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HIGHER FRACTIONS THEORY OF FRACTIONAL HALL EFFECT *

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ABSTRACT

A theory of fractional quantum Hall effect is generalized to higher fractions. n-particle model interaction is used and the gap is expressed through n-particles wave function. The excitation spectrum in general and the mean field critical behaviour are determined. The Hall conductivity is calculated from first principles.

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1. Introduction

The Fractional Quantum Hall Effect (FQHE) has been intensively studied in the past two years following its discovery by Tsui, Stormer and Gossard [1]. These authors found Hall conductivity σ_{xy} equal with high accuracy to fundamental constants $\frac{e^2}{h}\nu$ with $\nu = 1/3$ and $2/3$. Here the filling factor ν is defined as the ratio of the total number of electrons N_0 to the total magnetic degeneracy of the Landau level N_B. Subsequently, other fractions were observed [2] like 1/3, 5/3, 1/5, 2/5, 3/5, 2/7, 4/7 with odd denominators. A minimum in σ_{xx} at $\nu = 5/2$ was observed indicating even fraction quantization but the situation is still uncertain [3]. Much theoretical work was devoted to this exciting phenomenon. One of the most promising frameworks of a theory was suggested by Laughlin [4,5] who discovered a set of new quantum states of the 2-D electronic liquid in strong magnetic field. The fractional filling was attributed to the fractional charge transport originally introduced by Su and Schrieffer [6]. This line of research was developed by Halperin [7], by Arovas, Schrieffer and Wilczek [8], and by Arovas, Schrieffer, Wilczek and Zee [9], who constructed excitations with fractional statistics. Another possibility was the charge density waves approach studied by Fukuyama, Platzman and

Anderson [10], and subsequently by Rashba [11], by Anderson [12], and also by Tao and Thouless [13].

Meanwhile Levine, Libby and Pruisken [14] found a third order term in the field theory of localization [15,16] and constructed a theory of the integral QHE, which was developed by Khmelnitskii [17].

In a communication [18], hereafter referred to as I, one of the authors (I.Z.) of the present work suggested order parameters describing the new quantum state of the 2-D electronic liquid. The role of such order parameters is played by the macroscopic wave function of 3, 5, 7, etc. particles. In this type of theory at fractional fillings $\nu = 1/3, 2/3, 1/5, \ldots$ the system is unstable against forming multiples of charge carriers. The triplication of the electrons reduces the magnetic flux quantum $\Phi_0 = \frac{hc}{e}$ to $\Phi_0/3$. The magnetic degeneracy $N_B = \Phi/\Phi_0$ increases 3 times and the fractional filling 1/3 corresponds to a total integral filling of the Landau level. It could be mentioned that this approach explores the other possibility of conductivity fractionization - instead of the fractional charge transport, the fractional flux quanta are considered. The field theory of localization can be developed in this scheme and will be published elsewhere. Here is given a generalization of the theory I, operating with 5, 7, etc. multiples. The multiparticle's order parameters are introduced and the excitation spectrum is found to have a gap as in I. Here the dependence of the gap and the type of fractional quantization is found. Similarly to I, the multiparticle's ground state is constructed and the temperature and filling factor dependencies on the type of quantization are found. Finally the Ginzburg - Landau equation and the Hall conductivity are derived.

§2. Excitation spectrum.

The magnetic degeneracy discussed in I is related to the instability against particle's multiplication mentioned already. For fractional fillings $\nu = m/n$ the set of eigenfunctions spanning the Landau level splits in n subsets realizing representation of C_n with the same energy, and these n subsets remain degenerate whatever potential is applied. In order to lift this degeneracy, a multiparticle model interaction is introduced similarly to I

$$H_{int}^{(n)} = \frac{g_n}{n!} \int \psi^+_{\alpha_1}(\vec{r}) \ldots \psi^+_{\alpha_n}(\vec{r}) \psi_{\alpha_n}(\vec{r}) \ldots \psi_{\alpha_1}(\vec{r}) \, d\vec{r} \tag{1}$$

The indices α here run over 1,2,...,n, the $\psi_\alpha(\vec{r})$, $\psi^+_\beta(\vec{r})$ being destruction and creation operators. The strength of the interaction g_n is proportional to $\left(\frac{e}{\sqrt{\varepsilon_0}}\right)^n$.

