PERTURBATIVE CURRENT QUARK MASSES IN QCD

by

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Abstract

Neutral PCAC current quark masses follow from the covariant light plane or QCD requirement that $\langle\pi|(\bar{u}u)_M|\pi\rangle \propto \hat{m}(M)$, which is not inconsistent with the spontaneous breakdown of chiral symmetry. The resulting current quark mass ratio $(m_s/\hat{m})_{curr} \simeq 5$ and scale $\hat{m}_{curr} \simeq 62$ MeV at $M \simeq 2$ GeV are compatible with the observed πN σ-term, the Goldberger-Treiman discrepancy, the low-lying 0^-, $1/2^+$, 1^-, $3/2^+$ hadron mass spectrum, the flavor independence of the dynamically generated quark mass and the perturbative weak binding limit. (author)

I. Introduction

Even though it appears that quarks are condemned to be bound in hadrons, it is now well understood that there are two distinctly different types of quark masses: a nonperturbative, flavor-independent, dynamically generated mass m_{dyn} and *perturbative, flavor-dependent current* quark masses m^i_{curr}. In chiral field theories such as QCD, it is also clear that these (running) masses are momentum dependent and that $m_{curr} \equiv m_{curr}(M \simeq 2$ GeV) vanishes in the chiral limit while m_{dyn} does not, but the current quark mass ratio

$(m^i(M)/m^j(M))_{curr}$ remains constant for any value of M. Despite this good start it has always amazed us that so many physicists faithfully believe in and do so much hard work employing the "strong PCAC" [1] current quark mass paradigm (SPCAC)

$$m_\pi^2 \to -2f_\pi^{-2}\,\hat{m}_{curr}\,\langle 0|\bar{q}q|0\rangle \tag{1a}$$

$$(m_s/\hat{m})^{SPCAC}_{curr} = 2(m_K^2/m_\pi^2) - 1 = 25\ , \quad \hat{m}^{SPCAC}_{curr} = 5\ \mathrm{MeV}\ , \tag{1b}$$

based on so few (no) pieces of independent corroborating experimental evidence. In fact, we do not understand the dynamical origin of the current quark masses, so it is not at all clear what the mass scales (1b) — right or wrong — really mean.

In our opinion, of far greater significance and utility for the theory of chiral symmetry and low energy QCD is the nonperturbative, dynamically generated quark mass scale [2-4] $m_{dyn} \simeq 315$ MeV $\sim m_N/3$ obtained in at least eight independent ways [5]. Nevertheless, there now exists two pieces of experimental evidence based on the pioneering work of G. Höhler and collaborators [6] which should have a bearing on the current quark masses — the Goldberger-Treiman discrepancy $\Delta_{\pi NN}$ and the πN σ-term:

$$\Delta_{\pi NN} \equiv 1 - \left(m_N g_A/f_\pi g_{\pi NN}\right) = 0.06 \pm 0.01 \tag{2a}$$

$$\sigma_{\pi N} = 65 \pm 5\ \mathrm{MeV}\ , \quad \sigma_{\pi N}/m_N = 0.07 \pm 0.01\ . \tag{2b}$$

Qualitatively these "large" chiral symmetry-breaking effects both are significantly greater than the very small SPCAC mass scale of (1b), in terms of which one might expect

$$\Delta^{SPCAC}_{\pi NN} \sim (\sigma_{\pi N}/m_N)^{SPCAC} \sim O(\hat{m}_{curr}/\hat{m}_{con}) \sim 0.015\ , \tag{3}$$

where $\hat{m}_{con} = 1/2(m_u+m_d)_{con} \simeq 340$ MeV is the accepted average nonstrange constituent quark mass scale, only slightly larger than m_{dyn}.

For the past nine years I (along with H. F. Jones [7,8], J. F. Gunion and P. C. McNamee [9], and N. H. Fuchs [4,10,11]) have struggled with the incompatibility between (1)-(3) and have concluded that there is an alternative chiral-breaking scheme to SPCAC which is reasonably consistent with the data (2) and the fundamental perturbation theory — QCD formulae for the perturbative chiral-breaking hamiltonian density $\mathcal{H}' = \bar{q}\,\mathcal{M}_{curr}q$:

$$m_\pi^2 = \frac{1}{2}\langle\pi|\mathcal{H}'|\pi\rangle = \hat{m}_{curr}(M)\,\langle\pi|(\bar{u}u)_M|\pi\rangle \tag{4a}$$

$$m_K^2 = \frac{1}{2}\langle K|\mathcal{H}'|K\rangle = \frac{1}{2}\left[\hat{m}_{curr}(M)\,\langle K|(\bar{u}u)_M|K\rangle + m_{s,curr}(M)\,\langle K|\bar{s}s)_M|K\rangle\right]. \tag{4b}$$

This alternative, which we now call "neutral PCAC" [3-5,11,12], is <u>not</u> consistent with the additional SPCAC <u>assumption</u> which, e.g., converts (4a) to (1a) <u>while neglecting PCAC corrections</u>. The QCD renormalization M-dependence on the RHS of (4) suggests that such PCAC corrections are <u>of order unity</u> — whence the association of "strong PCAC" with (1). On the other hand, if one does <u>not</u> invoke the additional PCAC operation to convert (4a) to (1a) (neutral PCAC), the natural quark model mass dependence of the (mass dimension one) flavor-dependent matrix elements in (4) are the complete flavor-dependent and M-dependent QCD quark masses themselves:

$$\langle\pi|(\bar{u}u)_M|\pi\rangle \propto \hat{m}(M) \tag{5a}$$

$$\langle K|(\bar{u}u)_M|K\rangle \propto \hat{m}(M)\quad,\qquad \langle K|(\bar{s}s)_M|K\rangle \propto m_s(M)\ . \tag{5b}$$

Then in the high M <u>perturbative</u> region where the quark masses in (5) are primarily of the <u>current</u> type, (4) reduces to [3,4,9-12]

$$m_\pi^2 \propto 2\hat{m}^2_{curr}\quad,\qquad m_K^2 \propto (m_s^2 + \hat{m}^2)_{curr} \tag{6a}$$

$$(m_s/\hat{m})^{NPCAC}_{curr} = \left[2(m_K^2/m_\pi^2) - 1\right]^{1/2} \simeq 5\ ,\quad \hat{m}^{NPCAC}_{curr} \simeq 62\ \mathrm{MeV}\ . \tag{6b}$$

In a global sense we suggest that the <u>quadratic</u> NPCAC mass formulas (6) (for mesons and for quarks) point to a consistent theoretical picture whereby the <u>massless chiral limit</u>, $m_{\pi,K} \to 0$, $\hat{m}_{curr}$, $m_{s,curr} \to 0$ is achieved in the <u>relativistic</u> infinite momentum frame with $E_\pi = p_\pi[1 + m_\pi^2/2p_\pi^2]$, etc. While the hybrid SPCAC mass formulas (1) (quadratic in meson mass, linear in quark mass) do not provide such a picture, an approximate linearized version of (6) (linear in meson and in quark masses [13]) leads to essentially the same (NPCAC) current quark mass ratio as (6b).

