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EFFECTIVE EINSTEINIAN GRAVITY FROM POINCARÉ GAUGE FIELD THEORY

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ABSTRACT

The Poincaré gauge theory of gravity should apply in the microphysical domain. Here we investigate its implications for macrophysics. Weakly self double dual Riemann-Cartan curvature is assumed throughout. It is shown that the metrical background is then determined by Einstein's field equations with the Belinfante-Rosenfeld symmetrized energy-momentum current amended by spin squared terms. Moreover, the effective cosmological constant can be reconciled with the empirical data by absorbing the corresponding constant curvature part into the dynamical torsion of recently found exact solutions. Macroscopically this extra torsion remains undetectable.

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I. Introduction

Nowadays unified theories of particle interaction are set up within the framework of gauge field theories and the question arises , whether or not gravitational interactions can be incorporated into this scheme.

To be sure , it is commonplace that general relativity is a highly satisfactory theory of gravitationally coupled macroscopic systems. On the other hand , systematic work of the Cologne group , cf.F.W.Hehl [1], [2] , E.W.Mielke [3] , P.Baekler [4] and H.J.Lenzen [5] , has shown how to formulate a viable gauge theory of gravity for microscopic systems. Disregarding a possible boson-fermion supersymmetry , this theory is based on local Poincaré invariance ; therefore it can provide a consistent coupling to the mass and spin of fundamental particles. The geometrical counterparts are to be found in the torsion and the curvature of a Riemann-Cartan spacetime.

As has been proved within the lagrangian formalism , the dynamics of the Poincaré gauge field theory (PG) is determined by elegant Yang-Mills-type equations [6]. This poses the questions of the correspondence of the PG theory to macroscopic gravity , the physical rôle of dynamical torsion in explicit solutions and of the origin of the cosmological constant.

For the PG theory , restricted by duality conditions for the curvature , definite and physically important answers are obtained.

II. Riemann-Cartan Geometry

In a Riemann-Cartan spacetime (U_4) the independent geometrical variables are the basis frame one-forms (tetrads) $e^\alpha := e_i{}^\alpha dx^i$ and the connection one-forms $\omega^{\alpha\beta} := \Gamma_i{}^{\alpha\beta} dx^i$, which are antisymmetric with respect to the Lorentz indices.

In the following i , j , k... are world indices, and α, β, γ... are (anholonomic) Lorentz indices which both run from 0...3 .

In the Poincaré gauge field approach [1] these geometrical objects are identified as translational and (Lorentz-) rotational gauge field potentials , respectively. The corresponding gauge field strenths are the torsion

$$F_{ij}{}^\alpha := 2D_{[i}e_{j]}{}^\alpha = 2(\partial_{[i}e_{j]}{}^\alpha + \Gamma_{[i|\gamma}{}^\alpha e_{|j]}{}^\gamma) \qquad (1)$$

and the curvature

$$F_{ij\alpha}{}^\beta := 2(\partial_{[i}\Gamma_{j]\alpha}{}^\beta + \Gamma_{[i|\epsilon}{}^\beta \Gamma_{|j]\alpha}{}^\epsilon) \qquad (2)$$

of a U_4.

D_i denotes the exterior covariant derivative , $e := det(e_i{}^\alpha)$ the determinant of the tetrad coefficients , $F_{\alpha\beta} := F_{\mu\alpha\beta}{}^\mu$ the Ricci tensor and $F := F_{\mu\sigma}{}^{\sigma\mu}$ the curvature scalar .

