I-25060

12

DE-15504

UCID--20646

DE86 006933

ELECTRON BEAM BRIGHTNESS WITH FIELD IMMERSED EMISSION

J. K. Boyd
V. K. Neil

December 3, 1985

Lawrence Livermore National Laboratory

ELECTRON BEAM BRIGHTNESS WITH FIELD IMMERSED EMISSION*

J. K. Boyd and V. K. Neil
Lawrence Livermore National Laboratory
University of California
P. O. Box 808
Livermore, CA 94550

December 3, 1985

ABSTRACT

The beam quality or brightness of an electron beam produced with field immersed emission is studied with two models. First an evelope formulation is used to determine the scaling of brightness with current, magnetic field and cathode radius, and to examine the equilibrium beam radius. Second, the DPC computer code is used to calculate the brightness of two electron beam sources.

I. INTRODUCTION

Coherent radiation has been generated by transferring kinetic electron beam energy into electromagnetic radiation. The viability of this process depends on the density of electron beam current in phase space or beam brightness. Thus, it is useful to obtain an estimate of the brightness of possible electron beam sources. In addition knowledge of brightness infers a propagating beam radius if the effective beam current is known. The electron

---

*Work performed jointly under the auspices of the U. S. Department of Energy by the Lawrence Livermore National Laboratory under contract No. W-7405-ENG-48 and for the Department of Defense under DARPA, ARPA Order No. 4395 and 5316, monitored by Naval Surface Weapons Center under document number N60921-85-POW0001; SDIO/BMD-ATC MIPR #W31-RPD-53-A127; and SDIO/NSWC reference number B00014-85-WR35514.

beam source parameters of this investigation are a current of 100 kA and an energy of 8 MeV. It is assumed the beam is axisymmetrically emitted from a field immersed source with field strengths in the neighborhood of 20 kG.

In an axisymmetric beam configuration the canonical angular momentum, P_θ is conserved. This means a beam emitted with finite P_θ in a magnetic field rotates when it leaves the field. As shown previously [1] a finite P_θ results in an effective beam emittance. The brightness scales like the beam current divided by emittance squared. Thus, the brightest beam is obtained from the smallest P_θ. For magnetic field immersed emission a minimum magnetic field is desired in order to transport a beam at a reasonable radius. A minimum magnetic field implies a minimum P_θ and correspondingly a minimum effective beam emittance. Thus, a maximum beam brightness is imposed by the necessity of a particular magnetic field strength.

The nominal behavior of a finite P_θ beam is examined in Section II using the envelope equation. The envelope formulation is used to determine the scaling of brightness with current, magnetic field and cathode radius. From the steady-state envelope equation the equilibrium radius of an immersed emission beam is determined in and out of a magnetic field. Two computational results are presented in Section III. The DPC computer code is used to calculate the brightness of two electron beam sources in the parameter regime of interest. These calculations provide a measure of the validity of the envelope formulation predictions for brightness. They also indicate the magnitude of brightness which can be expected for the parameter range under investigation.

II. ENVELOPE EQUATION TREATMENT

The rms envelope equation [2] for beam current I_b is below.

$$\frac{d^2R}{dz^2} + \left(\frac{I_b(\text{amp})}{17{,}000}\right)\frac{\Gamma}{\gamma R} + \frac{k_c^2 R}{4} - \frac{[E(\text{rad-cm})]^2}{\gamma^2 R^3} = 0 \quad . \tag{1a}$$

Equation 1(a) assumes no scattering, no energy loss and $\beta \approx 1$ where $\beta = v/c$ and $\gamma = (1 - \beta^2)^{-1/2}$. The following definitions apply to Eq. (1a)

$$\langle\ \rangle = N^{-1} \sum_{i=1}^{N} \tag{1b}$$

$$R = \langle r_i^2 \rangle^{1/2} \tag{1c}$$

$$V = \langle v_i^2 \rangle^{1/2} \tag{1d}$$

$$L = \langle r_i v_{\theta i} \rangle \tag{1e}$$

$$k_c = qB/(\gamma mc^2) \tag{1f}$$

$$P_\theta = m\gamma L + \frac{1}{2} m\gamma c k_c R^2 \tag{1g}$$

$$E = \left(\gamma^2 R^2 \left(V^2 - c^2\left(\frac{dR}{dz}\right)^2 - \left(\frac{L}{R}\right)^2\right)\frac{1}{c^2} + (P_\theta/mc)^2\right)^{1/2} \tag{1h}$$

$$\Gamma = 1 - f_m - \beta^{-2}(1 - f_c) \quad . \tag{1i}$$

where q is the particles' charge, m is rest mass, c is the speed of light, E is the effective normalized rms emittance (rad-cm), and f_c, f_m are charge and current neutralization fraction respectively. For a charge neutral beam ($f_c = 1$) ΓI_b simply becomes the net current $I_{net} = I_b(1 - f_m)$. In vacuum ($f_c = 0$, $f_m = 0$) we have $\Gamma I_b = - I_b/\gamma^2$ so the second term in Eq. (1a) scales like γ^{-3}. For various degrees of charge and current neutralization the scaling of the second term in Eq. (1a) ranges from γ^{-1} to γ^{-3}. The term → 0 and changes sign when $\beta^2(1 - f_m) = 1 - f_c$.

