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THERMAL STABILITITY OF A THERMONUCLEAR PLASMA FOR DIFFERENT CONFINEMENT SCALING LAWS

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ABSTRACT

The thermal stability of the ignition curve is investigated using a simple 0D model for a temperature dependent energy confinement time ($\tau_E \propto 1/T^\gamma$).

The stability limit in the ($n\tau_E$,T) plane is also calculated for a plasma with external heating.

The degradation of confinement time with increasing temperature is found to be favorable for divergence temperature and minimum temperature for stable ignition. It also decreases the external power per unit volume necessary to reach divergence. On the contrary, it is extremely unfavorable for the required $n\tau_E$ for divergence and ignition.

Detailed results are given for the special case of the Kaye-Goldston scaling (γ = 1.38).

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INTRODUCTION

• We consider a deuterium-tritium thermonuclear plasma within the approximation of a simple zero-dimension (0D) model with no profile corrections. The following simplifying assumptions are also made:

$$n_D = n_T = \frac{n}{2} \tag{1}$$

$$T_e = T_i = T \tag{2}$$

where n is the electron density (the density of the ^4He ashes is supposed to be negligible).

We only take into account the D-T nuclear reaction

$$D + T \longrightarrow {}^4He(3.561\ MeV) + n(14.03\ MeV) \qquad [17.59\ MeV]\ , \tag{3}$$

where the total energy release has been calculated using precise mass default values and the repartition between the reaction products takes into account relativistic corrections.

• In section I of the paper, We first recall the equation of the ignition curve in the $(n\tau_E,T)$ plane. The problem of thermal stability of the ignited equilibrium is then addressed for a temperature dependent total energy confinement time of the form

$$\tau_E \propto \frac{1}{T^\gamma}\ . \tag{4}$$

As a representative value for γ, we will consider the one deduced from the Kaye-Golston scaling for neutral-beam-heated Tokamaks [1]

$$\tau_E \propto \frac{1}{P_{tot}^{0.58}}\ , \tag{5}$$

where P_{tot} is the total power coupled to the plasma ($P_{Joule} + P_{add}$). Using

$$P_{tot} = P_{loss}\ , \tag{6}$$

where P_{loss} is the total power lost by the plasma, and the definition of the total energy confinement time

$$P_{loss} = \frac{W_{th}}{\tau_E}\ , \tag{7}$$

where W_{th} is the total stored thermal energy

$$W_{th} = 3\, nkTV \,, \qquad (8)$$

(V is the plasma volume, k the Boltzmann constant), we get

$$\tau_E \propto \frac{1}{T^{1.38}} \,. \qquad (9)$$

We will suppose that such a dependence remains valid when P_{tot} includes fusion power coupled to the plasma.

• In section II, the curve limiting stable operation in the $(n\tau_E, T)$ plane for an externally heated plasma is calculated within the hypothesis where the power coupled to the plasma is independent of temperature. Minimum $n\tau_E$ and T values necessary for a thermal divergence leading to stable ignition operation are also plotted as well as the corresponding coupled external power.

• As a practical application, explicit expressions are given in section III for the reactor radius and external power needed for minimum divergence in the case of Kaye-Goldston scaling.

• Unless explicitly specified, units will be MKSA except temperature which is always expressed in keV (this means that we take $k = 1.6022 \times 10^{-16}$ J/keV).

• All the numerical results are given with three decimal digits to allow accurate cross-checking within the model. This precision reflects in no case the accuracy of our (very crude) model.

I. THERMAL STABILITY OF THE IGNITION OPERATION

1. The ignition curve

The ignition regime is reached when the plasma is self-sustained, i.e when the power given to the plasma by alpha particles exactly balances[1] the thermal losses. The ignition equilibrium then writes

$$P_{source} = P_{loss} , \tag{10}$$

where

$$P_{source} = P_{fus} = \frac{n^2}{4} \overline{\sigma v}(T) f_\alpha E_\alpha V \tag{11}$$

is the fusion power coupled to the plasma by the fraction f_α of trapped alpha particules (E_α = 3.561 MeV). $\overline{\sigma v}(T)$ denotes the average value of σv for two maxwellian D and T distributions with equal temperatures, and P_{loss} is given by (7,8).

The equation of the $(n\tau_E, T)$ ignition curve is then

$$(n\tau_E)_{ig} = \frac{12k}{f_\alpha E_\alpha} \frac{T}{\overline{\sigma v}(T)} . \tag{12}$$

It is represented in fig.2 for $f_\alpha = 1$ using the Brunelli expression for $\overline{\sigma v}$ [2] which will be used throughout this paper:

$$\overline{\sigma v} = \overline{\sigma v}_M \exp\left[-\alpha \left| \ln\frac{T}{T_M} \right|^{9/4}\right] \qquad 2 \text{ keV} < T < 150 \text{ keV} , \tag{13}$$

where

$$\overline{\sigma v}_M = 9 \times 10^{-22} \text{ m}^3/\text{sec}, \; T_M = 69 \text{ keV and } \alpha = \frac{1.35}{(\ln 10)^{5/4}} \simeq 0.476 .$$

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[1] It is shown in the Appendix that the Joule power given to the plasma by the confining plasma current is generally negligible at ignition.

2. Thermal stability of the ignition operation

The thermal stability condition of this 0D equilibrium simply writes

$$\frac{dP_{source}}{dT} < \frac{dP_{loss}}{dT} . \tag{14}$$

With the monomial law (4) for the decrease of energy confinement time versus temperature, condition (14) writes

$$n\tau_E < (n\tau_E)_{st} \qquad \text{for} \quad \frac{d\overline{\sigma v}}{dT} > 0 \quad (T < 69 \text{ keV}) , \tag{15}$$

$$n\tau_E > (n\tau_E)_{st} \qquad \text{for} \quad \frac{d\overline{\sigma v}}{dT} < 0 \quad (T > 69 \text{ keV}) , \tag{16}$$

where

$$(n\tau_E)_{st} = \frac{12k}{f_\alpha E_\alpha} \frac{1+\gamma}{d\overline{\sigma v}/dT} . \tag{17}$$

Remarks

- For $\gamma = 0$ (τ_E independant of T), it is easy to show that the curve (17) intersects the ignition curve (12) at its minimum (25.8 keV). In this case, only the part of the ignition curve with positive derivative (called increasing part) corresponds to stable equilibria.