The definitions of the field strengths (1) and (2) imply the two Bianchi identities for torsion and curvature .

$$D_{[i}F_{jk]}{}^\alpha = F_{[ijk]}{}^\alpha \qquad (3)$$

and

$$D_{[i}F_{jk]\alpha}{}^\beta = 0 \qquad (4) ,$$

or equivalently

$$D_j(e^* F^{*ij}{}_{\alpha\beta}) = 0 .$$

In terms of the (anholonomic) Levi-Civita symbol , the double dual of the curvature tensor is defined by

$$^*F^*_{\alpha\beta}{}^{\gamma\delta} = -\frac{1}{4}\eta_{\alpha\beta}{}^{\mu\nu}\eta^{\gamma\delta}{}_{\rho\sigma}F_{\mu\nu}{}^{\rho\sigma} \qquad (5)$$

which in U_4 with Minkowski signature, ensures an involutive operation [7].

Further analysis will be facilitated by decomposing the torsion and the curvature into irreducible parts with respect to the proper Lorentz group $SO_0(1,3)$. Accordingly, the torsion splits into the trace-free tensor torsion, the vector torsion $F_\mu := F_{\mu\rho}{}^\rho$ and the axial vector part :

$$F_{\mu\nu\alpha} = \sum_{i=1}^{3} {}^{(i)}F_{\mu\nu\alpha} = {}^{(1)}F_{\mu\nu\alpha} - (2/3)\eta_{[\mu|\alpha}F_{|\nu]} + F_{[\mu\nu\alpha]} . \tag{6}$$

In a similar fashion, the curvature can be decomposed irreducibly into six independent pieces :

$$F_{\alpha\beta}{}^{\gamma\delta} = \sum_{k=1}^{6} {}^{(k)}F_{\alpha\beta}{}^{\gamma\delta} \tag{7}$$

which, in consecutive order, are the tracefree Weyl tensor, the pair commutator the pseudo-scalar, the tracefree symmetric Ricci tensor, the antisymmetric Ricci tensor and the scalar curvature. In a purely Riemannian spacetime (V_4), i.e. for vanishing torsion, ${}^{(2)}F_{\alpha\beta}{}^{\gamma\delta}$, ${}^{(3)}F_{\alpha\beta}{}^{\gamma\delta}$ and ${}^{(5)}F_{\alpha\beta}{}^{\gamma\delta}$ are identically zero, cf. K.Hayashi [8], P.Baekler et al. [10], H.J.Lenzen [11]. This completes the geometrical frame of the Riemann-Cartan geometry and we are now prepared to investigate the dynamics of the PG theory.

III. Dynamical structure of the Poincaré gauge field theory

Following the notions of Yang-Mills gauge theories, we adopt

$$v = \frac{1}{4}[F_{ji}{}^{\alpha}H_\alpha{}^{ij} + F_{ji}{}^{\alpha\beta}H_{\alpha\beta}{}^{ij}] + \frac{eF}{4\chi l_0^2} + \frac{e\Lambda}{l_0^2} \tag{8}$$

as the general gauge field lagrangian, which is at most quadratic in torsion and curvature. This implies that the field momenta

$$H_\alpha{}^{ij} := \frac{\partial v}{\partial \partial_j e_i{}^\alpha} \tag{9}$$

$$H_{\alpha\beta}{}^{ij} := \frac{\partial v}{\partial \partial_j \Gamma_i{}^{\alpha\beta}}$$

which are canonically conjugated to the translational and rotational gauge potentials are of the following quasi-linear form :

$$H_\alpha{}^{ij} = \frac{e}{l_0^2}\sum_{n=1}^{3} a_n\, {}^{(n)}F^{ji}{}_\alpha \tag{10}$$

$$H_{\alpha\beta}{}^{ij} = \frac{e}{l_0^2}\frac{1}{\chi}e^i{}_{[\alpha}e^j{}_{\beta]} + \frac{e}{\kappa}\sum_{n=1}^{6} b_n\, {}^{(n)}F^{ji}{}_{\alpha\beta} \tag{11}$$

Besides the three "primary" dimensionless coupling constants χ, κ and Λ/l_0^2, there occur (3+5) "secondary" constants a_i, b_i weighting the irreducible parts of the field strengths, where b_1 can be scaled to 1. Since the resulting lagrangian (8), written explicitly, contains a boundary term of the Euler-type, one of the b_i's could always be put equal to zero.

