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OR
THE SEARCH FOR QUARK-GLUON PLASMAS

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Abstract

This paper reviews some aspects of the physics of ultra-relativistic heavy ion collisions. The qualitative changes expected in the properties of hadronic matter at high temperature and/or large baryon density are described in terms of simple models. We discuss a scenario giving the space-time evolution of a quark-gluon plasma. Finally we address the difficult question of the possible signatures of the formation of a quark-gluon plasma in heavy ion collisions.

1. INTRODUCTION

There is a growing interest in the study of ultra relativistic heavy ion collisions with center of mass energy in the range of 10 to 100 GeV per nucleon. The motivations of such studies are numerous.

First of all, there is the hope to be able to test experimentally some important predictions of the theory of strong interactions, namely Quantum Chromodynamics (QCD). The basic constituents of QCD, the quarks and the gluons have been identified, although somewhat indirectly. Furthermore, whenever perturbation theory is applicable, QCD leads to quantitative predictions which have been fairly well verified. However, one of the most spectacular predictions of the theory, namely the existence of a deconfinement transition at sufficiently high energy density remains to be checked experimentally.

The exploration of the properties of matter under extreme conditions is interesting in its own. The knowledge of these properties is necessary for the understanding of some astrophysical phenomena, in particular for the physics of the early universe.

As far as the physics of heavy ions is concerned, it has to be emphasized that ultra relativistic collisions represent totally new, and unexplored, regimes of collisions. It is therefore not unreasonable to expect that new phenomena might be observed. It would certainly be very interesting to observe collective phenomena involving explicitly quark and gluon degrees of freedom.

A further motivation, which is actually at the origin of the growing interest in the field, is the fact that relevant experiments can be performed with minor modifications of existing facilities. In particular, a series of fixed target experiments is planned in 1986, both at CERN (^{16}O beam with $E/A \sim 200$ GeV) and at Brookhaven (^{32}S beam with $E/A \sim 15$ GeV).

Finally it is an important aspect of this new field of physics that it lies at the border between nuclear and particle physics and one may expect fruitful developments from the interactions between these disciplines.

There exists numerous reviews on this rapidly developing field. See for example Refs. 1-4, where further references can be found.

2. PROPERTIES OF MATTER WITH HIGH ENERGY DENSITY

The properties of strongly interacting matter are believed to be described by Quantum Chromodynamics, a theory of interacting quarks and gluons. This theory has a few distinguishing properties. First of all its elementary constituents, the quarks and the gluons, cannot propagate in the ordinary vacuum; only color singlet objects can be observed. This property is known as color confinement. Secondly, to the extent that current quark masses can be neglected, chiral symmetry is an exact symmetry of the QCD Lagrangian. As well known the ordinary vacuum is not invariant under chiral transformations, i. e. chiral symmetry is spontaneously broken, as attested for example by the existence of very low mass pions. The last important property of QCD which is important to mention in the present context, is that of asymptotic freedom. It states that the running coupling constant becomes smaller and smaller as the energy scale of the process one considers increases beyond a value $\Lambda \sim 200$ MeV.

In matter at high temperature or large baryon density, i. e. in matter with large energy density, the elementary interactions involve large momenta and therefore, as consequence of asymptotic freedom, small coupling constants. In this regime one can calculate the properties of matter using perturbation theory. It follows that, if the energy density is large enough, matter dissolves into a gas of weakly interacting quarks and gluons. This expectation

is confirmed by detailed calculations on a lattice⁵⁾. These calculations also indicate that chiral symmetry is restored at high temperature, that is, the expectation value of $\langle \bar{\psi}\psi \rangle$ vanishes. The qualitative features of lattice calculations at finite temperature are illustrated in Fig. 1, which displays the energy density as a function of the temperature. There is still considerable debate as to whether the rapid variation observed in the energy density corresponds to an actual phase transition or not (in the presence of light quarks); also the precise value of the temperature at which this transition takes place is not accurately known. Typically one finds $T_c \sim 200$ MeV and $\Delta\epsilon \sim 2-10$ GeV/fm³ for the jump in the energy density. These orders of magnitude may be understood in terms of simple models⁶⁾, as discussed in the next section.