Besides the relativistic quadratic mass structure of the NPCAC current quark formulas (6), a second natural feature of NPCAC is the weak binding current quark mass scale in (6b), i.e., $\hat{m}_{curr} \sim m_\pi/2$. Since the current quark mass scale is set for p^2 above the confinement region of 300-500 MeV (i.e., at $p^2 = (2\ \mathrm{GeV})^2$), one can apply perturbative QCD for which $\alpha_s(p^2)$ is small and weak binding is therefore an obvious consequence. Given then $\hat{m}_{curr} \sim m_\pi/2$, we make the qualitative chiral symmetry breaking estimates

$$\Delta_{\pi NN}^{NPCAC} \sim (\sigma_{\pi NN}/m_N)^{NPCAC} \sim O(\hat{m}^2_{curr}/\hat{m}^2_{con}) \sim 0.041 \ , \tag{7}$$

which are between two and three times the SPCAC estimates (3) and more in line with the phenomenological values (2).

In this work we shall attempt to clarify and quantify the above differences between neutral and strong PCAC in the context of QCD. More specifically in Sec. II we review the argument [3,4,12] that in spite of the perturbative quadratic relation $m_\pi^2 \propto \hat{m}^2_{curr}$ in the chiral-broken world, QCD allows NPCAC to be consistent with nonperturbative spontaneous symmetry breakdown in the chiral limit. Then in Sec. III we review the quantitative theoretical and phenomenological determinations of the current quark mass ratio and suggest that only NPCAC provides a consistent picture. Next in Sec. IV we examine the SPCAC and NPCAC derivations of the current quark mass scale as set by m_π and independently by the low-lying 0^-, $1/2^+$, 1^-, $3/2^+$ hadron mass spectrum. Again only the NPCAC masses $\hat{m}_{curr} \approx 62$ MeV, $m_{s,curr} \approx 310$ MeV are compatible with phenomenology. In Sec. V we independently investigate the *momentum dependence* of the constituent, current and dynamically generated quark masses and show that the flavor independence of the latter leads uniquely to the NPCAC current quark masses. Finally in Sec. VI we formulate an SU(3) x SU(3) alternative to chiral perturbation theory based upon (NPCAC) perturbative weak binding in QCD rather than on very small (SPCAC) current quark masses.

II. Compatibility of Neutral PCAC with Spontaneous Symmetry Breakdown

The greatest drawback to the NPCAC quadratic mass dependence $m_\pi^2 \propto \hat{m}^2_{curr}$ is the SPCAC assumption (1a), the latter having its rigorous origins in the vacuum sum rule [1]

$$-\hat{m}_{curr} \langle 0|\bar{u}u + \bar{d}d|0\rangle = f_\pi^2 m_\pi^2 + i\hat{m}^2_{curr} \int d^4x\, e^{iq\cdot x} \langle 0|T(v^\pi(x),v^\pi(0))|0\rangle \,, \quad (8)$$

where $v^\pi = \bar{q}\,\lambda^3\,\gamma_5\,q$. Then if $\hat{m}_{curr}$ were "small enough" (we believe this is *not the case* in the real world), the background integral could be neglected, leading to (1a) along with the "NPCAC fiasco" $\langle 0|\bar{q}q|0\rangle \propto \hat{m}_{curr}$ which appears to violate the spontaneous symmetry breakdown condition $\langle\bar{q}q\rangle_o \neq 0$ in the chiral limit.

However it is QCD which saves the day and voids the above argument. What is important to stress in the fundamental chiral-breaking relations (4) is the renormalization M-dependence of the RHS. Once the corresponding M-dependence of (5) is established and $\hat{m}(M)$ is understood as the running QCD quark mass which is the sum of perturbative and nonperturbative parts [14]

$$\hat{m}(M) = \hat{m}_{curr}(M) + m_{dyn}(M) \,, \quad (9)$$

then two distinct limits may be achieved:

$$\langle\pi|(\bar{u}u)_M|\pi\rangle \propto \hat{m}(M) \approx \begin{cases} m_{dyn}(M) \text{ in the chiral limit} & (10a) \\ \hat{m}_{curr}(M) \text{ for } M \simeq 2 \text{ GeV} \,. & (10b) \end{cases}$$

Again we repeat that $M \simeq 2$ GeV is chosen high enough to suppress the nonperturbative dynamically generated quark mass $m_{dyn}(p^2)$ in (9) which in QCD falls off rapidly for large p^2 — like [14] $m_{dyn}(p^2) \propto (p^2)^{-1}\,(\ell n p^2)^{d-1}$ with $d = 1/2$. On the other hand the perturbative current quark masses *fall off* much slower as [15] $\hat{m}_{curr}(p^2) \propto (\ell n p^2)^{-d}$ so that (10b) holds in the high M perturbative region as expected.

Then in the chiral-broken world where (10b) is operative, (4a) recovers the NPCAC mass dependence

$$m_\pi^2 \propto \hat{m}_{curr}(M)\,\langle\pi|(\bar{u}u)_M|\pi\rangle \propto \hat{m}^2_{curr}(M) \text{ for } M \simeq 2 \text{ GeV} \,, \quad (11a)$$

while near the chiral limit (4a) combined with (10a) leads to

$$m_\pi^2 \propto \hat{m}_{curr}(M)\,\langle\pi|(\bar{u}u)_M|\pi\rangle \propto \hat{m}_{curr}(M)\, m_{dyn}(M) \,. \quad (11b)$$

Thus the QCD M-dependence and (9) allows us to achieve the quadratic current quark mass dependence of (11a) while also formally obeying the linear current quark mass dependence (11b). The latter result is precisely what is needed so that the vacuum sum rule (8) is satisfied in the chiral limit with $\langle\bar{q}q\rangle_o \neq 0$.

The final step in the argument is to remark that the NPCAC quadratic current quark mass dependence in (8) is recovered not by artificially demanding $\langle 0|\bar{u}u|0\rangle \propto \hat{m}_{curr}$ on the LHS of (8), but instead follows naturally from the $\hat{m}^2_{curr}$ dependence of the RHS background integral. This is not a violation of PCAC in (4), because the PCAC background corrections are expected to behave like

$$m_\pi^2 = -f_\pi^{-2} \langle 0|[Q_5^\pi,[Q_5^\pi,\mathcal{H}]]|0\rangle + O(\hat{m}^2_{curr}(M)/\hat{m}^2(M)) \,. \tag{12}$$

While the first current algebra term in (12) corresponds to the LHS of (8), the second PCAC correction can be identified with the background term in (8) and cannot be neglected because it is of order unity at $M \simeq 2$ GeV where $\hat{m}(M) \simeq \hat{m}_{curr}(M)$.

