I shall discuss the possibility of generalizing the 't Hooft's \[1\] decoupling conditions to the case of hypercolour theories with nonreal preon content. The problem arises if one attempts to construct models with massless composite fermion states. These are interesting physically because if the leptons or quarks are composite objects, they must be very light with respect to the confinement scale of their constituents.

The models are constructed as follows. Consider a gauge theory with local $G_c$ symmetry /the hypercolour/ and a set of massless elementary fermions /the "preons"/. The hypercolour is confined and the physical states are $G_c$ singlets. The theory has a global chiral symmetry $G_p$. Assume that $G_p$ is not broken by confinement. Then it is also present in the physical spectrum and keeps some of the composite fermions massless.

This is contrary to what happens in QCD and one may wonder whet-
ther the assumption in consistent. The problem has been investigated by 't Hooft, who has formulated two requirements to be satisfied by massless composite fermion spectrum. These are: the anomaly requirement, which states that the $J_3$ triangle anomaly must be the same for preons and for compositors, and the decoupling condition, which requires that when any of the originally massless preons is given a hypercolour invariant mass, the remaining chiral symmetry must not prevent any composite containing this preon from becoming massive, so that it may decouple in the infinite preon mass limit.

The 't Hooft's constraints are very restrictive and in many cases no solutions exist. However, they are less restrictive when we assume that the preons (which are all taken to be left-handed) are in a nonreal reducible hypercolour representation. Then some of preon mass terms are forbidden by the hypercolour symmetry and there are no corresponding 't Hooft's decoupling constraints. If the preon content contains no real subset of preons, the $G_c$ symmetry forbids all masses and the 't Hooft's constraints reduce to the anomaly equations only. But are these conditions complete?

I would like to examine here a possible generalization of the 't Hooft's decoupling condition, which takes into account the possibility of spontaneous breaking of the $G_c$ symmetry. If $G_c$ is broken, some of the originally forbidden preon mass terms shall be allowed by the remaining symmetry. The symmetry breaking may be dynamical (by a preon condensate) or due to an elementary scalar field.
Suppose that the hypercolour breaks down spontaneously to some subgroup $H_c$. Simultaneously, the global symmetry breaks down to $H_2$.

If I want to apply the generalized decoupling condition, I have to know what happens to the composite spectrum during the transition to the Higgs phase. The problem simplifies if the Higgs field responsible for the symmetry breaking is in a fundamental hypercolour representation. According to Fradkin and Shenker, there is no phase transition between the Higgs and symmetric phases; i.e., both descriptions are "complementary". Actually, in the case I consider they may differ by global symmetry.

I shall work with a particular example, which is the model proposed by Bars and Yankielowicz. It has $G_c = \text{SU}(2)$ and the left-handed preons in the $[\bar{3}] + (\bar{N} + 4 - K)[1] + N[1]$ reducible representation. The global symmetry is $G_p = \text{SU}(2) \times \text{SU}(4-K)$, and the quantum numbers of preons under $G_c \times G_p$ are:

- $P_1 = ([\bar{3}], 1, 1, -2K, 2(\bar{N} + 4 - K))$
- $P_2 = ([1], 1, \square, -N, \bar{N} + 4 - 2K)$
- $P_3 = ([1], \square, 1, \bar{N} + K, -(\bar{N} + 4 - K))$

The massless/left-handed/composite fermions, which agree with all 't Hooft's constraints, are:

- $f_1 = (1, 1, \square, 0, 2K)$
- $f_2 = (1, \square, \square, -K, -K)$
- $f_3 = (1, \square, 1, 2K, 0)$

In terms of preons:

- $a_b(f_1) = a_b(P_1 P_2 P_3^+) = (P_1^+ \alpha^\beta a(P_2^+) b(P_3^+))_\beta$
- $a(f_2) = a(P_1^+ P_2 P_3^+) = (P_1^+ \alpha^\beta a(P_2^+) b(P_3^+))_\beta$
In this model any mass terms for the $F_1$ preons are forbidden. But let us assume that the $\tau_c$ symmetry is broken dynamically: a condensate appears in the representation

$$f_H^\alpha = (F_1)^{\alpha \beta} (F_3)_\beta = ([1], 0, 1, K-1, H^4-K)$$

so that the complementarity applies. There are two possible patterns of symmetry breaking. The first one corresponds to

$$\left< \tau_c^H \right> \neq 0$$

with $H_c = SU/K-1$ and $H_p = SU/3/1 \times SU/4-K \times U_2'/1 \times U_2'/1 \times U_2'/1$. Here the $U/1$ groups are generated by combinations of the original $G_1$ charges and broken diagonal $\tau_c$ and $SU/3$ generators. The second is (we assume $K > N$):

$$\left< \tau_c^H \right> = \left< \tau_c^{-1} \right> = \ldots = \left< \tau_c^1 \right> \neq 0$$

with $H_c = SU/K-N$ and $H_p$ isomorphic to $G_2$.

