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ABSTRACT

The local scale transformation method is used for the construction of an Adiabatic Time-Dependent Hartree-Fock approach in terms of the local density distribution. The coherent density fluctuation relations of the model result in a particular case when the "fluctон" local density is connected with the plane wave determinant model function by means of the local-scale coordinate transformation. The collective potential energy expression is obtained and its relation to the nuclear matter energy saturation curve is revealed.

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1. INTRODUCTION

Recently, it has been shown /1/ that short-range nucleon-nucleon correlations, effectively incorporated into the Coherent Density Fluctuation Model /CDFM/, lead mainly to two important predictions. Firstly, as a distinction from other conventional models, the nucleon momentum distribution, calculated in CDFM, shows high momentum component, which is in accordance with the experimental data in inelastic nucleon processes /2/. Secondly, the existence of extreme breathing nuclear states has been predicted in the framework of CDFM, whose energies are comparable with the nuclear binding energies. These highly-excited states are presumably realized in the deep-inelastic proton-nuclei scattering /3/.

The Coherent Density Fluctuation Model has been successfully used for the description of different nuclear phenomena /4/ and along this attempts have been made for a better understanding of the physical nature of the model /5/ and its justification /6/. For instance, calculations in the Jastrow correlation method and in CDFM have been compared in /6/. The calculated nuclear momentum distribution in CDFM takes into account the essential part of the short-range correlations, contained in the Jastrow method.

The connection between the Coherent Density Fluctuation Model and the Generator Coordinate Method /GCM/ has largely been discussed in paper /5/. It has been concluded that the main relations in CDFM can be obtained as an appropriate limit to GCM, if the "potential energy" and the "effective mass", which depend on the non-equilibrium nuclear density, would be identified with those from the nuclear matter theory. Thus the problem of

formal derivation of CDFM from the first principles and relations with existing nuclear theoretical approaches has been tackled.

The present work deals with the problem of justification of CDFM. We show that the general relations of CDFM can be obtained in the framework of the local-scale limit to the ATDHF approach, recently suggested in /7/. Thus the connection between CDFM and ATDHF gives us a more transparent understanding of the underlying CDFM relationships.

2. LOCAL-SCALE VERSION OF ATDHF

The main point of the local-scale version of ATDHF approach lies in the way of introducing collective coordinates $\mathcal{U}=(\mathcal{U}_1,\mathcal{U}_2,\dots)$ in a Slater determinant wave function $\overline{\Phi}(\vec{r}_1,\vec{r}_2,\dots,\vec{r}_A)$ or in the one-body density matrix, respectively

$$\rho_{\mathcal{U}}(\vec{r},\vec{r}') = A\int \Phi_{\mathcal{U}}(\vec{r},\vec{r}_2,\dots,\vec{r}_A)\,\Phi^{*}_{\mathcal{U}}(\vec{r}',\vec{r}_2,\dots,\vec{r}_A)\,d\vec{r}_2\dots d\vec{r}_A, \qquad /2.1/$$

where $\vec{r}_1,\vec{r}_2,\dots,\vec{r}_A$ are the coordinates of nucleons.

The wave function $\Phi_{\mathcal{U}}(\vec{r}_1,\dots,\vec{r}_A)$, considered as a solution of the stationary Hartree-Fock problem, can be related to a certain many-particle model wave function $\overline{\Phi}(\vec{r}_1,\dots,\vec{r}_A)$ by means of the local-scale transformation /8/:

$$\overline{\Phi} = \overline{\Phi}(\vec{r}_1,\vec{r}_2,\dots,\vec{r}_A) = (|A|!)^{-1}\det\left|\overline{\varphi}_i(\vec{r}_j)\right|, \qquad /2.2/$$

$$\Phi_{\mathcal{U}} = \Phi(\vec{r}_1,\vec{r}_2,\dots,\vec{r}_A;\mathcal{U}) = D^{1/2}(\vec{r}_1)\times\dots\times D^{1/2}(\vec{r}_A)\,\overline{\Phi}\left(\vec{f}_{\mathcal{U}}(\vec{r}_1),\dots,\vec{f}(\vec{r}_A)\right), \qquad /2.3/$$

where $\vec{f}_{\mathcal{U}}(\vec{r}) = \vec{r}_0 f_{\mathcal{U}}(\vec{r})$, $\vec{r}_0 = \vec{r}/r$, $r=|\vec{r}|$ and

$$D(\vec{r}) = \frac{D(f_1,f_2,f_3)}{D(x,y,z)} = \frac{\partial f_{\mathcal{U}}(\vec{r})}{\partial r}\,\frac{f^2_{\mathcal{U}}(\vec{r})}{r^2}. \qquad /2.4/$$