The Euler-Lagrange equations, which can be obtained by independent variation of (8) with respect to the tetrad and the connection, are Yang-Mills-like :

$$D_j H_\alpha{}^{ij} - \epsilon_\alpha{}^i = e\Sigma_\alpha{}^i , \tag{12}$$

$$D_j H_{\alpha\beta}{}^{ij} - H_{[\beta\alpha]}{}^i = e\tau_{\alpha\beta}{}^i . \tag{13}$$

The sources of the right hand side of (12) and (13) are the canonical currents of energy-momentum and spin of the matter fields [1]. The nonlinear contribution

$$\epsilon_\alpha{}^i := e^i{}_\alpha v - F_{\alpha j}{}^\gamma H_\gamma{}^{ji} - F_{\alpha j}{}^{\gamma\delta}H_{\gamma\delta}{}^{ji} \tag{14}$$

to the first field equation comprises the energy-momentum currents of the gravitational gauge fields, whereas $H_{[\beta\alpha]}{}^i$ in (13) can be identified with the spin current of the translational gauge fields.

Local Poincaré invariance * implies the two Noether identities

$$D_i(e\Sigma_\alpha{}^i) = F_{\alpha i}{}^\beta \Sigma_\beta{}^i + F_\alpha{}^{i\beta\gamma}\tau_{\beta\gamma i} \tag{15}$$

$$D_i(e\tau_{\alpha\beta}{}^i) - e\Sigma_{[\alpha\beta]} = 0, \tag{16}$$

provided the equations of motions for the matter fields are fulfilled.

IV. Solving the second field equation

The problem of finding exact solutions of Yang-Mills-type field equations is considerably simplified by the duality ansatz of the type as found by A.A.Belavin et al. [12], cf. also A.Actor [13]. In order to transfer this methodological approach to PG theory, the modified double duality ansatz

$$H_{\alpha\beta}{}^{ij} = \frac{e\xi}{\kappa}\,{}^*F^{*ij}{}_{\alpha\beta} + 2\gamma\frac{e}{l_0^2}e^i{}_{[\alpha}e^j{}_{\beta]} \tag{17}$$

will be used, following earlier work P.Baekler et al. [7], I.M.Benn et al. [14], and E.W.Mielke [20]. Since part of the second field equation (13) is thereby transformed into the second Bianchi identity (4), such an ansatz reduces a field equation of second order to one of first order in the gauge potentials, provided γ is a suitable chosen constant, cf. also R.P.Wallner [31].

The additional term $e^i{}_{[\alpha}e^j{}_{\beta]}$ in (17) is vital, since it gives rise, after covariant differentiation, to the modified torsion tensor

$$T_{\alpha\beta}{}^i = F_{\alpha\beta}{}^i + 2e^i{}_{[\alpha}F_{\beta]} . \tag{18}$$

Then the second field equation (13) reduces to an algebraic relation which, after a resolution for the translational momenta, reads

$$H_{\alpha\beta\gamma} = \frac{-e\gamma}{l_0^2}(2T_{\gamma\beta\alpha} + 3T_{[\gamma\alpha\beta]}) + e(\tau_{\gamma\beta\alpha} + \tau_{\gamma\alpha\beta} + \tau_{\alpha\beta\gamma}) , \tag{19}$$

Provided $\gamma \neq 0$, a non-trivial torsion is expected, even for spinless matter. More information can be extracted from (19) by using the irreducible decomposition (6) and a related decomposition of the spin tensor. Then (13) splits into the independent relations

$$(2\gamma + a_1)\,{}^{(1)}F_{\alpha\beta\gamma} = 2l_0^2\,{}^{(1)}\tau_{\alpha\beta\gamma}, \tag{20}$$

$$(4\gamma - a_2)F_{\alpha\delta}{}^\delta = -2l_0^2\,\tau_{\alpha\delta}{}^\delta ,$$

$$(\gamma - a_3)F_{[\alpha\beta\gamma]} = l_0^2\,\tau_{[\alpha\beta\gamma]} .$$

* There is a delicacy concerning the interpretation of local translations since they implement diffeomorphisms of the affine bundle, cf. E.W.Mielke [3]