The above NPCAC line of reasoning preserves the integrity of the fundamental relations (4) and (12) along with the spontaneous breakdown condition $\langle\bar{q}q\rangle_o \neq 0$. However it does suggest that the SPCAC "identity" (1a) is only an assumption which is in fact incorrect if indeed $\langle\pi|(\bar{q}q)_M|\pi\rangle \propto \hat{m}(M)$. Since $\hat{m}(M) \simeq \hat{m}_{curr}(M)$ at $M \simeq 2$ GeV, one cannot expect to determine this "small" current quark mass via a (chiral) mass perturbation expansion when the "large" mass scale is itself $\hat{m}(M)$.

Turning the argument around, we see no fundamental reason why the SPCAC relation linking the perturbative chiral symmetry breaking parameters m_π^2 and $\hat{m}_{curr}$ to the spontaneous breakdown order parameter $\langle\bar{q}q\rangle_o$ has any validity. Rather, QCD only relates the various order parameters: m_{dyn} to f_π and m_{dyn} to $\langle\bar{q}q\rangle_o$. Connecting any of these three, however, to perturbative current masses as an identity may be beyond the realm of QCD or any other chiral-invariant field theory.

III. Current Quark Mass Ratio

The central issue for the perturbative current quark masses therefore has nothing to do with nonperturbative spontaneous symmetry breakdown. Rather it is to reaffirm or deny the NPCAC flavor dependence of $\langle\pi|\bar{q}q|\pi\rangle$ and $\langle K|\bar{q}q|K\rangle$ both theoretically in QCD and phenomenologically from experiment. There are three ways to reaffirm this flavor dependence, and all are related to the quark mass behavior of $\langle\pi|(\bar{q}q)_M|\pi\rangle \propto \hat{m}(M)$:

a) Light Plane-Parton model

The quark density $\bar{q}q$ is a "bad" opreator on the light plane [16]. In terms of good and bad light plane wave functions it can be expressed as $\bar{q}q \sim \phi^\dagger \chi + \chi^\dagger \phi$. Employing the Dirac equation to eliminate the bad fields χ, the quark mass m then appears as

$$\bar{q}q = m\, \phi^\dagger\, \nabla_-^{-1}\phi + \text{flavor-dependent spin flip terms.} \tag{13}$$

Thus the (spin non-flip) pseudoscalar matrix elements of (13) receive no contribution from the spin flip terms and $\langle\pi|\bar{q}q|\pi\rangle \propto \hat{m}$. The parton model (i.e., infinite momentum frame physics) also scales the hadron matrix elements of (bad) quark-parton operators with this additional power of quark mass [9,17].

Another way to motivate (13) is via the mismatch between the hadron matrix elements of vector and axial-vector charges [3,9]. Single particle hadron states transform simply on the light plane [18] with $Q^i_{5LP}|0\rangle = 0$ and Q^i_{5LP}|single hadron> $\propto$ |single hadron>. On the other hand, spontaneous breakdown of chiral symmetry involves the static axial charges in the (SU(2)) Goldstone condition $Q^i_5|0\rangle = (-i/2)f_\pi|\pi^i\rangle$. This mismatch is resolved by expressing bad quark operators such as $\bar{q}q$ back in terms of good two component fields ϕ (according to which single hadron states transform irreducibly), as in (13).

b) QCD quark loop

The QCD quark loop involving running quark masses is depicted in Fig. 1. Since $\mathrm{Tr}(\not{p}+\hat{m})^2\,(\not{p}-\hat{m}) = \hat{m}(p^2-\hat{m}^2)$, it is clear in the soft pion limit that [4]

$$\langle\pi|(\bar{q}q)_M|\pi\rangle \propto \int^M \frac{d^4p\ \mathrm{Tr}[(\not{p}+m)^2\ \gamma_5(\not{p}+\hat{m})\gamma_5]}{(p^2-\hat{m}^2)^3} \propto \hat{m}(M)\ . \tag{14}$$

Even with $\hat{m} \to \hat{m}(p^2)$, the integral over p in (14) scales out $\hat{m}(M)$, where M is the QCD renormalization point mass implied on the LHS of (14). Clearly it is the mass of the quark propagators in Fig. 1, i.e., the total quark mass (9) which appears in (14) (and in (13)).

c) Dimensional analysis

The matrix element $\langle\pi|\bar{q}q|\pi\rangle$ has the dimensions of mass as does the (nonstrange) flavor-dependent quark mass and the flavor-independent meson decay constant f_π and quark condensate $\langle\bar{q}q\rangle_0^{1/3}$. We suggest that the natural dimensional link is the flavor-dependent NPCAC relation $\langle\pi|\bar{q}q|\pi\rangle \propto \hat{m}$ rather than the flavor-independent SPCAC relation $\langle\pi|\bar{q}q|\pi\rangle \propto \langle\bar{q}q\rangle_0/f_\pi^2$ if only because the term $\hat{m}\,\bar{q}q$ breaks the chiral symmetry in the QCD lagrangian and $\hat{m}$ is the natural dimension-one dynamical mass scale. A possible flavor-independence of $\langle\pi|(\bar{q}q)_M|\pi\rangle$ for (SPCAC) small values of M and p^2 in the loop integral (14) depends upon the intricate details of <u>infrared confinement</u> in QCD and even then the SPCAC "identity" $\langle\pi|\bar{u}u|\pi\rangle = -\langle\bar{u}u\rangle_0/f_\pi^2$ is yet to be verified.

Turning then to the implications of the (NPCAC) flavor-dependence of $\langle\pi|\bar{q}q|\pi\rangle$ and $\langle K|\bar{q}q|K\rangle$, one obtains in the chiral-broken world from (4) and (5) the quadratic mass formulae (6a) and the current quark mass ratio of (6b), $(m_s/\hat{m})_{curr} \approx 5$ as found from the <u>pseudoscalar</u> mass spectrum. Independent tests of this ratio come from the <u>baryon</u> mass spectrum coupled with the observed πN σ-term or the πNN Goldberger-Treiman discrepancy. However one must be careful then <u>not</u> to assume the SPCAC SU(3) transformation properties for $\bar{q}\,\lambda_i\,q$ [9].

