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ISAJET 5.02:
A Monte Carlo Event Generator for pp and $\bar{p}p$ Interactions

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Perturbative QCD provides a very good description of all available data on hard interactions, those with momentum transfers large compared to 1 GeV. But perturbative QCD is formulated in terms of quarks and gluons, not the observed hadrons. The hadrons are presumed to be formed by nonperturbative aspects of QCD characterized by small momentum transfers, implying the creation of a jet of hadrons with limited p_T from each quark or gluon. This nonperturbative hadronization is not understood in any fundamental way, but it can be described by a variety of phenomenological models. Since the pioneering work of Field and Feynman,[1] Monte Carlo programs based on a combination of perturbative QCD and nonperturbative models have played an increasingly important role in particle physics. They are now widely used to extract results on jets from experimental data, to study signatures and backgrounds for various processes, and to correct data for detector effects.

Any QCD Monte Carlo program must describe the complete range of Q^2 from the initial hard scattering to the formation of hadrons at $Q^2 \leq 1\,\mathrm{GeV}^2$. The largest values of Q^2 are described by the appropriate cross section calculated in QCD perturbation theory. The small values are in the range for which confinement is important, so models must be used. For intermediate values higher order QCD processes must be taken into account. Partons (quarks and gluons) of a given Q^2

will in general radiate additional partons at lower Q^2, giving rise to a cascade. This radiation is most important when the radiated partons are nearly collinear; in this limit the probability for each additional radiation is given by a factor,

$$\sigma = \sigma_0 \left[\frac{\alpha_s(Q^2)}{2\pi Q^2} P(z) \right],$$

where z is the momentum fraction carried by one of the radiated partons and $P(z)$ is an Altarelli-Parisi function.[2] A reasonable approximation is to keep this simple factorized form for the cross section but to use exact, noncollinear kinematics. This approximation was introduced some time ago by Fox and Wolfram[3] for final state partons, but it has appeared to be more difficult to implement for the initial-state radiation present in hadronic reactions.

The first programs[4,5] to incorporate initial state gluon radiation clearly demonstrated its significance, but they either were slow[4] or required pretabulation of shower configurations.[5] Recently, Gottschalk[6] and Sjostrand[7] have proposed ways to simulate initial state QCD radiation which are both straigtforward and efficient. The essential idea is first to generate the primary hard scattering with the largest Q^2 and then to work backwards to the initial protons. This approach has been used to incorporate initial state radiation in ISAJET 5.02. With this addition ISAJET gives a reasonably good description of the main features of hadronic reactions, including in particular the data on jet and W production from the S$\bar{p}$pS. Of course, many aspects of QCD perturbation theory are still not incorporated, and nonperturbative hadronization is not well understood.

ISAJET simulates events in four distinct steps incorporating both perturbative QCD and nonperturbative models for hadronization:

(1) A primary hard scattering is generated according to one of several available cross sections combined with nonscaling structure functions.

(2) QCD radiation is added from both initial and final partons, thus producing scaling violations in jet fragmentation and also multiple jet events.

(3) Hadrons are produced from each parton using the independent fragmentation ansatz. (This ansatz adequately describes the fast hadrons in a jet, although its treatment of the slow hadrons is not very good.)

(4) Hadrons from the spectator beam jets are added, assuming that these are indistinguishable from a minimum bias event at the reduced energy.

All four steps will be discussed in turn below.

Hard Scattering

The first step in simulating an event is to generate the primary hard scattering. This is done according to a cross section $\hat{\sigma}$ calculated to leading order in QCD perturbation theory and convoluted with structure functions incorporating leading-log scaling violations. That is, the hard scattering cross section has the standard form for the QCD-improved parton model,

$$\sigma = \hat{\sigma} F(x_1, Q^2) F(x_2, Q^2),$$

where x_1 and x_2 are the usual momentum fractions. The default parameterization of the structure functions is Solution 1 of Eichten, Hinchliffe, Lane and Quigg (EHLQ).[8]

The following hard-scattering processes have been incorporated in ISAJET 5.02:

Minimum Bias: No hard scattering at all, so that the event consists only of beam jets. The term minimum bias suggests that these events are representative of the total non-diffractive cross section. At high energy this is not true, since the structure functions become large for $x \to 0$, causing the jet cross sections to become large. In fact, for $\sqrt{s} = 40\,\mathrm{TeV}$, lowest order QCD gives

$$\int_{3\,\mathrm{GeV}}^{\infty} dp_T \left(\frac{d\sigma}{dp_T}\right) \approx 200\,\mathrm{mb}.$$

While this result is not quantitatively reliable, recent work[9] suggests that the domain of validity of perturbation theory does extend down to rather small x and that events with jets will be common rather than rare at high energy. This may be quite important for estimating backgrounds from minimum bias interactions.

QCD Jets: All of the usual $O(\alpha_s^2)$ processes for two-body QCD scattering, including

$$g+g \to g+g, \qquad g+q \to g+q, \qquad g+g \to q+\bar{q}, \ldots.$$

These processes all give two jets in lowest order, but they can produce multiple jets after initial-state and final-state QCD gluon radiation is included. Quark masses are neglected for c and lighter quarks. Note that the EHLQ structure functions include heavy quarks, so that processes like $g+t \to g+t$ are also included. The algorithm for initial state radiation insures that the branching $g \to t+\bar{t}$ always occurs, so that heavy quark quantum numbers are conserved.

An optional fourth generation of quarks can be produced by gluon or quark fusion. The fourth generation is not included in the structure functions or in the QCD jet evolution.

Drell-Yan: Production and decay of a W, meaning any of γ, W^+, W^-, or Z^0 in the standard model. The leading order process is

$$q+\bar{q} \to W \to \ell+\bar{\ell},\ q+\bar{q}.$$

While to lowest order this process gives a W with $p_T = 0$, transverse momentum will of course be generated by the initial state gluon radiation. Alternatively, the $O(\alpha_s)$ processes

$$g+q \to W+q, \qquad q+\bar{q} \to W+g$$

can be simulated. These are the dominant processes at high p_T, but they give a $1/p_T^2$ singularity as $p_T \to 0$. To obtain a cross section which gives sensible results for all p_T, a cutoff is introduced for this singularity:

$$p_T^{-2} \to \left(p_T^4 + p_{T0}^4(M)\right)^{-1/2}, \qquad p_{T0}^2(M) = (.1\,\mathrm{GeV})M.$$

The form of this cutoff is roughly that obtained[10,11] from calculating the p_T distribution by summing the leading double logarithms of QCD perturbation theory, while the coefficient is chosen to get an integrated cross section about equal to the standard Drell-Yan result. Thus the $O(\alpha_s)$ cross section with the cutoff can be used for all p_T, although it does not give quite the right rapidity distribution.