The envelope equation and Eq. (1) definitions can be used to determine formulas for brightness and equilibrium radius. In the situation under investigation the beam is emitted from rest in a magnetic field B. For a cathode radius of r_{cath} Eq. (1g) and Eq. (1h) yield,

$$P_\theta = \frac{\gamma m c k_c}{4} r_{cath}^2 \tag{2a}$$

$$E = \frac{\gamma k_c}{4} r_{cath}^2 \tag{2b}$$

A "working" definition of brightness, $\mathcal{J}$ [3] for uniform phase space is

$$\mathcal{J} = \frac{2 I_b(\text{amp})}{9[E(\text{rad-cm})]^2} \ . \tag{3}$$

(The general definition of $\mathcal{J}$ is given in Section III.) Substituting Eq. (2b) and Eq. (1f) into Eq. (3) gives $\mathcal{J}$ (amp/(rad-cm)^2) in terms of I_b, r_{cath} and B.

$$\mathcal{J} = 1 \times 10^7 \ \frac{I_b(\text{amp})}{[B(\text{gauss})]^2 \ [r_{cath}\ (\text{cm})]^4} \tag{4}$$

In equilibrium $d^2R/dz^2 = 0$ and Eq. (1a) gives the equilibrium envelope radius, R_{eqin} in the magnetic field.

$$R_{eqin} = [2(E^2 k_c^{-2} \gamma^{-2} + (\Gamma I_b/17{,}000)^2 \gamma^{-2} k_c^{-4})^{1/2} - 2\,\Gamma\,(I_b/17{,}000)\,\gamma^{-1} k_c^{-2}]^{1/2} \quad (5)$$

If ΓI_b is small compared to $17{,}000\ \gamma k_c^2$ then $R_{eqin} = r_{cath}/\sqrt{2}$. Outside the magnetic field k_c is set to zero in Eq. (1a) and the equilibrium radius then depends on k_c through the emittance, Eq. (2b).

$$R_{eqout} = \frac{k_c\, r_{cath}^2}{4} \left(\frac{\gamma\ 17{,}000}{\Gamma I_b}\right)^{1/2} \quad (6a)$$

Substituting Eq. (1f) gives,

$$R_{eqout} = .15\, r_{cath}^2 \left(\frac{B(kG)}{10}\right) \left(\frac{16}{\gamma}\right)^{1/2} \left(\frac{100{,}000}{\Gamma I_b(amp)}\right)^{1/2} \quad (6b)$$

Envelope formulas for brightness and rms equilibrium radius given by Eq. (4), (5) and (6) can be used to obtain quantities as a function of cathode radius. For example, choosing B = 17 kG and $\gamma = 16$, parameter values are given in Table I for cathode radius ranging from 1.2 cm to 9 cm. The brightness is calculated for $I_b = 10^5$A. The values of R_{eqin} are obtained by setting $f_c = f_m = 0$ so that $\Gamma I_b = -I_b/\gamma^2$. The values of R_{eqout} are calculated for $f_c = 1$ and $f_m = 0.3$ so that $\Gamma I_b = 0.7 I_b$.

r_{cath} (cm)	$\mathcal{J}$ (amp/[rad-cm]^2)	R_{eqin} (cm)	R_{eqout} (cm)	Current density (kA/cm^2)
9.00	0.53	6.36	24.69	0.4
2.32	119.00	1.64	1.64	6.0
1.36	1000.00	0.96	0.56	17.2
1.20	1670.00	0.84	0.44	22.1

TABLE I. Envelope Formulation

The brightness values are optimistic for the chosen parameters since a finite emission temperature increases E which then results in a smaller brightness. In order to get brightness above 10^3 amp/(rad-cm)^2 it is necessary to have a cathode radius less than 1.36 cm which implies a current density greater than 17.2 kA/cm^2. For the parameter values chosen a matched radius results with r_{cath} = 2.32 cm for which $R_{egin} = R_{eqout}$ = 1.64 cm.