- For $\gamma > 0$ (τ_E decreasing with T), a part of the decreasing branch of the ignition curve becomes thermally stable.

- For $\gamma < 0$ (τ_E increasing with T), a part of the increasing branch of the ignition curve becomes thermally unstable.

The equation giving the temperature limit T_{igst} for stable ignition is obtained from (12) and (17). It writes

$$T_{igst} \frac{d\overline{\sigma v}}{dT}(T_{igst}) - (1+\gamma)\overline{\sigma v}(T_{igst}) = 0 . \tag{18}$$

Using the expression (13) for $\overline{\sigma v}$, this equation can be solved exactly, giving

$$T_{igst}(\gamma) = T_M \exp\left[-\text{sign}(1+\gamma)\left[\frac{4|1+\gamma|}{9\alpha}\right]^{4/5}\right] . \tag{19}$$

Note that this temperature is a universal function of γ, it just depends of the $\overline{\sigma v}$ function. It is represented in fig.3.

The corresponding $n\tau_E$ of the ignition curve can be deduced from (12):

$$(n\tau_E)_{igst}(\gamma) = \frac{12k}{f_\alpha E_\alpha} \frac{T_M}{\overline{\sigma v}_M} \exp\left[\left[\frac{4|1+\gamma|}{9\alpha}\right]^{4/5}\left[\frac{4|1+\gamma|}{9} - \mathrm{sign}(1+\gamma)\right]\right] . \qquad (20)$$

It is represented in fig.4 for $f_\alpha = 1$.

For $\gamma=0$, $T_{igst} = 26.8$ keV, $(n\tau_E)_{igst} = 1.53 \times 10^{14}$ cm^{-3} x sec , (21)

for $\gamma=1.38$, $T_{igst} = 10.4$ keV, $(n\tau_E)_{igst} = 2.88 \times 10^{14}$ cm^{-3} x sec . (22)

The degradation of τ_E with temperature (with $\gamma = 1.38$) permits a stable ignition regime at a minimum temperature of 10.4 keV instead of 26.8 keV (for $\gamma = 0$), but the required $n\tau_E$ for this operation almost doubles.

The dotted parts of the T_{igst} and $(n\tau_E)_{igst}$ curves correspond to points situated in the zone of the $(n\tau_E, T)$ plane forbidden by bremsstrahlung (see II.3) ($T = 4.28$ keV, $n\tau_E = 1.86 \times 10^{15}$ cm^{-3} x sec obtained for $\gamma = 2.85$).

II. THERMAL STABILITY LIMIT IN THE ($n\tau_E$,T) PLANE FOR AN EXTERNALLY HEATED PLASMA

1. The stability limit

In the presence of an externally supplied power P_{ext} effectively coupled to the plasma, the thermal equilibrium condition writes

$$\frac{n^2}{4}\,\overline{\sigma v}(T) f_\alpha E_\alpha V + P_{ext} = \frac{3nkT}{\tau_E} V \qquad (23)$$

where

$$\tau_E \propto \frac{1}{T^\gamma} \, . \qquad (24)$$

Supposing that the external power supply is independent of the plasma temperature, the stability condition of this equilibrium (14) reduces to condition (15-17). Now the curve (17) represents the stability limit in the ($n\tau_E$,T) plane for stable operation at a given γ. This curve must be stopped at its intersection with the ignition curve above which no operation is allowed.

Validity of the model

- P_{ext} represents ohmic plus additionnal heating

$$P_{ext} = P_{Joule} + P_{add} \, . \qquad (25)$$

P_{add} can be considered independent of T, but P_{Joule} is a function of temperature for fixed current and magnetic field. Considering here only regimes where

$$P_{add} >> P_{Joule} \, , \qquad (26)$$

we have neglected the P_{Joule} temperature dependence.

- For the small $n\tau_E$ for which present scaling laws have been derived, the bremsstrahlung[1] power loss P_B is negligible compared to the conduction-convection power loss, so that the total energy confinement time τ_E is identical to the non radiative energy confinement time τ_E^*. Netherthe-less, if extrapolable, the confinement time scaling clearly concerns the non

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[1] The cyclotron radiation losses are considered negligible for the relatively low temperatures that we discuss in this paper.

radiative energy confinement time. Then, the right member of (23) should be replaced by

$$P_{loss} = P_B + \frac{3nkT}{\tau_E^*} V \; , \qquad (27)$$

where

$$P_B = C_B \, n^2 \, T^{1/2} \, V \qquad (28)$$

and τ_E^* is given by (24). The correct temperature dependence of P_B should then be considered. So, for our approximation to be valid, we must check that the $(n\tau_E, T)$ points we consider correspond to small values of the ratio

$$\frac{P_B}{P_{loss}} = \frac{C_B}{3k} \frac{n\tau_E}{T^{1/2}} = 0.111 \, \frac{n_{14}\tau_E}{T^{1/2}} \; , \qquad (29)$$

where the Born approximation [4] for C_B has been taken ($C_B = 0.5355 \times 10^{-36}$ MKSA + keV) and n_{14} is the density expressed in 10^{14} cm^{-3} unit.

2. Graphical illustration

To get a qualitative idea of this thermal stability concept, let us consider the case $\gamma = 0$ (τ_E independent of T). Equation (23) can now be written

$$\overline{\sigma v}(T) = B \, \frac{T}{n\tau_E} - C \, P_{ext} \; , \qquad (30)$$

where

$$B = \frac{12k}{f_\alpha E_\alpha} \quad \text{and} \quad C = \frac{4}{f_\alpha E_\alpha n^2 V} \qquad (31)$$

are constants for our problem.

