson, the Gor'kov integral equations are solved assuming constant values of the order parameter on both sides of the barrier. The current through the barrier takes the form

$$J = J_1 \sin\phi$$

where ϕ is the phase difference of the two fluida. The time dependence of ϕ is determined by the Gor'kov-Josephson relation

$$\hbar\frac{\partial\phi}{\partial t} = (2k+1)\Delta\mu \quad , \quad k = 1, 2, 3 \ldots$$

Here $\Delta\mu$ is the chemical potential difference on both sides of the barrier. In the presence of potential $V = V_0 + V_1 \cos(\omega_d t + \theta_0)$ the phase depends on t according to

$$\phi(t) = (2k+1)\frac{eV_0}{\hbar}t + (2k+1)\frac{eV_1}{\hbar\omega_d}\sin(\omega_d t + \theta_0)$$

giving rise to a.c. and d.c. quantum interference.

XV. CONCLUSIONS

Concluding, we emphasize the results of the present work. The main point is the introduction of order parameters describing the macroscopic coherent quantum states of the 2-D electronic liquid corresponding to fractional quantization. The macroscopic multiparticle wave function enters a mean field type theory taking the role of order parameter much in a BCS manner. We have calculated the coefficients in the Ginsburg-Landau equation without scattering. The first principles calculation of the Hall conductivity not only lead to exact quantization, but also determines the accuracy of the effect. The mean field behaviour of the $\sigma_{xx}(T)$ and $\sigma_{xy}(T)$ as predicted here is visible from the data of Chang et al., demonstrating the temperature dependence of the Hall steps width. The fractional flux quantum found here is a consequence of the particle's triplication. It describes the phase transitions in the Laughlin quantum liquid states as an instability upon fractionization of the magnetic flux quantum. In the fractional flux state the number N_B^* of states increases 3,5, etc. times, leading to integral filling of the new bands.

One of the main predictions of the present theory is the multielectron tunnelling and multiparticles quantum interference effects. The fluctuations and the disorder can be taken into account in the framework of the Wegner's Q-matrix approach[23-25]. The discussion of the filling fractions with even denominators will be given elsewhere.

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FIGURE CAPTIONS

Fig. 1. Rectangular 2-D sample rolled onto a cylinder. The radial magnetic field is provided by a magnetic monopole wire.

Fig. 2. The three sectors represent the three degenerate subsets $\alpha = 1,2,3$. Each large sector contains P small sectors of N states each.

Fig. 3. Excitation spectrum with a gap $2E_g = \frac{4\sqrt{2}}{3}\Delta$.

Fig. 4. Density of states $N(\omega)$. One-dimensional singularities are reduced to finite maxima by the disorder.

Fig. 5. From Chang et al[4]. Both $\rho_{x_1}(\nu_2)$ and $\rho_{x_2}(\nu_2)$ clearly show a gap increasing at lower temperatures. The curve $\Delta(\nu_2)$ behave as order parameter. The range of validity of the present theory is in the region of the maximum.

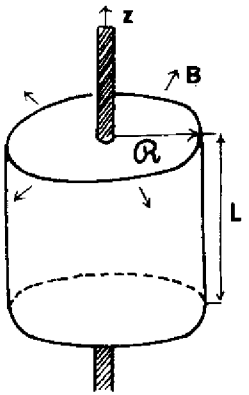

Fig.1

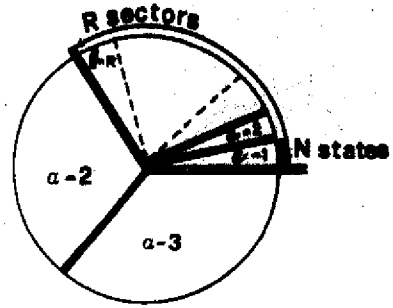

Fig.2

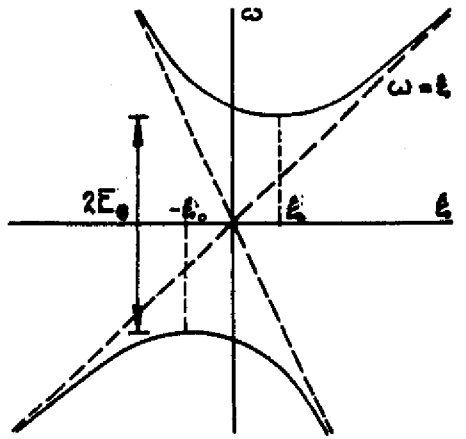

Fig.3

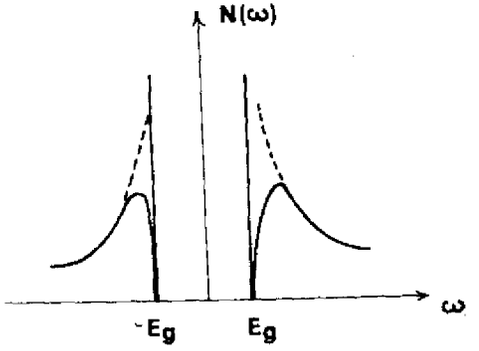
N(ω)
ω
-Eg
Eg

Fig.4

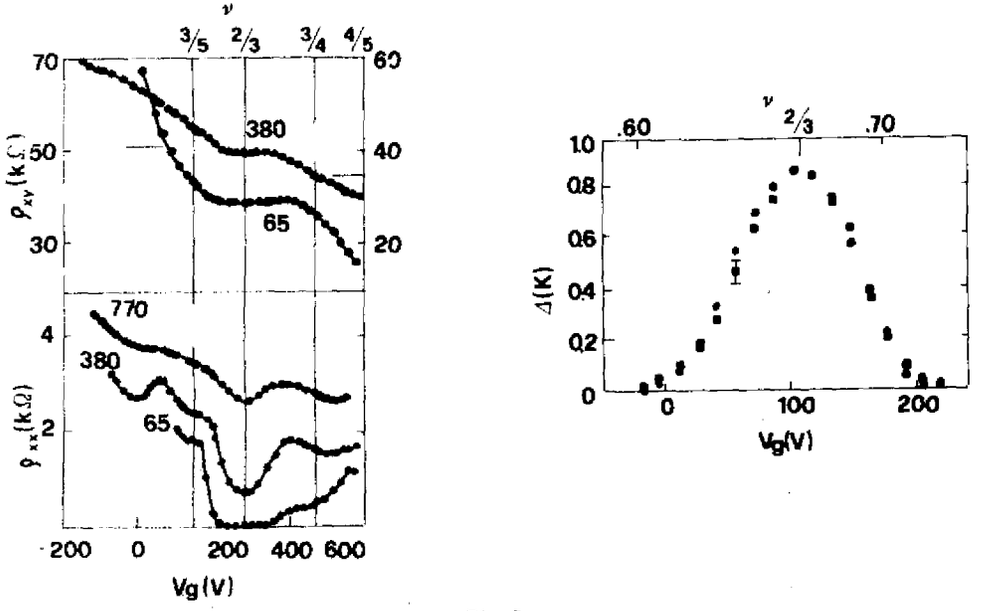
ν
3/5
2/3
3/4
4/5
70
50
30
60
40
20
380
65
ρxy (kΩ)
770
380
65
4
2
ρxx (kΩ)
-200
0
200
400
600
Vg (V)
ν
.60
2/3
.70
1.0
0.8
0.6
0.4
0.2
0
Δ(K)
0
100
200
Vg(V)

Fig.5