Looking first at the πN σ-term, the nucleon matrix elements of the operator identity $\sigma = \hat{m}_{curr}\,(\bar{u}u + \bar{d}d)$ can be combined with the (quadratic) baryon matrix elements of the semi-strong hamiltonian density $\mathcal{H} = \mathcal{H}_0 + \hat{m}_{curr}\,(\bar{u}u + \bar{d}d) + m_{s,curr}\,\bar{s}s$ and the Zweig (quark line) rule $\langle N|\bar{s}s|N\rangle = 0$ to give $m_N^2 = m_0^2 + 2m_N\,\sigma_{\pi N}$ and

$$(m_s/\hat{m})_{curr} = \left[\frac{3\Delta m_N^2}{m_N\,\sigma_{\pi N}} + 1\right]^{1/2} = 4.8 \pm 0.2\,, \qquad (15)$$

where $\Delta m_N^2 \approx 0.46\ \mathrm{GeV}^2$ is the nucleon mass shift from the SU(3) value of $(1.158\ \mathrm{GeV})^2$ and $\sigma_{\pi N} = 65 \pm 5$ MeV is the phenomenologically deduced σ-term [6]. Stated in terms of the old (linear) chiral-breaking language of [1] $\mathcal{H} = \mathcal{H}_0 + u_0 + c u_8$, then $\sigma_{\pi N} = (\sqrt{2}+c)\,(\sqrt{2}\,u_0 + u_8)_N/3 = 3\hat{m}_{curr}\,(u_8)_N$ and the linear version of (15) is recovered. Only if $\sigma_{\pi N}$ were ~ 25 MeV would the SPCAC current quark mass ratio of 25 be obtained. Alternatively the quark model Zweig rule would have to be violated as $\langle N|\bar{s}s|N\rangle / \langle N|\bar{u}u|N\rangle \sim 0.7$ to recover the SPCAC ratio of 25 from this analysis. We regard these possibilities as highly unlikely. Needless to say, without tampering with the phenomenology, the deduced ratio (15) is consistent with the theoretical NPCAC prediction.

Note that the qualitative arguments involving the current quark mass scale (3) and (7) and the quantitative formula (15) for the current quark mass ratio both suggest that the NPCAC πN σ-term should be slightly more than twice the SPCAC σ-term. This is a consequence of the quadratic quark mass feature of NPCAC formulae vs. linear SPCAC formulae (i.e., $\Delta m_N^2 = 2m_N \Delta m_N$ vs. Δm_N). The fact that $\sigma_{\pi N}$ appears to be over twice the SPCAC value is only confirming that perturbative current quark mases must be viewed from the (quadratic mass formula) infinite momentum frame as in NPCAC.

The second baryon measure of the current quark mass ratio follows from the pion-nucleon Goldberger-Treiman discrepancy. Consider the tadpole-dominance mechanism [19] of Fig. 2 which immediately leads to [8]

$$f_\pi g_{\pi NN}\, \Delta_{\pi NN} = -\, \hat{m}_{curr}\, \bar{v}_{\pi NN}\ , \tag{16}$$

where $\langle N|\bar{q}\gamma_5\lambda_3 q|N\rangle_{nonpole} = \bar{v}_{\pi NN}\ \bar{N}\tau_3\gamma_5 N$ is the non-pole background pseudoscalar density and (16) properly indicates that the Goldberger-Treiman relation becomes exact ($\Delta_{\pi NN} \to 0$) in the chiral limit $\hat{m}_{curr} \to 0$. Having removed the pion pole term in (16), the $\bar{v}_{\pi NN}$ can be generalized to include its SU(3) partners so that it can be eliminated in (16) in favor of $m_{s,curr}$. Then ignoring the SU(3) breaking in the very insensitive ratio $(-\sqrt{3}\ g_{KN\Lambda} + g_{KN\Sigma})/2g_{\pi NN} \simeq 1$ and likewise in the analog axial-vector ratio, (16) can be converted to [8]

$$\left(\frac{m_s}{\hat{m}}\right)_{curr} = \frac{2}{\Delta_{\pi NN}} \left[\frac{f_K}{f_\pi} - \left(1 - \Delta_{\pi NN}\right)\frac{m_\Sigma + m_\Lambda + 2m_N}{4m_N}\right] - 1 \simeq 5 \tag{17}$$

for $\Delta_{\pi NN} \simeq 0.06$ and $f_K/f_\pi \simeq 1.19$.

To reconfirm this prediction (17), we may account for the SU(3) breaking in the BBP coupling constants by accepting the phenomenologically determined values [6,20]

$$\frac{g^2_{\pi NN}}{4\pi} = 14.3\pm0.1,\quad \frac{g^2_{\pi\Sigma\Lambda}}{4\pi} = 11.8\pm2.0,\quad \frac{g^2_{\pi\Sigma\Sigma}}{4\pi} = 12.5\pm2.0,\quad \frac{g^2_{KN\Lambda}}{4\pi} = 13.9\pm2.0 \tag{18}$$

along with the axial-vector d to f ratio obtained from semileptonic weak decays of $(d/f)_A \simeq 1.8$ and $g^A_{NN} \simeq 1.254$. This leads to the GT-discrepancy

$$\Delta_{KN\Lambda} = 1 - \frac{1}{2}(m_\Lambda + m_N)g^A_{\Lambda N}/f_K g_{KN\Lambda} = 0.13 \pm 0.07. \qquad (19)$$

Combining this result with (16) and its kaon analog then leads to the current quark mass ratio

$$\left(\frac{m_s}{\hat{m}}\right)_{curr} = \frac{2f_K g_{KN\Lambda}\,\Delta_{KN\Lambda}\,\bar{v}_{\pi NN}}{f_\pi g_{\pi NN}\,\Delta_{\pi NN}\,\bar{v}_{KN\Lambda}} - 1 = 3.9 {+4.9 \atop -3.1}. \qquad (20)$$

Note that once the meson tadpole term has been removed from $\langle B_i|v_j|B_k\rangle$, as in Fig. 2, then the background matrix elements can be assumed to transform simply via SU(3) with the quark model value $(d/f)_{\bar{v}} = 3/2$ so that $\bar{v}_{KN\Lambda}/\bar{v}_{\pi NN} = -3\sqrt{3}/5$ in (20).

From either (17) or (20) we reaffirm that the baryon GT discrepancies are consistent with the NPCAC meson value of $(m_s/\hat{m})_{curr} \simeq 5$. As in the qualitative estimate (3), only if $\Delta_{\pi NN}$ were ~1% would the SPCAC current quark mass ratio of ~25 be recovered from the quantitative ratios (17) or (20).