Two Green functions are necessary for the description of the system, the normal single particle one

$$G_{\alpha\beta}(x,x') = -i \langle T(\psi_\alpha(x)\psi^+_\beta(x')) \rangle \tag{2}$$

and the anomalous n-particle's propagator

$$F^+_{\alpha_1\alpha_2\ldots\alpha_n}(x,x') = \langle N | T(\psi^+_{\alpha_1}(x) \ldots \psi^+_{\alpha_{n-1}}(x)\psi^+_{\alpha_n}(x')) | N-n \rangle \tag{3}$$

Much in the same way as in I the Gor'kov' equations are derived and the excitation spectrum is found to be

$$\omega^{\pm}(p) = \frac{1}{2}\left[-(n-2)\xi_p \pm \sqrt{(n\xi_p)^2 + 4\Delta_n^2}\right] \tag{4}$$

Here $\xi_p = \varepsilon_p - \mu$ is linear in $(p - p_F)$, and $\varepsilon(p)$ is the 2-D dispersion relation of the electrons in one Landau subband. The gap parameter Δ_n is defined as

$$\Delta_n^2 = |g_n|^2 \cdot |F_n(0)|^2 \cdot n_0^{n-2} \tag{5}$$

where n_o is the concentration of the electrons in the last partially filled Landau level. The spectrum is presented in fig.1 , the parameters ξ^*_n - minimum position, and $E^{(n)}_G$ - gap value being the following

$$\xi^*_n = \frac{n-2}{n\sqrt{n-1}} \cdot \Delta n \tag{6}$$

$$E^{(n)}_G = \frac{n}{\sqrt{n-1}} \cdot \Delta n \tag{7}$$

The density of states N(E) has 1-D singularities situated at $E = \pm \frac{E^{(n)}_G}{2}$, resembling the experimentally observed N(E).

The solution of the gap equation is found of the type

$$\Delta_n = n \cdot \omega_o \cdot \exp\left(-\frac{1}{\frac{2}{n} \cdot |g_n| \cdot n_o^{n-2} \cdot \rho_F}\right) \tag{8}$$

where ω_o is the cut-off energy. The gap is n-dependent and it depends as well on the concentration n_o. The coherent ground state is of the type

$$|\psi_o\rangle = \prod_p \left(u_p + v_p \cdot \frac{\varepsilon_{\alpha_1 \alpha_2 \cdots \alpha_n}}{n!} \cdot a^+_{\vec{p}_1 \alpha_1} \cdot a^+_{\vec{p}_2 \alpha_2} \cdots a^+_{\vec{p}_n \alpha_n}\right)|0\rangle \tag{9}$$

where the u, v parameters are

$$\left.\begin{matrix} u^2 \\ v^2 \end{matrix}\right\} = \frac{1}{2}\left(1 \pm \frac{n\xi}{\sqrt{(n\xi)^2 + 4\Delta_n^2}}\right) \tag{10}$$

Here the product is taken over the momentums $p_1, p_2, \ldots, p_n$ of the electrons with equal energies, satisfying the condition $p_1 + p_2 + \ldots + p_n = 0$.

§3. Mean field behaviour.

