UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01498 - SÃO PAULO - SP
BRASIL

PUBLICAÇÕES

IFUSP/P-549

ON THE EQUIVALENCE OF DILUTE ANTIFERROMAGNETS AND FERROMAGNETS IN RANDOM EXTERNAL FIELDS: CURIE-WEISS MODELS

by

J. Fernando Perez, L.F. Pontin and J.A. Baeta Segundo

Instituto de Física, Universidade de São Paulo

Outubro/1985

ON THE EQUIVALENCE OF DILUTE ANTIFERROMAGNETS AND FERROMAGNETS IN RANDOM EXTERNAL FIELDS: CURIE-WEISS MODELS

J. Fernando Perez[1], L.F. Pontin[2] and J.A. Baeta Segundo[2,3]

Instituto de Física, Universidade de São Paulo
Caixa Postal 20516, 01498 São Paulo, SP, Brazil

ABSTRACT

Using a method proposed by van Hemmen ~~and compute~~ the free energy of the Curie-Weiss version of the site-dilute antiferromagnetic Ising model ~~in the presence~~ incorrectly of an uniform magnetic field. The solution displays an exact correspondence between this model and the Curie-Weiss version of the Ising model in the presence of a random magnetic field. The phase diagrams are discussed and a tricritical point is shown to exist. (Author)

---

(1) Partially supported by CNPq.

(2) Permanent Address: Instituto Básico da Escola Federal de Engenharia de Itajubá, Caixa Postal 50, 37500 Itajubá, Brazil.

(3) Partially supported by CAPES.

Considerable attention has been devoted in the literature to the study of the Ising model in the presence of random magnetic fields (RMF) [1,6,7]. The critical behavior and the phase diagram of this model are believed to be related to those of a site dilute antiferromagnetic Ising model in the presence of an applied uniform magnetic field (DAF) [2,3]. This is specially relevant since site-dilute magnetic materials can be prepared and the magnetic-ordering transition as a function both of the external field and of the dilution of the magnetic species has been experimentally investigated [4].

In this letter we use an approach introduced by van Hemmen [5] to compute the free energy and the phase diagram of a Curie-Weiss mean field version of a DAF model. We then compare our results to those obtained by Salinas and Wrezinski [6] for the Curie-Weiss version of a RMF model and establish an exact correspondence between the parameters and phase diagrams of the two-models, as to show their complete equivalence.

Our mean field DAF model is described in a finite-volume $\Lambda \subset \mathbb{Z}^\nu$ by the hamiltonian

$$H_{DAF} = -\frac{J}{N}\sum_{i,j\in\Lambda_e} \epsilon_i\epsilon_j\sigma_i\sigma_j - \frac{J}{N}\sum_{i,j\in\Lambda_o} \epsilon_i\epsilon_j\sigma_i\sigma_j$$

$$+\frac{J}{N}\sum_{\substack{i\in\Lambda_e\\ j\in\Lambda_o}} \epsilon_i\epsilon_j\sigma_i\sigma_j + H\sum_{i\in\Lambda}\epsilon_i\sigma_i \qquad (1)$$

where $\Lambda_e = \Lambda\cap\mathbb{Z}^\nu_e$, $\Lambda_o = \Lambda\cap\mathbb{Z}^\nu_o$ with $\mathbb{Z}^\nu_e\,(\mathbb{Z}^\nu_o)$ being the sublattice of $\mathbb{Z}^\nu$ for which the sum of coordinates of each site are even (odd) integers. The interaction is antiferromagnetic ($J > 0$) between sites in different sublattices and ferromagnetic between sites in the same sublattice. The random

variables $\epsilon_i \in \{0,1\}$ describe the site dilution and they are taken to independent and identically distributed, with

$$\epsilon_i = \begin{cases} 1, & \text{probability } p \\ 0, & \text{probability } 1-p \end{cases}$$

The spin variables σ_i are, for simplicity, taken to be of Ising type: $\sigma_i = \pm 1$. The external magnetic field H is uniform and deterministic. N denotes the number of points in Λ.