W Pairs: Production of W pairs in the standard model. Only the processes

$$q + \bar{q} \to W^+ + W^-,\ Z^0 + Z^0,\ W^\pm + \gamma,\ W^\pm + Z^0$$

are included. In particular, contributions from Higgs bosons and from processes like

$$q + q \to q + q + W^+ + W^-$$

are ignored, even though the latter are important at high masses. The full matrix element for the decay of the W bosons is included in the narrow resonance approximation. Pairs of W bosons are expected to be one of the most important signatures for new physics in very high energy hadronic collisions.

Supersymmetry: Production of pairs of supersymmetric particles in the simplest model with global supersymmetry. The scalar partners of left-handed and right-handed quarks are taken to be degenerate, the $\tilde{\gamma} - \tilde{Z}^0$ and $\gamma - Z^0$ mixings are assumed to be identical, and mixings of gauginos and Higgsinos are ignored. Then the cross sections for producing supersymmetric particles are completely determined by the masses and the standard model. Both the $O(\alpha_s^2)$ processes

$$g + g \to \tilde{g} + \tilde{g}, \qquad g + q \to \tilde{g} + \tilde{q}, \ldots$$

and the $O(\alpha\alpha_s)$ processes,

$$g + q \to \tilde{W} + \tilde{q}, \qquad q + \bar{q} \to \tilde{W} + \tilde{g}$$

are included, where $\tilde{W}$ can be $\tilde{\gamma}$, $\tilde{W}^\pm$, or $\tilde{Z}^0$.

In general the user can specify the kinematic limits and the types for any of the primary partons. Then ISAJET finds a simple bound for the cross section within these limits and uses this bound to generate initial guesses for the kinematic points, improving the efficiency if a large number of events are generated in a run. Within a given run unweighted events are then produced by rejecting events with cross sections smaller than a random number times the bound used to generate the initial kinematic point.

QCD Radiative Corrections

For e^+e^- reactions a complete calculation[12] for the 2-jet, 3-jet, and 4-jet cross sections is available and gives a reliable approximation to QCD perturbation theory at PEP and PETRA energies. For hadronic reactions only the 2-jet and 3-jet cross sections and the $O(\alpha_s)$ loop corrections to the simplest 2-jet subprocess have so far been calculated, although more complete calculations are in progress.[13] Furthermore, the Q^2 scale for the S$\bar{\text{p}}$pS, and certainly for the Tevatron, is larger than that at PEP or PETRA, implying that multiple parton configurations are more important. Thus for hadronic reactions it is necessary to adopt a simple approximation which includes multiple jet states.

Consider the radiation of one extra gluon from a quark line,

$$q(p) \rightarrow q(p') + g(p'').$$

This radiation is most important in the collinear limit, $p^2 \rightarrow 0$, since it is this region which produces the leading-log scaling violations. From QCD perturbation theory, as $p^2 \rightarrow 0$ the cross section is given by the cross section σ_0 without the extra gluon times a simple factor,

$$\sigma = \sigma_0 \left[\frac{\alpha_s(p^2)}{2\pi p^2} P(z) \right],$$

where $P(z)$ is an Altarelli-Parisi function,[2]

$$P_{q\rightarrow qg}(z) = \frac{4}{3} \left(\frac{1+z^2}{1-z} \right),$$

and z is the momentum fraction,

$$z = \frac{p'_+}{p_+} = \frac{p'_0}{p_0} = \frac{p'_0 + |\vec{p}\,'|}{p_0 + |\vec{p}|} = \ldots .$$

The various choices for z are equivalent for collinear radiation but give different results when continued to large angles. The same form with different functions $P(z)$ holds for the other possible cases, $g \rightarrow gg$ and $g \rightarrow q\bar{q}$. Thus the most important part of the QCD radiation can be expressed in terms of cross sections or probabilities rather than amplitudes. By using exact noncollinear kinematics multiple jet states can be included, at least approximately. This provides a natural basis for a Monte Carlo algorithm called the branching approximation.[3]

The branching approximation correctly describes the leading-log scaling violations for the structure functions and for jet fragmentation. In fact the Altarelli-Parisi equations sum up just these pieces of the cross section with the masses p^2 strongly ordered. The branching approximation also gives correctly the structure of jets in QCD perturbation theory, since the typical mass of a jet is small, $M^2 = O(\alpha_s p_T^2) \ll p_T^2$, so that nearly collinear radiation dominates. Finally, the branching approximation turns out to reproduce the leading order three-jet cross section within a factor of about two over all of phase space.[14] This is good enough for many purposes, although it is certainly inadequate for extracting a quantitative measurement of α_s.

One defect of the branching approximation is that it overestimates the multiplicity of a jet at large Q^2. This occurs because while it correctly sums the leading series in $\ln Q^2$, it does not sum the $\ln(1/x)$ terms, and these are of comparable importance for the calculation of the multiplicity.[15] More physically, the leading series in $\ln Q^2$ comes from independent radiation of gluons from each parton. Many of the gluons which contribute to the multiplicity have low momentum, so they are not independent but couple to the total color charge. The correct calculation can be carried out and gives the same form but with the restriction that the emission angles as well as the masses are ordered. This has been implemented in a Monte Carlo program by Marchesini and Webber[16] but has not yet been incorporated in ISAJET. The correct calculation gives an asymptotic multiplicity growth[15]

$$\bar{n} \sim \exp\left\{\left[2\left(\frac{36}{33-2n_f}\right)\ln Q^2\right]^{1/2}\right\},$$

compared to the leading-log result

$$\bar{n} \sim \exp\left\{\left[4\left(\frac{36}{33-2n_f}\right)\ln Q^2\right]^{1/2}\right\}.$$

The difference is about a factor of two in the parton multiplicity, and less in the hadron multiplicity, for $Q \sim 1\,\mathrm{TeV}$.

Final State Radiation: The branching approximation was introduced for final state radiation by Fox and Wolfram.[3] Consider for simplicity only gluon radiation from a quark line; the general case is essentially identical. The approximate cross section for the emission of n gluons is

$$\frac{d\sigma}{dp_1^2 dz_1 \ldots dp_n^2 dz_n} = \sigma_0 \frac{1}{n!} \prod_i \left[\frac{\alpha_s(p_i^2)}{2\pi p_i^2} P(z_i)\right],$$
$$Q^2 > p_1^2 > p_2^2 > \cdots > p_n^2.$$

Note that this contains both collinear singularities, the explicit $1/p_i^2$, and infrared singularities, the $1/(1-z_i)$ in the $P(z_i)$.