IV. Current Quark Mass Scale

Unfortunately there is no clean phenomenological determination of the current quark mass scale, e.g., the magnitude of the nonstrange current quark mass. This is not surprising, however, because the fundamental perturbative current quark mass is not fixed in the determination of m_π^2 but instead is renormalization M-dependent in QCD,

$$2m_\pi^2 = \langle\pi|\mathcal{H}'|\pi\rangle = 2\hat{m}_{curr}(M)\,\langle\pi|(\bar{u}u)_M|\pi\rangle\,. \qquad (21)$$

As $M \to \infty$, $\hat{m}_{curr}(M)$ vanishes according to the deep euclidean QCD behavior [15]

$$m^i_{curr}(p^2) = m^i_{curr}(M^2)\left(\frac{\ln M^2/\Lambda^2}{\ln p^2/\Lambda^2}\right)^d \qquad (22)$$

for quark flavor index i, where $d = 12(33-2n_f)^{-1} = 1/2$. Extracating the additional NPCAC quark mass from $\langle\pi|(\bar{u}u)_M|\pi\rangle$, we define $\tilde{h}(M)$ via

$$\langle\pi|(\bar{u}u)_M|\pi\rangle \equiv 2\hat{m}(M)\tilde{h}(M) \qquad (23)$$

and then for $M = 2$ GeV, (21) gives

$$m_\pi^2 = 2\hat{m}_{curr}(M)\hat{m}(M)\bar{h}(M)$$
$$= 2\hat{m}^2_{curr}(M)\bar{h}(M) \ . \qquad (24)$$

Thus as $M \to \infty$, we have $m^2_{curr}(M) \sim (\ln M)^{-1} \to 0$ so that $\bar{h}(M) = (\ln M) \to \infty$ such that the product in (24) is the renormalization-independent value m_π^2.

The above observations suggest that the magnitude of $\hat{m}_{curr} = \hat{m}_{curr}$ (2 GeV) can be set by the scaling integral over the pion structure function h(x):

$$\bar{h}(M) = \int_0^1 \frac{dx}{x} h_M(x) \quad \text{with} \quad \int_0^1 h_M(x)dx = 1 \ . \qquad (25)$$

Clearly as $M \to \infty$, the nonvanishing quark distribution at $x \simeq 0$ forces $\bar{h}$ to diverge in (25) as anticipated in (24). However for M "small enough" at $M = 2$ GeV, the $x \simeq 0$ quark pairs and sea are suppressed so that $\bar{h}$(2 GeV) converges in (25). The QCD dependence [21] of Fig. 3(a), $h(x) \sim (1-x)^2$ as $x \to 1$, should then be symmetrized as $h_M(x) = 30x^2(1-x)^2$ which leads to the structure function scale [10]

$$\bar{h}(2 \text{ GeV}) = 30 \int_0^1 x(1-x)^2 dx = \frac{5}{2} \ . \qquad (26)$$

Note that (26) is near the weak binding structure function ($\delta(x - 1/2)$) limit of Fig. 3(b),

$$\bar{h}_{WBL} = \int_0^1 \frac{dx}{x} \delta\left(x - \frac{1}{2}\right) = 2 \ . \qquad (27)$$

An alternative but equivalent approach is to compute $\bar{h}(M)$ directly from the QCD triangle graph of Fig. 1 and (14). Reinstating the constant factors including the color factor of 3 and the pion-quark coupling constant [3] $g_{\pi qq} = \hat{m}_{con}/f_\pi \simeq 3.6$, we see that (14) becomes at $q = 0$ and $M = 2$ GeV [4] ,

$$\langle\pi|(\bar{u}u)_M|\pi\rangle = -ig^2_{\pi qq}\ \frac{2\cdot3\cdot4}{(2\pi)^4}\int_{1\ \mathrm{GeV}}^{M}\frac{d^4p\ \hat{m}_{curr}(p^2)}{[p^2-\hat{m}^2_{curr}(p^2)]^2} \propto \hat{m}_{curr}(M)\ \ln M^2, \quad (28)$$

where the lower bound in (28) corresponds to where the QCD coupling constant is expected to freeze out [22]. We see from (28) that m^2_π in (21) is essentially M independent and moreover the scale of $\tilde{h}(M)$ in (23) is given by [4] (which justifies our dropping the slight p^2 and M dependence of $g_{\pi qq}$ in (28)):

$$\tilde{h}(2\ \mathrm{GeV}) = \frac{3}{4\pi(1-d)}\ g^2_{\pi qq}\ \ln\frac{M^2}{1\ \mathrm{GeV}^2} \simeq 2.7. \quad (29)$$

Light plane wave functions can also be used to find $\tilde{h}$ [4], with roughly the same conclusion, $\tilde{h}(2\ \mathrm{GeV}) \simeq 3.0$. The approximate agreement between (26), (27) and (29) is satisfying and we henceforth accept the value (26).

Finally then we may compute the current quark mass scale from (24) and (26) as [10]

$$\hat{m}_{curr} = m_\pi/(2\tilde{h}(M))^{1/2} \simeq m_\pi/\sqrt{5} = 62\ \mathrm{MeV}\ . \quad (30)$$

Note that (30) is quite close to the <u>weak binding limit</u> as obtained from (24) and (27): $\hat{m}_{curr}$ (WBL) $= m_\pi/2 \simeq 69$ MeV. We suggest that this is not an accident but instead a key feature of <u>perturbative</u> QCD where $\alpha_s(p^2)$ is small for large p^2 (in the $p^2 \simeq (2\ \mathrm{GeV})^2$ region where $\hat{m}_{curr}$ is defined). This result is reflected in the WBL of Fig. 1 and (28), where $\langle\pi|(\bar{u}u)_M|\pi\rangle_{WBL} = 4\ \hat{m}_{curr}(M)$ requires [4]

$$(g_{\pi qq})_{WBL} = [(3/8\pi^2)\ln M^2/1\ \mathrm{GeV}^2]^{-1/2} \simeq 4.4\ , \quad (31)$$

also close to the on-shell value $g_{\pi qq} = \hat{m}_{con}/f_\pi \simeq 3.6$. Moreover (31) is consistent with $g_{\pi NN}/3 \simeq 4.5$, as one might expect.

Just as the current quark mass ratio can be scaled to the pseudoscalar mass spectrum and tested via the nucleon σ-term and baryon GT discrepancies, so too the magnitude of the current quark mass can be scaled to m_π as in (30) and also tested via the baryon mass spectrum and the nucleon GT discrepancy. To obtain the latter results we must also employ structure function scaling integrals which are the baryon analogs of (25)-(27). For $1/2^+$ and $3/2^+$ baryons we write [9]

$$\langle p|(\bar{u}u)_M|p\rangle = 2\hat{m}(M)\bar{f}_u(M)\ , \quad \langle p|(\bar{d}d)_\mu|p\rangle = 2\hat{m}(M)\bar{f}_d(M) \tag{32a}$$

$$\langle\Delta^{++}|(\bar{u}u)_M|\Delta^{++}\rangle = 2\hat{m}(M)\bar{f}_D(M)\ , \tag{32b}$$

with valence normalizations

$$\int_0^1 dx\ f_u(x,M) = 2\ , \quad \int_0^1 dx\ f_d(x,M) = 1\ , \quad \int_0^1 dx\ f_D(x,M) = 3\ . \tag{33}$$

The *corresponding weak binding scaling integrals* $\langle x^{-1}\rangle$ *are* [10]

$$\bar{f}_u^{WBL} = \int_0^1 \frac{dx}{x}\, 2\delta(x - \tfrac{1}{3}) = 6\ , \quad \bar{f}_d^{WBL} = \int_0^1 \frac{dx}{x}\, \delta(x - \tfrac{1}{3}) = 3, \tag{34a}$$

$$\bar{f}_D^{WBL} = \int_0^1 \frac{dx}{x}\, 3\delta(x - \tfrac{1}{3}) = 9\ , \tag{34b}$$

so it is not surprising that the QCD $\langle x^{-1}\rangle$ baryon analogs of (26) are at M = 2 GeV [10],

$$\bar{f}_u = 8\ , \quad \bar{f}_d = 5\ , \quad \bar{f}_D = 12\ . \tag{35}$$

Note that (35) are about 30% greater than the WBL scales (34), just as (26) and (29) are 30% greater than the WBL scale (27).

