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Relativistic quarkonium dynamics

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Abstract

We present, in the framework of relativistic quantum mechanics of two interacting particles, a general model for quarkonium systems satisfying the following four requirements : confinement, spontaneous breakdown of chiral symmetry, soft explicit chiral symmetry breaking, short distance interactions of the vector type. The model is characterized by two arbitrary scalar functions entering in the large and short distance interaction potentials, respectively. Using relationships with corresponding quantities of the Bethe-Salpeter equation, we also present the normalization condition of the wave functions, as well as the expressions of the meson decay coupling constants. The quark masses appear in this model as free parameters.

I. Introduction

This paper is devoted to the study and determination of the structure of the relativistic wave equations which might govern, in relativistic instantaneous approximations, the dynamics of quarkonium systems. The formalism which is used here is that of covariant relativistic quantum mechanics of two interacting particles and was presented by the author in a previous article [1].

In this formalism the fermion-antifermion wave function satisfies two independent wave equations which are generalizations of the Dirac equations relative to particles 1 and 2. They are of the form :

$$H_1\psi \equiv [\gamma\cdot p_1 - m_1 - (-\eta\cdot p_2 + m_2)V]\,\psi = 0 \quad , \qquad (1.1a)$$

$$H_2\psi \equiv [\eta\cdot p_2 + m_2 + (\gamma\cdot p_1 + m_1)V]\,\psi = 0 \quad , \qquad (1.1b)$$

where the wave function ψ is a 4x4 matrix function :

$$\psi = \psi_{\alpha_1\alpha_2}(x_1, x_2) \quad , \quad (\alpha_1, \alpha_2 = 1, \ldots, 4) \quad , \qquad (1.2)$$

and the matrices γ and η are the Dirac matrices acting on the fermion and antifermion spinor indices, respectively, (labeled by sub-indices 1 and 2) :

$$\gamma_\mu \psi \equiv \gamma_{1\mu} \psi = (\gamma_\mu)_{\alpha_1\beta_1} \psi_{\beta_1\alpha_2} \quad ,$$

$$\eta_\mu \psi \equiv \psi \gamma_{2\mu} = \psi_{\alpha_1\beta_2} (\gamma_\mu)_{\beta_2\alpha_2} \quad . \tag{1.3}$$

The potential V is a Poincaré invariant function of the coordinates, momenta and Dirac matrices. The compatibility condition of the two wave equations requires that V depend on the relative coordinates x through the transverse components x^T, with respect to the total momentum p :

$$V = V(x^T, p_1, p_2, \gamma, \eta) \quad , \tag{1.4}$$

with :

$$p = p_1 + p_2 \quad , \quad v = \frac{1}{2}(p_1 - p_2) \quad ,$$

$$X = \frac{1}{2}(x_1 + x_2) \quad , \quad x = x_1 - x_2 \quad ,$$

$$r_\mu \equiv x^T_\mu = x_\mu - (\hat{p}\cdot x)\hat{p}_\mu \quad , \quad x^L_\mu = (\hat{p}\cdot x)\hat{p}_\mu \quad ,$$

$$x_L = \hat{p}\cdot x \quad , \quad \hat{p}_\mu = p_\mu/(p^2)^{1/2} \, , \, (p^2 > 0) \quad . \tag{1.5}$$

Eqs.(1.1) completely determine the longitudinal relative coordinate $-x_L-$ dependence of the wave function through the equation

$$(p_1^2 - p_2^2)\psi = (m_1^2 - m_2^2)\psi \quad , \tag{1.6}$$

which is a consequence of Eqs.(1.1), and the solution of which is, for eigenfunctions of the total momentum p :

$$\Psi(x_1, x_2) = e^{-ip\cdot X} \, e^{-i(m_1^2 - m_2^2)\, p\cdot x/(2p^2)} \, \psi(x^T) \, . \tag{1.7}$$

The dynamics of the relative motion is therefore three-dimensional through the coordinates x^T.

It was outlined in Ref.1 that the wave function ψ and the potential V are in direct connection with the wave function ϕ and the kernel D of the Bethe-Salpeter equation, for the sector of "normal" solutions of the latter. When a relativistic instantaneous approximation is made for the kernel D, taken in the ladder approximation, then the above relationships become completely explicit and take the following form, in the equal-mass case $m_1 = m_2 = m$:

$$V = -\frac{i}{4}\int dx_L \, D(x_L, x^T)\left[1 + \frac{i\gamma\cdot\hat{p}\,\gamma\cdot\hat{p}}{4(m^2 - V^{T2})^{1/2}}\int D(x_L', x^T)\, dx_L'\right]^{-1} \frac{1}{(m^2 - V^{T2})^{1/2}} , \tag{1.8}$$

$$\Phi(X, x_L = 0, x^T) = e^{-ip\cdot X} \frac{(p^2)^{1/2}}{2(m^2 - V^{T2})^{1/2}}\left[1 + \gamma\cdot\hat{p}\,\gamma\cdot\hat{p}\, V\right] \psi(x^T) \, . \tag{1.9}$$

The more general relation between Φ and Ψ for any x_μ is presented in Ref.1 (formula (6.27)). It allows one to obtain the normalization condition of Ψ from that of Φ (in the C.M. frame) :

$$\int d^3\vec{x}\, \frac{1}{4}\, Tr\left[\psi^+(\vec{x})\, \frac{1}{(m^2+\vec{v}^2)^{1/2}}\,\psi(\vec{x}) - \psi^+(\vec{x})\, V^+\, \frac{1}{(m^2+\vec{v}^2)^{1/2}}\, V\,\psi(\vec{x})\right] = \tag{1.10}$$

where p^2 independent kernels have been assumed.