The infrared singularities cancel in the usual way with those from the virtual graphs. This can be implemented simply by treating $P(z)$ as a distribution containing a $\delta(1-z)$ term such that

$$\int_0^1 dz P(z) = 0,$$

which is just the condition needed to ensure energy and momentum conservation in the Altarelli-Parisi equations. The collinear singularities do not cancel; they are needed to build up the leading-log QCD scaling violations. They are handled by introducing a cutoff $p_i^2 = t_i > t_c$ and assuming that QCD below the cutoff is described by the nonperturbative hadronization model. If t_c is relatively large, then the model must explicitly produce jets, but if $t_c \sim 1\,\mathrm{GeV}^2$, then it can be as simple as cluster decay with two body phase space. The choice made in ISAJET is to take $t_c = (6\,\mathrm{GeV})^2$ and to use independent fragmentation for the nonperturbative model.

The basic quantity needed to set up the Monte Carlo algorithm is the probability $\Pi(t_0, t_1)$ for evolving from an initial mass t_0 to a final mass t_1 emitting no gluon radiation greater than the cutoff. That is, $\Pi(t_0, t_1)$ is the probability for emitting no radiation which is to be treated explicitly rather than being included in the hadronization model. The formula is simple if the cutoff is taken to be not $t > t_c$ but rather $z_c < z < 1 - z_c$. Then the t and z integrations separate, and the result is[3]

$$\Pi(t_0, t_1) = \left[\frac{\alpha_s(t_0)}{\alpha_s(t_1)}\right]^{2\gamma(z_c)/b_0}, \qquad \gamma(z_c) = \int_{z_c}^{1-z_c} dz P(z).$$

Since the nonperturbative scale should be set by the mass at which QCD becomes strong, a cutoff on z is not very physical. But a t cutoff together with two-body kinematics for the decay implies a z cutoff. With the choice used in ISAJET

$$z = \frac{p_0' + |\vec{p}\,'|}{p_0 + |\vec{p}|},$$

the physical cutoff $t > t_c$ implies

$$z_c = \tfrac{1}{2}\left[1 - \sqrt{1 - 4t_c/t}\right].$$

This choice treats radiation from all lines symmetrically.

For a fixed z cutoff the whole Monte Carlo algorithm can be determined in terms of $\Pi(t_0, t_1)$. Since $\Pi(t_0, t_1)$ is by definition the probability for no resolvable radiation, its derivative $\Xi(t_1) = \partial\Pi(t_0, t_1)/\partial t_1$ gives the distribution for the mass t_1 at which the first resolvable radiation occurs. An explicit expression for $\Xi(t_1)$ is not needed, since to generate t_1 according to $\Xi(t_1)$ it is sufficient to solve

$$\frac{1 - \Pi(t_0, t_1)}{1 - \Pi(t_0, t_c)} = \xi,$$

where ξ is a uniformly distributed (pseudo)random number with $0 < \xi < 1$. For the given form of $\Pi(t_0, t_1)$ this equation can be explicitly solved for t_1 in terms of ξ. After t_1 for the first branching is chosen, a z for the branching is selected according to $P(z)$, and the masses of the two new partons are evolved starting from zt_1 and $(1 - z)t_1$ respectively. Given the masses and z, the momenta can be calculated. The whole procedure is then iterated.

To obtain a fixed t cutoff rather than a fixed z cutoff, the minimum value of z_c, corresponding to the initial value of t, is calculated, and a branching is generated as described above. Then the value of z is compared with the z_c calculated for the new mass t_1. If z is outside the allowed range, the branching is discarded and the evolution is continued from the new mass. In this way the simple form of $\Pi(t_0, t_1)$ can be used to generate events corresponding to the more complicated, but physicaly more reasonable, cutoff.[3]

The algorithm just described would be adequate if the final masses were always small compared to the initial mass, as is true in the leading log approximation.

Sometimes, however, the branching approximation gives rise to large final masses, as it must if it is to reproduce three-jet events. To handle these cases, it is necessary to introduce a variety of *ad hoc* modifications to prevent the program from failing on a few percent of the events. For example, in solving for the center of mass decay angle θ^* in terms of z, the larger of z or $1-z$ is used to calculate θ^* or $\pi - \theta^*$ respectively. All of these modifications correspond to non-leading terms in α_s and so can be legitimately introduced, but they do imply that what is actually simulated is not simply leading-log QCD.

The extension to include the branchings $g \to gg$ and $g \to q\bar{q}$ is straightforward. It is only necessary to include all of the possible terms in the definition of $\gamma(z_c)$ and to select among the various possibilities. For $t \gg t_c$, z_c is small, so that

$$\gamma_{g\to gg}(z_c) \sim 2c_A \ln(1/z_c)$$

dominates over

$$\gamma_{g\to\bar{q}q}(z_c) \sim n_q/3$$

Nevertheless, there is still significant heavy quark production in gluon jets. This seems to be the dominant real background for signatures involving leptons or missing p_T plus jets, so it is essential to consider it.

Initial State Radiation: There are two apparent problems in extending the branching approximation to initial state gluon radiation. The virtual masses are spacelike, so that kinematics alone does not order them, and the kinematic limits are set only by the total energy. More seriously, the QCD evolution starting at the proton almost never gives the correct momentum, making the algorithm very inefficient. As has been recently suggested by Gottschalk and by Sjostrand, the solution to both problems is to do the evolution backwards from the desired hard scattering, forcing the ordering of the virtual masses t_i by hand. In Gottschalk's approach, the QCD radiation is used to produce the scaling violations for the structure functions. In Sjostrand's approach, the nonscaling structure functions are assumed to be known and are used to calculate the probabilities for radiation. This allows the initial hard scattering to be generated according to the cross section with nonscaling structure functions, so it has been adopted in ISAJET 5.02.

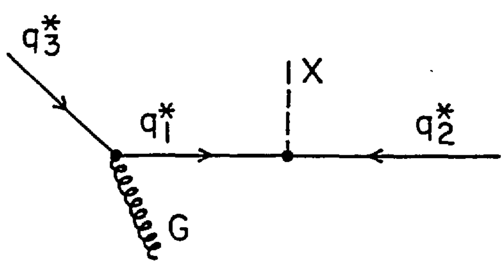

Fig. 1: Radiation of one extra initial state gluon.

Consider the emission of one extra gluon from a process producing a state X with mass s_{12}. As for final state radiation, all definitions of the momentum fraction z are equivalent if the gluon is collinear, but a choice must be made for noncollinear gluons. The choice for which the branching approximation reproduces first order QCD is[6]

$$z = s_{12}/s_{13}.$$

Then the cutoff $|t| > t_c$ plus two-body kinematics for the process $2 + 3 \to X + g$ implies that the upper limit on z is

$$z_{\max} = \frac{1}{1 + t_c/s_{12}}.$$

while the lower limit is set by the available beam momentum.