The local-scale transformation function $f_{\mathcal{U}}(\vec{r})$ satisfies the equation

$$\rho_{\mathcal{U}}(\vec{r}) = \frac{f^2_{\mathcal{U}}(\vec{r})}{r^2}\,\frac{\partial f_{\mathcal{U}}(\vec{r})}{\partial r}\,\overline{\rho}(\vec{f}_{\mathcal{U}}) \qquad /2.5/$$

or its equivalent

$$F(f_{\mathcal{U}}) \equiv \int_0^r \rho_{\mathcal{U}}\,r'^2dr' - \int_0^{f_{\mathcal{U}}}\overline{\rho}\,r'^2dr' = 0, \qquad /2.6/$$

where $\rho_{\mathcal{U}}(\vec{r})$ and $\overline{\rho}(\vec{r})$ are the local-density distributions, obtained by means of the wave functions $\Phi_{\mathcal{U}}$ and $\overline{\Phi}$, respectively.

Equation /2.5/ offers the possibility of relating the collective parameters to the moments of the density $\rho_{\mathcal{U}}(\vec{r})$ or to its proper characteristics, such as the half-radius R, the surface diffuseness parameter B and so on.

Bearing in mind, the constrained Hartree-Fock method for defining the collective parameters, one can mention the simplicity of the wave function /2.1/ for practical applications and flexibility in the choice of suitable parameters. The usefulness of this density constrained method has been demonstrated in studying the breathing mode exitations in nuclei /9/.

As has been shown in /7/, when the density $\rho_{\mathcal{U}}(\vec{r})$ is fixed and depends on the time t by means of its parameters $\mathcal{U}=\mathcal{U}(t)$, the local-scale transformed single-particle wave functions

$$\psi_i = \psi_i(\vec{r},t) = \left\{\frac{f^2_{\mathcal{U}}}{r^2}\,\frac{\partial f_{\mathcal{U}}}{\partial r}\right\}^{1/2}\overline{\varphi}_i(\vec{f}_{\mathcal{U}}), \qquad /2.7/$$

which are involved in $\Phi_{\mathcal{U}}$, satisfy the generalized scaling property

$$\frac{d\psi_i}{dt} = -\frac{1}{2}\psi_i(\nabla\cdot\vec{v}) - \vec{v}\cdot(\nabla\psi_i). \qquad /2.8/$$

The collective velocity $\vec{v}$ in /2.8/ is expressed by the local-scale transformed function f_u as

$$\vec{v} = \vec{v}(\vec{r}, t) = - \vec{r}_0 \, \dot{f}_u / f'_u \,, \qquad /2.9/$$

with $\dot{f}_u = df_u/dt$ and $f'_u = \vec{r}_0 . (\nabla f_u)$. Using eqs./2.5/ or /2.6/, we can rewrite the velocity /2.9/ as

$$\vec{v} = - \vec{r}_0 \sum_i v_i^* \dot{u}_i \,, \qquad /2.10/$$

where

$$v_i^* \equiv v_i^*(\vec{r}, t) = \left\{ \int_0^r \frac{\partial \rho_u}{\partial u_i} \, r'^2 dr' \right\} / \left(\rho_u \, r^2 \right). \qquad /2.11/$$

Thus, using the continuity equation the local-scale version of the ATDHF approach, formulated in /7/, leads to the Schrödinger equation:

$$\hat{\mathcal{H}}(u) \Psi(u) = E \Psi(u) \,, \qquad /2.12/$$

with a collective Hamiltonian

$$\hat{\mathcal{H}}(u) = - \frac{\hbar^2}{2m} |M(u)|^{-1/2} \sum_{ij} \frac{\partial}{\partial u_i} |M(u)|^{1/2} m_{ij}^{-1}(u) \frac{\partial}{\partial u_j} + V(u) \qquad /2.13/$$

and an inertial tensor

$$m_{ij}(u) = m \int \rho_u(\vec{r}) \, v_i^* \, v_j^* \, d\vec{r} \,, \qquad /2.14/$$

where v_i^* are given by eqs./2.11/, $M(u)$ is the determinant of the matrix $m_{ij}(u)$, and m_{ij}^{-1} is the matrix inverse of $m_{ij}(u)$.