III. COMPUTATIONAL RESULTS

The DPC computer code [4] has been used to determine the beam brightness for two cases in the 8 MeV, 100 kA, 17 kG parameter range. DPC is a particle code which solves the relativistic equation of motion with fields obtained in the Darwin approximation. DPC calculates brightness using the following general formula.

$$\mathcal{J} = \frac{\pi^2 I_b}{(\gamma\beta)^2 V_4} \qquad (7)$$

In Eq. (7) I_b is the current enclosed in the V_4 four dimensional phase space volume. The general definition of brightness is not standard. Thus, note should be made of the π^2 factor in Eq. (7) which other researchers may not have in their definitions.

Both cases run by DPC are diode configurations with 8 MV anode voltage and a uniform dc magnetic field of 17 kG. In the first case a 9 cm radius flat cathode was used with an 8.5 cm A-K gap. The cathode radius was sized to yield an average emission density of 400 amp/cm^2. This is significantly higher than operational accelerator injectors at Livermore but is conceivable. The A-K gap was selected to draw 100 kA with space charge limited emission. It should be noted the stress exceeds 1 MV/cm which is unrealistic if emission from structures near the anode must be avoided. The beam radius as a function of z along with potential lines is shown in Fig. 1a. The cathode and anode are indicated with shading. The beam radius is held nearly constant by the 17 kG field. The scalloping behavior occurs at approximately the 10 cm cyclotron wavelength which is expected. The current density as a function of radius at z = 1.5 cm is shown in Fig. 1b, and ranges from 200 to 600 amp/cm^2. Emission preferentially occurs at large radius which is nearer the anode and thus the beam tends to have a hollowed current profile. The brightness versus enclosed current for this example is approximately constant with value .3 amp/(rad-cm)^2 at z = 58 cm.

It is clear from Eq. (4) a smaller cathode radius increases brightness. To emphasize this effect in the second example a 1.2 cm radius flat cathode was used with a 1.4 cm A-K gap. The average emission density for 100 kA is 22,000 amp/cm^2. A density this large causes material surface problems and also raises the issue of whether the diode could be discharged more than once. The electric field stress corresponding to 8 MV across 1.4 cm is unthinkable if it is necessary to prevent emission from the structure around the cathode. The beam radius as a function of z along with potential lines is plotted in Fig. 2a. The cathode and anode are indicated with shading. It was necessary to limit the axial length of the anode to .6 cm at a radius of 1.2 cm to avoid spilling current. For this case the beam tends to pinch strongly moving away from the cathode and then suddenly diverge moving past the anode. The edge radius swells to 2 cm before rolling over and collapsing toward the origin. A near virtual cathode structure develops at $z = 8$ cm where the potential is depressed to 12% of the anode value. The beam conditions are far from equilibrium and the radius is modulated by 50%. This is much more drastic than the first case where only gentle scalloping resulted. The current density as a function of radius at $z = .3$ cm is shown in Fig. 2b. The emission density varies from 5,000 to 25,000 amp/cm^2. Again, the beam has a hollow current profile with preferential emission nearest the anode. The brightness versus enclosed current is constant at a value of approximately 10^3 amp/(rad-cm)^2 at $z = 10$ cm. The brightness definition assumes a phase space with a constant energy value. For this case there is a substantial energy variation due to the potential depression. Thus the 10^3 value, calculated with an average energy is valid only within a factor of two.

The DPC results are summarized in Table II.

r_{cath} (cm)	Stress (MV/cm)	(amp/[rad-cm]^2)	Current (kA/cm^2) density
9.0	~ 1.0	0.3	0.2 to 0.6
1.2	~ 6.0	1000.0	5.0 to 25.0

TABLE II DPC Results

CONCLUSION

The envelope formulation has been used to obtain the scaling of brightness for immersed emission of an electron beam. The brightness scales linearly with beam current and inversely with magnetic field squared. The brightness scales inversely as the fourth power of the cathode radius. In the parameter regime of interest the envelope formulation predicts a matched equilibrium radius ($R_{eqin} = R_{eqout}$ = 1.64 cm) with a brightness of 120 amp/(rad-cm)^2. To get brightness above 10^3 amp/(rad-cm)^2 it is necessary to have a radius less than 1.36 cm which implies a current density greater than 17.2 kA/cm^2.

The predictions of the envelope formulation have been further investigated with the DPC computer code. Two 8 MeV cases were examined with configurations designed to generate 100 kA. The first case is thought to be buildable, but having an undesirably large electric field stress. This case with a 9 cm radius cathode resulted in an unacceptably poor brightness of .3 amp/(rad-cm)^2. In the second case the cathode radius was reduced to yield higher brightness. Between these two cases the inverse radius to the fourth scaling worked well and a 1.2 cm radius cathode yielded a brightness of 10^3 amp/(rad-cm)^2. This level of brightness is only marginally interesting. However, it is not believed that such a configuration could be operated in a repetitive mode experimentally. It may be possible to improve brightness by a factor of 2 or 3 by employing multiple electrodes. In a multiple electrode scheme the first gap stress must be large enough to draw the required 100 kA current Succeeding gaps must have the same or increasing stress levels to avoid de-focusing. Thus, again an experimentally possible configuration is unlikely.