3. SIMPLE MODELS OF TRANSITION FROM HADRONIC TO QUARK MATTER.

In this section we consider simple models to describe the transition from hadronic to quark matter, this in order to get quick estimates of the magnitude of the parameters. We shall in fact consider two distinct models, one appropriate for the transition at zero baryon density, the other to describe the transition at zero temperature.

At zero baryon number, we assume that hadronic matter is composed only of pions which, for simplicity, are taken to be massless and non interacting. The pressure of the system is then given by:

$$P_\pi = 3 \frac{\pi^2}{90} T^4 \quad (1)$$

and the energy density by:

$$\epsilon = 3 P. \quad (2)$$

At very high temperature, quarks and gluons are deconfined

and non interacting. The pressure of this quark-gluon plasma is given by:

$$P_{QG} = 37 \frac{\pi^2}{90} T^4 - B \quad (3)$$

where B , the bag constant, is the pressure exerted by the vacuum on the plasma, and 37 is the effective number of degrees of freedom in a plasma with u and d quarks ($37 = 2 \cdot 8 + \frac{7}{8} \cdot 2 \cdot 2 \cdot 2 \cdot 3$). The energy density of the plasma is:

$$\varepsilon = 37 \frac{\pi^2}{30} T^4 + B \quad (4)$$

By comparing the expressions (1) and (3), one sees that there exists a temperature T_c where the pressures become equal:

$$T_c = \left(\frac{45}{17\pi^2} \right)^{1/4} B^{1/4} \approx 0.72 B^{1/4} \quad (5)$$

When $T < T_c$, the phase of maximum pressure is the hadronic phase, while for $T > T_c$, it is the plasma which has the largest pressure. For a "typical" value of the bag constant B , $B^{1/4} \sim 200$ MeV, one obtains $T_c \sim 150$ MeV.

Note that in this model, the transition is of first order, with a latent heat which can be expressed in terms of the difference between the energy densities of the plasma and the hadronic phase:

$$\Delta\varepsilon = B + \frac{\pi^2}{30} \frac{45 B}{17 \pi^2} (37-3) = 4 B \quad (6)$$

that is $\Delta\varepsilon \sim 1 \text{ GeV}/\text{fm}^3$ for $B^{1/4} \sim 200$ MeV.

At zero temperature, hadronic matter is composed mostly of nucleons. When the density increases, the nu-

cleons starts to overlap, which eventually leads to the deconfinement. The density n_c at which the transition takes place is related to the density n_0 of ordinary nuclear matter by $n_c/n_0 = (r_0/r_n)^3$ where r_n is the effective radius of the nucleon (confinement radius) and r_0 is the average distance between the nucleons at ordinary density ($r_0 \sim 1.2$ fm). For $r_n \sim 1$ fm, one obtains $n_c \sim 2 n_0$, while $n_c \sim 15 n_0$ for $r_n \sim 0.5$ fm

There exists many other heuristic arguments which lead to these orders of magnitude. For example we could have determined the temperature at which the pion gas transforms into a ~~quark~~ quark gluon plasma by using an argument similar to the one we just used to describe the transition at zero temperature. Thus one may consider that the transition takes place when all space is filled with pions, that is when $4\pi/3 r_\pi^3 n_\pi = 1$. Since the number of pions is $n_\pi \sim 3 T^3 \zeta(3)/\pi^2 \sim 0.365 T^3$, one obtains this way $T_c \sim 0.87/\pi^2$, that is $T_c \sim 285$ MeV for $r_\pi \sim 0.6$ fm. This temperature is slightly higher than the one obtained before, which gives an idea of the uncertainty which plagues these estimates.

