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L'ÉNERGIE ATOMIQUE DU CANADA LIMITÉE

THE APPLICATION OF MVPACK TO THE DESIGN OF MULTIVARIABLE CONTROL SYSTEMS FOR A NUCLEAR STEAM GENERATOR

Emploi de MVPACK pour concevoir des systèmes de commande multivariable pour les chaudières nucléaires

H.W. HINDS

Presented at the Power Plant Digital Control and Fault-Tolerant Microcomputers Seminar, Phoenix, Arizona, 1985 April 9-12

Chalk River Nuclear Laboratories

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Résumé

Les systèmes de commande de la plupart des centrales nucléaires sont conçus au moyen de méthodes classiques monovariables qui traitent chaque boucle de commande indépendamment. Les interactions entre les boucles sont réalisées empiriquement en réglant les contrôleurs de façon à obtenir les résultats voulus. Alors que la taille et la complexité des centrales augmentent et que ces dernières doivent fonctionner le plus près possible de leur capacité maximale pour avoir un rendement économique, ces interactions dynamiques prennent beaucoup d'importance et rendent peu fiable l'extrapolation des méthodes classiques. Cependant, un réglage amélioré peut être obtenu en ayant recours à des méthodes multivariables pour la conception et l'analyse des systèmes de commande. Ces méthodes considèrent la centrale fondamentalement comme un système d'interaction ayant de nombreuses entrées et sorties rivalisant pour répondre à des objectifs souvent contradictoires.

MVPACK est un logiciel facilement utilisable ayant une forte capacité pour concevoir et analyser des systèmes de commande multivariable, sous forme de fonctions de transfert ou d'espace des états. Il est constitué par un ensemble de modules interactifs que l'utilisateur voit comme une calculatrice de niveau élevé. Il comprend une base de données, des méthodes d'interaction, une bibliothèque mathématique et une importante collection d'algorithmes conceptuels comme ceux-ci: réduction de la dimension des systèmes matricials des modèles linéaires; positionnement de pôles avec rétroaction des sorties; contrôle modal; contrôle optimal avec estimation aléatoires des états; et la méthode du critère de Nyquist inversé.

L'objet de ce rapport est la conception de contrôleurs multivariables pour les chaudières nucléaires. Les résultats disponibles du modèle du 15^{e} ordre sont la pression de la vapeur et le niveau d l'eau; les paramètres de commande sont le débit de la vapeur et le débit de l'eau d'alimentation. On décrit dans les grandes lignes quatre techniques conceptuelles parmi celles qui peuvent être employées dans le module MVPACK et on les applique au modèle.

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ABSTRACT

Control systems for most nuclear power plants are designed using conventional single-variable methods which treat each control loop independently. Interactions between loops are accounted for empirically by tuning the controllers to achieve the desired results. As plants increase in size and complexity, and are required to operate closer to maximum capacity for greatest economic returns, these dynamic interactions become more important and render the extrapolation of these techniques unreliable. However, improved regulation can be achieved when multivariable design and analysis methods are used, which view the plant as a fundamentally interacting system with numerous inputs and outputs vying to meet often conflicting objectives.

MVPACK is a user-friendly software package that combines a powerful capability to design and analyze complex multivariable control systems, both in state space and transfer function form, with an ease of application. MVPACK is a set of interactive modules that appears to the user as a high-level calculator. It is composed of a database, interaction methods, a mathematical library, and an extensive collection of design algorithms including: order reduction of linear models; pole shifting with output feedback; modal control; optimal control with stochastic state estimation; and the inverse Nyquist array method.

The subject of this paper is the design of multivariable controllers for a nuclear steam generator. The available outputs of the 15th-order model are steam pressure and water level; control inputs are steam and feedwater flows. Four of the design techniques implemented in the MVPACK module are outlined and then applied to this model.

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NOMENCLATURE

A	plant dynamics matrix
a	coefficient of the open-loop characteristic polynomial
B	plant input matrix
C	plant output matrix
D	direct input-output matrix
d	radii of Gershgorin circles
F	diagonal feedback matrix
G	plant open-loop transfer function matrix, dimension (mxp)
H	(1) system closed-loop transfer function matrix, dimension (mxm) (2) Kalman filter gain
I	identity matrix
i,j	indices
J	cost function
K	controller matrix
K_1,K_2,K_d	factors of K
k^*	vector (1xm) to convert outputs to a single output
L	post-compensator matrix
m	number of outputs
N	number of encirclements of origin
n	number of states
p	number of inputs
p_o	number of unstable open-loop poles
Q	(1) compensated open-loop frequency response matrix (2) output cost matrix
q	vector (px1) to convert single input into actual inputs
R	input cost matrix
r	(1) reference value vector of dimension p or n (2) dominance ratios
s	Laplace variable (s^{-1})
t	time (s)
U	dual eigenvector matrix, $U^T=V^{-1}$
u	input vector of dimension p
V	eigenvector matrix of A
W	weight vector
x	state vector of dimension n
y	output vector of dimension m

