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TIME-DEPENDENT THOMAS-FERMI APPROACH TO NUCLEAR MONOPOLE OSCILLATIONS+

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TIME-DEPENDENT THOMAS-FERMI APPROACH TO NUCLEAR MONOPOLE OSCILLATIONS[+]

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Abstract

A Time-dependent Thomas-Fermi (TDTF) method has been used to study nuclear monopole oscillations and in particular, the semiclassical strength function S(E). An analysis of the S(E) moments, obtained by suitable integrations, shows that these results are in good agreement with data previously obtained from static Thomas-Fermi calculations.

In the last years, Nuclear Hydrodynamics has been widely applied to static and dynamic problems (see ref. 1 and references therein). It is possible to cast the hydrodynamical equations in a form quite similar to the one adopted for the standard time-dependent Hartree-Fock equations [2-5] .

In the present note we want to emphasize how that procedure can be applied and delineate some of its potential capabilities, rather than compare its theoretical results with the corresponding experimental data. The equations in question will only be applied to small amplitude, zero temperature monopole oscillations.

Our computational procedure is based on the well known fluid dynamics equations (see sect. 13.3.4 of ref. 1 for notation):

$$\dot{\rho} = -\vec{\nabla}\left[\rho\vec{\nabla}S\right] \tag{1}$$

$$-m\dot{S} = \frac{1}{2}m\left(\vec{\nabla}S\right)^2 + \frac{\delta e[\rho]}{\delta\rho} \tag{2}$$

where $e[\rho]$ is the <u>modified</u> BKN potential energy density [6] plus the Thomas-Fermi (TF) expression for the kinetic energy, including the Weizsäcker correction. Introducing the <u>fictitious</u> wave function

$$\Psi(r,t) = \rho^{1/2}(r)\, e^{i\frac{m}{\hbar}S(r,t)} \tag{3}$$

eqs. 1 and 2 can be combined [2-5] into a single time-dependent

Schrödinger equation:

$$i\hbar \frac{\partial}{\partial t} \Psi(r,t) = -\frac{\hbar^2}{2m} \Delta \Psi(r,t) + V \Psi(r,t) \qquad (4)$$

where the single particle potential V is given by

$$V = \frac{\delta e[\rho]}{\delta \rho} + \frac{\hbar^2}{2m} \frac{\Delta \rho^{1/2}}{\rho^{1/2}} \qquad (5)$$

Obviously, eqs. 1-5 hold for both protons and neutrons. In order to compute the pertinent isoscalar (IS) monopole oscillations with this dynamical method one uses, as initial condition, the static self-consistent equilibrium solution obtained with the same force as the one used in the dynamical case, pertaining to the constrained Hamiltonian

$$H + \lambda \langle r^2 \rangle \equiv H + \lambda \int r^2 \rho(r)\, d\vec{r} \qquad (6)$$

Coulomb interaction has not been included and we have restricted ourselves to N=Z nuclei. The value of λ has to be small enough so that linear response theory applies. We used $\lambda = 0.02$ MeV fm^{-2} and verified that the results obtained were insensitive to its precise value. The static solution was then allowed to evolve in time according to eq. 4, using a time-step of $\Delta t = 0.25 \times 10^{-23}$ s for a time of about 2.5×10^{-21} s. The calculations were performed within a simulated sphere of 20 fm and an r-step of $\Delta r = 0.10$ fm.

Figures 1 and 2 show the time evolution of $\langle r^2 \rangle / A$ for ^{16}O and ^{40}Ca respectively. The weakly damped oscillations observed in

the figures present unambiguous periodic behaviour. For ^{40}Ca, an almost pure harmonic oscillation of period $\sim 1.3 \times 10^{-22}$ s corresponding to a collective energy of $\hbar\omega \sim 31.4$ MeV can be clearly distinguished. For ^{16}O, small beats can be discerned in the oscillation, but one can still extract a value of $\hbar\omega \sim 35$ MeV by simple inspection for the energy of the mode. This rather high value for the energy of the IS monopole resonance (or breathing mode) is due to the high value of the compressibility of the BKN force (~ 350 MeV), a behaviour displayed by all Skyrme and similar forces currently used in TDHF calculations [7,8]. Similar qualitative results were obtained in ref. 9 using TDHF.