The second term on the right-hand side of eq./2.13/ is the collective potential energy

$$V(u) = \langle \Phi_u \mid H \mid \Phi_u \rangle \,. \qquad /2.15/$$

This is just the expectation value of the nuclear Hamiltonian H taken on the LST wave functions, determined by eqs./2.2/ - /2.5/.

The solution of the Schrödinger equation /2.12/ gives the collective wave functions $\Psi_k(u)$ and energies E_k for ground /k = 0/ and excited /k ≠ 0/ states. Then, taking into account that the collective functions $\Psi_k(u)$ are normalised according to

$$\langle \Psi_k(u) \mid \Psi_k(u) \rangle = \int |M(u)|^{1/2} \, |\Psi_k(u)|^2 \, du_1 du_2 \ldots = 1 \qquad /2.16/$$

it is possible to obtain a number of important nuclear quantities. For instance, the local density is

$$\rho(\vec{r}) = \langle \Psi_0(u) \mid \rho_u(\vec{r}) \mid \Psi_0(u) \rangle \qquad /2.17/$$

and the transition densities are

$$\rho_{kk'}(\vec{r}) = \langle \Psi_k(u) \mid \rho_u(\vec{r}) \mid \Psi_{k'}(u) \rangle \,. \qquad /2.18/$$

One can obtain the nuclear momentum distribution, as well:

$$n(\vec{q}) = \langle \Psi_0(u) \mid n(u, \vec{q}) \mid \Psi_0(u) \rangle \,. \qquad /2.19/$$

In summary, it is obvious that once the model Slater determinant $\Phi(\vec{r}_1, \ldots, \vec{r}_A)$ and the density $\rho_u(\vec{r})$ in the basic collective state $\Phi_u(\vec{r}_1, \ldots, \vec{r}_A)$ are given, the relevant nuclear quantities can be obtained by the suggested local-scale version of ATDHF.

3. JUSTIFICATION OF ODFM

Let us now assume density distributions $\rho_u(\vec{r})$ to be determined in terms of θ-like densities

$$\rho_u(\vec{r}) \equiv \rho_x(r) = \rho_0(x) \, \theta(x - r) \,; \quad \theta(y) = \begin{cases} 1 & y \le 0 \\ 0 & y > 0 \end{cases} \qquad /3.1/$$

i.e. the nucleons in the basic collective state /2.1/ are uniformly distributed in the sphere with radius X and density

$$\rho_0(x) = \frac{3A}{4\pi x^3} = \frac{A}{\Omega_x} \,, \qquad /3.2/$$

where A is the mass number of the nucleus. Consequently, eqs./2.6/ and /3.1/ immediately lead to the LST function $f_\chi(r)$, which is labelized by the same radius χ.

The substitution of $f_\chi(r)$ into /2.1/ gives the collective basic wave function $\Phi_\chi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A)$. In fact, the function describes the state of this abstract nuclear object, which is a "piece" of nuclear matter in the volume Ω_χ with the constant density $\rho_0(\chi)$. In other words, the assumption /3.1/ introduces the general CDFM picture of the vibrating nucleus, where such virtual configuration with the density $\rho_0(\chi)$ has been called "flucton" /1/.

Further, one can obtain the dynamical equation of motion through the local-scale version of ATDHF, since the basic wave functions Φ_χ have been obtained in accordance with the wave functions /2.1/. In the case of the density /3.1/, using eqs./2.11/ /2.15/, the collective Hamiltonian $\mathcal{H}(\chi)$ can be expressed in the following simple form:

$$\mathcal{H}(\chi) = -\frac{\hbar^2}{2m}\frac{1}{\sqrt{m(\chi)}}\frac{d}{d\chi}\frac{1}{\sqrt{m(\chi)}}\frac{d}{d\chi} + U(\chi), \qquad /3.3/$$

where the effective mass $m(\chi)$ is obtained after the substitution of the density $\rho_\chi(r)$ in eqs./2.11/ and /2.14/.

The collective potential energy $U(\chi)$ in /3.3/ can be expressed from eq./2.15/ as

$$U(\chi) = \langle \Phi_\chi | H | \Phi_\chi \rangle \quad , \qquad /3.4/$$

where H is the nuclear Hamiltonian. Thus, we have to consider the collective Schrödinger equation /see eq./2.12//:

$$\left(-\frac{\hbar^2}{2m_{\mathcal{H}}}\frac{d^2}{d\chi^2} + U(\chi)\right)\Psi(\chi) = E\,\Psi(\chi). \qquad /3.5/$$

The effective mass $m_{\mathcal{H}} \equiv m(\chi) = \frac{\partial m}{\partial}A$ turns out to be a simple function of the mass number A, independent of χ.