In order to get for the potential a local expression in x^T, it is necessary to approximate the operator $(m^2 - v^{T^2})^{-1/2}$ by some local function. For instance one might replace the operator v^{T^2} by its mean value in the state Ψ, or simply by a common constant for all the states :

$$\left(m^2 - v^{T^2}\right)^{-1/2} \simeq \left(m^2 - \langle v^{T^2}\rangle\right)^{-1/2} . \tag{1.11}$$

Furthermore, in order to work in the usual free scalar product of st*a*
(in the C.M. frame)

$$(\chi, \psi) = \int d^3\vec{x}\; Tr\,(\chi^+\psi) , \tag{1.12}$$

it is necessary to apply on Ψ the additional transormation

$$\psi \rightarrow \left[1 - (\gamma\cdot\hat{p}\;\gamma\cdot\hat{p}^{\,T'})^2\right]^{-1/2}\psi , \tag{1.13}$$

valid in the local approximation (1.11) and for hermitian p^2 independent potentials V (and also in the unequal mass case) ; the normalization condition of ψ then becomes (in the C.M. frame and in the equal-mass case) :

$$\int d^3\vec{x}\ \mathrm{Tr}\left[\psi^+ \frac{1}{4(m^2-\langle V^{T2}\rangle)^{1/2}}\,\psi\right] = 1 \quad . \tag{1.14}$$

We notice that in order transformation (1.13) to be finite for finite values of x it is necessary that

$$\frac{1}{4}\ \mathrm{Tr}\,(\gamma\cdot\hat{p}\ \gamma\cdot\hat{p}\ V)^2 < 1 \quad , \tag{1.15}$$

which shows that the potential V must be appropriately bounded and in particular regularized at finite values of x. Confining interactions may occur when the upper bound 1 of (1.15) is reached for some particular value of $|x^{T\,2}|$ (in general when $|x^{T2}| \to \infty$).

The relationship (1.8) between the potential V and the Bethe-Salpeter kernel permits one to classify the former according to its tensor structure in the momenta and the Dirac matrices in a parallel way with that of an interaction Lagrangian. If V is chosen arbitrarily, then the Bether-Salpeter kernel will be that of an effective field theory with effective propagators.

Finally, the physical Hilbert space is defined by the subspace of solutions of Eqs.(1.1) which correspond to positive eigenvalues of both $\beta.p_1$ and $\beta.p_2$, the latter being also related by Eq.(1.6).

In the present work we consider potentials V which *are local functions* of x^T (not involving integral operators).

It is the aim of this paper to exhibit the tensor structure of the interaction potentials which are necessarily present for the description of quarkonium systems. This is achieved by imposing on them the following four theoretical requirements, which are expected to be true in QCD :

i) Confinement ;

ii) Spontaneous breakdown of chiral symmetry, in the sense that, when the quark masses tend to zero, the bound state mass spectrum displays a massless pseudoscalar meson, with a non-zero value of its decay coupling constant ;

iii) Soft explicit chiral symmetry breaking, in the sense that the bound state spectrum remains stable under small variations of the quark masses, in particular around the value zero of the latter ;

iv) Short distance interactions of the vector type.

We shall show that these conditions restrict enough the tensor structure of the interaction potentials V in the wave equations (1.1). In particular it turns out that the general class of potentials which are relevant for the first three conditions is of the pseudoscalar type. The above conditions leave still free the precise $(x^T)^2$ dependences of the potentials. The latter could be determined only after some additional theoretical requirements are imposed on the properties of the potentials or after a more detailed analysis of the bound state mass spectra and transition amplitudes is done. Furthermore, there also remains some freedom in the choice of the structure of the potentials in the intermediate distance region. The study of these aspects of the problem is left for future work.

II. Confinement and chiral symmetry breaking

In this section, we study the implications of confinement and spontaneous breakdown of chiral symmetry on the structure of the interaction potentials.

In quantum field theory the phenomenon of spontaneous breakdown of chiral symmetry manifests itself by the non-invariance of the vacuum under the action of the corresponding axial charge, while the equations of motion remain invariant under the chiral transformations, except for the mass terms. In quantum mechanics we do not have an explicit equivalent state of the field theory vacuum, and therefore the non-invariance of the latter has to be expressed through the non-invariance of the wave equations themselves. However, one has also to guarantee that the matrix elements of the axial vector current divergences are still proportional to the masses of the constituent particles.

Among the scalar, pseudoscalar and vector interactions we considered in Ref.1 (Sec.VII) only the first two satisfy the requirements of non-invariance of the wave equations under chiral transformations. We therefore have to search for the confining potentials within these two types of interactions.

The scalar interactions correspond to the case where V is independent of the Dirac matrices and of the relative momenta :

$$V = V(r^2, p^2) \quad . \tag{2.1}$$

(Since it is the function V which appears in the wave equations and not the kernel D (1.8) itself, we have preferred to classify the interactions according to the tensor structure of the former ; in this case the tensor structure of D will be more complicated than that of V).

After bringing the operators $\eta.p_2$ and $\gamma.p_1$ in Eqs.(1.1) on the right of V and using again the equations of motion, the wave equations become, after the change of function (1.13) :

$$H_1\psi \equiv \left\{ \gamma\cdot p_1 - m_1 - \frac{1}{(1-V^2)}\left[m_2 V + m_1 V^2 + i\hbar \dot{V}\eta\cdot r\right]\right\}\psi = 0 \ , \tag{2.2a}$$

$$H_2\psi \equiv \left\{ \eta\cdot p_2 + m_2 + \frac{1}{(1-V^2)}\left[m_1 V + m_2 V^2 + i\hbar \dot{V}\gamma\cdot r\right]\right\}\psi = 0 \ , \tag{2.2b}$$

where

$$\dot{V} = \frac{\partial V}{\partial r^2} \quad . \tag{2.3}$$

The pseudoscalar interactions correspond to the case where V is proportional to the matrices $\gamma_5\eta_5$:

$$V = W(r^2, p^2)\gamma_5\eta_5 \quad , \tag{2.4}$$

the γ_5 type matrices being defined by :

$$\gamma_5\psi \equiv \gamma_{15}\psi \quad , \quad \eta_5\psi \equiv \psi\gamma_{25} \quad ,$$

$$\tilde{\gamma}_\mu \equiv \gamma_\mu\gamma_5 \quad , \quad \tilde{\eta}_\mu = \eta_\mu\eta_5 \quad , \quad \tilde{\eta}_\mu\psi = \psi\gamma_{25}\gamma_{2\mu} \quad ,$$

$$\gamma_5 = \frac{i}{4!}\varepsilon_{\mu\nu\kappa\rho}\gamma^\mu\gamma^\nu\gamma^\kappa\gamma^\rho \quad , \quad \varepsilon_{0123} = 1 \; . \tag{2.5}$$

The wave equations become after the change of function (1.13) :

$$H_1\psi \equiv \left(\gamma\cdot p_1 - m_1 + i\hbar A\gamma_5\tilde{\eta}\cdot r\right)\psi = 0 \quad , \tag{2.6a}$$

$$H_2\psi \equiv \left(\eta\cdot p_2 + m_2 - i\hbar A\tilde{\gamma}\cdot r\,\eta_5\right)\psi = 0 \quad , \tag{2.6b}$$

$$A = -\frac{2\dot{W}}{1 - W^2} \quad , \tag{2.7a}$$

$$W(r^2) = -\frac{\left[th\left(\frac{1}{2}\int_{r_0^2}^{r^2} A(r'^2)dr'^2\right) - W(r_0^2)\right]}{\left[1 - W(r_0^2)\,th\left(\frac{1}{2}\int_{r_0^2}^{r^2} A(r'^2)dr'^2\right)\right]} \quad . \tag{2.7b}$$

(Notice that the effective potential A determines the original potential W up to an integration constant).