The probability for one additional radiation is obtained by noting that the structure functions satisfy the Altarelli-Parisi equations,[10]

$$\frac{df_b(x,t)}{dt} = \frac{\alpha_s(t)}{2\pi t} \sum_a \int_x^1 \frac{dx'}{x'} f_a(x',t) P_{a\to bc}\left(\frac{x}{x'}\right)$$

Hence the probability that b disappears in a backwards evolution through an infinitesimal step dt is

$$\begin{aligned} dP_b &= \frac{1}{f_b(x,t)} \frac{df_b(x,t)}{dt} dt \\ &= \frac{\alpha_s(t)}{2\pi t} dt \sum_a \int_x^1 \frac{dx'}{x'} \frac{f_a(x',t)}{f_b(x,t)} P_{a\to bc}\left(\frac{x}{x'}\right) \end{aligned}$$

The probability for a finite Δt is just the exponential of the infinitesimal probability. With $z = x/x'$ the probability for b to survive is[7]

$$S_b = \exp\left\{ -\int_{|t_1|}^{|t_0|} dt' \frac{\alpha_s(t')}{2\pi t'} \sum_a \int_{z_{\min}}^{z_{\max}} dz \left[\frac{(x/z) f_a(x/z,t)}{x f_b(x,t)}\right] P_{a\to bc}(z) \right\}.$$

This formula is the basis for the initial state Monte Carlo algorithm. It is rather similar to the final state formula for $\Pi(t_0, t_1)$ except for the ratio of structure functions in the square brackets. This ratio, which is derived from the Altarelli-Parisi equations, is physically very reasonal . It implies that a branching can occur only if the structure function $(x/z)f_a(x/z,t)$ for the new initial parton a is large enough so that finding it in the incoming hadron is not improbable.

In generating events according to S_b it is assumed that the structure functions have the usual Regge behavior as $x \to 0$, so that

$$\begin{aligned} &x f_g(x,t), \\ &\sqrt{x} f_v(x,t), \qquad v = u, \bar{u}, d, \bar{d}, s, \bar{s}, \\ &x f_h(x,t), \qquad h = c, \bar{c}, b, \bar{b}, t, \bar{t}, \end{aligned}$$

are monotonically decreasing functions of x. Then the ratio of structure functions, with a factor of $\sqrt{z}$ removed for the valence quarks, is bounded by its value at

$z = 1$. For gluons and light quarks values of t and z for a branching are generated just as for the final state case, using as the effective anomalous dimensions

$$\gamma_{g\to gg} = \int_{z_{\rm min}}^{z_{\rm max}} dz P_{g\to gg}(z),$$
$$\gamma_{q\to gq} = \frac{f_q(x,t_0)}{f_g(x,t_0)} \int_{z_{\rm min}}^{z_{\rm max}} dz P_{q\to gq}(z)/\sqrt{z},$$
$$\gamma_{q\to gq} = \int_{z_{\rm min}}^{z_{\rm max}} dz P_{q\to gq}(z)/\sqrt{z},$$
$$\gamma_{g\to q\bar q} = \frac{f_g(x,t_0)}{f_q(x,t_0)} \int_{z_{\rm min}}^{z_{\rm max}} dz P_{g\to q\bar q}(z).$$

Then the branching is rejected if the ratio of structure functions is less than a random number times the bound.

For a heavy quark the $f_b(x,t)$ vanishes at the threshold $t_q^* = 4m_q^2 x/(1-x)$, making the ratio of structure functions infinite and so forcing the branching $g \to q\bar q$ to occur. Hence a heavy quark in the initial state will be accompanied by an associated antiquark. In this case the evolution is restricted to be between $|t_0|$ and $\sqrt{|t_0 t_q^*|}$, the ratio is evaluated at the lower limit, and the process is repeated if no branching occurs. A branching is forced if none occurs before $t_q^* + t_c$.

Once values of t and z have been found, a mass k^2 for the radiated parton is generated by the final state evolution algorithm starting at the kinematic limit

$$k^2_{\rm max} = -t\left[\frac{1}{z(1-t/s)} - 1\right].$$

Then given t, z, k^2, and the azimuthal angle ϕ, which is generated uniformly, it is possible to solve for the four components of k with the new initial partons 2 and 3 having zero transverse momentum. The whole procedure is then iterated with the system X containing one additional parton.

Jet Fragmentation

Once partons are generated by the hard scattering and the QCD jet evolution, hadrons must be formed. In ISAJET 5.02 this is done using the independent fragmentation ansatz originally proposed by Field and Feynman.[1] To fragment a quark q of momentum p, a $q'\bar q'$ pair is generated with $u : d : s = .4 : .4 : .2$. (To incorporate baryons, diquark pairs are generated with a total probability of .08 and with the same mixture of flavors.) The q' and $\bar q'$ are given transverse momenta $\pm\vec{k}_T$, with $< k_T >= .35\,{\rm GeV}$. Then the $\bar q'$ is combined with the original q to form a 0^- or 1^- meson with equal probability, which is about right for light mesons but probably underestimates the fraction of heavy vector mesons. The momentum $p'_+ = E + p_L$ of the meson is taken to be $p'_+ = zp_+$, where z is generated according to

$$f(z) = 1 - a + a(b+1)(1-z)^b, \qquad a = .77,\ b = 2$$

for light quarks and

$$f(z) = (b+1)z^b, \qquad b = 0 \text{ for } c,\ b = 2 \text{ for } b,\ b = 4 \text{ for } t,$$

for heavy quarks. Hadrons with negative p_L are discarded. The procedure is then iterated for the new quark q' with momentum $(1-z)p$. A gluon is fragmented like a light quark or antiquark with $u:d:s = .4:.4:.2$.

Independent fragmentation is very simple and correctly incorporates the most important features of jet fragmentation, particularly for the fast particles in the jet, but it has a number of obvious defects. Since a massless parton is fragmented into massive hadrons, energy and momentum cannot be conserved exactly. Energy and momentum conservation are enforced in ISAJET by boosting the system of fragmented jets to their rest frame, rescaling all the three-momenta by a factor, and recalculating all the energies. Similarly, flavor is not conserved, since hadrons with $p_L < 0$ and the final quark are discarded. Finally, since jets are fragmented independently, a collinear branching of a quark into a quark and a gluon gives a larger multiplicity than a single quark, even if the quark-gluon mass is so small that they could not possibly be resolved. Branchings down to some fixed cutoff must be included in the QCD jet evolution to get the correct scaling violations. The problem is that the structure of the events depends on the cutoff. This problem is minimized in ISAJET by taking a relatively high cutoff, $t_c = (6\,GeV)^2$, for which independent fragmentation is more reasonable.