Let us consider the first case. The massive preons are $(F_1)^{\alpha \chi}$ and $(F_3)_\chi$, $\chi = 1 \ldots K-1$. Suppose that their mass is increased so that they should decouple from the low energy sector, which composite states should disappear? All of them contain the $(F_3)^{\alpha \chi}$ preon. For example, if $\alpha, \beta = 1 \ldots K-1$:

$$f_1 = (F_1)^{\alpha \beta} (F_2^+)_\alpha (F_2^+)_\beta + (F_1)^{\alpha \chi} (F_2^+)_\chi (F_2^+)_\chi$$

To see, however, that it appears only as an admixture, and the $f_1$ state can be constructed from massless preons alone. Now let us take the states which include also the $(F_3)_\chi$ preons.
\[ a^H(f_2) = (\bar{\sigma}_1^\alpha \bar{\sigma}_2^\beta (P_2^+)^{\alpha \beta} + (P_1^+)^{\alpha \beta}) \left[ \bar{\sigma}_2 (P_2)^{\alpha \beta} (P_3)^{K+} + (P_2)^{\alpha \beta} \bar{\sigma}_3 (P_3)^{K+} \right] \]

\[ N_2(f_3) = (P_1^{\alpha \beta}) \bar{\sigma}_3 (P_3)^{\alpha \beta} + (P_1)^{\alpha \beta} \left[ \bar{\sigma}_2 (P_2)^{\alpha \beta} (P_3)^{K+} + (P_3)^{\alpha \beta} \bar{\sigma}_3 (P_3)^{K+} \right] \]

\[ N_3(f_3) = \tau(N_2) (P_3)^{K+} \]

\[ N_H(f_3) = \tau(N_3) (P_3)^{K+} \]

\[ /g=1...N/. Here all terms include massive preons. But the \( H \) symmetry forbids any masses for these states. Is the decoupling condition violated? The answer is no, because these states are just the complementary representations of the liberated massless preons:

\[ a^H(f_2) \sim \sigma(P_2)^{K+} \]

\[ N_2(f_3) \sim \tau(P_3) (P_3)^{K+} \]

\[ N_H(f_3) \sim \tau(P_3)^{K+} \]

so that, in the Higgs picture, they can again be constructed without the massive preons. In result, no composite states are required to decouple. This remains true if we choose the second pattern of symmetry breaking.

So far the model agrees with the generalized decoupling condition. But let us now consider a special case when the hypercolour group is SU(5), that is we set \( K=5 \). Then we can have one more preon condensate in the fundamental representation:

\[ h_\alpha = \varepsilon_\alpha \bar{\sigma}_2 \bar{\sigma}_3 \bar{\sigma}_5 (P_2)^{\alpha \beta} (P_3)^{K+} \]

One possible pattern of symmetry breaking is then:

\[ \langle H^5 \rangle \neq 0 \]

\[ \langle h_2 \rangle \neq 0 \]

\[ \langle h_\alpha \rangle \] aligns with \( \langle H^{5 \alpha} \rangle /\). We then have \( H_\alpha = SU(4) \) and \( H_F \) is SU(5)xSU(4)xU(1)xU(1). All components of the \( P_1 \) preons are now massive, together with \( N(P_3) \), \( /\alpha=1...4/. \) Consequently, all composites include massive preons in an essential way. Some of them mix with the liberated preons:

\[ a^H(f_2) \sim \sigma(P_2)^{5} \]

\[ f_2(f_3) \sim \tau(P_3) (P_3)^{5} \]

\[ N_3(f_3) \sim \tau(P_3)^{5} \]

\[ /f=1...N-1/ \] but the \( f_1 \), \( a^H(f_2) \) and \( f_2(f_3) \) states are required to ob-
tain a mass. However, the $SU(N-1) \times SU(N-1)$ subgroup of $H_P$ keeps these composite fermions massless and the decoupling condition is violated. The exception is the case $N=2$ (two $P_3$ flavours). In this case the $SU(N-1) \times SU(N-1)$ symmetry is absent and the decoupling condition is satisfied again.

To summarize, we have seen that when some preons become massive due to the spontaneous hypercolour symmetry breaking, the composite fermions may behave in three different ways consistent with the generalized decoupling condition:

1. Remain massless, because the massive preons are only an admixture;
2. Remain massless by mixing with the liberated massless fermions;
3. Obtain a $H_P$ invariant mass.

The 4th alternative is that the generalized decoupling condition is violated.

There is a relation between this scheme and the "light composite fermions" construction by Dimopoulos, Raby, and Susskind. Their procedure is as follows: condensates in the fundamental representation are used to break the hypercolour symmetry until all remaining massless preons are liberated. Then, by complementarity, the corresponding composite states are constructed in the symmetric picture. The generalized decoupling condition works in an opposite direction: starting from the composite states one looks for their interpretation in the Higgs picture. However, not all models which satisfy the decoupling condition must be equivalent to those obtained by the DR3 procedure. One example is the $K=5, N=2$ model discussed here.

We conclude that the generalized decoupling conditions (at least
with the restriction to the case of the symmetry breaking by a fundamental representation/ can be imposed and are not trivial and not too restrictive. Some questions are:

1. Must these conditions be satisfied for all possible patterns of symmetry breaking? /A related question is: how unique is the DRS construction? /

2. Can they be generalized beyond the fundamental representation? /The DRS construction can 5/. 

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