One may obtain the IS monopole strength function by Fourier-analyzing figures 1 and 2 [7]. Introducing the IS monopole strength function

$$S(\omega) = \frac{1}{\hbar} \sum_k |\langle 0| r^2 |k\rangle|^2 \, \delta(\omega - \omega_k) \qquad (7)$$

where $|0\rangle$ is the ground state and $|k\rangle$ an excited state of energy $\hbar\omega_k$, it can be shown [4,7] that

$$S(\omega) = \frac{-\omega}{\lambda \eta} \int_0^\infty \left(\langle r^2(t) \rangle - \langle r^2(0) \rangle \right) \cos \omega t \, dt \qquad (8)$$

where r(t) is the nuclear radius at time t. It is worth emphasizing that eq. 8 was obtained using first order perturbation theory [4]. An identical expression is obtained when the state $\Psi(r,t)$ which evolves in time, is the TDHF state and the static states $|k\rangle$ and energies $\hbar\omega_k$ are RPA quantities [7]. In these cases, $S(\omega)$ would be the <u>exact</u> or the RPA strength function, respectively. No

such theorem (eq. 8) is known to relate TDTF and static TF frameworks. The validity of eq. 8 in the TF case is taken for granted at this stage although subsequent moment calculations of $S(\omega)$, discussed below, will justify this a priori choice.

Figures 3 and 4 show S(E) for ^{16}O and ^{40}Ca respectively. As we have already inferred by inspection of figure 2, the strength function of ^{40}Ca is concentrated in a very narrow, single peak whereas for ^{16}O an additional enhancement, located at about 8 MeV above the main peak can be seen. This second peak should not be considered as the second giant monopole state which, in a pure harmonic model, would lie at much higher energy ($\sim 2\hbar\omega$ in principle). Since our analysis is semiclassical in its very nature, we cannot assert whether the second peak is the echo of some underlying quantum strength or not. We want to point out, however, that other macroscopic models also give rise to several IS monopole resonances [10].

We have carried out a moment analysis of S(E). Some of these moments, defined as

$$m_k \equiv \int_0^\infty E^k S(E) dE \qquad (9)$$

can be computed from static calculations. Of special interest are the m_{-1}, m_1 and m_3 moments (sum rules)[11], which can easily be obtained in the TF approximation[12]. From these moments, two average energies and an estimate of the variance σ of S(E) can be defined:

$$E_3 = (m_3/m_1)^{1/2}$$
$$E_1 = (m_1/m_{-1})^{1/2} \qquad (10)$$

$$\sigma_{31}^2 = (E_3^2 - E_1^2)/4$$

Table 1 collects the results corresponding to ^{16}O and ^{40}Ca. Besides E_3, E_1 and σ_{31} we also show the average energy $\bar{E}$ and the variance, both computed directly from S(E) as follows:

$$\bar{E} = m_1/m_o \qquad (11)$$
$$\sigma^2 = m_2/m_o - (m_1/m_o)^2$$

In the table, the row labeled "dynamic" corresponds to the results obtained frum brute force integrations of eq. 9, whereas the "static" results were obtained as indicated in ref. 12. One can see from the table that the overall agreement is good.

We have also performed some preliminar calculations at finite temperature [4], following the methods proposed in refs. 2-4 and 12. In this case, the agreement between static and dynamic calculations turns out to be as good as at zero temperature.