For e^+e^- interactions the Ali et al.[17] and Hoyer et al.[18] independent fragmentation models, the Lund string model,[19] the Gottschalk incoherent cluster model,[20] and the Webber coherent cluster model[21] all fit most of the data from PEP and PETRA very well. However, the Lund and Webber models, which include coherent effects between jets, do a better job of describing the slow particles in the events.[22] Since there are other sources of soft particles in hadronic reactions, independent fragmentation has been chosen for ISAJET on the basis of simplicity.

Beam Jet Fragmentation

After a hard scattering event has been generated, something must be done with the remaining constituents of the proton. While factorization in QCD perturbation theory implies that high p_T jets must be treated like jets in e^+e^- reactions, there is very little theoretical guidance on what to do with the beam jets. The assumption made in ISAJET is that they are identical to a minimum bias event at the reduced energy, but this is surely too simplistic.

All of the standard pictures of multiparticle production are based on the idea of pulling pairs of particles out of the vacuum, leading to only short-range rapidity correlations and essentially a Poisson multiplicity distribution. The minimum bias data clearly shows long-rang correlations and a broad multiplicity distribution. A very attractive resolution for this conflict was proposed by Abramovskii, Kanchelli, and Gribov (AKG).[23] Their idea is that the basic amplitude is a single chain, or cut Pomeron, having only short range correlations and giving an average multiplicity $\bar{n}$ with Poisson fluctuations. But unitarity then requires that this amplitude be iterated, leading to graphs with several cut Pomerons giving multiplicity

$2\bar{n}$, $3\bar{n}$,.... A different discontinuity of the two Pomeron graph gives the leading contribution to the elastic cross section, so the probability for $2\bar{n}$ should be of order $\sigma_{\rm el}/\sigma_{\rm tot} \approx .20$. Furthermore, events with high multiplicity in one region generally have several cut Pomerons and hence high multiplicity everywhere. All of this is in general agreement with experimental data on minimum bias interactions.

A simplified version of the AKG scheme modified to take account of leading particles has been implemented in ISAJET. A number K of cut Pomerons is selected. For the left and right sides of the event an x_0 for the leading baryon and an x_i for each Pomeron are generated uniformly between 0 and 1, and the sum $x_1 + \cdots + x_K$ is rescaled to $1 - x_0$. Then each cut Pomeron is hadronized in its own center of mass using a modified independent fragmentation model. The probabilities for the different flavors are taken to be $u : d : s = .46 : .46 : .08$ to reproduce the observed fraction of K mesons. To incorporate the observed increase[24] in dn/dy the splitting function is made energy dependent:

$$f(x) = 1 - a + a(b(s) + 1)(1 - x)^{b(s)},$$
$$b(s) = b_0 + b_1 \ln s.$$

The probabilities P_K for K cut Pomerons are taken to be independent of energy and are adjusted to fit the experimental data. While it might appear more natural to make the P_K energy dependent, in the AKG analysis the single particle distribution is completely determined by the single chain graph. The energy dependence of $f(x)$ corresponds in Regge language to a Pomeron which is more singular than a simple pole, perhaps because of Pomeron interactions.

An alternative possibility, which is used in the Lund Monte Carlo PYTHIA,[7,25] is to treat the hadronization of beam jets like that of e^+e^- jets. Then it is necessary to introduce multiple scattering to reproduce the average multiplicity and the fluctuations seen in the minimum bias hadronic data. In PYTHIA, the number of scatterings is taken to be proportional to the QCD jet cross section,

$$\frac{dN}{dp_T} = \frac{1}{\sigma_0}\frac{d\sigma}{dp_T}$$

where σ_0 is a normalization which should be of order the total cross section. Physically, σ_0 gives the probability for any interaction, and dN/dp_T is the number of jets per interaction. At very high energies something like this approach is necessary, since events with jets comprise a large fraction of the total cross section. At S$\bar{\rm p}$pS energies it offers the possibility of relating the beam jets to e^+e^- physics. However, it loses the connection with unitarity built into the AKG approach, and unitarity should be important for beam jet physics.

At SSC energies it is important to include low-p_T QCD jets when studying the effects of minimum bias events. The best way to do this with ISAJET is probably to generate a sample of jet events with the lower limit on p_T adjusted to give a cross section equal to the total cross section. This handles intermediate values of p_T correctly, although it does not include the effects of several very soft jets in one event.

Results and Discussion

Jet Shape: QCD jet events were generated for the S$\bar{p}$pS with $\sqrt{s} = 540\,\mathrm{GeV}$ and $p_T > 35\,\mathrm{GeV}$ and were put through a trivial calorimeter simulation with cells $\Delta y = .1$ and $\Delta\phi = 5°$ and energy resolutions $.15/\sqrt{E}$ for electrons and photons and $.70/\sqrt{E}$ for hadrons. Jets were found by the UA1 algorithm, clustering all cells of the calorimeter with $E_T > .5\,\mathrm{GeV}$ and within

$$\Delta R = \left[(\Delta y)^2 + (\Delta\phi)^2\right]^{1/2} < 1.$$

For all jets with $p_T > 20\,\mathrm{GeV}$ the scalar transverse energy flow dE_T/dy was plotted versus $y - y_{\mathrm{jet}}$. The results and the data from the UA1 Collaboration[26] are shown in Fig. 2. Compared to the data, ISAJET 5.02 gives slightly too narrow jets, but the transverse energy flow away from the jets is about right. This is in contrast with previous versions which did not include initial state QCD radiation and which gave too little dE_T/dy by a factor of two away from the jets.

Results from PYTHIA including only single scatterings[7] are also shown in Fig. 2. This model gives too low a value for dE_T/dy away from the jets, even though it treats initial state QCD radiation in the same way as ISAJET. There are two sources for the difference. PYTHIA uses Lund string fragmentation, which tends to merge two nearly collinear jets and so gives fewer hadrons from the same initial state gluons. For ISAJET the average transverse momentum of the initial state partons is about 4 GeV, which is much greater than the strong interaction scale but not big enough to make independent fragmentation obviously a good approximation. Also, PYTHIA treats the beam jet as just another string system, giving a minimum bias multiplicity significantly below the data at S$\bar{p}$pS energies.

PYTHIA can obtain agreement[25] with the data by including multiple hard scatterings with $p_T > 1.6\,\mathrm{GeV}$ with a normalization cross section $\sigma_0 = 10\,\mathrm{mb}$. The value of σ_0 seems rather small, and the fit to the minimum bias data requires $\sigma_0 = 40\,\mathrm{mb}$, so that the jet model does not extrapolate smoothly to low p_T. This may be a detail, or it may indicate that the physics of beam jets is really different, perhaps because unitarity is important.

There is no reason to believe that the underlying beam jet must be the same in a hard scattering event as in a minimum bias event. Indeed, one might expect that a hard scattering would disrupt the leading proton and so change the beam jet, and there is some data which suggests that this in fact occurs. Since string fragmentation seems to work better than independent fragmentation for e^+e^- interactions, it may be that less of the dE_T/dy should be attributed to gluon radiation and more to soft physics than is the case in ISAJET. Nevertheless the agreement is satisfying.