Introducing the sub-lattice magnetizations:

$$m_e = \frac{1}{N} \sum_{i \in \Lambda_e} \epsilon_i \sigma_i \quad , \quad m_o = \frac{1}{N} \sum_{i \in \Lambda_o} \epsilon_i \sigma_i$$

the hamiltonian (1) can be written in the form

$$H_{DAF} = -N \frac{J}{2} (m_e - m_o)^2 + NH(m_e + m_o) \tag{2}$$

As proved by van Hemmen [5] the quenched free energy density $f(\beta)$ of the system at inverse temperature β is given by:

$$-\beta f(\beta) = \max_{\vec{m} \in \mathbb{R}^2} \{Q(\vec{m}) + c^*(\vec{m})\} \tag{3}$$

where $\vec{m}$ is the two-component vector $\vec{m} = (m_e, m_o)$,

$$Q(\vec{m}) = -\frac{\beta}{N} H_{DAF} \tag{4}$$

$$c^*(\vec{m}) = -\sup_{\vec{t} \in \mathbb{R}^2} (\vec{m} \cdot \vec{t} - c(\vec{t})) \tag{5}$$

and

$$c(\vec{t}) = \lim_{N\to\infty} \frac{1}{N} \ln\left[\frac{1}{2^N} \sum_{\{\sigma_i=\pm 1\}} \exp N\vec{t}\cdot\vec{m}\right] \tag{6}$$

We then compute:

$$c(\vec{t}) = \frac{1}{2} \lim_{N\to\infty} \left\{\frac{2}{N} \sum_{i\in\Lambda_e} \ln\cosh(t_e \epsilon_i) + \frac{2}{N} \sum_{i\in\Lambda_o} \ln\cosh(t_o \epsilon_i)\right\}$$

$$= \frac{1}{2}\left\{\overline{\ln\cosh(t_e \epsilon)} + \overline{\ln\cosh(t_o \epsilon)}\right\} \tag{7}$$

where $\bar{A}$ denotes the average of the quantity $A(\epsilon)$ with respect to the probability distribution of the random variable ϵ. The last equality in (7) is a consequence of the strong law of large numbers, which is a key ingredient in the technique of van Hemmen. Therefore

$$c(\vec{t}) = \frac{p}{2} \ln\cosh t_e + \frac{p}{2} \ln\cosh t_o \tag{8}$$

If we now introduce the order parameter $m = m_e - m_o$, the free energy is given by

$$f_{DAF}(\beta, J, p, H) = \frac{1}{2} J m^2 - \frac{p}{2\beta} \ln\left[\cosh\beta(Jm-H)\right]$$

$$- \frac{p}{2\beta} \ln\left[\cosh\beta(Jm+H)\right] \tag{9}$$

with m determined by the equation

$$\frac{2m}{p} = \tanh\left[\beta(Jm+H)\right] + \tanh\left[\beta(Jm-H)\right] \tag{10}$$

or equivalently

$$f_{DAM}(\beta, J, p, H) = \inf_m g_{DAM}(\beta, J, p, H, m) \tag{11}$$

where g_{DAM}, the free energy density in the magnetization ensemble, is defined by the r.h.s. of (9).

Let us now consider the Curie-Weiss RMF hamiltonian

$$H_{RMF} = -\frac{J}{N}\sum_{i,j\in\Lambda}\sigma_i\sigma_j + \sum_{i\in\Lambda}h_i\sigma_i \tag{12}$$

where the random variables h_i, $i\in\Lambda$ are independent and identically distributed, being equal to $\pm h$ $(h>0)$ with probability $\frac{1}{2}$. Applying the method of van Hemmen to H_{RMF}, the free energy density can be computed [6]:

$$f_{RMF}(\beta,J,h) = \frac{1}{2}Jm^2 - \frac{1}{2\beta}\ln[\cosh\beta(Jm-h)] - \frac{1}{2\beta}\ln[\cosh\beta(Jm+h)] \tag{13}$$

with the order parameter m determined by

$$2m = \tanh[\beta(Jm-h)] + \tanh[\beta(Jm+h)] \tag{14}$$

or equivalently

$$f_{RMF}(\beta,J,h) = \inf_m g_{RMF}(\beta,J,h,m) \tag{15}$$

with g_{RMF} defined as the r.h.s. of (13).