Fig.1 shows the curve $\overline{\sigma v}(T)$ and the straight line representing the second member of equation (30). The slope of this line is inversely proportional to $n\tau_E$ and the distance between the origin and its intersection with T=0 is proportional to P_{ext}. For a given $n\tau_E$, the straight line translates upwards when P_{ext} is increased, its intersection with the curve $\overline{\sigma v}(T)$ giving the operation temperature. The first operating points are stable because the slope of the source ($\overline{\sigma v}$) is lower than the slope of the loss (the straight line). If $n\tau_E$ is sufficiently large, the solution disappears for the temperature T_d, the decreasing of P_{ext} giving unstable equilibria. In fact, if the injected power stays constant as well as τ_E, the system spontaneously evolves towards the stable operating point T_f because of alpha particle heating. The power can then be switched off, the system goes to the stable ignition point T_{ig} of the stable portion of the ignition curve corresponding to the given $n\tau_E$.

3. The thermal stability curve

Fig.2 represents the thermal stability curves for $\gamma = 0$ and $\gamma = 1.36$. These curves intersect the ignition curve at the minimum temperature stable ignition points, given by (19,20) for the $\overline{\sigma v}$ of eq. 13.

Also represented is the trajectory of a given plasma when the temperature (or external power) is increased. These trajectories ($n\tau_E = 1/T^{\gamma}$) appear as straight lines on the log-log plot. It is easy to show that the particular trajectory that intersect the ignition curve at the minimum stable point is tangential to the ignition curve. The intersection of this trajectory with the stability limit curve (17) will be called the minimum divergence point. It represents the minimum $n\tau_E$ [denoted $(n\tau_E)_{dm}$] giving a thermal divergence leading to the minimum stable ignition point, and the corresponding temperature T_{dm}.

The upper straight line in fig.2 represents the $(n\tau_E, T)$ points for which the power loss is completely dominated by bremsstrahlung ($P_{loss} = P_B$). The equation of this curve writes

$$n\tau_E = \frac{3k}{C_B} T^{1/2} \quad . \tag{32}$$

The region above this curve is forbidden, its intersection with the ignition curve gives the minimum possible ignition temperature (4.28 keV).

4. Temperature and external power requirements at minimum divergence

- The equation to be solved to get T_{dm} writes

$$\frac{1}{T_{dm}^{\gamma}} \frac{d\overline{\sigma v}}{dT}(T_{dm}) - (1+\gamma) \frac{1}{T_{igst}^{1+\gamma}} \overline{\sigma v}(T_{igst}) = 0 \quad . \tag{33}$$

For $\gamma > -1$, the equation writes explicitely using (13) and (19)

$$\frac{9\alpha}{4}\,\frac{(\ln\frac{1}{x})^{5/4}}{x^{1+\gamma}}\,\exp\left[-\alpha(\ln\frac{1}{x})^{9/4}\right] - (1+\gamma)\,\exp\left[\frac{5}{9}(\frac{4}{9\alpha})^{4/5}(1+\gamma)^{9/5}\right] = 0 \;, \qquad (34)$$

where $x = \dfrac{T_{dm}}{T_M}$.

Note that T_{dm} is also a universal function of γ.

T_{dm} is represented in fig.3, $(n\tau_E)_{dm}$ in fig.4 for $f_\alpha = 1$.

For $\gamma = 0$, T_{dm} = 8.14 keV, $(n\tau_E)_{dm} = 1.53 \times 10^{14}$ cm^{-3} x sec ,

for $\gamma = 1.38$, T_{dm} = 5.17 keV, $(n\tau_E)_{dm} = 7.54 \times 10^{14}$ cm^{-3} x sec .

We see that the degradation of the energy confinement time lowers the temperature of thermal divergence leading to stable ignition, but considerably increases the required $n\tau_E$.

The dotted parts on the large γ side of the T_{dm} and $(n\tau_E)_{dm}$ curves correspond to points situated in the part of the $(n\tau_E, T)$ plane forbidden by bremsstrahlung (T = 3.94 keV, $n\tau_E = 1.78 \times 10^{15}$ cm^{-3} x sec obtained for γ = 2.00). The dotted part on the small γ side correspond to poor precision in the Brunelli approximation of $\overline{\sigma v}$ (T < 2 keV).

• The external power coupled to the plasma at the minimum ignition point is deduced from (23), it writes

$$\frac{(P_{ext})_{dm}}{V} = n^2 f_\alpha E_\alpha\,\frac{1}{4}\left[(\frac{T_{dm}}{T_{igst}})^{1+\gamma}\;\overline{\sigma v}(T_{igst}) - \overline{\sigma v}(T_{dm})\right] . \qquad (35)$$

A practical formula is obtained when expressing n in units of 10^{14} cm^{-3} (n_{14}):

$$\frac{(P_{ext})_{dm}}{V} = f_\alpha n_{14}^2\,P(\gamma) \;, \qquad (36)$$

where $P(\gamma)$ is a universal function of γ. It is represented in fig.5 for the Brunelli $\overline{\sigma v}$. It is a rapidly decreasing function of γ.

For $\gamma = 0$, $P = 1.64 \times 10^{-1}$ MW/m^3,

for $\gamma = 1.38$, $P = 1.07 \times 10^{-2}$ MW/m^3.