$W^\pm$ Transverse Momentum: Events with $W^\pm \to e^\pm\nu$ were generated for the S$\bar{p}$pS with $\sqrt{s} = 540\,\mathrm{GeV}$ and with transverse momentum produced by the initial state QCD radiation. The missing p_T of the ν was reconstructed with the same trivial calorimeter simulation described above and the p_T of the $W^\pm$ was calculated. With just the p_T from the gluon radiation it originally was found that there were too many events at very low p_T, presumably because the rather high cutoff t_c

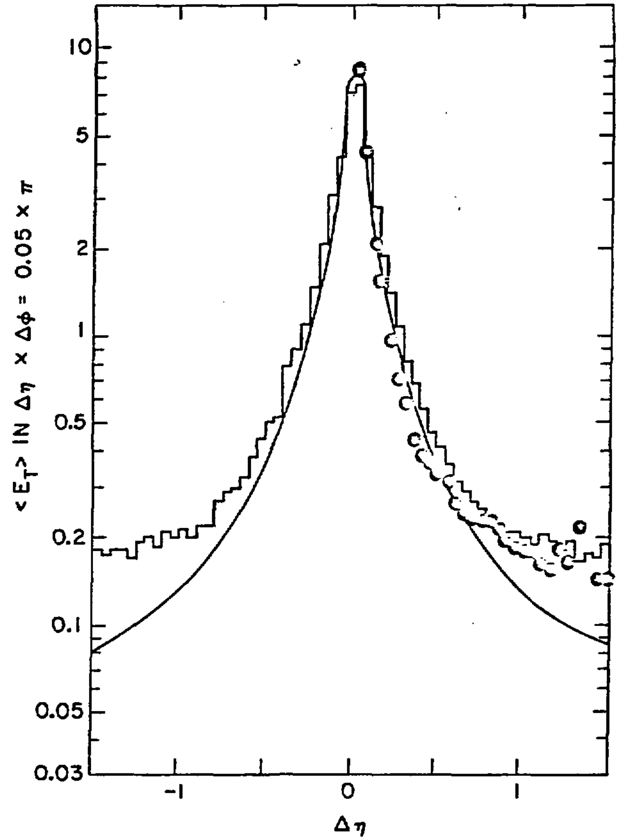

Fig. 2: Transverse energy flow dE_T/dy in $\Delta\phi < 90°$ vs. $y - y_{\rm jet}$. The histogram is data from the UA1 Collaboration,[26] the circles are from ISAJET 5.02, and the smooth curve is from PYTHIA.[7]

needed with independent fragmentation eliminates too much soft gluon radiation. To cure this, opposite transverse boosts are made to the hard scattering system and to the beam jets with

$$< p_T^2 >= (.1\,\mathrm{GeV})\sqrt{Q^2},$$

a form similar to the p_T dependence obtained by summing soft gluons.[10,11] Then the results are in good agreement with the data, as can be seen in Fig. 3.

Jets at SSC: At higher mass scales the perturbative QCD effects should dominate the nonperturbative hadronization. Fig. 4 shows the predicted transverse energy flow from ISAJET for jets with $p_T > 1\,\mathrm{TeV}$ at $\sqrt{s} = 40\,\mathrm{TeV}$. The value of dE_T/dy away from the jet increases almost in proportion to p_T. This is as expected: for $p_T \gg \Lambda_{\overline{MS}}$ the transverse momentum carried by the extra radi-

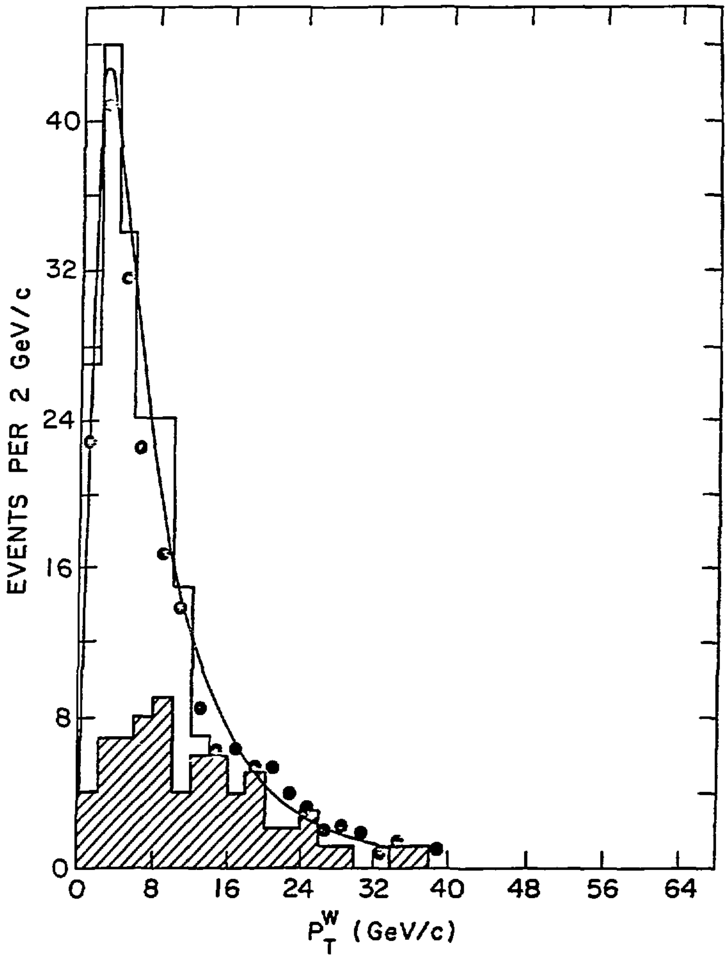

Fig. 3: Transverse momentum distribution for $W^{\pm}$. The histogram is the data from the UA1 Collaboration,[27] with the shaded area representing the events for which jets are found. The points are from ISAJET 5.02. The smooth curve is an analytic calculation done by summing soft gluons.[11]

ated gluons must be proportional to the primary p_T. Such an increase will have important consequences for experiments at very high energies.