We impose our third condition, which concerns the stabi ty of th ound state spectrum under small variations of the k mas in particular around the value zero of the latt This dition eliminates the scalar potentials as confin g inte ions. In effect, if one neglects the mass dependent erms i wave equations (2.2), then the confining interacti in the ro mass limit is represented by the last terms. Ho ver, wh he mass dependent terms are present, then the domin t parts the latter at large distances behave as the expon tials o he previous confining terms and thus they completely modify bound state mass spectrum. The zero quark mass limi is the re not a smooth limit if the confining interactio s are esented by scalar type potentials. The latter mi t still present in intermediate distance regions, but a detailed ana s of the bound state spectrum is needed in order o reach efinite conclusion. In the present work, where we inly c rate on qualitative theoretical features of the i eractio otentials, this type of potentials will no longer e considered.

As to the pseudoscalar type interactions, Eqs.(2.6), hey do display any explicit sensitive dependences on the ark mas s and could consequently be chosen as independen or w ly dependent, functions of them. We are therefore eft wi the pseudoscalar type potentials for the realizatio of con ement and spontaneous breakdown of chiral symmetry. n the n section, we study the properties of these potentials nd show at they actually realize these two goals.

III. Pseudoscalar potentials as prototypes of confining interactions

For the pseudoscalar type potentials the wave equation operators H_1 and H_2 - Eqs.(2.6)-satisfy the compatibility condition in the strong sense :

$$[H_1 , H_2] = 0 \quad . \tag{3.1}$$

The "square" of Eqs.(2.6) yields the generalized Klein-Gordon type equation :

$$\tilde{H}\psi \equiv (H_1 + 2m_1)H_1\psi = (H_2 - 2m_2)H_2\psi \quad , \tag{3.2}$$

$$\tilde{H}\psi = \Big\{ \tfrac{1}{4}p^2 - \tfrac{1}{2}(m_1^2 + m_2^2) + \frac{(m_1^2 - m_2^2)^2}{4p^2} + v^{T2} + \hbar^2 A^2 r^2 - \frac{4A}{p^2}\gamma\cdot\hat{p}\,\gamma'\cdot\hat{p}\,W_{1s}\cdot W_{2s} - \frac{8\dot{A}}{p^2}\gamma\cdot\hat{p}\,\gamma'\cdot\hat{p}\,W_{1s}\cdot r\,W_{2s}\cdot r \Big\}\psi = 0 \,, \tag{3.3}$$

where W_{1s} and W_{2s} are the Pauli-Lubanski spin operators of particles 1 and 2, respectively :

$$W_{1s\alpha} = -\frac{\hbar}{4}\varepsilon_{\alpha\beta\mu\nu}P^{\beta}\sigma^{\mu\nu} \,,$$

$$W_{2s\alpha} = -\frac{\hbar}{4}\varepsilon_{\alpha\beta\mu\nu}P^{\beta}\xi^{\mu\nu} \,, \qquad (\varepsilon_{0123} = 1),$$

$$W_s = W_{1s} + W_{2s} \,,$$

$$\sigma_{\mu\nu} = \frac{1}{2i}[\gamma_\mu , \gamma_\nu] \,, \quad \xi_{\mu\nu} = \frac{1}{2i}[\gamma'_\mu , \gamma'_\nu] \,, \tag{3.4}$$

and the transverse components v^T are defined as in (1.5) for x^T.

A particular feature of Eq.(3.3) is that the operator $\tilde{H}$ commutes with the longitudinal matrices $\gamma.\hat{p}$ and $\eta.\hat{p}$, and therefore its solutions can be classified according to the eigenvalues of these matrices ; the positive energy solutions, both in $p_1.\hat{p}$ and $p_2.\hat{p}$, will correspond to the eigenvalues +1 of $\gamma.\hat{p}$ and of $-\eta.\hat{p}$.

If $\tilde{\Psi}$ is an eigenfunction of P_μ, of Eq.(3.3) and of $\gamma.\hat{p}$ and $\eta.\hat{p}$, then the solution of Eqs.(2.6) is given by an inverse Foldy-Wouthuysen transformation of $\tilde{\Psi}$:

$$\Psi = (H_1 + 2m_1)(H_2 - 2m_2)\tilde{\Psi}/(-4m_1m_2) \;,$$
$$\tilde{H}\tilde{\Psi} = 0 \;,$$
$$\gamma\cdot\hat{p}\,\tilde{\Psi} = -\eta\cdot\hat{p}\,\tilde{\Psi} = \tilde{\Psi} \;. \tag{3.5}$$

This is a direct consequence of the fact that the operators H_1 and H_2 commute strongly -Eq.(3.1)- and of the definition of $\tilde{H}$ - (3.2).

The wave function $\tilde{\Psi}$ above, satisfies, like Ψ, condition (1.6)-(1.7) ; let $\tilde{\psi}(r)$ represent the corresponding internal wave function :

$$\tilde{\Psi}(x_1, x_2) = e^{-ip\cdot X} \, e^{-i(m_1^2 - m_2^2)\, p\cdot x/(2p^2)} \, \tilde{\psi}(r) \; . \tag{3.6}$$

In terms of $\tilde{\psi}$, Eq.(3.3) becomes :

$$\Big\{ \frac{1}{4}p^2 - \frac{1}{2}(m_1^2 + m_2^2) + \frac{(m_1^2 - m_2^2)^2}{4p^2} + V^{T2} + \hbar^2 A^2 r^2$$
$$+ A\left(\frac{2}{p^2} W_S^2 + 3\hbar^2\right) + 2\dot{A}\Big[\frac{2}{p^2}(W_S\cdot r)^2 + \hbar^2 r^2\Big]\Big\} \tilde{\psi}(r) = 0 , \tag{3.7}$$

where we have introduced the total spin operator (3.4).