Discussion. It is well known that the energy of the monopole mode can be obtained from many different methods besides the costly, detailed continuum RPA calculations. In this letter, our main point has been to show that, in the small amplitude limit, our TDTF model does reproduce the nuclear monopole oscillations, the collective energies being those one should expect from the nuclear hamiltonian we employ . The use of Nuclear Hydrodynamics means that our description of the monopole mode is a first sound one [1] . As a consequence, the description of higher multipolarity resonances is beyond its scope (a possible exception might by the dipole resonance in the Goldhaber-Teller model).

Concerning the monopole case, where the (existing) deformations of the Fermi sphere does not seem to play a relevant role, we expect our static sum rule calculations to be reliable for the following reasons. It has been pointed out [13] that the hydrodynamical (HD) model should be a reliable method for obtaining the m_{-1} and m_1 sum rules, even in the case of non-spherical, static constraining fields [14]. For m_3, the reliability of the HD model in the case of the monopole mode, comes from the accidental fact that the quantum kinetic energy and the TF one change in the same way when scaling the wave functions (and densities, respectively) [11]. Thus, it is not surprising that static TF calculations have yielded good results in the study of the breathing mode [12,15,16]. Here, we have tried to go a step further, presenting a circunstancial evidence (fulfillment of usual sum rules) about the reliability of the dynamical HD model for the study of monopole oscillations. We want to stress the fact that, triggered by a small perturbation, the response of the system has remained small too, without developping anharmonicities. It may be seen as a trivial point, but within the RPA theory, the response of the system to a small perturbation is forced, by construction, to be small. In our calculations, as in those TDHF of refs 7 and 9, the system is let to evolve freely and by increasing the value of the constraining parameter λ one would eventually reach a regime where the dynamical method would be still valid whereas RPA would not.

Obviously, we do not claim the present method be an alternative to more fundamental RPA calculations. For example, an apparent shortcoming of HD is its poor estimate of the resonance escape width as compared with the RPA estimate. We work in coordinate space and conse-

quently, the continuum states are automatically included in our calculation. It seems, thus, that within the present model, the coupling with the continuum states is too weak. Moreover, the continuum being different in TF and HF theories, we cannot expect TDTF to yield results close to those obtained from continuum RPA calculations. The Fluid Dynamics method of Holzwarth and Eckart [17] gives escape widths in better agreement with RPA.

Acknowledgments.

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Table captions.

Table 1.- Characteristics of the isoscalar monopole resonance for ^{16}O and ^{40}Ca

Table 1

	^{16}O		^{40}Ca	
	Static	Dynamic	Static	Dynamic
m_{-1} ($MeV^{-1}fm^4$)	5.94	5.31	35.1	33.8
m_o (MeV)		1.98×10^2		1.06×10^3
m_1 (MeV fm^4)	8.12×10^3	7.52×10^3	3.50×10^4	3.31×10^4
m_2 (MeV^2fm^4)		2.95×10^5		1.04×10^6
m_3 (MeV^3fm^4)	1.23×10^7	1.20×10^7	3.61×10^7	3.30×10^7
E_3 (MeV)	38.9	40.0	32.1	31.6
E_1 (Mev)	37.0	37.6	31.6	31.3
$\bar{E}$ (MeV)		38.0		31.3
σ_{31} (MeV)	6.0	6.7	2.8	2.1
σ (MeV)		6.9		2.2

Figure captions.

Figure 1.- $\langle r^2 \rangle / A$ as a function of time for ^{16}O.

Figure 2.- $\langle r^2 \rangle / A$ as a function of time for ^{40}Ca.