They remined us of the second field equation of the Einstein - Cartan - theory

$$T_{\alpha\beta}{}^{i} = 2l_0^2\, \tau_{\alpha\beta}{}^{i} \tag{21}$$

where torsion is coupled algebraically to the spin current.

From the duality ansatz we find the algebraic restrictions on the curvature :

$$\frac{1}{\kappa}\,{}^{(1)}F_{\alpha\beta\gamma\delta}(b_1+\xi)=0\,, \tag{22}$$

$$\frac{1}{\kappa}\,{}^{(2)}F_{\alpha\beta\gamma\delta}(b_2-\xi)=0\,,$$

$$\frac{1}{\kappa}\,{}^{(3)}F_{\alpha\beta\gamma\delta}(b_3+\xi)=0\,,$$

$$\frac{1}{\kappa}\,{}^{(4)}F_{\alpha\beta\gamma\delta}(b_4-\xi)=0\,,$$

$$\frac{1}{\kappa}\,{}^{(5)}F_{\alpha\beta\gamma\delta}(b_5+\xi)=0\,,$$

$$\frac{Fl_0^2}{\kappa}\,{}^{(6)}F_{\alpha\beta\gamma\delta}(b_6+\xi)=6(2\gamma-\frac{1}{\chi})$$

Observe , that the curvature scalar F is constant provided $\xi \neq -b_6$. Before considering the algebraic constraints in more detail , we turn to the implications of the duality ansatz (17) for the first field equation.

V. Reduction of the first field equation

In analogy to Einstein's field equations , in (12) the energy-momentum currents of matter and gauge fields are completely balanced . In the anholonomically written form

$$D_j H_{\alpha\beta}{}^{j} + \frac{e}{l_0^2}X_{\alpha\beta} - {}^{rot}\epsilon_{\alpha\beta} = e\Sigma_{\alpha\beta} \tag{23}$$

the current of the gravitational energy-momentum has , for later convenience , been decomposed into a translational part $X_{\alpha\beta} = (l_0^2/e)\ {}^{tr}\epsilon_{\alpha\beta}$ quadratic in torsion and a remaining rotational part ${}^{rot}\epsilon_{\alpha\beta}$ depending on the curvature , for details compare , e.g. , E.W.Mielke [20] and P.Baekler [4] .

Insertion of the Cartan-type relation (19) yields , after some reshifting of terms , the equation

$$-2\gamma D_j(eT^j_{(\alpha\beta)}) - \gamma D_j(eT_{\alpha\beta}{}^{j}) - l_0^2\ {}^{rot}\epsilon_{\alpha\beta} + eX_{\alpha\beta} = l_0^2\Sigma_{\alpha\beta} - l_0^2 D_j[e(\tau^j{}_{\beta\alpha} + \tau^j{}_{\alpha\beta} + \tau_{\alpha\beta}{}^{j})]\,. \tag{24}$$

For a further reduction the identity

$$\frac{1}{e}D_j(eT^j{}_{(\alpha\beta)}) - G_{(\alpha\beta)}(\Gamma) + G_{\alpha\beta}(\{\}) + Y_{\alpha\beta} = 0\,, \tag{25}$$

involving the Einstein tensors $G_{\alpha\beta} := F_{\alpha\beta} - (1/2)F\eta_{\alpha\beta}$ in a U_4 and V_4 , respectively , the contracted first Bianchi identity