The problem now is very similar to that of non-relativistic quantum mechanics with two spin-$\frac{1}{2}$ particles. The eigenfunctions of Eq.(3.7) can be expanded on the basis of the eigenfunctions of the internal angular momentum operators W_S^2 (total spin), W_L^2 (orbital angular momentum), W^2 (total angular momentum) and $\vec{W}.\vec{P}/|\vec{P}|$ (helicity) :

$$W_{L\alpha} = \varepsilon_{\alpha\beta\mu\nu}\, p^\beta r^\mu V^\nu \; ,$$
$$W = W_L + W_S \; . \tag{3.8}$$

We designate by $\tilde{\mathcal{Y}}^{m++}_{\ell sj}$ the (r^2 independent) eigenfunctions of these operators and of $\gamma.\hat{p}$ and $-\eta.\hat{p}$; they satisfy the eigenvalue equations :

$$\gamma\cdot\hat{p}\,\tilde{\mathcal{Y}}^{m++}_{\ell sj} \quad -\gamma\cdot\hat{p}\,\tilde{\mathcal{Y}}^{m++}_{\ell sj} = \tilde{\mathcal{Y}}^{m++}_{\ell sj} \quad ,$$

$$W_s^2\,\tilde{\mathcal{Y}}^{m++}_{\ell sj} = -\hbar^2 p^2 s(s+1)\,\tilde{\mathcal{Y}}^{m++}_{\ell sj} \quad , \quad s=0,1 \; ,$$

$$W_L^2\,\tilde{\mathcal{Y}}^{m++}_{\ell sj} = -\hbar^2 p^2 \ell(\ell+1)\,\tilde{\mathcal{Y}}^{m++}_{\ell sj} \quad , \quad \ell=0,1,\ldots \; ,$$

$$W^2\,\tilde{\mathcal{Y}}^{m++}_{\ell sj} = -\hbar^2 p^2 j(j+1)\,\tilde{\mathcal{Y}}^{m++}_{\ell sj} \quad , \quad |\ell-s|\leq j\leq \ell+s \; ,$$

$$\frac{\vec{W}\cdot\vec{p}}{|\vec{p}|}\,\tilde{\mathcal{Y}}^{m++}_{\ell sj} = \hbar\,(p^2)^{\frac{1}{2}} m\,\tilde{\mathcal{Y}}^{m++}_{\ell sj} \quad , \quad m=-j,-j+1,\ldots,j \; . \tag{3.9}$$

In the C.M. frame $\tilde{\mathcal{Y}}^{m++}_{\ell sj}$ reduces essentially to the well known function $\mathcal{Y}^{m}_{\ell sj}$:

$$\tilde{\mathcal{Y}}^{m++}_{\ell sj}\Big|_{\vec{p}=0} = \begin{pmatrix} 0 & \mathcal{Y}^{m}_{\ell sj} \\ 0 & 0 \end{pmatrix} \quad , \tag{3.10}$$

where the representation in which γ_0 is diagonal has been used.

The eigenstates of p^2 have definite values of j, s, m but may not be eigenstates of W_L^2. If in Eq.(3.7) the term $(W_S.r)^2$ were absent, then they would also have a definite value of ℓ. In this case for a given ℓ, one would have four kinds of states (putting aside the helicity and the radial quantum numbers) : $s = 0$, $j = \ell$; $s = 1$, $j = \ell$ and $j = \ell \pm 1$. In the presence of the term $(W_S.r)^2$, the states characterized by $s = 0$, $j = \ell$ and $s = 1$, $j = \ell$ are still eigenstates of p^2, but the two other types of states get mixed. The parity and charge conjugation (for the equal mass case $m_1 = m_2$) quantum numbers are given by $P = (-1)^{\ell+1}$ and $C = (-1)^{\ell+s}$, respectively.

For confining interactions it is the term A^2r^2 which dominates at large space-like distances in the potential of Eq.(3.7), and the condition on A to yield confinement is that

$$- A^2 r^2 \xrightarrow[-r^2 \to \infty]{} \infty \quad . \tag{3.11}$$

We now show that for confining potentials the ground state of the spectrum of p^2 (with positive $p_1.\hat{p}$ and $p_2.\hat{p}$) is a massless particle when the quark masses m_1 and m_2 go to zero. We expect that such a state will have the quantum numbers $s = \ell = j = 0$ and $n = 0$ (no nodes in the radial wave function) ; these are precisely the quantum numbers of a pseudoscalar meson. Writing the wave function $\tilde{\psi}$ as :

$$\tilde{\psi}^{m=0}_{n=0,\ell=0,s=0,j=0} = F_{0000}(r^2)\,\tilde{\mathcal{Y}}^{0++}_{000} = (4\pi)^{-\frac{1}{2}} F_{0000}(r^2)\,\frac{1}{2}(1+\gamma\cdot\hat{p})\gamma_5 \quad ,$$

(3.12)

we get from Eq.(3.7) the following equation for the radial wave function F :

$$\Big\{\frac{1}{4}p^2 - \frac{1}{2}(m_1^2+m_2^2) + \frac{(m_1^2-m_2^2)^2}{4p^2} - \hbar^2\Big(6\,\frac{d}{dr^2} + 4r^2\frac{d^2}{(dr^2)^2}\Big)$$
$$+\hbar^2A^2r^2 + 3\hbar^2A + 2\hbar^2\dot{A}r^2\Big\} F_{0000} = 0 \quad .$$

(3.13)

This equation is of the same type as the one obtained by Witten in his quantum mechanical one-dimensional supersymmetric model [2]. It can be checked that the function

$$F_{0000} = C\,\exp\Big[\frac{1}{2}\int^{r^2} A\,dr^2\Big] \quad , \qquad (3.14)$$

where C is a constant and the asymptotic sign of A has been chosen positive (r^2 is negative), is a solution of Eq.(3.13) with the mass eigenvalue :

$$(p^2)^{1/2} = m_1 + m_2 \quad , \qquad (3.15)$$

which goes to zero when m_1 and m_2 go to zero.

One has also to check whether the other eigenstates of p^2 have masses greater than the value (3.15). This can be established with a wide class of potentials A. One has first to notice that the state $s = \ell = j = n = 0$ is indeed the ground state of the sector $s = \ell = j = 0$ (its radial wave function has no nodes). Then the radial wave functions of the other sectors ($\ell \neq 0$, $s = 0,1$, or $\ell = 0$, $s = 1$) can be expanded on the basis of radial functions of the sector $s = \ell = 0$ and the positivity of the additional terms appearing in the potential (due to $\ell \neq 0$ and/or $s \neq 0$) noticed. (For this a sufficient condition is that $A + 2r^2\dot{A} > 0$, with A satisfying (3.11)). Therefore the state (3.12), (3.14), (3.15) is actually the ground state of the spectrum of p^2.