Figure 3.- Isoscalar strength function S(E) of ^{16}O

Figure 4.- Isoscalar strength function S(E) of ^{40}Ca

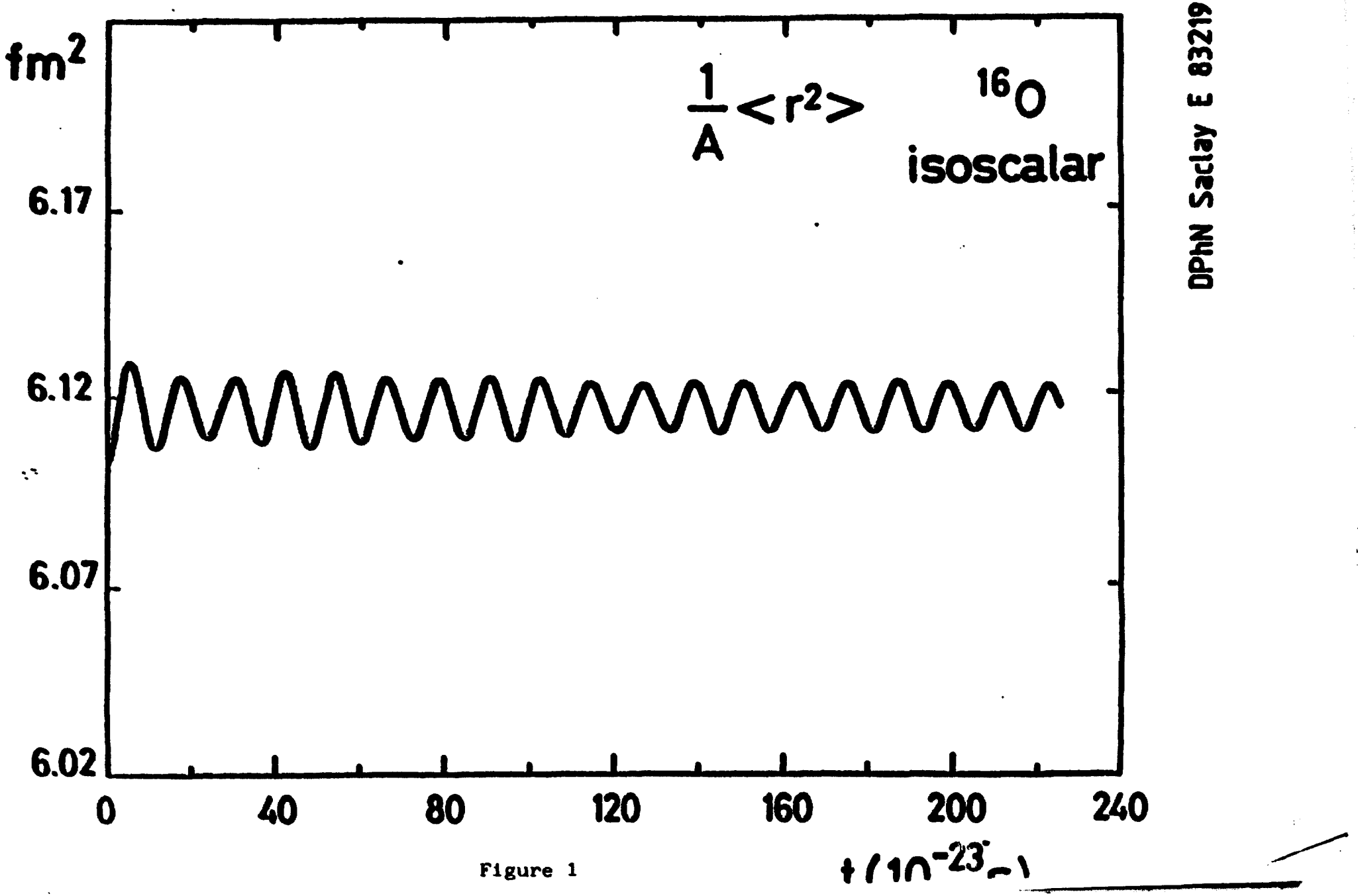
fm²
6.17
6.12
6.07
6.02
0
40
80
120
160
200
240
t (10⁻²³ s)
1/A <r²>
¹⁶O
isoscalar
DPhN Saclay E 8219

