LOW MOMENTUM PENGUIN CONTRIBUTIONS IN A CHIRAL THEORY

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It is reported that penguin diagram contributions corresponding to u-quark loop momenta below a scale $\Lambda_\chi \sim 1$ GeV are enhanced and could at least partly explain the $\Delta I = \frac{1}{2}$ rule. Thus a previos calculation within the bag model is confirmed. The present calculation is performed within an effective chiral theory with pions and kaons coupled to quarks. It is found that low momentum left-left loop contributions are important, while left-right contributions can be neglected.

In spite of the great success of the standard model, a satisfactory explanation of the $\Delta I = \frac{1}{2}$ rule in $K \to 2\pi$ decays still seems to be lacking. One should therefore pay attention to the solution of this problem. The relative enhancement of the Wilson coefficient of the weak $\Delta I = \frac{1}{2}$ operator compared to the corresponding $\Delta I = 3/2$ operator found a decade ago[1] was numerically only a partial success. Later, Shifman, Vainshtein and Zakharov (SVZ)[2] proposed that the socalled penguin diagram (see Fig.1) induced pure $\Delta I = \frac{1}{2}$ operators which could account for the observed $\Delta I = \frac{1}{2}$ enhancement. While the weak Hamiltonian of ref.1 contained products of two left-handed currents, the Hamiltonian of ref.2 in addition contained operators of left-right type due to the penguin diagram. There are, however, two problems with the approach of SVZ[2]. First, to get the correct chiral properties of the amplitude corresponding to left-right operators, one has to subtract the socalled anomalous commutator term corresponding to a tadpole diagram[3,4,5,6]. This makes the total left-right contribution smaller. Second, the "leading logarithm" $\ln(m_c^2/\mu^2)$ obtained from the penguin loop is hardly big enough to give a short distance behaviour of the penguin diagram within perturbative QCD (μ is the renormalisation point, normally taken as $\mu \lesssim 1$ GeV)[7]. Even for "optimistic values", the Wilson coefficients of the left-right operators are probably too small to account for the $\Delta I = \frac{1}{2}$ rule[4,5,6,8,9].

In this paper another approach to the penguin diagram is advocated. For the reasons mentioned above, we neglect the effect of penguin diagram induced left-right operators at $\mu \simeq 1$ GeV. Instead we look for low momentum penguin contributions below $\mu \simeq 1$ GeV outside the standard approach[1,2,3]. A leading logarithm $\ln(M^2/\mu^2)$ can be considered as the effect of loop momenta between μ and M. We know that the total amplitude for some process is independent of μ. But choosing a model to describe low non-perturbative loop momenta, the choice of μ for perturbative QCD corrections[1,2] should be chosen to be consistent with that model. That is, all possible loop momenta should be taken into account, and no momentum region should be counted twice.

In a previous paper[9] low momentum loop contributions were considered within the MIT bag model[10] by replacing free quark propagators by propagators written in terms of bag model wave functions. The loop momenta were assumed to be divided in a perturbative and a non-perturbative region at a scale $\mu = m_* \sim 1$ GeV. Sizeable penguin contributions corresponding to loop momenta below m_* were found for the $\Delta I = \frac{1}{2}$

$K \to \pi$ transition.(The u-quark and c-quark contributions above $\sim$ 1 GeV (see Fig.1) are considered to be in the perturbative region,and will approximately be cancelled due to the GIM-mechanism) The physical $K \to 2\pi$ amplitude can as usual be obtained from the $K \to \pi$ transition amplitude by means of PCAC and the soft pion approximation. It is well known that the bag model has trouble with chiral invariance.To be sure that the enhancement obtained in ref.9 is not an artifact of the bag model, a similar calculation should be performed in another model. In this paper a calculation within the effective chiral SU(3)xSU(3) field theory of Georgi and Manohar[11] is reported. This theory contains the octet 0^- mesons coupled to constituent quarks for momenta below the chiral symmetry breaking scale Λ_χ which is taken to be $\simeq$ 1 GeV (That a phenomenological scale $\simeq$ 1 GeV exists for non-leptonic decays of kaons is found[12]from analysis of $K \to 2\pi$ and $K \to 3\pi$ data) The effective Hamiltonian corrected for hard QCD effects is taken as that of ref.1 with $\mu = \Lambda_\chi$. That is, effects of penguin left-right operators are neglcted at $\mu = \Lambda_\chi$, as in ref.9 for $\mu = m_c$.