This spectrum can be exactly calculated in the particular case where A is a constant λ. Eq.(3.7) becomes :

$$\left\{ \frac{1}{4}p^2 - \frac{1}{2}(m_1^2+m_2^2) + \frac{(m_1^2-m_2^2)^2}{4p^2} + V^{T2} + \hbar^2\lambda^2 r^2 + \lambda\left(\frac{2}{p^2}W_S^2 + 3\hbar^2\right)\right\}\tilde{\Psi} = 0 \quad , \tag{3.16}$$

which corresponds to the harmonic oscillator case. Here the eigenstates of p^2 are also eigenstates of W_L^2 and one has :

$$\tilde{\Psi}^m_{n\ell sj} = F_{n\ell sj}(r^2)\,\tilde{Y}^{m++}_{\ell sj} \quad . \tag{3.17}$$

The spectrum of [illegible] is given by :

$$p^2 = (m_1^2 + m_2^2) + 4\hbar\lambda(\ell + 2s + 2n) + \left\{\left[(m_1^2 + m_2^2) + 4\hbar\lambda(\ell + 2s + 2n)\right]^2 - (m_1^2 - m_2^2)^2\right\}^{1/2} .$$

(3.18)

This spectrum was also obtained in Ref.3 and a similar one, differing only in the dependence on the spin quantum number, was given in Ref.4.

The solution (3.12), (3.14), (3.6) for the ground state of Eq. (3.7) can also be expressed in the Dirac representation by the inverse Foldy-Wouthuysen transformation (3.[illegible]). It turns out that also in this representation the ground state wave function is an eigenfunction of the matrices $\gamma.\hat{p}$ and $-\eta.\hat{p}$, and is given by the same function $\tilde{\psi}$ as above (this can also be checked directly on Eqs.(2.6)) :

$$\psi^0_{0000}(x_1, x_2) = \tilde{\psi}^0_{0000}(x_1, x_2) .$$

(3.19)

IV. Quark masses as parameters of explicit chiral symmetry breaking

At this stage we have to check that the quark masses m_1, m_2 play the same role as in quantum field theory, in order to be able to interpret the limits m_1, $m_2 \to 0$ as corresponding to the conservation of an axial vector current.

The field theory (or Bethe-Salpeter) wave function is defined as [5,6] :

$$\Phi_{\alpha_1\alpha_2}(x_1, x_2) = \langle 0 | T(\psi_{1\alpha_1}(x_1)\bar{\psi}_{2\alpha_2}(x_2)) | p \rangle \quad , \tag{4.1}$$

where $|p\rangle$ is the bound state with total momentum p. One then gets the following relations for the matrix elements of the axial vector current [7] :

$$\langle 0 | j_{\mu 5}(0) | p \rangle = \langle 0 | \bar{\psi}_2(0)\gamma_\mu\gamma_5\psi_1(0) | p \rangle = -Tr(\gamma_\mu\gamma_5\Phi(0,0)) , \tag{4.2a}$$

$$\langle 0 | \partial^\mu j_{\mu 5}(0) | p \rangle = -\left[(\partial_1^\mu + \partial_2^\mu) Tr\, \gamma_\mu\gamma_5 \Phi\right]_{x_1 = x_2 = 0} \quad . \tag{4.2b}$$

In order to evaluate the right-hand sides of Eqs.(4.2) we have to replace Φ in terms of the quantum mechanical wave function Ψ and use the corresponding wave equations in (4.2b). Since in the latter equation it is the derivatives with respect to the total variables X_μ which enter and since the dependence of Ψ on the relative longitudinal variable x_L is trivial -Eq.(1.7)- we can immediately put $x_L = 0$ and use the relationships (1.9), (1.11) between Φ and Ψ at $x_L = 0$. One also has to make the additional transformation (1.13) on Ψ. For simplicity we shall limit ourselves to the equal-mass case $m_1 = m_2 = m$, but the results which will be obtained remain qualitatively the same for the unequal-mass case.

We consider for $|p\rangle$ a pseudoscalar meson state, for which the decay coupling constant F_p is defined as :

$$\langle 0 | j_{\mu 5}(0) | p \rangle = F_p \, p_\mu \quad . \tag{4.3}$$

We then get :

$$\Phi(X=0, x_L=0, x^T) = \frac{(p^2)^{1/2}}{2(m^2 - \langle V^{T2} \rangle)^{1/2}} \cdot \frac{(1 + \tilde{\gamma}\cdot\hat{p}\;\tilde{\gamma}\cdot\hat{p}\,W)}{(1 - W^2)^{1/2}} \psi(x^T) \, , \tag{4.4}$$

$$\langle 0 | j_{\mu 5} | p \rangle = - \frac{(p^2)^{1/2}}{2(m^2 - \langle V^{T2} \rangle)^{1/2}} \left(\frac{1 + W(0)}{1 - W(0)} \right)^{1/2} \mathrm{Tr}\left(\gamma_\mu \gamma_5 \psi(0)\right) \, , \tag{4.5}$$

$$\langle 0|\gamma^\mu j_{\mu 5}|p\rangle = -2im \frac{(p^2)^{1/2}}{2(m^2-\langle v^{T^2}\rangle)^{1/2}} \left(\frac{1+W(0)}{1-W(0)}\right)^{1/2} \mathrm{Tr}(\gamma_5 \psi(0))$$

$$= -2im\left(\frac{1+W(0)}{1-W(0)}\right) \quad \mathrm{Tr}(\gamma_5 \Phi(0))$$

$$= 2im\left(\frac{1+W(0)}{1-W(0)}\right)\langle 0|\bar{\psi}_2(0)\gamma_5\psi_1(0)|p\rangle \quad . \tag{4.6}$$

(In getting Eq.(4.5), we have used the fact that $\mathrm{Tr}(\gamma_\mu\gamma_5\psi(0))$ is proportional to $\hat{p}_\mu$ according to (4.3)).

The last relation shows that the quark masses m play the same role, up to the multiplicative factor $(1+W(0))/(1-W(0))$, as the quark masses of the field theory with vector interactions.