Given these structure function integrals, one finds for $\hat{m}(M) = \hat{m}_{curr}$ at M=2 GeV

$$2m_N^2 = \langle p|\mathcal{H}|p\rangle = 2m_0^2 + 2\hat{m}^2_{curr}\ (\bar{f}_u + \bar{f}_d) \tag{36}$$

$$\Delta m_N^2 = \frac{1}{3}\ (m_s^2 - \hat{m}^2)_{curr}\ (\bar{f}_u + \bar{f}_d) \tag{37a}$$

$$= (1.158\ \text{GeV})^2 - m_N^2 = 0.46\ \text{GeV}^2. \tag{37b}$$

Although m_0 is unknown in (36) (unless we input the "debated" value of $\sigma_{\pi N}$ = 65 MeV), we can extract the current quark mass scale from the accepted value of

Δm_N^2 in (37) along with (35):

$$(m_s^2 - \hat{m}^2)_{curr} = \frac{3\Delta m_N^2}{(\bar{f}_u+\bar{f}_d)} = 0.106 \text{ GeV}^2 . \tag{38}$$

This scale compares favorably with the decuplet determination [9,10]

$$(m_s^2 - \hat{m}^2)_{curr} = \frac{3}{5\bar{f}_D} [m_\Omega^2 + m_{\Xi^*}^2 - 2m_\Delta^2] = 0.105 \text{ GeV}^2 , \tag{39}$$

and also the pseudoscalar and SU(6) vector determinations

$$(m_s^2 - \hat{m}^2)_{curr} = \begin{cases} (m_K^2 - m_\pi^2)/\bar{h} = 0.091 \text{ GeV}^2 & \text{(40a)} \\ (m_{K^*}^2 - m_\rho^2)/\bar{h} = 0.081 \text{ GeV}^2 . & \text{(40b)} \end{cases}$$

Note that (40a) is equivalent to the NPCAC values $(m_s/\hat{m})_{curr} = 5$ and $\hat{m}_{curr} = 62$ MeV.

We regard the striking agreement between (38)-(40) as the third indication (apart from the observed πN σ-term and GT discrepancy) that NPCAC is realized in nature. One can, in principle, also determine $\hat{m}_{curr}$ directly from the GT discrepancy (16). Aside from the usual $k_\parallel$ scaling integral, the latter determination also depends upon a $k_\perp$ integral which we have not yet been able to evaluate.

Finally one can in principle extract the scale of $\hat{m}_{curr}$ from threshhold pion photoproduction [23]. However present measurements only yield the wide range of values [24] $\hat{m}_{curr} = 70 \pm 70$ MeV, which do not distinguish between strong and neutral PCAC.

With regard to the SPCAC current quark mass scale, given the ratio $(m_s/\hat{m})_{curr} = 25$, one independently invokes the "rule" that $\bar{q}q$ counts constituent quarks of a given flavor. Stated covariantly for proton and rho matrix elements,

$$\langle p|\bar{u}u|p\rangle|_{SPCAC} = 2\hat{m}_{con}\cdot 2 = 1.36 \text{ GeV} \tag{41a}$$

$$\langle p|\bar{d}d|p\rangle|_{SPCAC} = \langle \rho|\bar{u}u|\rho\rangle = 2\hat{m}_{con}\cdot 1 = 0.68 \text{ GeV} , \tag{41b}$$

presumably at the mass scale M ~ 1-2 GeV. These numbers are roughly the order of our (more accurate) NPCAC scaling integral formulation at M = 2 GeV:

$$\langle p|(\bar{u}u)_M|p\rangle = 2\hat{m}_{curr}(M)\bar{f}_u(M) = 2(0.062\ \text{GeV})\cdot 8 = 0.99\ \text{GeV} \quad (42a)$$

$$\langle p|(\bar{d}d)_M|p\rangle = 2\hat{m}_{curr}(M)\bar{f}_d(M) = 2(0.062\ \text{GeV})\cdot 5 = 0.62\ \text{GeV} \quad (42b)$$

$$\langle p|(\bar{u}u)_M|p\rangle = 2\hat{m}_{curr}(M)\bar{n}(M) = 2(0.062\ \text{GeV})\cdot 5/2 = 0.31\ \text{GeV}\,. \quad (42c)$$

It is at this point that SPCAC and NPCAC touch base up to a factor of 2. Consequently the SPCAC splitting law [1] of ~140 MeV per strange quark in hadrons may be only a factor of 2 too low from (41b) vs. (42c) and the NPCAC scale of $m_{s,curr} \approx 5\cdot(62\ \text{MeV}) \approx 310$ MeV. However the SPCAC scale of $\hat{m}_{curr} \approx$ 140 MeV/5 ≈ 5 MeV is definitely not consistent with the NPCAC scale of $\hat{m}_{curr} \approx$ 62 MeV, presumably because of the quadratic nature of the perturbative chiral symmetry breaking relation $m_\pi^2 \propto \hat{m}^2_{curr}$.

Independent of these remarks, there is a vector dominance derivation of $\hat{m}_{curr} \approx 5$ MeV in the context of SPCAC [25]. Neutralizing this, however, is an analog derivation of $\hat{m}_{curr} \gtrsim 40$ MeV in the context of NPCAC [26].

V. Flavor Independence of Chiral Quark Wave Functions and Masses

The chiral bound meson-quark (light-plane) wave function, Goldberger-Treiman coupling $g_{\pi qq}$ or dynamically generated quark mass m_{dyn} are all expected to be flavor independent because they are created by flavor independent gluon exchanges. Thus the accepted SU(3) flavor-broken constituent quark masses [27]

$$\hat{m}_{con} = 340\ \text{MeV} \qquad m_{s,con} = 510\ \text{MeV} \quad (43)$$

can be used to predict other SU(3) flavor-broken quantities.