The qualitative features of the phase diagram of hadronic matter are illustrated in Fig. 2. The curve which separates the hadronic matter from the deconfined phase corresponds to a critical energy density between 1 and 10 GeV/fm³. It is useful to put this number in perspective. For comparison, let us recall that the energy density of nuclear matter at ordinary density is $\epsilon_0 \sim 0.15$ GeV/fm³.

4. SPACE TIME PICTURE OF AN ULTRA RELATIVISTIC HEAVY ION COLLISION

One of the important questions which motivates the study of ultra relativistic heavy ion collisions is whether during such collisions an extended zone can be formed with an energy density ten to hundred times larger than the

energy density of ordinary nuclear matter. The answer to that question requires a detailed understanding of the complex phenomena which are taking place during the collision. We are still very far from this situation. However, during the last few years, a plausible scenario of ultra relativistic heavy ion collisions has been elaborated. This scenario is useful as a unifying framework in which several phenomena may be described, and we are going to present it.

Clearly, central collisions are the most interesting in the present context. They are rare, but not that rare: a simple geometrical argument shows that the number of collisions with impact parameters $b \leq b_0$ and the total number of collisions are in the ratio $(b_0 / 2R)^2$ where R is the (common) radius of the colliding nuclei. For Uranium nuclei ($R \sim 7.4 \text{ fm}$) and an impact parameter $b_0 \sim 1 \text{ fm}$, one gets $(b_0 / 2R)^2 \sim 5 \cdot 10^{-3}$. Thus, the collisions with impact parameter smaller than 1 fm represent 0.5% of the total number of collisions. Such central collisions are easily identified from the very large multiplicity $\langle n \rangle$ of the produced particles; typically, one expects $\langle n \rangle \sim 10^3 - 10^4$.

The study of high energy proton-proton or proton-nucleus collisions has led to distinguish two regimes for the ultra relativistic heavy ion collisions. At moderate energy ($E_{1,ab} \sim 10 \text{ GeV/A}$) there exists an appreciable probability that multiple collisions lead to stopping the colliding nuclei in their center of mass. (Let us recall that the total number of collisions is of the order of $R/\lambda = Rn\sigma \sim 0.75 A^{1/3}$, where $R = 1.2 A^{1/3}$ is the radius of the nucleus, $n = 0.16 \text{ fm}^{-3}$ is the density, and $\sigma \sim 4 \text{ fm}^2$ a typical cross section at this energy; for a large nucleus $R/\lambda \sim 5$). In this regime, large baryon densities may be achieved ($n \sim 3-10 n_0$).

At higher energy ($E_{1,ab} \sim 1 \text{ TeV/A}$), transparency sets in and during the collision the nuclei go through each other, converting an appreciable fraction of their kinetic

energy into excitation energy. This energy is distributed in three regions which one distinguishes by the rapidity of the produced particles. The fragmentation regions are those regions of phase space where the rapidity of the produced particle is close to the rapidity of the projectile or the target. The central region contains the particles with low rapidities (in the center of mass frame). The central region contains no (or very few) baryons, but the energy density there may be quite substantial, of the order of a few GeV/fm^3 .

In the rest of this paper we shall discuss only the evolution of the central region. The description of the fragmentation region is complicated by the presence of baryons and the production of particles inside the nuclei. It is convenient to discuss the evolution of the central region with the help of a space-time diagram such as the one displayed in Fig. 3. In this space-time diagram, the various stages of the collision are bordered by hyperbolas, for reasons which will be explained shortly. Let us now describe these various stages.