Using the temperature Greenfunctions the gap temperature dependence is calculated. Close to the critical temperature for the n-phase $T_c^{(n)}$ defined as $\Delta_n(T_c^{(n)}) = 0$ one reaches the mean field dependence

$$\Delta_n(T) = n\pi T_c^{(n)} \cdot \sqrt{\frac{2}{7.(n-1).\zeta(3)}\left(1 - \frac{T}{T_c^{(n)}}\right)} \tag{11}$$

Here $T_c^{(n)}$ depends on n according to

$$T_c^{(n)} = \frac{2\sqrt{n-1}}{n\pi} \cdot \gamma \cdot \Delta_n(0), \qquad \ln\gamma = C = 0{,}577\ldots \tag{12}$$

The critical filling number ν^*_n is defined similarly by $\Delta_n(\nu^*_n) = 0$. The density of states at the Fermi level is approximated by a constant $\rho_F = N_B/E_o$, where E_o is of the order of the Landau level band width. The calculated ν^*_n and the $\Delta(\nu)$ -dependence in the vicinity of $T_c^{(n)}$ for the case of V_g control - B = const., are given by

$$\Delta_n(\nu) = n\pi T_c \sqrt{\frac{\lambda\, n(n-2)}{7\,\zeta(3)(n-1)}\left(1 - \frac{\nu^*_n}{\nu}\right)}, \qquad \lambda = \ln\frac{2\omega_o\gamma\sqrt{n-1}}{\pi T_c} \tag{13}$$

$$\nu^*_n = \nu_B \cdot \lambda^{-1/n-2}, \qquad \nu_B = \frac{hc}{eB}\left(\frac{n}{2|g|\rho_F}\right)^{\frac{1}{n-2}} \tag{14}$$

and for the case of B control one has (V_- = const.)

$$\Delta_n(\nu) = n\pi T_c \sqrt{\frac{\lambda\, n}{7\,\zeta(3)(n-1)}\left(1 - \frac{\nu}{\nu^*_n}\right)} \qquad \nu^*_n = \nu_g \cdot \lambda \tag{15}$$

$$\nu_g = \frac{2|g|\, n_o^{n-1} \cdot N}{E_o \cdot n} \tag{16}$$

Several thermodynamic quantities as a function of the temperature are also calculated. For example the deviation of the thermodynamic potential in the low temperature regime is

$$\Omega_o - \Omega_n = \rho_F\left[\frac{n}{n-1}\frac{\pi^2 T^2}{6} - \frac{\Delta^2}{n} - 2\Delta\sqrt{\frac{\pi\Delta T}{n\sqrt{n-1}}} \cdot e^{-\frac{2\sqrt{n-1}\,\Delta}{nT}} \cdot \left(1 + \frac{15\, nT}{16\sqrt{n-1}\,\Delta}\right)\right] \tag{17}$$

It is seen that for T=0 the thermodynamic potential has a negative term proportional to Δ^2. Thus, the coherent state energy is lower than the energy of the normal one.

In the case of inhomogeneous 2-D system the order parameter is no longer constant. Its position dependence is determined by the Ginzburg-Landau equation

$$\left[\frac{1}{2m^*}\left(-i\hbar\nabla+\frac{e^*}{\hbar c}\vec{A}\right)^2+\alpha_0\left(1-\frac{T}{T_c}-|\Psi|^2\right)\right]\cdot\Psi=0 \qquad (18)$$

where

$$\alpha_0=\frac{2n\,(\pi T_c)^2}{7\zeta(3)(n-1)^2\,\varepsilon_F} \qquad (19)$$

The effective mass is m^*=n.m , and the effective charge is e^*=n.e . The correlation length ξ is related to α_0 by

$$\xi^2=\xi_0^2\Big/\left(1-\frac{T}{T_c^{(n)}}\right), \qquad (20)$$

and

$$\xi_0^2=\frac{\hbar^2}{2m^*\alpha_0}=7\zeta(3)\left[\frac{(n-1)\hbar v_F}{2n\pi T_c}\right]^2 \qquad (21)$$

The current density is found in the form

$$\vec{j}(\vec{r})=\frac{e^*}{2m}\left[\Delta^*(\vec{r})\left(-i\hbar\nabla+\frac{e^*}{c}\vec{A}\right)\Delta(\vec{r})+c.c.\right]. \qquad (22)$$

Integrating this equation along a closed contour fractional flux quantization is reached

$$\Phi=k\Phi_0\,, \qquad \Phi_0=\frac{hc}{ne} \qquad (23)$$

where k is an integer.