Comparing Eqs.(4.5) and (4.3) for the case of the Goldstone boson and using the expression (3.12), (3.19) of the corresponding wave function, we get :

$$F_p = \left(\frac{1+W(0)}{1-W(0)}\right)^{1/2} \frac{F_{0000}(0)}{(4\pi)^{1/2}(m^2-\langle v^{T^2}\rangle)^{1/2}} \quad . \tag{4.7}$$

Recalling that the wave function ψ is normalized according to the condition (1.14), we deduce that $F_{0000}(0)$ and hence F_p do not vanish in the limit $m \to 0$. (We emphasize that the mean value $\langle v^{T^2}\rangle$ remains different from zero in that limit, because it is essentially given by the mass scale of the confining interaction rather than by m). This result completes the indication that the pseudoscalar type interactions reproduce the main effects of spontaneous breakdown of chiral symmetry, that are the existence, when $m \to 0$, of a massless pseudoscalar meson in the bound state mass spectrum, with a non-zero value of the corresponding decay coupling constant.

We end this section with the following remarks :

i) The value of F_P depends on $W(0)$ which cannot be determined from the mass spectrum of the bound states. Indeed, the relationships (2.7) between the potential W and the effective potential A, which governs the wave equations (2.6), show that the latter does not fix completely W.

ii) The fact that the almost -Goldstone boson mass depends linearly on the quark masses -Eq.(3.15)- shows that the order parameter of chiral symmetry breaking is given by a quantity like $\int d^4x \langle 0|T(\bar\psi(x)\psi(x)\bar\psi(0)\psi(0))|0\rangle$ rather than by $\langle 0|\bar\psi\psi|0\rangle$, which should vanish in the l mit $m \to 0$. This is seen by expanding (in field theory) the pseudoscalar meson masses in perturbation series in the quark masses and using low energy theorems.

iii) It may seem puzzling that Rel.(4.6) is obtained with the use of pseudoscalar type interactions. One would expect that the right-hand side of (4.6) make also appear a zeroth order term in m, which is typical for scalar and pseudoscalar interactions in quantum field theory. The result (4.6) is *however* qualitatively general and can be understood in the following way.

In order that the right-hand side of (4.6), defined by (4.2b), contain zeroth order terms in m it is necessary that the potential V of the wave equations (1.1) contain terms which are odd *in* the *Dirac* matrices. This is easily seen by starting from the expression $Tr(\gamma.p\,\gamma_5\psi(0))$ which is proportional to the right-hand side of (4.2b) and then using the wave equations (1.1). In the limits $m_1 = m_2 = 0$, one gets the relation :

$$Tr\left(\gamma\cdot p\,\gamma_5\,(1-V(0))\,\psi(0)\right) = 0 \quad . \tag{4.8}$$

V(0) is a scalar function of the matrices γ and η and of p. If V contains only terms which are even in the Dirac matrices, then it can be seen that the left-hand side of (4.8) is again proportional to $Tr\gamma.p\,\gamma_5\psi(0)$, which establishes the result.

In the ladder approximation of the Bethe-Salpeter kernel, the latter will always contain an equal *number of* γ and η matrices, and therefore the result (4.6) will remain qualitatively true for any kind of interaction (in its local approximation). It is only when vertex corrections (in local approximations) are taken into account that V may contain odd numbers of Dirac matrices. It is through such terms that the scalar and pseudoscalar interactions of quantum field theory could qualitatively modify the result (4.6).

Therefore if we wish to preserve the chiral symmetry properties of the presently considered confining interactions, we have to admit that they are equivalent to effective pseudoscalar theories only at the level of ladder type approximations.

V. Inclusion of short distance vector interactions

Vector type interactions correspond to potentials V which are proportional to the matrices $\gamma_\mu \eta_\nu$:

$$V = \gamma_\mu \eta_\nu C^{\mu\nu}(x^T, p) \quad . \tag{5.1}$$

The choice for $C_{\mu\nu}$ of the Feynman type gauge

$$C_{\mu\nu} = g_{\mu\nu} C(r^2, p^2) \tag{5.2}$$

leads to interactions possesing the structure of relativistic Coulomb like potentials, which are required to be present in short distance dominating vector interactions.

It was however pointed out in Ref.1 (Sec. VII.4) that V, as chosen in the form (5.1), leads to a rather complicated structure of the wave equations, after the operators $\eta \cdot p_2$ and $\gamma \cdot p_1$ are brought on the right of V in Eqs.(1.1) and transformation (1.13) is used. The wave equations considerably simplify if one adds to the potentials (5.1) a second term, of the axial vector type, which is of third order in C and which serves to cancel several terms of the resulting effective potential. Such a term can be thought of as representing some effective local approximation of high order diagrams of the Bethe-Salperter kernel.

The expression of V is then :

$$V = \gamma\cdot\eta\, C_0(r^2, p^2) + \tilde{\gamma}\cdot\tilde{\eta}\, \tilde{C}_0(r^2, p^2) \quad , \tag{5.3}$$

$$\tilde{C}_0 = - C_0^3 + O(C_0^4) \quad . \tag{5.4}$$

The corresponding wave equations are, after the change of function (1.13) :

$$H_1\psi \equiv \left\{\gamma\cdot p_1 - m_1 - \gamma_\mu\left[\tfrac{1}{2}Bp^\mu + Kv^\mu + \hbar\dot{K}\xi^{\mu\nu}r_\nu + i\hbar\dot{K}r^\mu\right]\right\}\psi = 0, \tag{5.5a}$$

$$H_2\psi \equiv \left\{\eta\cdot p_2 + m_2 - \eta_\mu\left[\tfrac{1}{2}Bp^\mu - Kv^\mu - \hbar\dot{K}\sigma^{\mu\nu}r_\nu - i\hbar\dot{K}r^\mu\right]\right\}\psi = 0, \tag{5.5b}$$

where

$$B = -\frac{2C}{1-C} \quad , \quad K = \frac{2C}{1+C} \quad ,$$

$$C = C_0 + O(C_0^3) \quad , \tag{5.6}$$

σ and ξ defined in (3.4) and $\dot{K}$ as in (2.3).

Because of condition (1.15), $|C|$ is bounded by $\frac{1}{4}$:

$$|C| < \frac{1}{4} \quad . \tag{5.7}$$

For attractive interactions C is positive.

Eqs.(5.5) satisfy the compatibility condition in the strong sense (3.1).