First consider the meson decay constants apearing in the GT relations at the quark level

$$f_\pi g_{\pi qq} = \hat{m}_{con}\,, \qquad f_K g_{Kqq} = \frac{1}{2}\,(m_s + \hat{m})_{con}\,, \quad (44)$$

so that flavor-independent couplings $g_{\pi qq} = g_{Kqq}$ and (44) lead to the SU(3)-breaking prediction [3,28]

$$\frac{f_K}{f_\pi} = \frac{1}{2}\left[\left(\frac{m_s}{\hat{m}}\right)_{con} + 1\right] = 1.25 \tag{45}$$

for $(m_s/\hat{m})_{con} = 1.5$ as found from (43). Likewise for the mixed η and η' pseudoscalar states defined relative to the quark basis as $|\eta\rangle = \cos\phi_P|\eta_{NS}\rangle - \sin\phi_P|\eta_S\rangle$ with the phenomenological value [29] $\phi_P = 42^\circ$, the analog ratio to (45) which becomes unity as $(m_s/\hat{m})_{con} \to 1$ is

$$\frac{f_\eta}{f_\pi} = \left[\sin^2\phi_P\left(\frac{m_s}{\hat{m}}\right)_{con} + \cos^2\phi_P\right] = 1.22 \; . \tag{46}$$

Both of the predicted ratios (45) and (46) are quite close to the observed values of $f_K/f_\pi = 1.19$ and [29] $f_\eta/f_\pi = 1.18 \pm 0.09$.

For our purposes a more significant example of flavor independence is associated with the dynamically generated quark mass, which in the deep euclidean region obeys the QCD formula [14]

$$m_{dyn}(p^2) = m_{dyn}(M^2)\,\frac{M^2}{p^2}\left(\frac{\ln M^2/\Lambda^2}{\ln p^2/\Lambda^2}\right)^{1-d} . \tag{47}$$

The momentum dependence of (47) is employed in the chiral-limiting calculation of f_π, leading to the successful prediction [2,4,5] $g_{\pi qq} = 2\pi/\sqrt{3} = 3.6$. Such a computation makes use of the presently deduced small QCD energy scale [30]

$$\Lambda = 150 \pm 50 \text{ MeV} , \tag{48}$$

which suggests that we can apply the deep euclidean formulae (22) and (47) down to the constituent quark mass region (43) with still $p^2 \gg \Lambda^2$.

In particular, in the nonstrange quark sector

$$\hat{m}(p^2) = \hat{m}_{curr}(p^2) + m_{dyn}(p^2) , \tag{49}$$

we take the on-shell condition [3,31] $\hat{m}(p^2 = \hat{m}^2_{con}) = \hat{m}_{con} \simeq 340$ MeV along with the NPCAC mass scale $\hat{m}_{curr}$ (p = 2 GeV) = 62 MeV. The latter scales up according to (47) as a logarithm until the freeze out at $p^2 \sim 1\ \mathrm{GeV}^2$ so that $\hat{m}_{curr}$ (p = 340 MeV) = 72 MeV. Then from (49) we deduce that

$$m_{dyn}(p = 340\ \mathrm{MeV}) = 340\ \mathrm{MeV} - 72\ \mathrm{MeV} = 268\ \mathrm{MeV}\ . \tag{50}$$

Since below $p^2 \simeq 1\ \mathrm{GeV}^2$ the QCD coupling freezes out, the p^2 dependence of (47) becomes $m_{dyn}(p^2) \propto 1/p^2$ and (50) then requires the on-shell value [3-5]

$$m_{dyn}(p = 314\ \mathrm{MeV}) = m_{dyn} = 314\ \mathrm{MeV}\ . \tag{51}$$

On the other hand, in the strange quark sector

$$m_s(p^2) = m_{s,curr}(p^2) + m_{dyn}(p^2)\ , \tag{52}$$

so that combining $m(p^2 = m^2_{s,con}) = m_{s,con} \simeq 510$ MeV with the NPCAC strange current mass $m_{s,curr}$ (3 GeV) = 310 MeV which scales up via (22) to m_s(p = 510 MeV) = 386 MeV, we find from (52)

$$m_{dyn}(p = 510\ \mathrm{MeV}) = 510\ \mathrm{MeV} - 386\ \mathrm{MeV} = 124\ \mathrm{MeV}\ . \tag{53}$$

Again employing $m_{dyn}(p^2) \propto 1/p^2$ in this region, (53) scales up to the on-shell value [3-5]

$$m_{dyn}(p = 318\ \mathrm{MeV}) = m_{dyn} = 318\ \mathrm{MeV}\ . \tag{54}$$

The fact that (51) and (54) are essentially the same mass verifies the flavor independence of m_{dyn}. Stated in reverse, there are seven other ways to determine m_{dyn} and all lead to [5] $m_{dyn} \simeq 315$ MeV. Combining the latter mass with the constituent masses (43) then uniquely reproduces the NPCAC masses, $\hat{m}_{curr} \simeq 62$ MeV and $m_{s,curr} \simeq 310$ MeV. This QCD-inspired analysis is completely independent of the light plane and scaling computations which produced the NPCAC current masses in Secs. II-IV.

VI. Chiral Perturbation Theory vs. the Weak Binding Limit

The quark model manifests the $(3,\bar{3}) + (\bar{3},3)$ equal time algebra of (bad) current density operators [32] $u_i = \bar{q}\,\lambda_i\,q$ and $v_i = \bar{q}\,\lambda_i\,\gamma_5\,q$:

$$[Q_i,u_j] = if_{ijk}\, u_k\,, \qquad [Q_i,v_j] = if_{ijk}\, v_k \tag{55a}$$

$$[Q_i^5,u_j] = -id_{ijk}\, v_k\,, \qquad [Q_i^5,v_j] = id_{ijk}\, u_k\,. \tag{55b}$$

Realizing this algebra in practice, however, is a different matter because it requires use of the SPCAC chain rule for the pseudoscalar matrix elements:

$$\langle P_i|u_j|P_k\rangle \to (-i/f_i)\,\langle 0|[Q_i^5,u_j]|P_k\rangle \to (1/f_i\cdot f_k)\,\langle 0|[[Q_i^5,u_j],Q_k^5]|0\rangle\,. \tag{56}$$

This chain rule in turn would make sense as a "chiral perturbation theory" [33] _if_ the nonstrange current quark mass were very small, $\hat{m}_{curr} \leq 5$ MeV, i.e., if [1] SU(3) x SU(3) would break down to SU(2) x SU(2) rather than to SU(3).