§4. Conductivity quantization.

The calculation of the Hall conductivity can be done from first principles. In a coherent state the gap $\Delta\neq 0$ and at T=0 the longitudinal conductivity $\sigma_{xx}=0$. The nondissipative Hall current is calculated in two steps:

(1) In the reference frame where the electric field is $\mathcal{E}=0$ the Hall current is also zero. With this condition the phase of the wave function ψ is fixed.

(2) The transition to the actual reference frame ($\mathcal{E}\neq 0$) according to the Galilean transformations is

$$\psi_{\mathcal{E}}=\psi_{\mathcal{E}=0}\cdot e^{\frac{im^*\vec{v}}{\hbar}(\vec{r}+\vec{v}t)}\,, \qquad |\vec{v}|=c\,\frac{\mathcal{E}}{B}\,. \qquad (24)$$

Substituting this function in the current density relation (22) one reaches the remarkable expression

$$\vec{j}=e^*|\psi_{\mathcal{E}=0}|^2\cdot c\cdot\frac{\vec{\mathcal{E}}}{B}=\frac{e^2}{h}\,\nu_c\cdot\vec{\mathcal{E}} \qquad (25)$$

This relation determines ν_c not only for T=0, $\nu=1/n$, but also for T≠0. Specifically for low enough temperatures and exact 1/n quantization ($\nu_c=1/n$ when T=0) one has

$$\sigma_{xy}=\frac{e}{h}\cdot\frac{1}{n}\left(1-2\left(\frac{\pi T}{n\sqrt{n-1}\,\Delta_n}\right)^{1/2}\cdot e^{-\frac{2\sqrt{n-1}\,\Delta_n}{nT}}\cdot\left(1-\frac{n.T}{16\sqrt{n-1}\,\Delta_n}\right)\right)\cdot \qquad (26)$$

This expression accounts for the accuracy of the fractional quantization at very low temperatures.

Another type of temperature dependence can be found close to $T_c^{(n)}$. In this situation Δ^2 is proportional to $T_c^{(n)}-T$ and the conductivity vanishes linearly in $T_c^{(n)}-T$

$$\sigma_{xy}\propto\left(T_c^{(n)}-T\right). \qquad (27)$$

The gap Δ is clearly related to the plateau width and the plateau length temperature dependence is in accordance with the mean field predictions. The calculation of σ_{xx} will be given elsewhere.

§5. Conclusions.

In the present work the (2n+1)-particles wave function was introduced as an order parameter for describing the (2n+1) fractional quantum state of the 2-D electronic liquid. The excitation spectrum was found for an arbitrary n to have a gap in the case of effective attraction. The temperature dependence of the gap and the thermodynamic quantities were found as well as the critical fillings. The Hall conductivity was calculated from first principles and the accuracy of the fractional quantization was evaluated.

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FIGURE CAPTION

Fig.1 - Gap and excitations spectrum close to m/n falling with odd n.

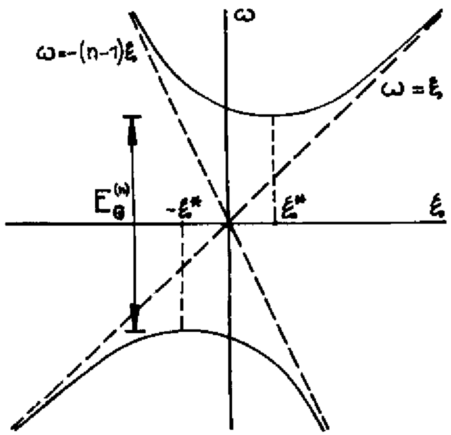

fig.1.