• <u>The limit $\gamma = -1$</u>

- When $\gamma \rightarrow -1$, the whole part of the $(n\tau_E,T)$ plane corresponding to $T < 69$ keV becomes thermally unstable. We have the relations

$$T_{dm} \rightarrow 0 \ , \ (n\tau_E)_{dm} \rightarrow 0 \quad \text{when } \gamma \rightarrow -1 \ . \tag{37}$$

In the case of Brunelli $\overline{\sigma v}$, it can be shown that

$$\lim_{\gamma \rightarrow -1} \frac{(P_{ext})_{dm}}{V} = \frac{1}{4} n^2 f_\alpha E_\alpha \overline{\sigma v}_M \tag{38}$$

corresponding to $P(-1) = 1.28$ MW/m^3 as is illustrated in fig 5. Nevertheless, this result is not general since the T dependence of the Brunelli $\overline{\sigma v}(T)$ when $T \rightarrow 0$ is not correct.

The part of the plane corresponding to $T > 69$ keV is thermally stable.

- For $\gamma < -1$, a thermally stable zone does exist for $T > 69$ keV corresponding to the points located above $(n\tau_E)_{st}$ which is now positive (eq. 16). Any equilibrium with $T < 69$ keV is thermally unstable as can be seen directly from equation (23) which shows that P_{source} is an increasing function of temperature whereas P_{loss} is a decreasing one.

III. REACTOR PARAMETERS FOR THE KAYE-GOLDSTON SCALING

1. Dimension and external power at minimum divergence

• With MKSA + keV units, the Kaye-Golston scaling for a circular Tokamak with flat profiles (our model) writes

$$\tau_E = 3.82 \times 10^{-11} \frac{I_p^{1.24}\, n^{0.26}\, A^{1.65}\, a^{1.16}}{B_T^{0.09}\, P_{tot}^{0.58}} , \quad (39)$$

where $A = R/a$ is the aspect ratio. Expressing I_p in terms of B_T, q_a (the edge safety factor) and a by the relation

$$I_p = \frac{2\pi}{\mu_0} \frac{B_T}{Aq_a} a , \quad (40)$$

and P_{tot} in terms of τ_E, n, T by relations (6,7,8), we get the equivalent scaling

$$n\tau_E = 2.18 \times 10^{14} \frac{B_T^{2.74}\, n^{0.238}\, a^{1.57}}{q_a^{2.95}\, A^{0.405}} \frac{1}{T^{1.38}} . \quad (41)$$

• Using the relation giving the required $n\tau_E$ for minimum divergence:

$$(n\tau_E)_{dm} = \frac{12k}{f_\alpha E_\alpha} \frac{1+\gamma}{\frac{d\overline{\sigma v}}{dT}(T_{dm})} , \quad (42)$$

and using $T_{dm} = 5.17$ keV for $\gamma = 1.38$, we get

$$(n\tau_E)_{dm} = \frac{7.54 \times 10^{20}}{f_\alpha} . \quad (43)$$

• Combining equation (41) for $T = T_{dm}$ and (43), we get the following condition for minimum divergence:

$$n^{0.238}\, a^{1.57} = 3.34 \times 10^{7} \frac{q_a^{2.95}\, A^{0.405}}{f_\alpha\, B_T^{2.74}} . \quad (44)$$

• The external power that must be coupled at minimum divergence writes from (35) with $\gamma = 1.38$

$$(P_{ext})_{dm} = 2.11 \times 10^{-35} n^2 f_\alpha A a^3 . \quad (45)$$

• Solving (44, 45) for a and $(P_{ext})_{dm}$, we get the practical formulas for reactor dimension and external power at minimum divergence:

$$a = 57.2 \frac{q_a^{1.66} A^{0.258}}{n_{14}^{0.152} f_\alpha^{0.537} B_T^{1.74}} , \quad (46)$$

$$(P_{ext})_{dm} = 3.95 \times 10^{10} \frac{n_{14}^{1.545} q_a^{5.64} A^{1.77}}{f_\alpha^{0.909} B_T^{5.23}} . \quad (47)$$

2. An example

• For the numerical application, we suppose that the maximum density allowed by the Murakami condition is reached. Taking the Hugill [3] expression for the Murakami limit:

$$n_{14} = \frac{2B_T}{q_a R} , \quad (48)$$

we obtain the following expressions:

$$a = 104 \frac{q_a^{2.39} A^{0.462}}{f_\alpha^{0.75} B_T^{2.23}} , \quad (49)$$

$$(P_{ext})_{dm} = 8.78 \times 10^7 \frac{f_\alpha^{0.25} q_a^{0.393}}{B_T^{0.232} A^{0.515}} . \quad (50)$$

We see that low values of q_a, small A (fat torus) and strong B_T are favorable for reactor dimension.

• As an illustrative exemple, we choose the very favorable parameters:

$$f_\alpha = 1, \ q_a = 2, \ A = 3, \ B_T = 10T .$$

We find:

$$a = 5.44 \text{ m giving } R = 16.3 \text{ m and } V = 9550 \text{ m}^3 ,$$

$$(P_{ext})_{dm} = 39.2 \text{ MW} ,$$

$$n = 0.612 \times 10^{14} \text{ cm}^{-3} .$$

We have also

$$(I_p)_{dm} = 45.4 \text{ MA} ,$$

$$(\tau_E)_{dm} = 12.3 \text{ sec} .$$

Recall that the temperature is $T_{dm} = 5.17$ keV.

• We can also compute the fusion power coupled to the plasma at this point. Using (11), we get

$$(P_{fus})_{dm} = 79.9 \text{ MW} .$$

The total power lost by the plasma at the minimum divergence point is then

$$(P_{tot})_{dm} = (P_{fus})_{dm} + (P_{ext})_{dm} = 118 \text{ MW} .$$

At this point of the $(n\tau_E, T)$ plane, the ratio (29) of bremsstrahlung to total power loss is found to be 16%. This is compatible with the approximation of our model (eq. 23).