Envelope formulation predictions of brightness are more optimistic than DPC computational results. There are at least two aspects of the envelope equation which can lead to this discrepancy. First, the envelope equation does not account for the dynamic evolution of the real beam profile. The envelope equation has no knowledge of radial mixing of charge. This effect has a great impact on brightness. Second, the envelope equation loses validity when the axial velocity is comparable to the transverse velocity. This difficulty is especially pronounced near a region of potential reduction such that $\beta\gamma$ is not approximately equal to γ.

REFERENCES

[1] V. K. Neil, "Emittance and Transport of Intense Hollow Electron Beams," Lawrence Livermore National Laboratory, Rept. UCID 17673, Nov. 1977.

[2] E. P. Lee, R. K. Cooper, Particle Accelerators, 7, 83 (1976).

[3] D. Prosnitz, private communication.

[4] J. K. Boyd, G. J. Caporaso, A. G. Cole, "Numerical Simulation of an Accelerator Injector," IEEE Trans. Nuc. Sci., Vol. NS-32, p. 2602, Oct. 1985.

JKB:jma-0124j

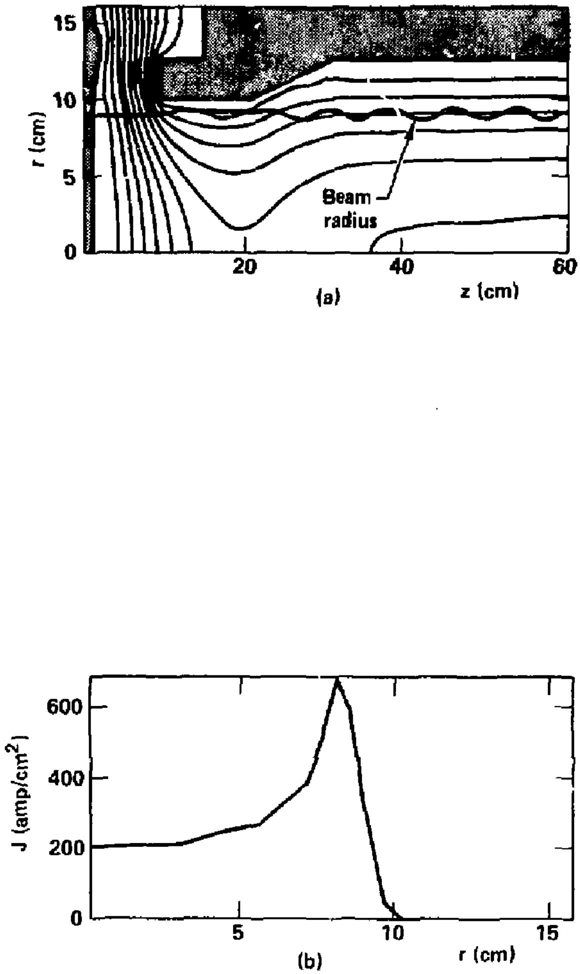
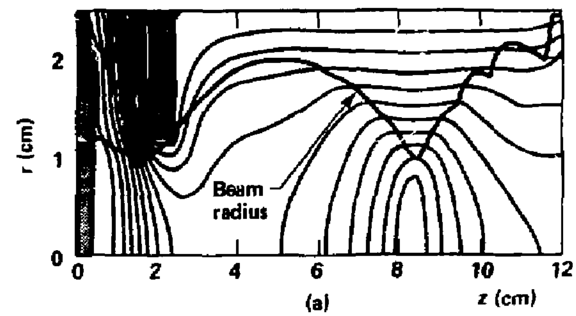


Figure 1. DPC result with 9 cm radius cathode: a) Beam radius and potential lines with 667 kV spacing; b) Current density versus radius at z = 1.5 cm.



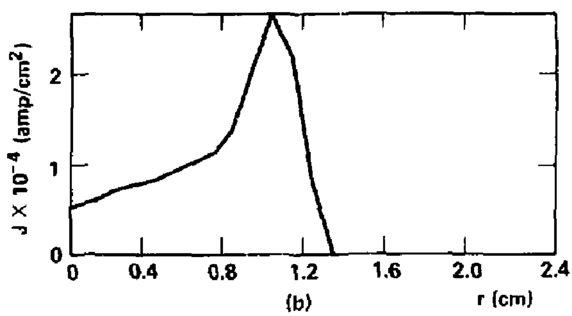


Figure 2. DPC result with 1.2 cm radius cathode: a) Beam radius and potential lines with 667 kV spacing; b) Current density versus radius at z = .3 cm.