$$(1/e)D_j(eT_{\alpha\beta}{}^{j}) = 2G_{[\alpha\beta]} \tag{26}$$

the relation

$$(l_0^2/e)\ {}^{rot}\epsilon_{\alpha\beta}(**) = -2\gamma G_{\alpha\beta} + [\Lambda + \frac{F}{4}(\frac{1}{\chi} - 2\gamma)]\eta_{\alpha\beta} \tag{27}$$

and the second Noether identity (16) are instrumental. The "linearization" of ${}^{rot}\epsilon_{\alpha\beta}$ comes about after employing the duality ansatz (17) cf. P.Baekler [4] , H.J.Lenzen [5] and E.W.Mielke [20] . $Y_{\alpha\beta}$ consists of terms quadratic in torsion [7] .

A comparison of the relations (24-27) reveals that $G_{\alpha\beta}(\Gamma)$ drops out completely . As a result the first field equation reduces to an Einstein-type field equation

$$2\gamma(G_{\alpha\beta}(\{\}) + \Lambda_{eff}\eta_{\alpha\beta}) = l_0^2\sigma_{\alpha\beta}(\{\}) + l_0^4 s_{\alpha\beta} \tag{28}$$

with an effective "cosmological constant"

$$\Lambda_{eff} = \frac{1}{2\gamma}[-\Lambda + \frac{F}{4}(2\gamma - \frac{1}{\chi})] \tag{29}$$

residing in a Riemannian background . As sources there occur

$$\sigma_{\alpha\beta} = \Sigma_{\alpha\beta}(\{\}) - \frac{1}{e}D_j^{\{\}}(e\tau^j{}_{(\alpha\beta)}) \tag{30}$$

and a nonlinear contribution

$$s_{\alpha\beta} = \frac{3}{4}c_2(\tau^{\mu\nu}{}_{\alpha}\tau_{|\mu\nu\beta|} + \tau^{\mu\nu}{}_{\beta}\tau_{|\mu\nu\alpha|}) - \eta_{\alpha\beta}(\sum_{i=1}^{3} c_i\ {}^{(i)}\tau_{\mu\nu\lambda}\ {}^{(i)}\tau^{\lambda\mu\nu}) + Z_{(\alpha\beta)} \tag{31}$$

due to spin.Here the abbreviations $c_1 := 2/(2\gamma + a_1)$, $c_2 := 2/(a_2 - 4\gamma)$ and $c_3 := 1/(\gamma - a_3)$ have been introduced for convenience and $Z_{(\alpha\beta)}$ is the contortional part of $\sigma_{\alpha\beta}$ and vanishes e.g. for Dirac fields , cf. [16] .

VI. Effective gravity

The reduction by means of duality is a decisive step towards the understanding of the interrelation of microscopic and macroscopic gravity . Formally , the resulting metrical background is determined by the Einstein-type field equations (28) , which couples to the Belinfante-Rosenfeld symmetrized energy-momentum tensor $\sigma^{\{\}}_{\alpha\beta}$. As is well-known F.W.Hehl [16] , the latter can be identified with the metrical energy-momentum tensor.

The spin of matter , due to $s_{\alpha\beta}$, is acting as an additional source . Due to the l_0^4-coupling , however , this contribution is in macroscopic matter distributions diminutively small , like in Einstein-Cartan theory , cf. A.Trautman [17] , J.Nester [18] . Only at exceedingly high spin densities , or after a renormalization of coupling constants these terms are expected to become important .

Moreover , the macroscopical correspondence with Einstein's theory requires $\gamma = 1/2$. For spinless matter , this requirement implies dynamical torsion for non-trivial configurations . This is a consequence of the Cartan-type relation (19) , in which essentially two cases are to be distinguished :

A : ${}^{(i)}F_{\alpha\beta\gamma} \neq 0$ necessarily leads to the so-called Einstein-choice for i = 1 , 2 , 3 .

$$a_1 = -1\,, a_2 = 2\,, a_3 = 1/2 \tag{32}$$

B : Merely $F_{[\alpha\beta\gamma]} = 0$ implies the so-called viable set

$$2a_1 + a_2 = 0 \tag{33}$$

Note that it contains the purely quadratic PG model of P.von der Heyde and F.W.Hehl [16] with $a_1 = -1$, $a_2 = -2$ and $a_3 = -1$.