• The fraction of ohmic power can also be computed. Supposing that the current is purely inductive and taking the simplified Spitzer resistivity

$$\eta = \frac{\eta_0}{T^{3/2}} \qquad \text{with } \eta_0 = 3 \times 10^{-8} \text{ (MKSA + keV)} , \qquad (51)$$

we find

$$(P_{Joule})_{dm} = 5.78 \text{ MW} .$$

The additionnal power needeed at minimum divergence is then

$$(P_{add})_{dm} = (P_{ext})_{dm} - (P_{Joule})_{dm} = 32.4 \text{ MW} .$$

P_{Joule} represents 15% of the total external power. This is compatible with the use of the Kaye-Goldston scaling law and of the constant P_{ext} approximation of our stability model (eq. 23).

• If we suppose that the plasma dimensions, density, current and magnetic field are kept constant (41), the thermal divergence makes the plasma evolve toward a stable ignition regime characterized by (22 and fig.2)

$$T_{igst} = 10.4 \text{ keV} , \quad (n\tau_E)_{igst} = 2.88 \times 10^{14} \text{ cm}^{-3} \times \text{sec} .$$

For our numerical example, we get the following parameters at ignition:

$$(\tau_E)_{igst} = 4.71 \text{ sec} ,$$

$$\beta = \frac{2nkT_{igst}}{B_T^2/2\mu_0} = 0.511 \% .$$

This low β value is due to the reduction of T_{igst} [due to the strong $\tau_E(T)$ degradation], to the relatively low value of the maximum density for this large R machine (48) and to the strong field necessary for divergence.

The total fusion power of the machine can also be computed at ignition:

$$(P_{fus-tot})_{igst} = \frac{n^2}{4} \overline{\sigma v}(T_{igst})(E_\alpha+E_n) \ V , \qquad (52)$$

where $E_\alpha+E_n = 17.59$ MeV. We get

$$(P_{fus-tot})_{igst} = 3060 \text{ MW} ,$$

giving a total power flow per unit surface of 0.87 MW/m^2 and a total power yield per unit volume of 0.32 MW/m^3.

IV. CONCLUSION

The thermal stability of a thermonuclear plasma has been considered using a simple 0D model with a monomial law for total energy confinement time versus temperature. The application to the case of Kaye-Goldston scaling with Murakami limitation on density has been made. It shows that, to reach thermal divergence leading to minimum stable ignition with realistic reactor dimensions, very strong toroidal fields must be used.

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APPENDIX

Ohmic power at ignition

We only consider the case where the current is maintained inductively. In this case, the Joule power density coupled to the plasma at ignition writes

$$\frac{P_{Joule}}{V} = \eta j^2. \tag{A.1}$$

Taking the simplified Spitzer resistivity (31) and expressing I_p in terms of B_T, A and q_a by (40), we get

$$P_{Joule} = \frac{4\eta_0}{\mu_0^2} \left(\frac{B_T}{q_a R}\right)^2 \frac{1}{T_{ig}^{3/2}} . \tag{A.2}$$

Since the q_a value must be kept finite in order to maintain equilibrium, the Joule power at ignition cannot be zero.

Using (11) for P_{fus}, we get

$$\left(\frac{P_{Joule}}{P_{fus}}\right)_{ig} = \frac{16\eta_0}{\mu_0^2} \frac{1}{f_\alpha E_\alpha} {}^{3}\left(\frac{B_T}{n q_a R}\right)^2 \frac{1}{T_{ig}^{3/2}\, \overline{\sigma v}(T_{ig})} . \tag{A.3}$$

If we suppose that the density at ignition is given by the Murakami limit (48), this ratio becomes a quasi-universal function of T_{ig} which writes

$$\left(\frac{P_{Joule}}{P_{fus}}\right)_{ig} = 1.33 \times 10^{-23} \frac{1}{f_\alpha} \frac{1}{T_{ig}^{3/2}\, \overline{\sigma v}(T_{ig})} . \tag{A.4}$$

This ratio is represented in fig.6 for the Brunelli $\overline{\sigma v}$ and $f_\alpha = 1$. For the minimum possible ignition temperature (T_{ig} = 4.28 keV), the ratio is of the order of 19% and rapidly decreases to negligible values for realistic ignition temperatures.

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FIGURE CAPTIONS

Fig.1 : Thermal stability illustration for a plasma with constant τ_E.

Fig.2 : Ignition curve, bremsstrahlung limit, thermal stability limit curves and plasma trajectories for $\gamma = 0$ and $\gamma = 1.38$ ($f_\alpha = 1$).

Fig.3 : Minimum divergence temperature and corresponding stable ignition temperature as a function of γ.

Fig.4 : Minimun divergence and corresponding stable ignition $n\tau_E$ as a function of γ ($f_\alpha = 1$).

Fig.5 : Coupled external power density necessary for minimum divergence as a function of γ for $n = 10^{14}$ cm^{-3} and $f_\alpha = 1$.

Fig.6 : Ratio of Joule to fusion power at ignition as a function of T_{ig}, for Murakami density and $f_\alpha = 1$.