In calculating the low momentum penguin loop contribution in the model of ref.11, the free quark lines at the left and the right side of the penguin diagram (Fig.1) are tied together to make a K and a π respectively,thus making a three loop diagram for the transition $K \to \pi$ (see Fig.2). This diagram can be calculated as a Feynman loop diagram because three level (derivative) axial vector couplings for $K^- \to s\bar{u}$ and $d\bar{u} \to \pi^-$(say) of order f_π^{-1} are contained in the theory[11]. It should be noted that the theory is non-renormalisable and that Λ_χ is considered as an explicite physical cut-off in the theory. During the three loop calculation it is an advantage to divide the coloured quark current at the lower quark-gluon vertex (point z in Fig.2)in a left-handed and a right-handed part(At the upper vertex;point x in Fig.2 ; the current is left-handed due to the weak interaction). Then Fierz transformations can be used for the left-left and the left-right parts of the three loop diagram. Having performed the Fierz-transformations, the calculation is straightforward(-but tedious).

The result of the calculation is that the left-left part of the three loop diagram behaves (-for Λ_χ considered as "big"-) as :

$$\frac{1}{\pi^4 f_\pi^2} \left[\Lambda_\chi^2 \ln(\Lambda_\chi^2) \right]^2 , \qquad (1)$$

where f_π is the π-decay constant. The left-right part is found to be suppressed by Λ_χ^{-4} compared to (1) and can therefore be neglected. The reason for the result (1) is that the ordinary one loop penguin diagram behaves basically as $q^2 \ln(\Lambda_\chi^2)$, q being the gluon four momentum. Thus, when the one loop penguin diagram is inserted in the rest of the three loop diagram, the momentum factor q^2 in the numerator will cause the formal divergence in (1). One should keep *in mind* that for loop momenta $< \Lambda_\chi$ there is no GIM-mechanism to cancel the behaviour (1),- the c-quark loop is "frozen" at $\mu = m_c > \Lambda_\chi$. Using the relation [11] $\Lambda_\chi = 4\pi f_\pi$, the low momentum penguin contribution (1) will be (-except for the logarithmic factor) of the same order as the vacuum insertion result for matrix elements of typical left-left operators in the weak effective Hamiltonian [2,3]:

$$\langle \pi^- | \bar{u}_L \gamma^\mu s_L \bar{d}_L \gamma_\mu u_L | K^- \rangle_{Vac.} = -\frac{f_K}{\sqrt{2}} \frac{f_\pi}{\sqrt{2}} \; p_K \cdot p_\pi \qquad (2)$$

where $p_K (p_\pi)$ is the kaon(pion) four momentum. It should be noted that the result of the three loop diagram is also proportional to $p_K \cdot p_\pi$ which is necessary to satisfy the requirements of chiral symmetry [4,5,8]

Combining (2) with (1) (-keeping the detailed version of (1)-) one obtains the following ratio between $\Delta I = 1/2$ and $\Delta I = 3/2$ amplitudes (further details will be given in a forthcoming paper):

$$\frac{A_{LL}(\Delta I = \frac{1}{2}) + A_{Peng.}}{A_{LL}(\Delta I = \frac{3}{2})} = \frac{1}{2}\left(1 + \frac{3}{2}\frac{C_-}{C_+}\frac{Z_-}{Z_+}\right) + \frac{9}{8 Z_+}\left(1 + \frac{C_-}{C_+}\right)\frac{\alpha_s}{\pi} g_A^2 \, F \qquad (3)$$

where $C_\pm$ are the well-known Wilson coefficients of the operators occurring in the effective weak Hamiltonian [1]. Numerically, $C_-/C_+ \simeq 4$. $Z_\pm$ are factors to correct for the vacuum insertion approximation (2). The A_{LL}'s are the weak amplitudes for $K \to \pi$ obtained from the effective Hamiltonian of ref.1. The factor