For negative times, the two ions are moving towards each other at a speed close to that of light. We have ignored in drawing Fig. 3 the thickness of the ions in the z direction. In fact the Lorentz contraction reduces this extension to $2R/\gamma$, where $\gamma = 1/\sqrt{1-v^2}$ and R is the radius of one ion (we are considering the collision of two equal size ions). Actually, this Lorentz contraction does not apply to the small momentum components of the nuclear wavefunction (small momentum referring to the center of mass frame), and it may be argued that the thickness of a nucleus in the z direction may not be considered as being smaller than typically 1 fm. The nuclei collide at $z=t=0$, producing a lot of excitations (quarks, anti-quarks and gluons). These excitations will materialize as well identified quanta after a characteristic time usually referred

to as the formation time. The detailed mechanism of this formation process is poorly understood. This first stage of the collision is usually described in terms of an inside-outside cascade model⁷⁾ which leads to a distribution of quanta which is invariant under longitudinal Lorentz boosts. Thus the quanta materialize on an hyperbola corresponding to constant "proper time" $\tau_{form} = \sqrt{t^2 - z^2}$. A particle which appears on this hyperbola at point (z, t) has there a velocity equal to z/t . In the simplified picture where one ignores the longitudinal thickness of the nuclei, one may consider that all the quanta are created at $z=t=0$, with a uniform velocity distribution. They then propagate without interacting. The slow ones materialize first (at $z=0, t=\tau_{form}$), the fast ones later (as a result of the Lorentz time dilation). The value of τ_{form} is not well determined. A commonly accepted value is $\tau_{form} \sim 1\text{fm}/c$.

The newly formed quanta start to propagate freely, but as their density decreases collisions become more and more important (remember that at very high density the collisions are inefficient due to asymptotic freedom). The net effect of these collisions is to transform the initial distribution of quanta into a distribution of local equilibrium⁸⁾:

$$n(r, p, t) = \frac{1}{e^{\epsilon_p / T(r, t)} \pm 1} \quad \epsilon_p = \sqrt{p^2 + m^2} \quad (7)$$

where $T(r, t)$ is the local temperature of the system. Let us remark that because of the longitudinal boost invariance T depends in fact only on the proper time τ . The estimates of the time at which matter is thermalized are very uncertain. Also it is not entirely clear whether the formation and thermalization of matter should be considered as two well separated stages of the collision. In order to remain specific in our description of the scenario, we shall

assume that after a proper time $\tau_0 = \tau_{form} + \tau_{th} \sim 1 \text{ fm}/c$, the system of quanta is in local equilibrium. Its evolution is then described by the equations of hydrodynamics.

Hydrodynamics is usually assumed to be valid when the mean free path of the particles is small in comparison with the typical length scale of the problem which, in the present case, may be taken to be the radius of a nucleus. If we assume that the matter at time τ_0 consists of a gas of thermalized quarks and gluons with equal numbers of u, \bar{u}, d and \bar{d} , and that the energy density is $2 \text{ GeV}/\text{fm}^3$, then the temperature is $T \sim 200 \text{ MeV}$ and the total number of quanta is $n_0 \sim 4 \text{ fm}^{-3}$. This large density of quanta implies that they have relatively short mean free path $\lambda = 1/n_0\sigma$. Taking for the cross section σ a typical value of $10 \text{ mb} = 1 \text{ fm}^2$, one gets $\lambda \sim 0.25 \text{ fm}$, which is indeed much smaller than a typical nuclear radius. This crude estimate gives some confidence in the validity of a hydrodynamic description of the collision.

The hydrodynamic equations may be written in the following compact form:

$$\partial_\mu T^{\mu\nu} = 0 \quad (8)$$

where

$$T^{\mu\nu} = (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu}. \quad (9)$$

In this formula, ε is the energy density, P the pressure, $g^{\mu\nu}$ the metric tensor and $u^\mu = \gamma(1, \mathbf{v})$ the quadri-velocity of the fluid. The form of the energy momentum tensor given above ignores possible viscous effect and the hydrodynamic equations which we have written conserve entropy. These equations need to be completed by an equation of state which relates the pressure and the energy density. We shall assume the simplest possible equation of state, namely that of an ultrarelativistic gas, $P = \varepsilon/3$ (to within the bag constant terms, as explained earlier; see Eqs. 1-4).