Figure 1

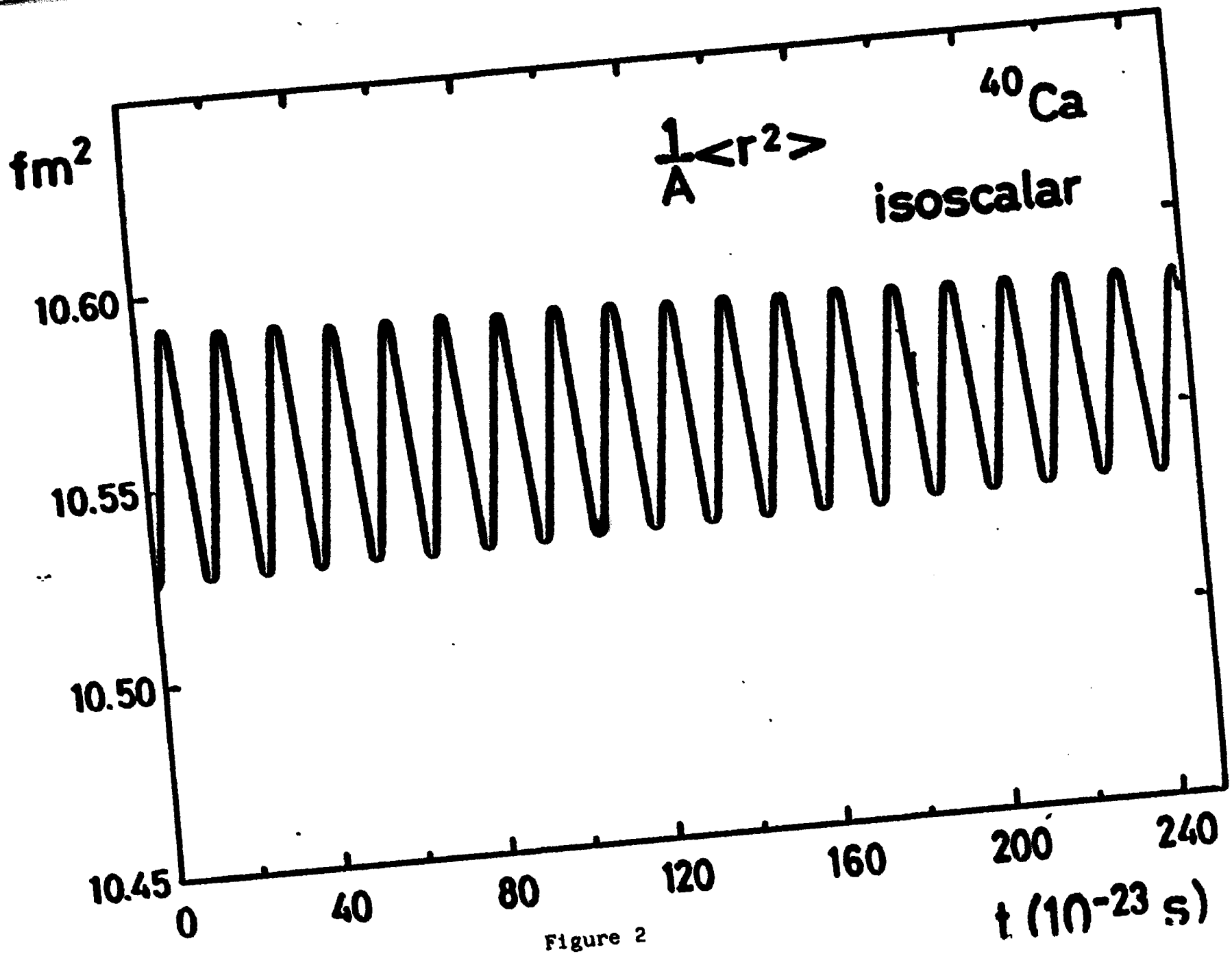
fm²
10.60
10.55
10.50
10.45
0
40
80
120
160
200
240
t (10⁻²³ s)
$\frac{1}{A}\langle r^2 \rangle$
⁴⁰Ca
isoscalar
DPhN Saclay E 83221