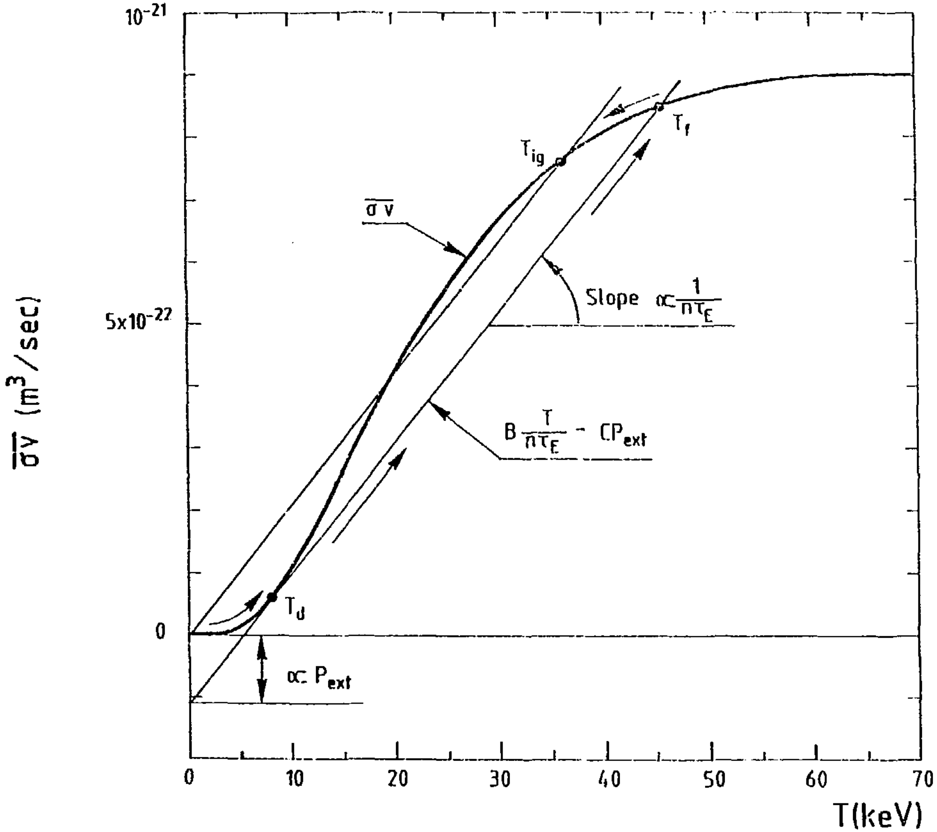

Fig. 1: THERMAL STABILITY ILLUSTRATION FOR A PLASMA WITH CONSTANT τ_E.

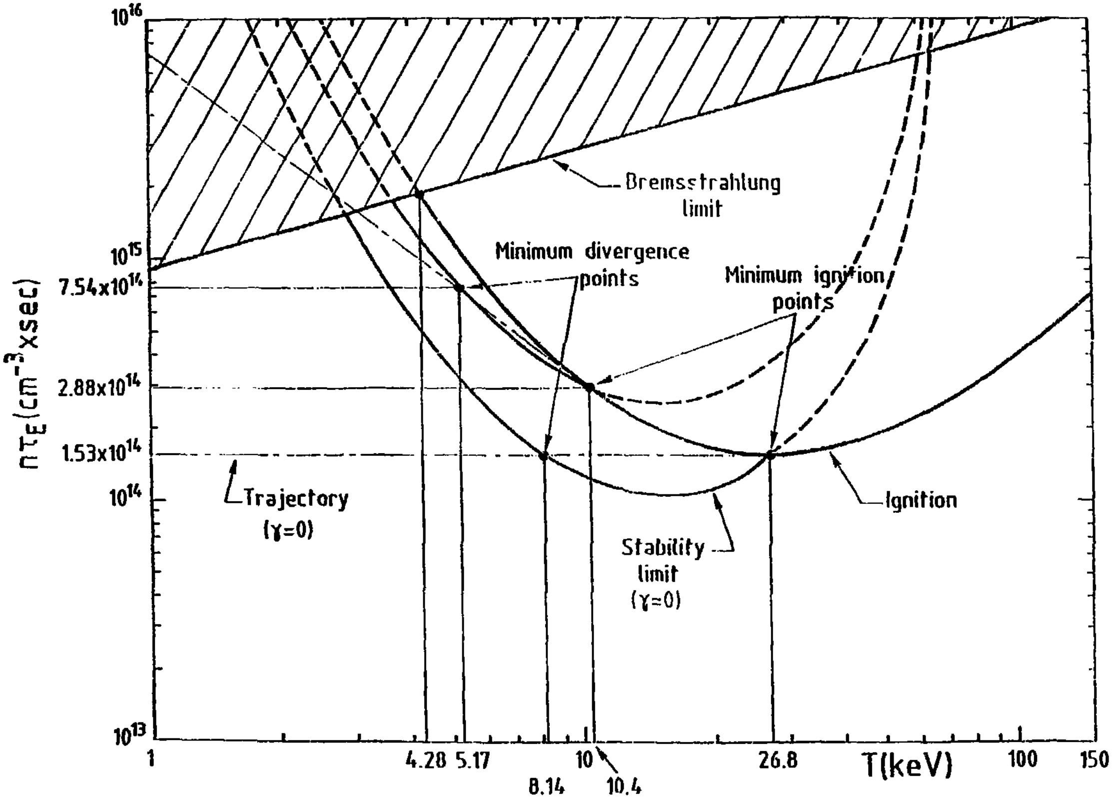

Fig. 2: IGNITION CURVE, BREMSSTRAHLUNG LIMIT, THERMAL STABILITY LIMIT CURVE AND PLASMA TRAJECTORIES FOR γ=0 AND γ=1.38 (f_α=1).