As a result , the duality restricted PG theory contains quadratic torsion parts in the effective lagrangian which reduces to that of teleparallelism models cf. [19] in the limit of vanishing U_4-curvature . The latter are known to be indistinguishable from Einstein's theory up to the 4^{th} post-Newtonian approximation . Moreover , the complete PG lagrangian in vacuum reduces to that of Einstein-Hilbert up to boundary terms , E.W.Mielke [20] and H.J.Lenzen [5] .

In the physically important case of the coupled PG-Maxwell system , the reduction via duality leads to an *effective Einstein-Maxwell system* . This is due to the fact that the U(1) gauge potentials *do not feel torsion since they do not carry dynamical spin* .

Consequently , for non-trivial torsion configurations , the equation which remains to be solved is the double duality ansatz (17) . Among the large class of solutions found so far , the physically most important configuration is a charged Taub-NUT-(anti-)deSitter solution [21] , [22] . Because of the appearence of non-trivial torsion , the NUT-parameter is , contrary to Einstein's theory , not a freely specifiable constant of integration,but is necessarily interrelated to the coupling constant $1/\kappa$ of the rotational Lorentz gauge fields . This has a considerable impact on the gravitational analogue of the Witten effect cf. O.Foda [23] and E.W.Mielke [24] .

VII. Extra torsion : A solution to the cosmological constant problem ?

In modern gauge field theories of particle interactions the vacuum energy density ϵ_{vac} gives rise to a huge cosmological constant $\Lambda = l_0^2\epsilon_{vac} > 0$.This is due to the fact that vacuum fluctuations feel all the complicated physics originating from the Higgs vacuum , fermion and gauge field condensates etc . On the other hand , the macroscopically observed effective cosmological constant Λ_{eff} is very small , if not zero .

Recently , V.A. Rubakov and M.E. Shaposhnikov [25] *have proposed an interesting "solution"* to this *problem* by *rising* the number of dimensions a la Kaluza and Klein (cf.A.Chodos [26] for a review) . They found a solution to a higher dimensional theory of gravity with non-compact internal dimensions on a spacetime obeying the standard Einstein equations in a V_4 with an arbitrary (possibly zero) value of Λ_{eff}.

Since the PG theory with torsion is a promising model even in higher dimensions (M.O. Katanayev and I.V. Volovich [27]) the following ramification concerning the cosmological constant problem suggests itself : For dually constrained PG theories , the value (29) of Λ_{eff} appearing in the metrical equation (28) depends on the choice of the coupling constants . Since (29) involves via (22) the coupling constant $1/\kappa$ of the Lorentz-"rotons" , Λ_{eff} is partially of microscopic origin . Superficially , it can be given any value , thus resembling the Rubakov-Shaposhnikov mechanism *of "compactification"* . *Then by an appropriate adjustment of the constants , the metrical background could remain* Ricci-flat.

Since PG theory resides in a Riemann-Cartan space , possible effects of torsion have to be taken care of , however . In exact generic solutions with O(3)-symmetry , the vacuum torsion rises asyptotically as $(1/6)(4\Lambda_{eff} - F)r$. Therefore a constant curvature part appears to be absorbed in the linearly rising contortion of the world . A related , but rather ad-hoc suggestion has been made by S.Hawking [28] by introducing antisymmetric third rank tensor potentials , cf. also [29] . In our case , however , the torsion remains concealed empirically . The reason being that , as Stoeger [29] has shown , macroscopic test bodies in cosmology , such as stars and galaxies , would not feel the linearly rising contortion .

Although these findings are by no means conclusive , a new *facet* is added to the *cosmological constant problem* by *admitting dynamical torsion* .

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