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Anomalies

by

William A. Bardeen
Theoretical Physics Department
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, IL 60137, USA

Abstract

Anomalies have a diverse impact on many aspects of physical phenemona. The role of anomalies in determining physical structure from the amplitude for π^0 decay to the foundations of superstring theory will be reviewed.

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Anomalies first appeared as an inconsistency in the application of meson theory to the two photon decay of the pion. Pseudoscalar and pseudovector couplings for the pion led to different predictions[1] for the π^0 decay amplitude when the use of the field equations appeared to require the same result from the two calculations. This discrepancy was not an artifact of the specific calculational procedure but was a result of a general phenomena now known as anomalies. Since this initial discovery, this phenomena of anomalies has had an impact on many aspects of physics. In this talk I will briefly review the role of anomalies in determining the structure of physical effects from the π^0 decay to superstrings.

Anomalies were originally discovered by applying perturbation theory methods to the calculation of various physical amplitudes. The famous triangle diagram contributes to the amplitude for the two photon matrix element of the axial vector current when computed in the one spinor loop approximation. The anomaly apppears as an extra contribution to the Ward - Takahashi for the triangle diagram. While the explicit computation[1,2] of this contribution was recognized in the early fifties, the true significance of the anomaly was not appreciated until much later in the work of Adler[3] and of Bell and Jackiw[4].

Adler studied the anomaly within the context of spinor electrodynamics and showed that the anomaly should be interpreted as an operator correction to the axial vector divergence equation valid to all orders in spinor electrodynamics. It is interesting to note that the paper of Bell and Jackiw attempted the construction of a gauge invariant and chiral invariant regularization scheme for chiral field theories which avoids the anomaly, the result which Adler had just proved impossible to achieve. Actually the "regularization" scheme of Bell and Jackiw did not regulate the spinor loops but was rather an anomaly cancelation scheme where a strongly coupled, heavy fermion was used to cancel the anomaly of the light fermion. As the mass of this regulator fermion becomes large it does not decouple but produces an effective "Wess - Zumino"-like effective action for the chiral field, a mechanism which has recently been rediscovered through the more general study of decoupling. From these early perturbative studies, the impact of anomalies has reached to practically every aspect of particle physics and beyond. I want to review the structure of anomalies and their impact on a broad sprectrum of physics issues.

The analysis of the abelian anomaly was soon extended to the full nonabelian case. The anomalous Ward-Takahashi identities were determined for an arbitrary spinor field interacting with general nonabelian external fields including vector, axial vector, scalar, and pseudoscalar fields[5]. This result showed that the anomaly was a property of the nonabelian gauge fields alone[6] . In modern language, the full gauge dependence of the spinor determinant (or effective action for the spinor loops) was computed for arbitrary external field couplings in the Dirac operator. The existence of the anomaly implies that the fermion determinant can not preserve the gauge covariance of the classical action, and the lack of gauge covariance is given precisely by the explicit form[5] computed for the anomaly. This original calculation has been reproduced many times in the literature using a wide variety of methods.

The implications of the spinor anomaly go far beyond the original perturbative calculation of the spinor loop. Wess and Zumino[7] showed that the anomaly must satisfy a set of consistency conditions and that these conditions could be precisely solved in some cases. This approach has been greatly developed through the use of the methods of differential geometry[8]. Wess and Zumino went on to apply the consistency conditions to determine the effective action for soft pion interactions which includes contributions of the anomalies. The anomalous terms, now called Wess-Zumino terms, include the amplitude for π^0 decay as well as corrections to the strong interaction amplitudes and many other processes. The significance of these terms was emphasized by Witten[9] and has recently been applied to many problems from skyrmions to superstrings.

The fundamental nature of the spinor loop anomalies was also determined through the existance of the "Adler-Bardeen" nonrenormalization theorems[10,11]. The use of various regularization methods established that the anomalies of the simple spinor loop were not modified by higher orders in perturbation theory. These results were also otained by Zee[12] using renormalization group methods. Hence the anomaly structure of the full theory could be determined completely from the known anomaly structure of the spinor loop. It is not frequent that exact results can be obtained in nontrivial, physical field theories. However, some aspects of the nonrenormalization theorems have remained controversial, particularly in supersymmetric theories[13].

For gauge field theories, the anomaly structure has three important implications:

[a] the dynamical currents must be fully gauge covariant and free of all anomalies. This may be achieved by requiring that there be no anomalies in the spinor loops, or by requiring that the anomalies cancel between different contributions to spinor loops. This was shown to occur in the Standard Model where the anomalies from the quarks cancel against the anomalies from the leptons. These cancellation conditions[14] must be imposed for the dynamical gauge theory to be consistent.

[b] flavor currents have charges which commute with the gauged, dynamical currents. However, the flavor currents may have dynamical anomalies where the conservation of the flavor current is modified due to anomalies involving the dynamical gauge fields. These anomalies imply the proton decay in the Standard Model[15], the solution to the U(1) problem in QCD[16], and the structure of axion couplings in models which solve the strong CP problem[17].

[c] flavor currents may have anomalies associated with other flavor currents. Since these anomalies are not renormalized, they must be reflected in the infrared structure of the exact theory. The Wess-Zumino terms[7] in the effective pion action of QCD are generated by the requirements of these anomalies. In composite models, these anomalies imply constraints on the bound state structure which are summarized by the 't Hooft conditions[18] on the bound state spectrum. These conditions provide practically the only firm information known for most bound state structures.