We have now to include this vector interaction into the previously considered pseudoscalar type confining interaction. The simplest way would correspond of course to adding the two potentials (2.4) and (5.3). But again the resulting wave equations appear to possess a rather complicated structure. By adding to the previous two potentials an interference term of the tensor type, which is of second order in C, one can, however, simplify the structure of the corresponding wave equations. Their effective potential turns out to be the sum, up to redefinition of functions, of the effective potentials of Eqs.(2.6) and (5.5). The result is :

$$V = W \gamma_5 \eta_5 + C_0 (1 - W^2) \gamma \cdot \eta + D_0 \sigma \cdot \xi + \tilde{C}_0 \tilde{\gamma} \cdot \tilde{\eta} \tag{5.8}$$

$$\begin{aligned} H_1 \psi \equiv \Big\{ & \gamma \cdot p_1 - m_1 + i\hbar A \gamma_5 \tilde{\eta} \cdot r \\ & - \gamma_\mu \Big[\tfrac{1}{2} B p^\mu + K v^\mu + \hbar \dot{K} \xi^{\mu\nu} r_\nu + i\hbar \dot{K} r^\mu \Big] \Big\} \psi = 0 \,, \end{aligned} \tag{5.9a}$$

$$H_2\psi \equiv \Big\{ \gamma\cdot p_2 + m_2 - i\hbar A\tilde{\gamma}\cdot r\gamma_5$$
$$- \gamma_\mu \big[\tfrac{1}{2} B p^\mu - K v^\mu - \hbar \dot{K} \sigma^{\mu\nu} r_\nu - i\hbar \dot{K} r^\mu \big] \Big\} \psi = 0 ,$$

(5.9b)

where B and K have the same definitions as in (5.6) and :

$$D_0 = C_0^2 W(1-W^2) + O(C_0^3) \quad ,$$
$$\tilde{C}_0 = O(C_0^3) \quad ,$$
$$A = -2\big[\dot{W} - 2C_0\dot{W} + 8WC_0\dot{C}_0(1-W^2)$$
$$+ 2C_0^2\dot{W}(3-2W^2)\big](1-W^2)^{-1} + O(C_0^3) .$$

(5.10)

Eqs.(5.9) satisfy the compatibility condition in the strong sense (3.1).

A further remark concerns the notion of short distances. We recall that the potential V is not equal to a propagator, but rather to its relativistic instantaneous approximation, essentially given by its integral over x_L (see formula (1.8)). While in field theory the notion of short distances is usually understood as the limit $x \to 0$, the above feature of the potential V shows that it always feels some part of large distance effects coming from the large values of $|x_L|$ in the integral. This means that in the present framework, where it is the variables x^T which appear in the dynamical quantities, rather than x, we cannot straightforwardly transpose the short distance results of field theory into the potentials or the wave equations. In particular we cannot state that the pseudoscalar potential W should vanish in the limit $x^T \to 0$ (as one would naively expect from asymptotic freedom), because this limit is not equivalent to $x \to 0$.

We could, however, impose a weak form of short distance dominance of vector interactions, by demanding, for instance, that in the limit $x^T \to 0$ the pseudoscalar interactions disappear from the wave equations. If the effective potential A satisfies the property

$$\lim_{x^T \to 0} (-r^2)^{1/2} A = 0 \quad , \tag{5.11}$$

then such a condition could be fulfilled.

Finally, the inequality (1.15) is satisfied, to second order in C_0, by the following sufficient conditions :

$$|W| \leq 1 \quad ,$$
$$-\frac{1}{4(1-W)} \leq C_0 \leq \frac{1}{4(1+W)} \quad . \tag{5.12}$$

We now turn to the study of the properties of Eqs.(5.9). We first note that by calculating the "square" of the operators H_1 and H_2 -Eq.(3.2)- one finds that the total interaction still remains confining provided A satisfies condition (3.11) and $|C_0|$ remains bounded by 1.

Secondly, the fact that vector interactions are explicitly chiral invariant means that Rel.(4.6) remains qualitatively unchanged (up to multiplicative zeroth order factors in m).

Next we check whether the Goldstone boson survives to the inclusion of vector interactions. Contrary to the pure pseudoscalar case, the "square" (3.2) of the operators H_1 and H_2 does no longer commute with the matrices $\gamma.\hat{p}$ and $\eta.\hat{p}$ and therefore the passage to the Foldy-Wouthuysen representation is less interesting here. Furthermore, Eqs.(5.9) cannot be solved explicitly for the ground state for arbitrary potentials K(B). However, we can expand these equations in the masses m_1 and m_2 (with respect to their explicit dependences on these parameters, but not for the kinematic term p^2 present in r_μ) and calculate the ground state solution to lowest order in m and the corresponding mass to first order in m. We again find a zero-mass bound state.

The solution is :

$$\psi(r) = (16\pi)^{-\frac{1}{2}}\left[(1+\gamma\cdot\hat{r})\gamma_5 F_{++}(r^2) + (1-\gamma\cdot\hat{r})\gamma_5 F_{--}(r^2)\right] + O(m) \;,$$

$$F_{\pm\pm} = \frac{1}{2}\left[a \pm b(1-K)\right] F \;,$$

$$F(r^2) = \frac{1}{(1-K)^2}\, \exp\left[\frac{1}{2}\int^{r^2}\left(\frac{A}{1-K}\right) dr^2\right] \;,$$

$$\frac{b}{a} = \left[\frac{\int d^3x^T F^2}{\int d^3x^T F^2(1-K)^2}\right]^{1/2} \;,$$

$$(p^2)^{\frac{1}{2}} = (m_1+m_2)\left[\frac{\int d^3x^T F^2(1-K)^2}{\int d^3x^T F^2}\right]^{1/2} + O(m^2) \;,$$

(5.13)

where a and b are constants and m designates a linear combination of m_1 and m_2. Notice that the calculation of the ratio b/a necessitates the knowledge of the mass formula to first order in m in order to pick up by the limiting procedure $m \to 0$ the solution which corresponds to the positive energy values of $\beta \cdot p_1$ and $\beta \cdot p_2$.

In case one replaces the potential K(B) by a constant mean value ($K = K_0$, $B = B_0$), then the solution can be calculated exactly :

$$(p^2)^{\frac{1}{2}} = (m_1 + m_2)(1 - K_0) \quad ,$$

$$\psi(r) = \frac{a}{(16\pi)^{1/2}} (1 + \gamma \cdot \hat{p}) \gamma_5 F(r^2) .$$

(5.14)

It is crucial to observe that $|K|$ must be bounded by 1 in order to have a normalizable solution. This is actually a consequence of the general inequality (1.15) imposed on the potentials, some aspects of which were met in formulae (5.7) and (5.12).