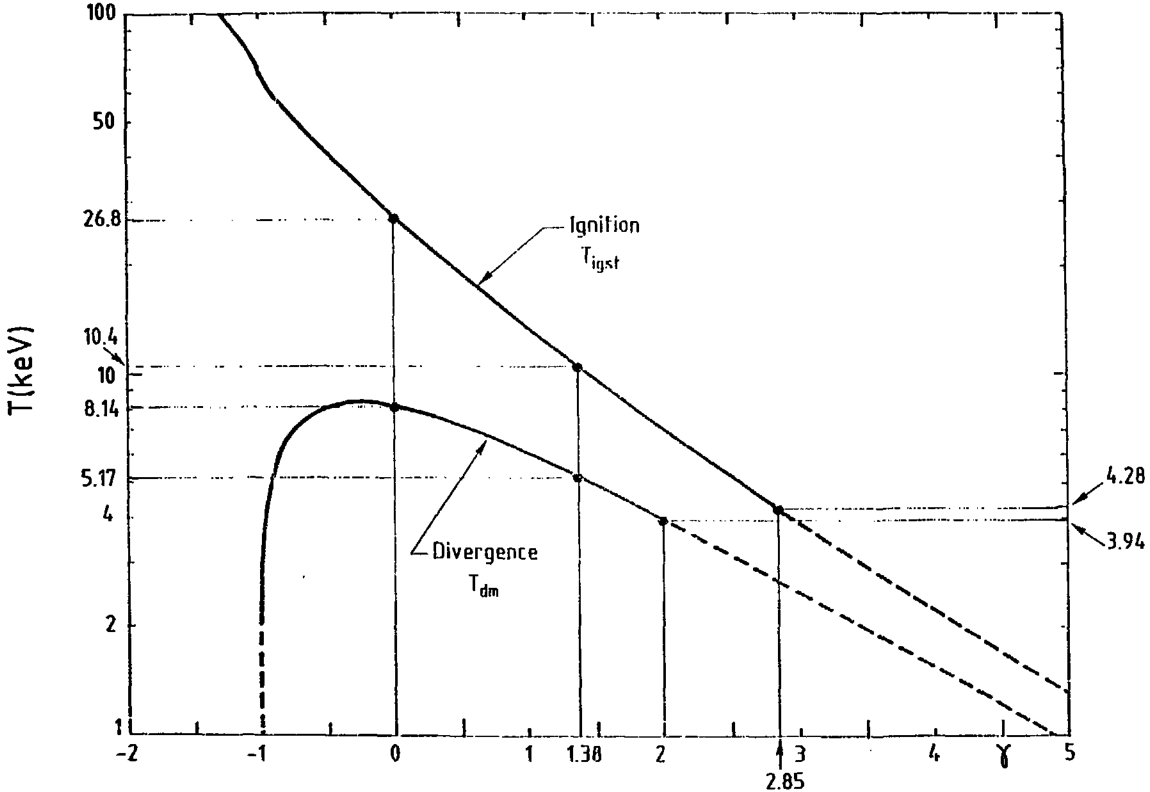

Fig. 3: MINIMUM DIVERGENCE TEMPERATURE AND CORRESPONDING STABLE IGNITION TEMPERATURE AS A FUNCTION OF γ.

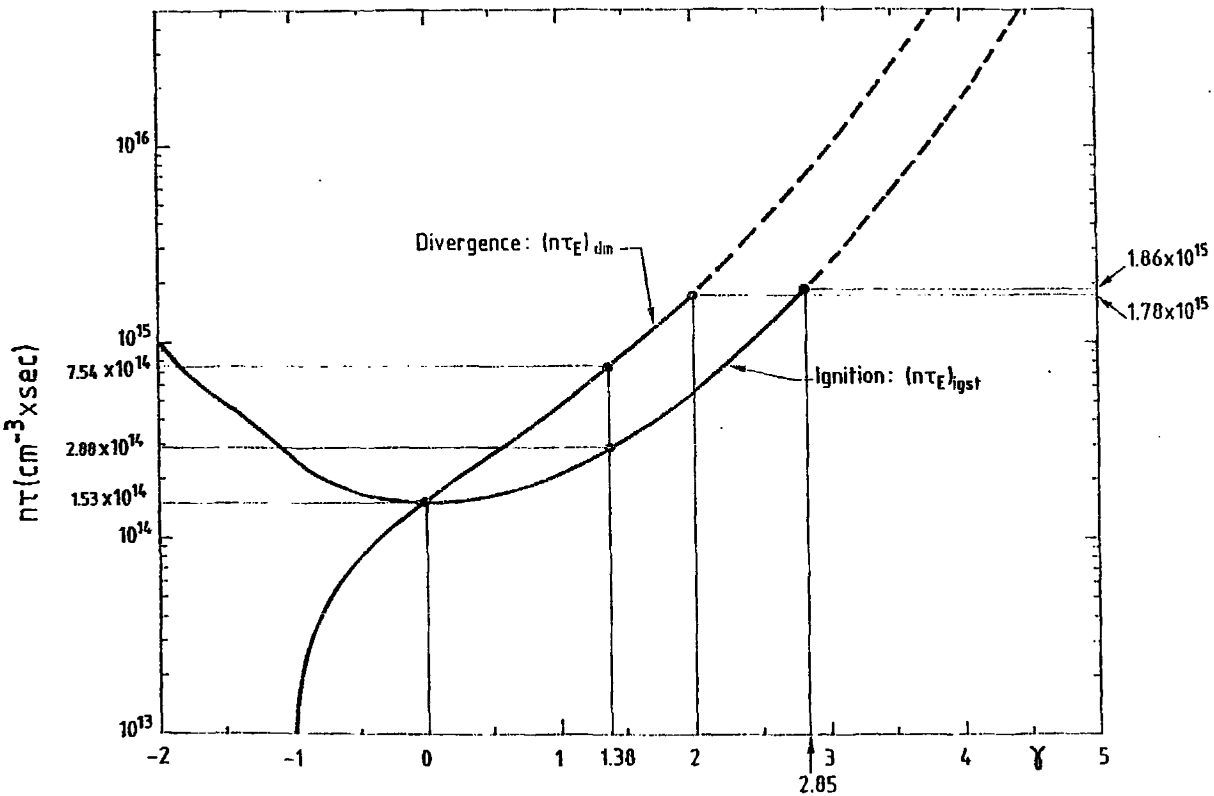

Fig. 4: MINIMUM DIVERGENCE AND CORRESPONDING STABLE IGNITION $n\tau_E$ AS A FUNCTION OF γ (=1).