Figure 2

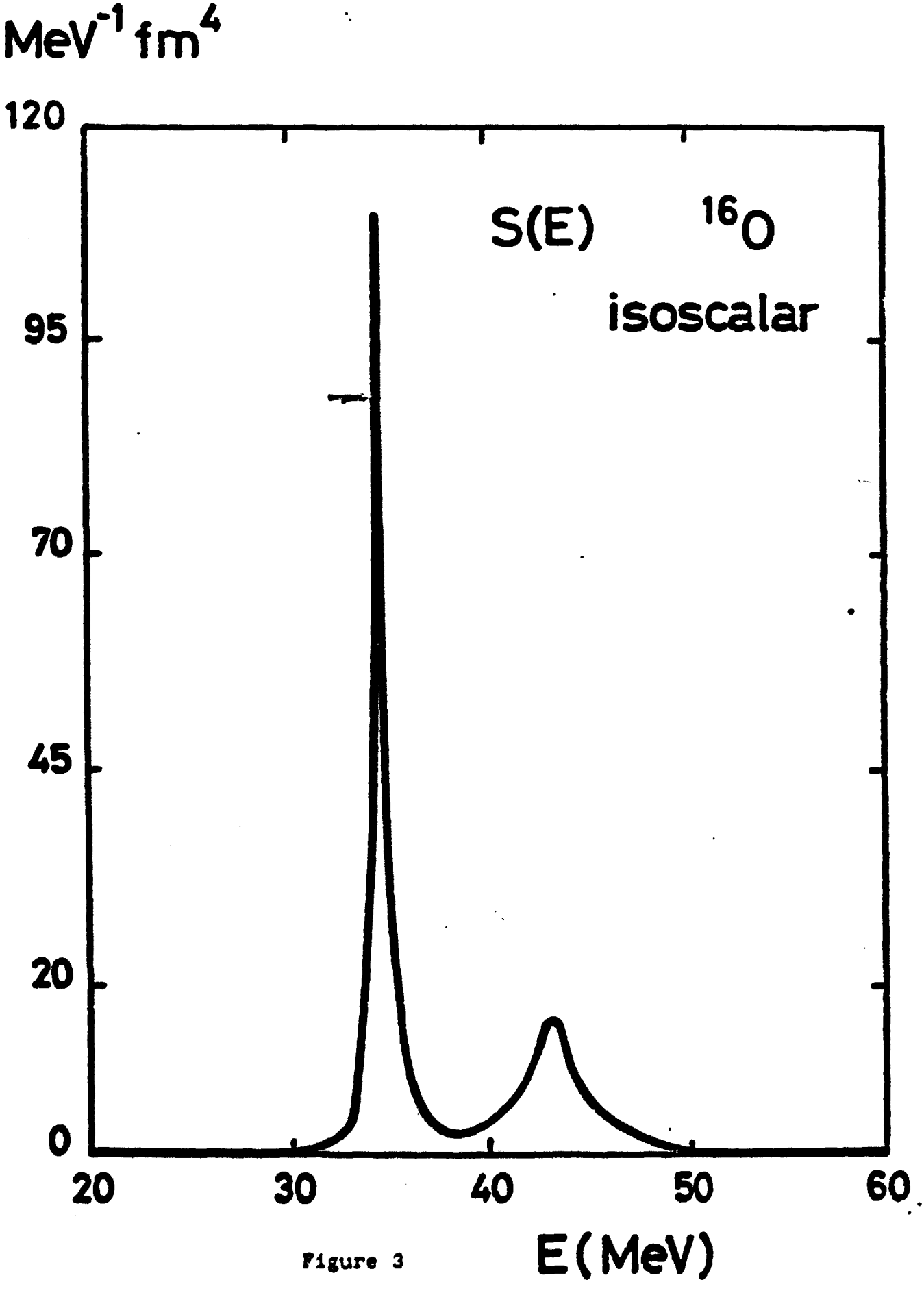

Figure 3