We end this section by calculating the expressions of the decay coupling constants of pseudoscalar and vector mesons. The relationship (1.9) becomes now with the potential (5.8), after transformation (1.13) is used and to second order in C_0 (for the equal-mass case $m_1 = m_2 = m$) :

$$\Phi(X=0, x_L=0, x^T) = \frac{(p^2)^{1/2}}{2(m^2 - \langle v^{T2} \rangle)^{1/2}} \frac{1}{(1-W^2)^{1/2}}$$

$$\Big\{ 1 + C_0 + 2C_0^2(1+2W^2) + C_0 W \tilde{\gamma} \cdot \tilde{\eta} - C_0 \sigma^{LT} \cdot \xi^{LT}$$

$$+ (1 + 6C_0^2) W \tilde{\gamma} \cdot \hat{p} \, \tilde{\eta} \cdot \hat{p} + C_0^2 W (\gamma^T \cdot \eta^T + \tilde{\gamma}^T \cdot \tilde{\eta}^T)$$

$$+ O(C_0^3) \Big\} \psi(x^T) . \qquad (5.15)$$

For the pseudoscalar meson states one gets :

$$\langle 0 | j_{\mu 5} | p \rangle = - \frac{(p^2)^{1/2}}{2(m^2 - \langle v^{T2} \rangle)^{1/2}} \cdot \left(\frac{1 + W(0)}{1 - W(0)} \right)^{1/2}$$

$$\left[1 - 2C_0(0) + 2C_0^2(0)(1 + 2W(0)) \right] Tr\left(\gamma_\mu \gamma_5 \psi(0) \right) ,$$

(5.16)

$$\langle 0 | \partial^\mu j_{\mu 5} | p \rangle = - 2im \frac{(p^2)^{1/2}}{2(m^2 - \langle v^{T2} \rangle)^{1/2}} i \left(\frac{1 + W(0)}{1 - W(0)} \right)^{1/2} \cdot$$

$$(1 - k(0)) \left[1 - 2C_0(0) + 2C_0^2(0)(1 + 2W(0)) \right] Tr\left(\gamma_5 \psi(0) \right)$$

$$= 2im \left(\frac{1 + W(0)}{1 - W(0)} \right) \left[1 - 8C_0(0) + 32C_0^2(0) \right.$$

$$\left. + \frac{6C_0^2(0)}{(1 - W(0))} + 8C_0^2(0) W(0) \right] \langle 0 | \bar{\psi}_2(0) \gamma_5 \psi_1(0) | p \rangle .$$

(5.17)

For the Goldstone boson (5.13), one gets for F_p (4.3) :

$$F_p = \left(\frac{1 + W(0)}{1 - W(0)} \right)^{1/2} \cdot \frac{(F_{++}(0) - F_{--}(0))}{(4\pi)^{\frac{1}{2}} (m^2 - \langle v^{T2} \rangle)^{1/2}} .$$

$$\left[1 - 2C_0(0) + 2C_0^2(0)(1 + 2W(0)) \right] . \qquad (5.18)$$

(The ultaneous use of formulae (5.16)-(5.17) gives nothing but a cons tency relation identically satisfied by solution (5.13)).

For the vector meson states one has :

$$\begin{aligned} \langle 0 | j_\mu | p,\lambda \rangle &\equiv \langle 0 | \bar{\psi}_2(0)\gamma_\mu \psi_1(0) | p,\lambda \rangle \equiv \varepsilon_\mu^\lambda(p) M_V F_V \\ &= - \mathrm{Tr}\left(\gamma_\mu \Phi(0)\right) \\ &= - \frac{(p^2)^{1/2}}{2\left(m^2 - \langle v^{T2} \rangle\right)^{1/2}} \cdot \left(\frac{1 + W(0)}{1 - W(0)}\right)^{1/2} \cdot \\ &\quad \left[1 + 2C_0(0) + 2C_0^2(0)\left(1 + 2W(0)\right)\right] \mathrm{Tr}\left(\gamma_\mu \psi(0)\right). \end{aligned} \tag{5.19}$$

Comparing formulae (5.19) and (5.16), we find that the multiplicative factor contained in the brackets is larger for the vector mesons than for the pseudoscalars, due to the positive sign of C_0 for attractive vector interactions.

VI. Summary and concluding remarks

We presented, in the framework of relativistic quantum mechanics of two interacting particles, a general model for quarkonium systems satisfying the expected four requirements of confinement, spontaneous breakdown of chiral symmetry, soft explicit chiral symmetry breaking and short distance interactions of the vector type. The model thus built up is minimal in the sense that only the necessary and simplest pieces for these requirements were retained. It is characterized by two arbitrary scalar functions entering in the large and short distance interaction potentials, respectively.

Using the relationships of the quantum mechanical potential and of the wave function with the corresponding quantities of the Bethe-Salpeter equation, as were previously established, we also displayed the normalization condition of the wave function. This permits us to calculate the decay coupling constants and the field theory quark masses in terms of the quantum mechanical quantities.

The quark masses appear in this model as free parameters and therefore could be adjusted according to the quark flavors. In order the model to be universal, that is applicable for any quark flavors with the only modification of the quark mass values, it is necessary to exhibit in an explicit way the quark mass dependences of the interaction potentials. This question was not studied in this work and merits a seperate analysis. The only hypothesis we made about it is that the potentials do not vanish when the quark masses tend to zero. When the interaction is perturbative, the relationship (1.8) between the potential and the Bethe-Salpeter kernel gives us a first answer to this question. To lowest order of the interaction the kernel D is the one particle exchange propagator and is independent of the quark masses and therefore the mass dependence of V comes from the explicit mass factors in Rel.(1.8). This should be the case for instance for the short distance vector potential.

For non-perturbative interactions, such as the confining ones, the above feature is no longer true. The kernel D is not a one particle exchange propagator but rather should be understood as a local approximation of a series of high order diagrams. In this case it may display a complicated quark mass dependence. In order to have a rough idea about such a dependence it seems indicated to study the non-relativistic behavior of the interaction potentials. Since in this limit the mass terms become dominant, then from the knowledge or the expected behavior of the confining potential in the non-relativistic limit one could guess, at least partly, the types of mass terms present in it.

Finally the forms of the potentials themselves should be fixed by a more detailed analysis of the spectroscopic and decay properties of meson bound states as well as with the help of other theoretical constraints.

References and footnotes

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7. Remarks : i) color indices are supposed to be summed over, the physical states being considered as color singlets ; ii) we are considering two fermions of different kinds and hence the axial vector current divergence matrix element is free of anomalies.