For the actual determination of the collective potential energy /3.4/, one has to make a choice of the model wave function which we have already accepted to be a certain parametrical function within the local-scale version of ATDHF. Bearing in mind the constant behaviour of the density /3.1/, one can realize the usefulness of the nuclear matter theory considerations at this point.

To calculate the property of nuclear matter, we ignore surface effects by passing to the limit $A \to \infty$ and $\Omega \to \infty$. The single particle wave functions, normalized over a volume Ω, are plane waves

$$\overline{\varphi_i} \equiv \varphi_{k_i}(\vec{r}) = \frac{1}{\sqrt{\Omega}}\, e^{i\vec{k}_i\cdot\vec{r}} \qquad /3.6/$$

and they are involved in the plane wave Slater determinant function

$$\overline{\Phi}(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A) = \frac{1}{\sqrt{A!}}\, det\left|\varphi_{k_i}(\vec{r}_j)\right|. \qquad /3.7/$$

Because of the periodic boundaries in Ω, even when Ω is a finite quantity Ω_χ, the wave function /3.7/ can be used as a model wave function within the local-scale version of ATDHF.

Making the choice /3.7/ for the model wave function $\overline{\Phi}$, it can be easily verified that the collective potential energy results in the saturation curve of the nuclear matter: $W(\rho_0(\chi)) = U(\chi)/A$. Such a form of the potential energy $\mathcal{E}(\chi) \equiv U(\chi) = A\,W(\rho_0(\chi))$ has been accepted in CDFM as well as the collective Schrödinger equation /3.5/, rewritten as

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{d\chi^2} + \mathcal{E}(\chi)\right)\mathcal{F}(\chi) = E\,\mathcal{F}(\chi). \qquad /3.8/$$

Let us now consider the interpretation of the collective wave function $\mathcal{F}(\chi) \equiv \Psi(\chi)$. We start from the definition of the density /see

eqs./2.17/ and /3.1//:

$$\rho(\vec{r}) = \int |\mathcal{F}(x)|^2 \rho_x(r)\, dx . \qquad /3.9/$$

Since the density $\rho_x(r)$ is normalized to the nucleon number A, $\mathcal{F}(x)$ can be interpreted as a weight-function for finding all A nucleons in the volume Ω_x in the state Φ_x with the constant density $\rho_x(x) = A/\Omega_x$. Such a virtual configuration, as we have already said, was called "flucton" in CDFM. Consequently, this notion and the meaning of $\mathcal{F}(x)$, which is specific for CDFM, quite rigorously arise in the present local-scale version of ATDHF.

The same conclusion for the transition densities

$$\rho_{kk'}(\vec{r}) = \int \mathcal{F}_k(x)\, \mathcal{F}_{k'}(x)\, \rho_x(r)\, dx \qquad /3.10/$$

and for the nucleon momentum distribution

$$n(q) = \int |\mathcal{F}(x)|^2 n_x(q)\, dx \qquad /3.11/$$

can be made. The "flucton" momentum distribution in /3.11/ is

$$n_x(q) = n_0(x)\, \Theta(K_F(x) - q), \qquad /3.12/$$

where $K_F(x)$ is the Fermi momentum $K_F(x) = \left\{\frac{3\pi^2}{2}\rho_0(x)\right\}^{1/3}$.

4. CONCLUSION

In conclusion, we have to mention that when the flucton picture of the vibrating nucleus has been accepted by the assumption of the density $\rho_x(\vec{r})$ and the model function Φ expressed by eqs./3.1/ and /3.7/, respectively, the whole nuclear dynamics in this picture follows from the local-scale limit to the ATDHF approach.

The most inconsistent point of CDFM is the problem of the effective mass entering the collective kinetic energy / the first term on the right-hand side of eq./3.3// and in the collective equation /3.2/. In fact, the effective mass is a free parameter in CDFM and its value has been determined by comparing with the experimental data. Unfortunately, this fitted value /3/ is in some way different from the hydrodynamical limit $m_{eff} = \frac{3}{5} m A$, obtained above. Both, the hydrodynamical effective mass value and the equation /3.9/ lack in the correct reproduction of the ground state energy.

The necessity of renormalisation of the effective mass value is not strongly motivated. Various reasons might presumably be raised for this inconsistency. One of them is the use of a Slater plane-wave determinant function at the periodic boundaries which contain a limited part of correlations among particles.

To conclude, it is now clear that the theoretical determination of the correct effective mass value is the only problem to be solved in the way of the complete justification of CDFM.

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