If one ignores the transverse expansion of the fluid,

one can easily solve the equations (8). There exists indeed a scaling solution⁹⁾ in which the fluid velocity at point $(z;t)$ is simply given by $v(z,t)=z/t$. In such a solution, the entropy density varies as:

$$s(\tau) = s(\tau_0) \tau_0 / \tau \quad (10)$$

This result reflects the fact that the entropy is conserved during the evolution of the system, and that the proper volume increases like the proper time τ . Using elementary thermodynamics, one easily shows that the energy density and the temperature also obey scaling laws:

$$\varepsilon(\tau) = \varepsilon(\tau_0) \left(\frac{\tau_0}{\tau} \right)^{4/3} \quad (11)$$

$$T(\tau) = T(\tau_0) \left(\frac{\tau_0}{\tau} \right)^{1/3} \quad (12)$$

The uniform expansion implied by the scaling solution is the dominant feature of the time development of the system. Superimposed on this longitudinal expansion is the transverse motion which begins with the inward propagation of a rarefaction wave. The underlying Lorentz invariance greatly simplifies the problem of coupling the longitudinal and transverse motions, by allowing one to determine the transverse motion in only one frame and then boosting to other frames. In this qualitative presentation we shall not describe in detail the transverse motion¹⁰⁾. In fact the hydrodynamics is largely dominated by the longitudinal expansion which causes a rapid cooling of the plasma. For example, the temperature at the point $z=0$ drops by 20% in a time of the order of $1 \text{ fm}/c$. For comparison, the time it takes the rarefaction wave to reach the collision axis is equal to $R/c_s = R\sqrt{3} \sim 12 \text{ fm}$ for a large nucleus (we have used the fact that the speed of sound in an ultrarelativistic medium is $c/\sqrt{3}$). After this time the temperature in $z=0$ has dropped to 40% of its initial value.

In the course of the hydrodynamic expansion, the plasma cools down and eventually the temperature crosses

the critical value corresponding to the deconfinement transition. The precise way in which the quark-gluon plasma will turn into ordinary hadronic matter is still not entirely understood. Several scenarios are possible: production of supercooled quark matter followed by a violent transition towards ordinary hadrons, formation of condensation discontinuities in the transverse rarefaction wave... It is also possible that the quark-gluon plasma turns smoothly into a hadron gas, with formation of an intermediate mixed phase¹¹⁾. Recent calculations indicate that the transverse rarefaction wave propagates very slowly into the mixed phase (the sound velocity vanishes in the mixed phase). In a first approximation one may ignore the transverse motion. It is then very easy to estimate the lifetime of the mixed phase. It is simply given by the ratio of the number of degrees of freedom in the plasma and in the hadron gas, respectively:

$$\tau_{had} / \tau_{tp} = 37/3$$

where τ_{tp} is the time at which the plasma reaches the critical temperature and τ_{had} is the time at which hadronization is completed. The time τ_{had} may be quite long. If one assumes that the initial temperature is 250 MeV and the transition temperature 200 MeV, one gets $\tau_{tp} \sim 2$ fm and $\tau_{had} \sim 25$ fm. In this scenario, the system will spend most of its lifetime in the mixed phase.

At the end of the hadronization process the particles which have been produced, mostly pions, will continue the hydrodynamic expansion. This hydrodynamic regime will last as long as the collisions between pions will be able to maintain the local equilibrium. It is usually assumed that this is the case as long as the temperature is higher than the pion mass¹²⁾. When $T \sim m_{\pi}$, the pion distributions freeze out and will subsequently evolve as the distributions of free particles.

The scenario which we have described in the previous section has led to well defined calculations. However, as we have mentioned several times, many problems remain open, either in the description of the initial stage of the collision where the plasma is produced, or in the analysis of the phenomena leading to the thermalization, or in the computation of viscous effects which may alter the hydrodynamic evolution, or still in the detailed description of the hadronization transition and the decoupling.

EXPERIMENTAL SIGNALS

In the last part of this talk, we discuss some of the experimental signals which may provide indications that a plasma has been formed during a collision. Owing to the complexity of the system under study, one should presumably not expect to find a clean, unambiguous signature of the plasma formation. Certainly the correlation of several observations will be necessary in order to get a reasonable understanding of what really goes on during a collision.