In Secs. II-V we have suggested that $\hat{m}_{curr}$ is _not_ small enough to support the SPCAC chain rule (56). Once $\hat{m}_{curr}$ is greater than ~7 MeV, then the QCD renormalization point quark mass dependence means that in the perturbative region M = 2 GeV the approximate equality

$$\hat{m}(M) \simeq \hat{m}_{curr}(M) \quad \text{at } M = 2 \text{ GeV} \tag{57}$$

holds. This equality means that PCAC corrections are of order _unity_ and SPCAC therefore cannot be trusted. More specifically the Heisenberg equation of motion $i\partial\cdot J_5 = [Q_5, \mathcal{H}]$ manifests the first step in the SPCAC chain rule (56) (at least for j = 0,3,8):

$$\langle P_i|u_j|P_k\rangle = (-i/f_i)\,\langle 0|[Q_i^5,u_j]|P_k\rangle = -d_{ijk'}\,\langle 0|v_{k'}|P_k\rangle\,. \tag{58}$$

However the quark mass (flavor) dependence of the bad operators u and v as in (13) or (14) destroys the simple SU(3) transformation properties: $\langle 0|v_{k'}|P_k\rangle \not\propto \delta_{kk'}$ and $\langle P_i|u_j|P_k\rangle \not\propto d_{ijk}$. Alternatively the second step in the PCAC chain (56) is _invalid_ because of (57); otherwise $\langle 0|v_{k'}|P_k\rangle \propto \delta_{k'k}$ and $\langle P_i|u_j|P_k\rangle \propto d_{ijk}$ would hold and the SPCAC current quark masses would follow.

What current quark masses are consistent with the perturbative QCD condition (57)? Scaling the dynamically generated quark mass from M = 315 MeV to the perturbative region M = 2 GeV, we find from (47), d = 0.48 and Λ = 150 MeV that

$$m_{dyn}(M = 2\text{ GeV}) = \frac{m_{dyn}^3}{M^2}\left(\frac{\ln 1\text{ GeV}^2/\Lambda^2}{\ln M^2/\Lambda^2}\right)^{1-d} = 7\text{ MeV} . \qquad (59)$$

This means that for $\hat{m}_{curr}$ (2 GeV) >> 7 MeV, the QCD perturbative condition and NPCAC current quark masses are valid. Alternatively if $\hat{m}_{curr}$ (2 GeV) << 7 MeV, then (57) is invalid and the SPCAC chain rule and current quark masses hold. The phenomenological 6-7% σ-term $\sigma_{\pi N}/m_N$ and GT discrepancy $\Delta_{\pi NN}$ are much too large to support the SPCAC assumption $\hat{m}_{curr}$ (2 GeV) << 7 MeV. On the other hand, these observed 6% values follow naturally from the perturbative QCD condition (57) and neutral PCAC.

What then is the NPCAC replacement for "chiral perturbation theory" and the SPCAC chain rule (56)? Along with the perturbative QCD condition (57) and the relativistic quadratic mass dependence $m_\pi^2 \propto \hat{m}_{curr}^2$, N. Fuchs and I suggest that the key physical concept underlying NPCAC is the weak binding limit. Not only is this notion compatible with an asymptotically free decreasing coupling α_s (p^2) which admits to perturbative QCD dynamics at M = 2 GeV, but the bound pion-quark coupling constant $g_{\pi qq} = \hat{m}_{con}/f_\pi \approx 2\pi/\sqrt{3} = 3.6$ is close to the WBL (31), i.e., $(g_{\pi qq})_{WBL} \approx 4.4$ and also $g_{\pi NN}/3 \approx 4.5$. More to the point for current quark masses, the QCD $\langle x^{-1}\rangle$ scaling value $\bar{h} = 5/2$ or equivalently the GT or light plane wave function values $\bar{h}$ (2 GeV) = 2.7-3.0 are quite near the weak binding limit $\bar{h}_{WBL} = 2$. This in turn forces the NPCAC current quark mass $\hat{m}_{curr} \approx 62$ MeV to be very close to the WBL $m_\pi/2 = 69$ MeV and likewise $(m_s + \hat{m})_{curr} \approx 370$ MeV to be moderately near $m_K \approx 495$ MeV.

Almost by definition then, perturbative QCD requires the chiral symmetry breaking mass of the pion to be approximately the sum of the perturbative nonstrange valence current quark masses -- if for no other reason than at M = 2 GeV both $m_{dyn}(M)$ and the quark sea and glue are suppressed. This weak binding model for the perturbative current quark masses of the pseudoscalar mesons must be superimposed upon the nonperturbative, strongly bound constituent quark, massless meson states. Together this relativistic picture for the 0^- mesons complements the standard SU(6) nonrelativistic picture of baryons and heavy 1^- mesons composed of weakly bound constituent quarks [27]. Moreover, as stressed in Sec. II, for $\hat{m}_{curr}$ >> 7 MeV, the perturbative QCD condition (57) and $m_\pi^2 \propto \hat{m}_{curr}^2$ are compatible with the spontaneous breakdown of chiral symmetry and $\langle\bar{q}q\rangle_0 \neq 0$.

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Figure Captions

Fig. 1. QCD triangle diagram for $\langle\pi^+|(\bar{q}q)_M|\pi^+\rangle$. The darkened circles on the internal quark lines represent gluon-dressed quark masses.

Fig. 2. Removal of the pion tadpole from the nucleon matrix elements of the pseudoscalar quark density operator $v_\pi = \bar{q}\,\tau_3\gamma_5\,q$.

Fig. 3. Pion structure function graph for (a) leading order QCD and (b) weak binding limit.

Fig. 1

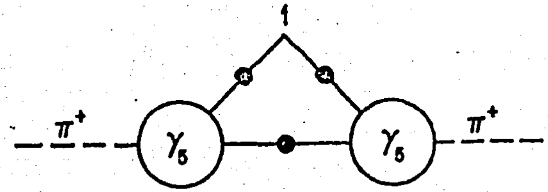


Fig. 2

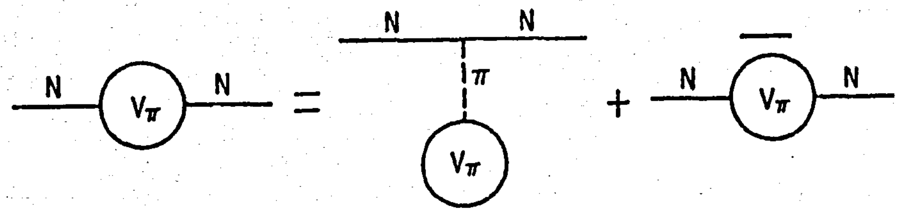
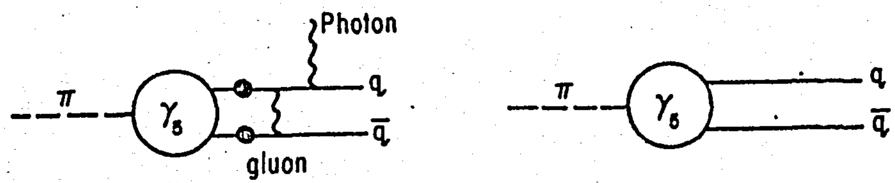


Fig. 3



(a) (b)