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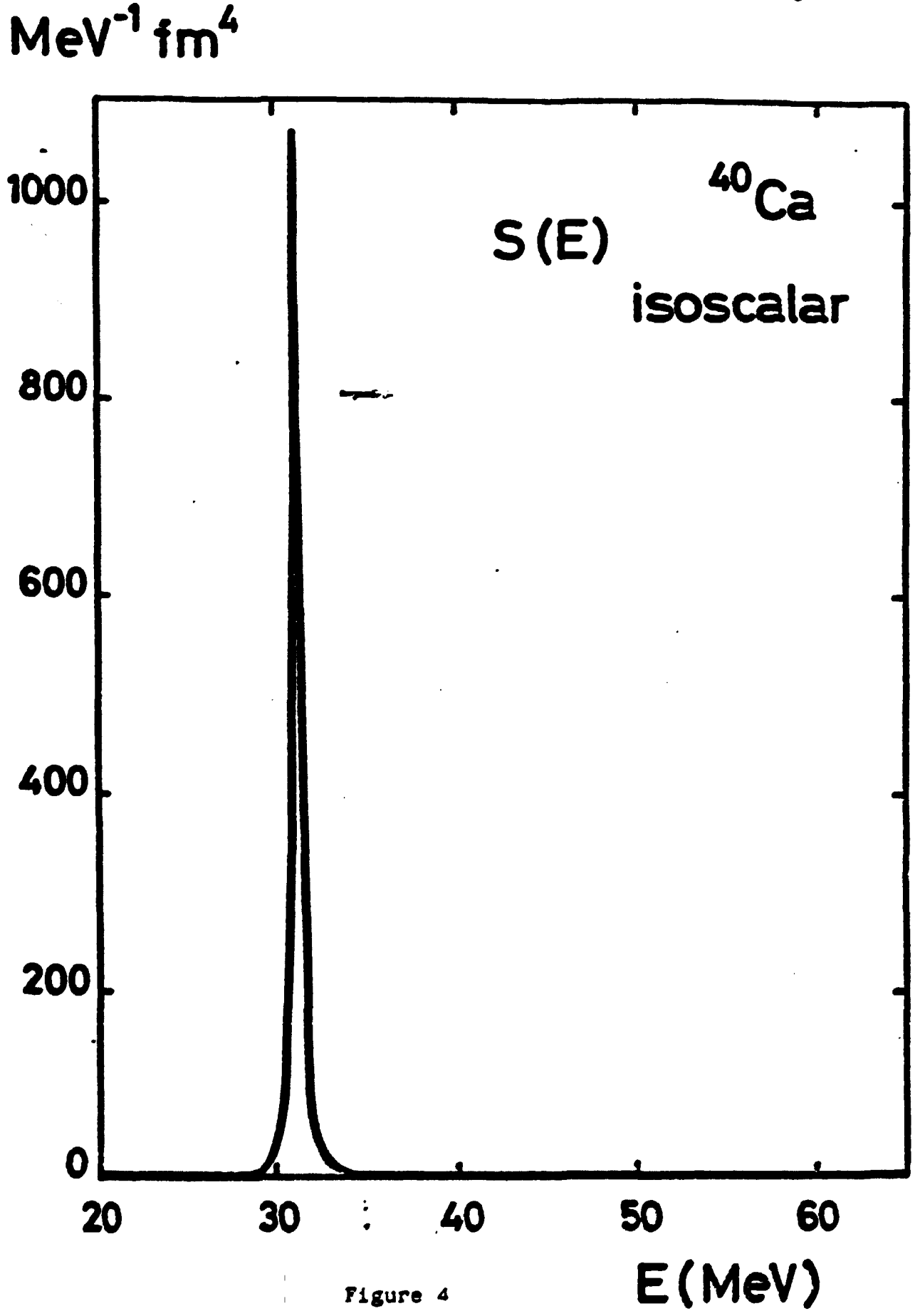

Figure 4