The gross features of the collisions can be deduced from measurements of the distribution of the produced particles. For example the knowledge of the multiplicity of the produced pions, of their transverse momenta and energies, may be used to determine the entropy, the energy density or the pressure of the system in the later part of the reaction. If one assumes that the evolution of the system, after the plasma has been formed, is isentropic, the knowledge of the final entropy allows the calculation of the entropy at the time τ_0 where the hydrodynamic expansion begins. As an illustration, let us write down the relation between the final multiplicity of pions and the initial entropy density. We shall assume for simplicity that all particles are ultrarelativistic. Then the entropy density is simply proportional to the number of particles: $s \sim 3.65 n$. In the expanding plasma the element of proper volume is

simply $d^2 x_{\perp} \tau dy$, where y is the rapidity. It follows that the number of particles per unit rapidity is given by:

$$\frac{dN}{dy} \simeq \frac{\pi R^2}{3.65} \tau s(\tau) = \frac{\pi R^2}{3.65} \tau_0 s(\tau_0)$$

where we have used equation (10) which expresses the conservation of the entropy.

The many pions which are produced in the collision carry with them many more informations than those mentioned above. For example their spectra may be used to infer the final temperature of the system. Interferometry measurements may provide indications about the size of the production region. Furthermore an analysis of the fluctuations in the multiplicity distributions may reveal hydrodynamic instabilities associated with a violent phase transition.

Many particles other than pions are produced during the collision. Among them, the dileptons are particularly interesting because of their long mean free path. Once formed, they escape from the interaction region without interacting with the surrounding medium. Therefore they carry information about the various stages of the collision at which they have been produced. A specific information which could be deduced from the analysis of dilepton spectra concerns the possible modification of the ρ -meson peak, which has been proposed as a signal identifying the restoration of chiral symmetry¹³⁾.

It has been suggested that the chemical composition of the matter could also provide information about the plasma formation. In particular the strangeness abundance may be modified. The same type of considerations may apply to the production of charmed particles. Also, fluctuations in the baryon number may lead to anomalously large production of antibaryons, or even antinuclei.

Many other probes have been suggested and the field

is rich of all kind of speculations. There is no doubt that the forthcoming experiments are utterly needed to clarify the situation.

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FIGURE CAPTIONS

- Figure 1 : Qualitative behaviour of the energy density versus temperature, as predicted by Quantum Chromodynamics.
- Figure 2 : Idealized phase diagram of hadronic matter ; the critical line which separates the confined and deconfined phases corresponds to an energy density $\epsilon \sim 1-10 \text{ GeV/fm}^3$.
- Figure 3 : Space-time diagram illustrating the evolution of the central region in an ultrarelativistic heavy-ion collision. The various hyperbolae, corresponding to lines of constant proper time, separates the various stages of the collision.