$$F \simeq \left[\ln(1 + \Lambda_\chi^2 / m^2) \right]^2 \qquad (4)$$

is due to the loop integration. m is the constituent quark mass(The SU(3) limit $m_s = m_{u,d} = m$ and $f_K = f_\pi = f$ is used). For $\Lambda_\chi / m \simeq 3$, $F \simeq 5.2$ However, this leading order value of F is increased to $\simeq 6.3$ when numerically important non-leading contributions are considered. g_A is the axial vector coupling constant for octet mesons coupling to quarks.(The total meson coupling to quarks is $\sim (g_A/f) \gamma \cdot p \gamma_5$; p being the meson momentum)

The numerical result obtained from (3) will of course depend on the numerical values of C_-/C_+, Z_-, and especially on Z_+, g_A, and the strong fine structure constant α_s. To fit the observed axial vector coupling $\simeq 1.25$ at nucleon level, the corresponding coupling g_A at quark level is taken as $\simeq$ 0.75 in ref.11. Moreover α_s is taken as low as $\simeq$ 0.3 to fit the mass splittings in the baryon spectrum. Qualitatively, one explanation for a low value of α_s could be that the gluons appearing in the effective theory below $\mu = \Lambda_\chi$ are perturbative gluons, while the non-perturbative strong interactions are taken care of by the non-linear chiral interactions. However, one may argue that the low values for α_s and g_A advocated in ref.11 are those to be used at $\mu \sim m$ or $\mu \sim \Lambda_{QCD}$ while the values just below $\mu = \Lambda_\chi$ are bigger. It is known that the vacuum insertion method (-as well as quark model calculations-) overestimates matrix elements of left-left operators[6,13,14]. The vacuum insertion approximation ($Z_\pm = 1$) constitutes an upper bound , that is, $Z_\pm$ should be less than one. One may hope that this will be confirmed by loop corrections to next order in $1/f_\pi^2$ (note that (1) is already of this order.). Corrections to weak non-leptonic matrix elements has recently been performed in chiral perturbation theory[15]. But these calculations neither involve an explicite cut-off Λ_χ nor octet mesons coupling to quarks.

Using "reasonable values" of α_s, g_A and $Z_\pm$ in (3), ($\alpha_s \simeq 0.5$ to 0.8, $g_A \simeq 0.8$ to 0.9, and $Z_\pm \simeq 0.5$ to 0.8 , say), one can account for numerical values $\sim$ 10 to 20 for the ratio between $\Delta I = 1/2$ and $\Delta I = 3/2$ amplitudes, while the experimental value is $\simeq$ 22. To draw a final conclusion concerning the $\Delta I = \frac{1}{2}$ rule within the framework of ref.11, one has to take into account the full theory including all loop effects to a given order in f_π^{-1}. An attemt in this direction, which shows a $\Delta I = \frac{1}{2}$ enhancement, has already been made[16]. However, a better understanding of the matching conditions for weak operators at $\mu = \Lambda_\chi$ seems to be needed.

To conclude, the previous result of ref.9 and the present result (3) indicates that a better understanding of the $\Delta I = \frac{1}{2}$ rule can be achieved in terms of low momentum penguin loop contributions. More general, non-leptonic decays can probably not be satisfactory described within the standard approach [1,2,3] alone. Long distance effects have to be included in some way[8,9,14,15,16,17,18,19].

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Figure captions:

Fig.1 : Penguin diagram for loop momenta above $\mu = \Lambda_{\chi}$ inducing left-right operators[2,3].

Fig.2 : Low momentum u-quark three loop penguin diagram for $K \rightarrow \pi$ within the effective chiral field theory of ref.11 .

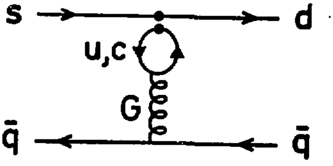

Fig.1

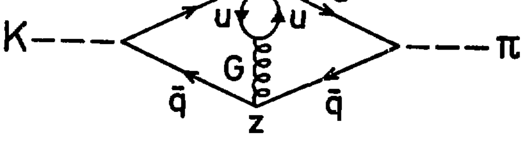

Fig. 2