The smooth curve shown in Fig. 4 is not really representative of the individual events, which contain many fluctuations. Fig. 5a-c show the transverse energy flow for three selected events with $p_T = 1\,\mathrm{TeV}$ at $\sqrt{s} = 40\,\mathrm{TeV}$. Fig. 5a looks almost like the naive parton model. Fig. 5b is 'typical' in that there are two clear primary jets, each fragmented into several subjets. This is what is expected from the renormalization group structure of QCD. Fig. 5c is the most complicated of

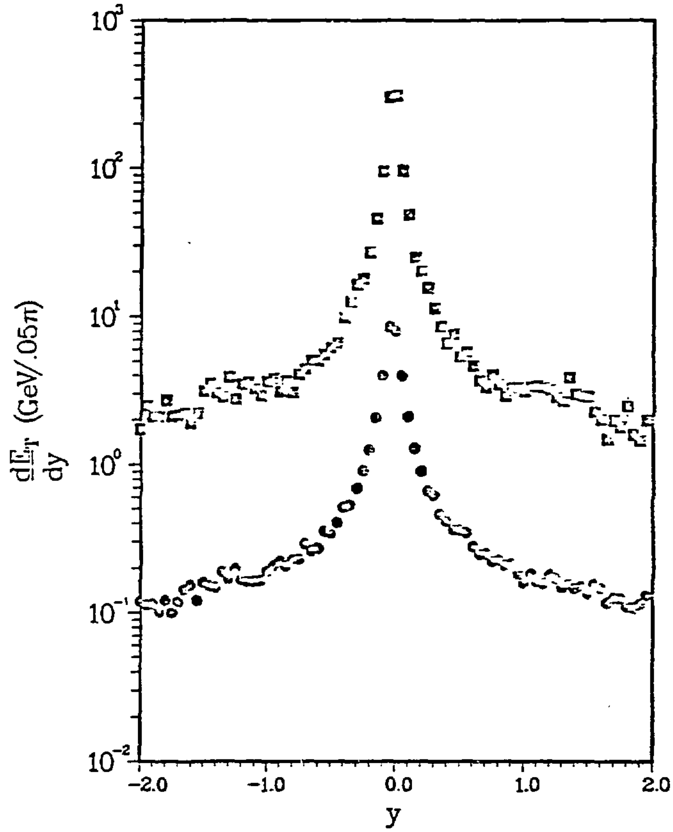

Fig. 4: Transverse energy flow dE_T/dy in $\Delta\phi < 90°$ vs. $y - y_{\rm jet}$ for jets with $p_T > 35\,\text{GeV}$ at $\sqrt{s} = 540\,\text{GeV}$ (circles) and for jets with $p_T > 1\,\text{TeV}$ at $\sqrt{s} = 40\,\text{TeV}$ (squares).

the first ten generated. Events from PYTHIA are somewhat less collimated than those from ISAJET. Because of fluctuations of this sort, multiple jets are not a good signature at SSC energies.

QCD Monte Carlo programs for e^+e^- have for some time been able to describe the data very well. With the incorporation of initial state gluon radiation the programs for hadronic reactions now reproduce at least all the main features, although they have certainly not reached the same level of precision. To make further progress it will be necessary to understand better the hadronization of jets and especially of beam jets. To this end it is important to have more experimental data on the low p_T particles in hard scattering events.

The authors wish to thank Tom Gottschalk and Torbjorn Sjostrand for many valuable discussions during the Supercollider Physics Topical Conference at the

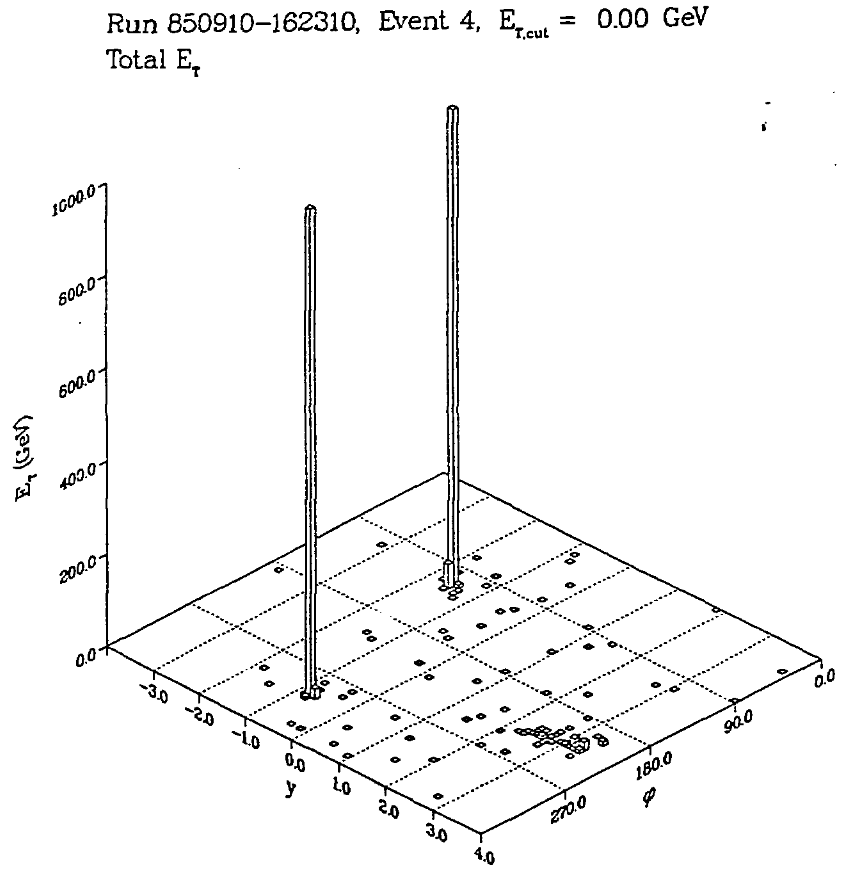

Fig. 5a: Transverse energy flow for an ISAJET jet event selected from the first ten generated at $p_T = 1\,\mathrm{TeV}$ and $\sqrt{s} = 40\,\mathrm{TeV}$.

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Appendix: Implementation

ISAJET is maintained with PATCHY, the CERN Program Library code management system, for CDC 7600 and CYBER 175, DEC VAX, and IBM 370 and 30xx computers. It is written mainly in ANSI standard FORTRAN 77, but there is some machine dependent code, especially in the CDC version.