Comparing expressions (9) and (13) we see that:

$$f_{DAF}(\beta,J,p,H) = p f_{RMF}(\beta,pJ,H) \tag{16}$$

and

$$g_{DAF}(\beta, J, p, H, m) = p\, g_{RMF}(\beta, pJ, H, \frac{m}{p}) \qquad (17)$$

which establishes the correspondence between the two models.

As in [6] the phase diagram of the DAF model can be analysed by expanding g_{DAM} in powers of m around m=0:

$$g_{DAF}(m) = A + Bm^2 + Cm^4 + Dm^6 + \cdots \qquad (18)$$

where

$$\begin{aligned} A &= -\frac{p}{\beta} \ln \cosh \beta H \\ B &= \frac{J}{2} - \frac{p}{2\beta}(\beta J)^2 \operatorname{sech}^2 \beta H \\ C &= \frac{p}{2\beta}(\beta J)^4 (\operatorname{sech}^2 \beta H)(1 - 3\tanh^2 \beta H) \\ D &= -\frac{p}{6\beta}(\beta J)^6 (\operatorname{sech}^2 \beta H)(\tanh^4 \beta H - \tanh^2 \beta H + \frac{2}{15}) \end{aligned} \qquad (19)$$

There is a critical line determined by the condition B=0 i.e.

$$p\beta J = \cosh^2 \beta H \qquad (20)$$

On this critical line, there is a tri-critical point determined by the conditions B=C=0 , D>0 :

$$p\beta J = \frac{3}{2} \quad , \quad \tanh^2 \beta H = \frac{1}{3} \qquad (21)$$

The critical line represents a first order phase transition

for $p\beta J > \frac{3}{2}$ (since $c < 0$) and a second order phase transition for $p\beta J < \frac{3}{2}$ (since $c > 0$).

It is a pleasure to thank S.R. Salinas for stimulating conversations.

REFERENCES

[1] Y. Imry and S.K. Ma, Phys. Rev. Lett. 35, 1399 (1975); M. Pytte, Y. Imry, and D. Mukamel, Phys. Rev. Lett. 46, 1173 (1981); and G. Grinstein and S.K. Ma, Phys. Rev. Lett. 49, 685 (1982).

[2] S. Fishman and A. Aharony, J. Phys. C12, L729 (1979); Z. Wong, S. von Molnar, and P. Dimon, J. Appl. Phys. 53, 954 (1982).

[3] S. Galam, Phys. Lett. 100A, 105 (1984); J.L. Cardy, Phys. Rev. 29B, 505 (1984); and J.T. Chayes, L. Chayes, and J. Fröhlich, Commun. Math. Phys. 100, 399 (1985).

[4] H. Yoshizawa, R.A. Cowley, G. Shirane, R.J. Birgenau, H.J. Guggenheim, and H. Ikeda, Phys. Rev. Lett. 48, 438 (1982); and D.P. Belanger, A.R. King, and V. Jaccarino, Phys. Rev. Lett. 48, 1050 (1982).

[5] J.L. van Hemmen, A.C.D. van Enter, and J. Conisius, Z. Phys. B50, 311 (1983).

[6] S.R. Salinas and W.F. Wreszinski, "On the Mean-Field Ising Model in a Random External Field", to appear J. Stat. Phys. 1985.

[7] L.A. Pastur, J. Stat. Phys. 27, 119 (1982); and J.F. Fernando Perez, W.F. Wreszinski, and J.L. van Hemmen, J. Stat. Phys. 35, 89 (1984).