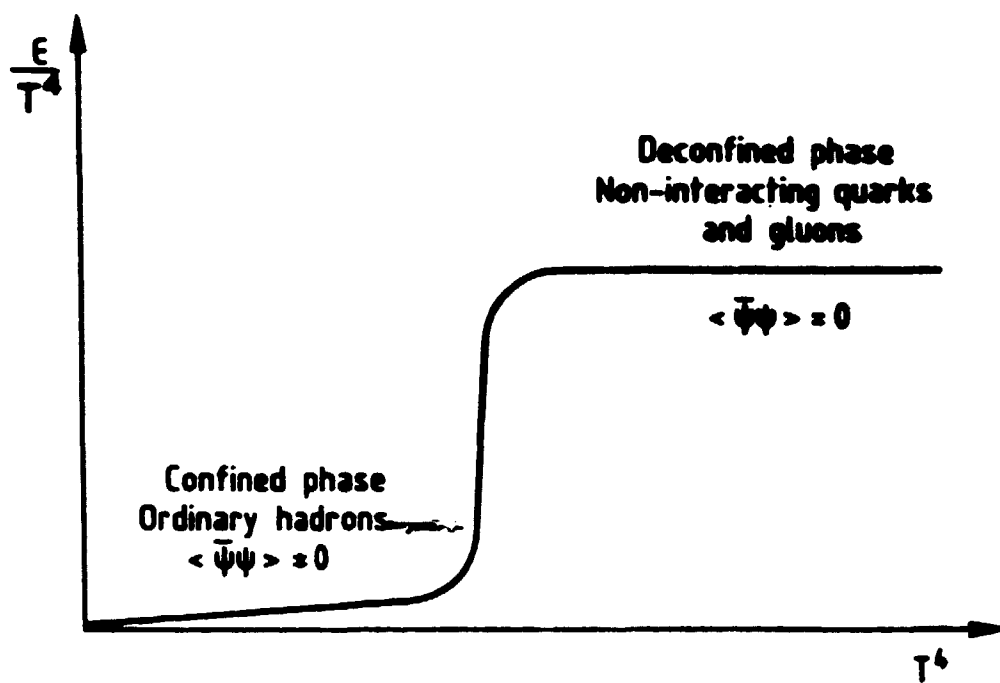


Figure 1

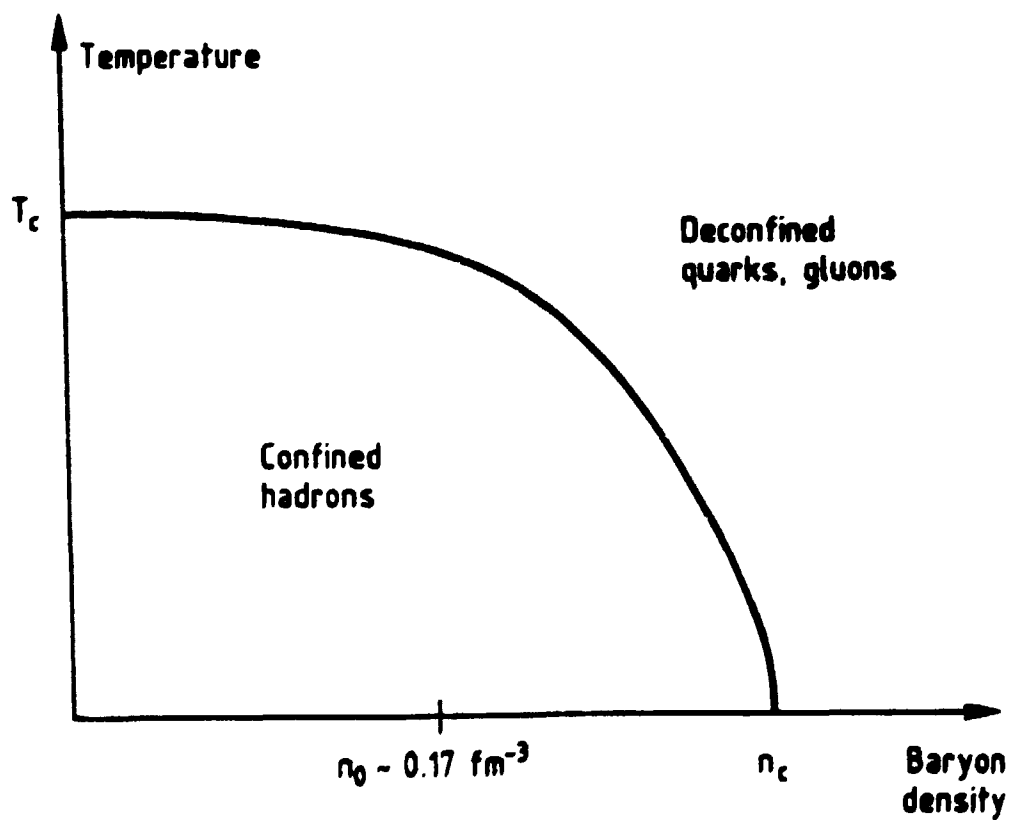