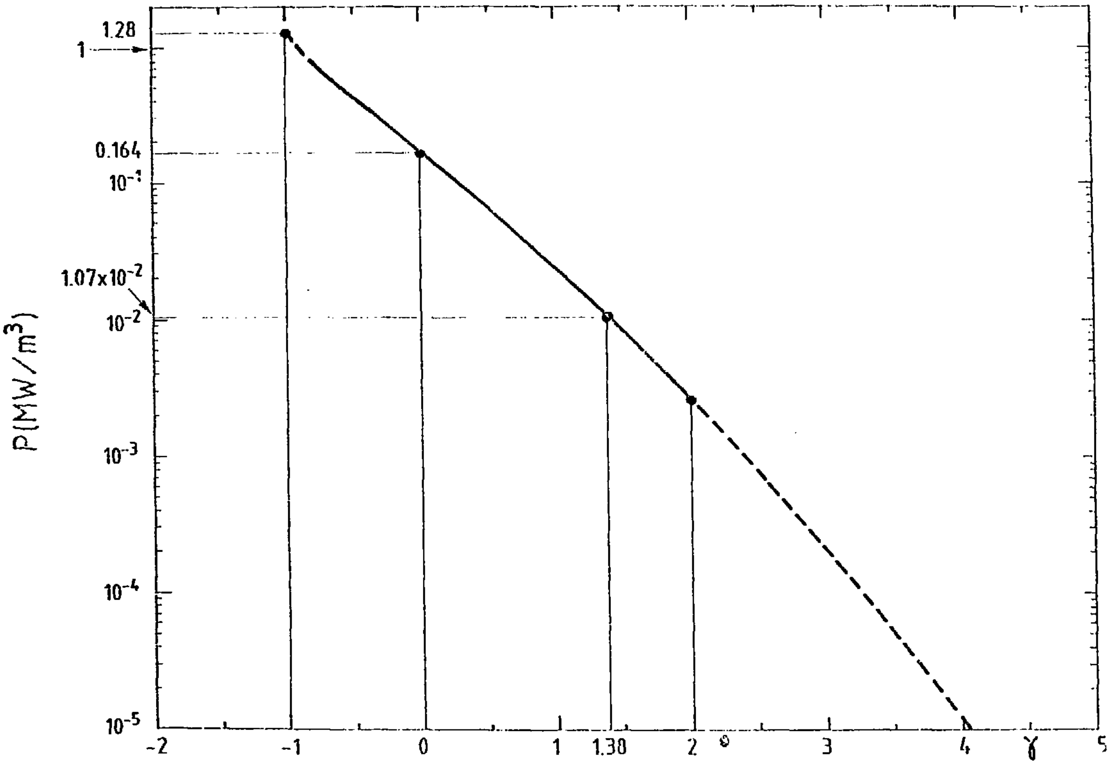

Fig. 5: COUPLED EXTERNAL POWER DENSITY NECESSARY FOR MINIMUM DIVERGENCE AS A FUNCTION OF γ FOR $n=10^{14} cm^{-3}$ AND $f_{\alpha}=1$.

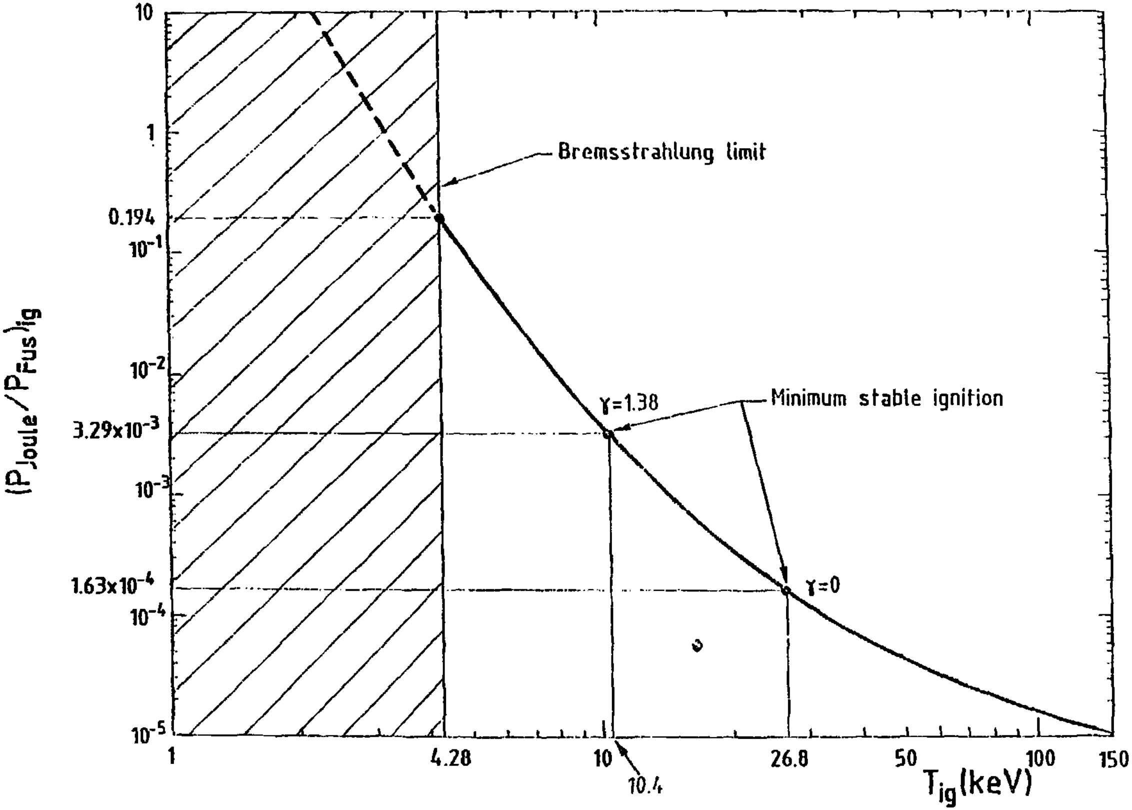

Fig. 6: RATIO OF JOULE TO FUSION POWER AT IGNITION AS A FUNCTION OF T_{ig}, FOR MURAKAMI DENSITY AND $f_{\alpha}=1$.