PATCHY allows several logically distinct files to be combined together into a single compressed binary file called a PAM (for program assembled material). The ISAJET PAM contains four major sections, each of which can be extracted

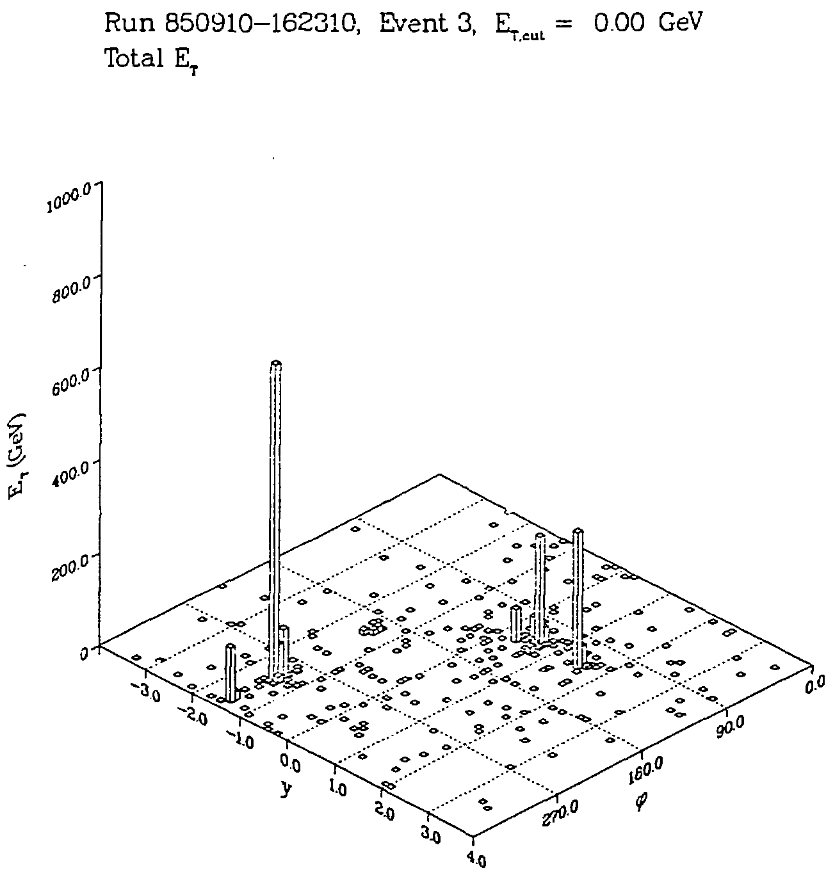

Fig. 5b: Transverse energy flow for an ISAJET jet event selected from the first ten generated at $p_T = 1\,\mathrm{TeV}$ and $\sqrt{s} = 40\,\mathrm{TeV}$.

separately:

- **ISAJET:** Code to generate, to write, and to read events.
- **ISATEXT:** Instructions for using the program.
- **ISADECAY:** Table of decay modes.
- **ISAPLT:** Analysis package.

In each section the machine dependent features are obtained by selecting the desired computer (CDC, IBM, or VAX). It is also possible to select use of the CERN Program Library for machine dependent utilities such as the random number generator.

ISAJET is primarily designed to generate events and to write them onto a file for later analysis. It is controlled by commands read from the input; the user must

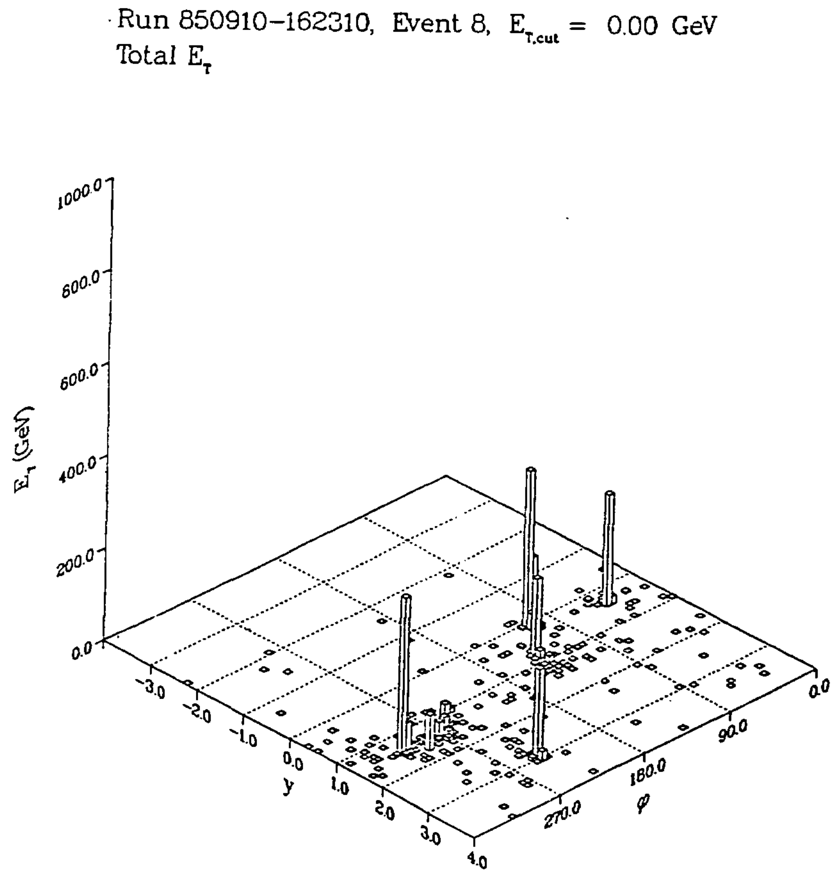

Fig. 5c: Transverse energy flow for an ISAJET jet event selected from the first ten generated at $p_T = 1\,\mathrm{TeV}$ and $\sqrt{s} = 40\,\mathrm{TeV}$.

specify the energy, the number of events, the process, and any kinematic limits or other parameters. Then the program writes a file containing an initial record, a record for each event, and a final record containing the integrated cross section for the run. Subroutines are also prc /ided to read this file and to restore the information to various common blocks. In the event record, the common block /PJETS/ contains the primary partons generated according to the QCD cross section, /JETSET/ contains the parton cascade from the QCD jet evolution, and /PARTCL/ contains the particles from both the jet fragmentation and the beam jet fragmentation. In each case the information consists of jet or particle types and momenta plus pointers to allow jet evolution, hadronization, and decays to be traced either forwards or backwards.

ISATEXT contains detailed instructions for using ISAJET. It is divided into the following sections:

1. Introduction.
2. Physics. A brief summary of the physics assumptions used in the program and described in this document.
3. Sample Job. A sample job for generating events on each computer.
4. PATCHY. A description of the organization of the ISAJET PAM and some brief comments on the use of PATCHY.
5. Main Program. The format for the main program, which must be supplied by the user.
6. Input. The complete list of input commands and the default values of all user parameters.
7. Output Tape. The format for the output file, and the list and description of the common blocks into which the information is restored.
8. Tape Reading. Usage of the subroutines which read an ISAJET tape or file.
9. Decay Table. Format for the decay table.
10. IDENT Codes. Description of the scheme for identitfying particles and a list of the codes.

ISATEXT should allow a novice user to work with the program.

ISAPLT contains the skeleton of a job for making properly weighted histograms of cross sections using the HBOOK package from the CERN Program Library. It also contains a trivial calorimeter simulation package and a jet finding algorithm, which may be useful for some applications.

ISAJET is available on request from the authors, and PATCHY and the instructions for using it are available from the DD Division of CERN.

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