Figure 2

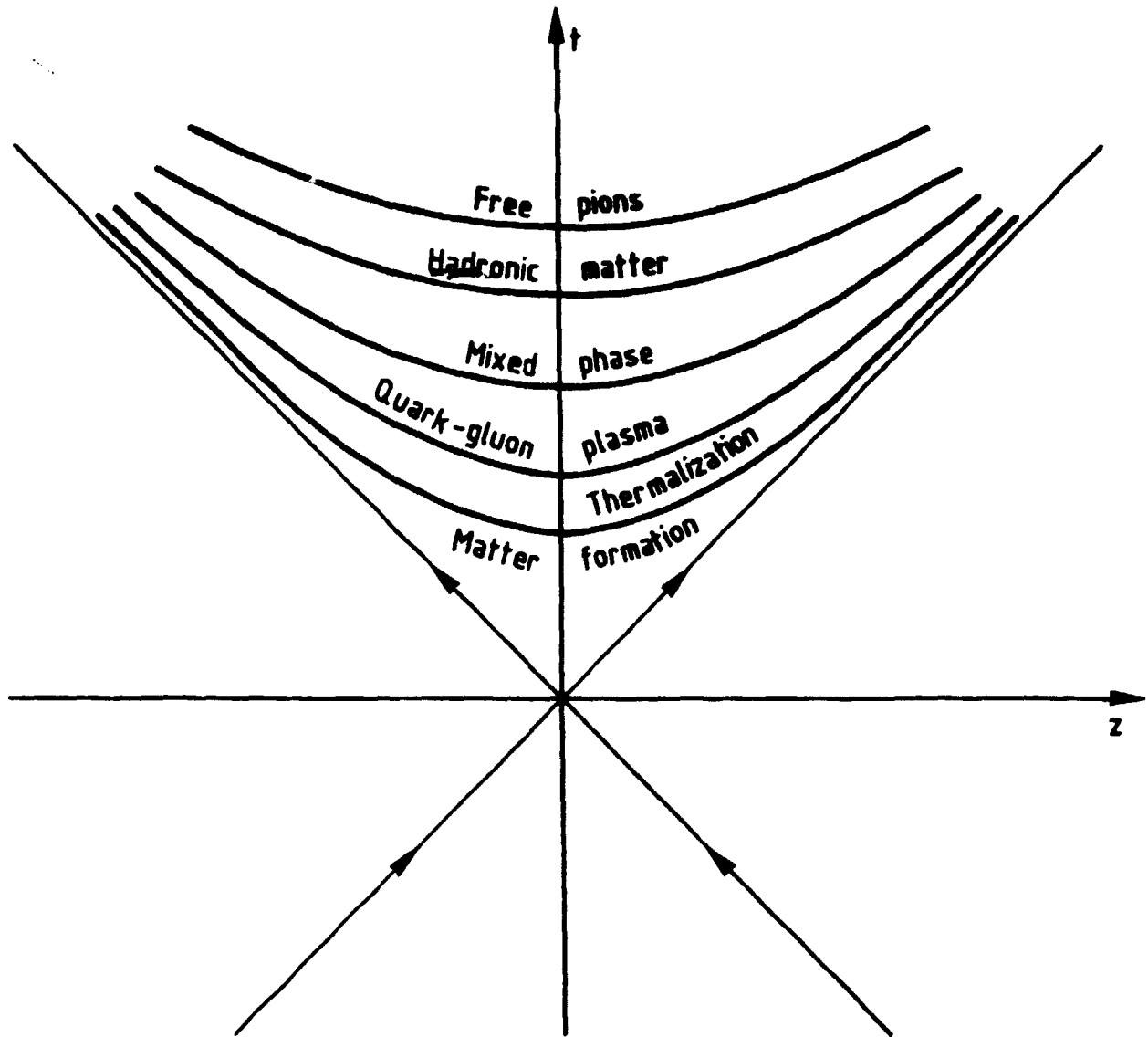


Figure 3