For most of the above applications, the perturbative knowledge of the anomaly structure combined with the appropriate nonrenormalization theorem is sufficient to determine the results. However, anomalies also are important when the background gauge fields have nontrivial topological structure.

Atiyah and Singer[19] established the existence of the anomaly through the study of an index associated with the dirac operator containing background gauge fields. These index theorems relate the topological structure of the background fields to invariant properties of the operator spectrum. Jackiw and Rebbi[20] were first to observe that topological structure of the background fields can produce anomalous, fractional charge for currents affected by the anomaly. Charge fractionalization was predicted[21] to occur for certain excitations of real solid state systems such as polyactylene; these anomalous effects have, in fact, been observed. The nature of the topological charge that is generated by solitons has both global and local aspects[22]. Index theory and spectral flow have also been exploited to analyze the mechanisms of charge fractionalization[23].

The particle intrepretation of topological structures was advocated in the original work of Skyrme[24]. Solitons of the chiral meson field are stabilized by certain higher derivative interactions, or Skyrme terms, of the effective action. Interest in these Skyrmions was renewed[25] by the discovery of the structure of the anomaly induced charges and spin of the solitons[26]. The application of these methods to the effective actions of the known mesons (pions, kaons, etc) which is implied by QCD in the limit of large color number has led to the interpretation of the observed baryon spectrum as solitions of the meson field. Although there are still some ambiguities in the meson theory, the preliminary attempts to develop a realistic phenomenology for baryons have been remarkably successful[27].

Anomalies have also have also had an impact on the dynamics of monopoles. Proton decay stimulated by monopoles was the dramatic result of the work of Rubakov[28] and Callan[29]. Their results stimulated a wide variety of work on the interaction of fermions with mononpoles including the conservation laws which follow from the anomalies related to the topological structure of the monopole field[30].

The interactions of fermions with the background gravitational field was also shown to induce chiral anomalies[31]. Alvarez-Gaumé and Witten[32] discovered that fermions interacting with the background gravitational field may also generate purely gravitational anomalies. These gravitational anomalies determine the consistency of theories which include couplings to the gravitational field, as in the case of the normal gauge anomalies. The gravitational couplings must obey the general coordinate and local lorentz symmetries. The gravitational anomalies may be analyzed by the methods developed for the usual gauge interactions due to the direct connection between the local symmetries of gravity and the local symmetries of the gauge field. The structure of the gravitational anomalies are found to be identical to the structure of the corresponding gauge anomalies[33]. For theories that include a consistent treatment of gravity, the full structure of the gravitational anomalies must be understood.

During the past year, superstrings have emerged as the unique candidate of the fundamental theory of all matter which creates a unified picture of all interactions including gravity. Anomalies have played a crucial role in developing these superstring theories and continue to provide the the essential tools for analyzing the theories. Through the study of gauge and gravitational anomalies in higher dimensions, Alvarez-Gaumé and Witten[32] found that supergravity models based on the type II superstrings were free of anomalies while those based on the superstring I theories contained both gauge and gravitational anomalies. Unfortunately, only the type I theories seemed to contain the rich gauge structure needed for the known phenomenology evenwhen the Kaluza-Klein effects were taken into account in the reduction from the natural ten dimensions of these theories to the physical four dimensions.

A careful examination of the type I theories by Schwarz and Green[34] proved that the loop anomalies could be cancelled by the introduction of additional anomalous terms involving the partners of the graviton field. These terms are similar to the Wess-Zumino terms of the chiral models and are actually already contained in the correct treatment of the superstring theory. This delicate cancellation mechanism works only for the gauge group, SO_{32}. Hence the anomaly structure demands an essentially unique unified fundamental theory of gauge and gravitational interactions. Actually the supergravity theory allows just one other gauge group, $E_8 \otimes E_8$. Gross, Harvey, Martinec, and Rohm[35] exploited this possibility and invented an entirely new closed string theory, the heterotic string, which could incorporate both the SO_{32} and $E_8 \otimes E_8$ gauge groups.

The analysis of the effective low energy theories produced by these superstring theories has been the subject on intense study. Again the anomalies, in the form of index theorems, play a crucial role in this analysis. The superstring theory must be formulated in ten dimensions of space-time. The observed four dimensions of space-time result when the remaining six spacelike dimensions are associated with a compact six dimensional manifold of Planck size. Most of the string states will then have masses of order the Planck scale, but some of the string states will remain light and can be identified with the observed quarks, leptons, gauge bosons, etc. The spectrum of light states depends on the topological structure of the compact manifold and can be determined from the appropriate index theorems. Although this analysis is quite complex, the heterotic string seems produce all the elements of the correct low energy phenomenolgy[36] as well as a finite theory of all interactions even beyond the Planck scale.

Anomalies began as a curious inconsistency of the meson theory when applied to the loop calculation of π^0 decay. Now anomalies seem to be fundamental to many aspects of particle theory including the low energy theorems for QCD, anomaly cancellation in gauge theories, the solution to the U(1) problem, the resolution of the strong CP problem through axions and their phenomenology, the 't Hooft conditions on composite models, the anomalous charge structure of solitons, the skyrmion interpretation of baryons, monopole dynamics, and the development of superstring theories and their phenomenological analysis. Even this long list does not exhaust all aspects of the anomaly, and this brief summary can not begin to record the diverse contributions of many people to our understanding of anomalies, their structure and their implications.

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