<u>Greek</u>

Λ	working matrix
κ	Vandermode matrix of the pre-assigned eigenvalues
Δi	characteristic polynomial evaluated at λ_i
λ	eigenvalue (s^{-1})
Ξ	state derivative noise covariance matrix
Θ	output noise covariance matrix
θ	damping angle
ω	imaginary part of eigenvalue, angular velocity ($rad \cdot s^{-1}$)
$-\sigma$	real part of eigenvalue (s^{-1})

INTRODUCTION

Control systems for most nuclear power plants have traditionally been designed by conventional single-variable methods which treat each control loop independently. Interactions between loops are accounted for empirically by tuning the controllers to achieve the desired results. The tuning may be done in the field by the operators or by the designer using his intuition while working with a plant simulation. As plants increase in size and complexity and are required to operate closer to the design values to maximize the economic returns, the dynamic interactions between loops become stronger and more limiting. The continued use of the traditional techniques becomes unreliable and leads to economic penalties.

Multivariable design and analysis methods have been developed over the years but have not achieved widespread practical use in the nuclear industry. These methods treat the plant as a fundamentally interacting system with numerous inputs, outputs and internal states. These design methods can be used to achieve specific response characteristics or to meet (at times conflicting) design objectives.

There are two reasons why the use of multivariable techniques is not more widespread:

- lack of understanding by control engineers, and
- lack of suitable software support packages.

The software support is essential as it reduces the time and effort required to solve an individual problem considerably, by freeing the designer from the details of mathematical algorithms and software generation. It also can reduce somewhat the level of understanding necessary to apply a particular method, as many of these details can be built into the package. Thus it can help in building the understanding of the designers.

MVPACK is a software package developed at the Chalk River Nuclear Laboratories to aid in designing multivariable control schemes. It appears to the user as a user-friendly high-level calculator with a wide variety of design and analysis algorithms as well as more basic mathematical operations.

The subject of this paper is the design, via four of the methods available in MVPACK, of multivariable controllers for a nuclear steam generator.

MULTIVARIABLE BASICS

The system under study is described by a linear model about the operating point chosen by the designer, with the state-space formulation

$$\frac{d}{dt} x = Ax + Bu \tag{1}$$

$$y = Cx + Du \tag{2}$$

A constant gain controller is usually expressed as

$$u = K(r-y) \tag{3a}$$

for output feedback. Some control methods require state feedback instead, which can be expressed as

$$u = K(r-x) \tag{3b}$$

Some of the design methods available use dynamic compensators instead of constant gain compensators; however, these are not covered in detail here.

Eqs. 1 and 2 can be Laplace transformed and rearranged to give the input-output transfer function matrix for the plant

$$y(s) = G(s)u(s) \tag{4}$$

where

$$G(s) = [C(sI-A)^{-1} B + D] \tag{5}$$

The closed-loop response is given by

$$y(s) = H(s)r(s) \tag{6}$$

where

$$H(s) = (I + G(s)K)^{-1} G(s)K \tag{7}$$

The block diagram of the complete plant with an output feedback controller is shown in Figure 1.

The object of multivariable design is to choose the values in K such that the desired response characteristics are achieved. These response characteristics may be expressed in either the time or frequency domains, for example:

- Stability. The closed-loop system should be asymptotically stable, that is, all poles of the closed-loop system should lie in the open left half-plane.
- Stability margin. Let λ be a pole of the closed-loop system such that $\lambda = -\sigma \pm j\omega$; then the angle $\theta = \tan^{-1} \sigma/\omega$ is a measure of the stability margin. Indeed, over a period $2\pi/\omega$, the damped oscillation has a decrement equal to $\exp(-2\pi\sigma/\omega)$. θ should be larger than 45° (or $\sigma > \omega$).
- Speed of response. The time taken for the system to respond to a step input indicates how effectively it will follow changes of the inputs. Because speed of response depends on the position of the dominant poles, these poles should not be too close to the origin.
- Sensitivity. The system's response should be insensitive to small perturbations or system parameter changes.

MVPACK

MVPACK is an interactive computer-aided design package for multivariable control systems. It acts like a high-level calculator working with the following types of data:

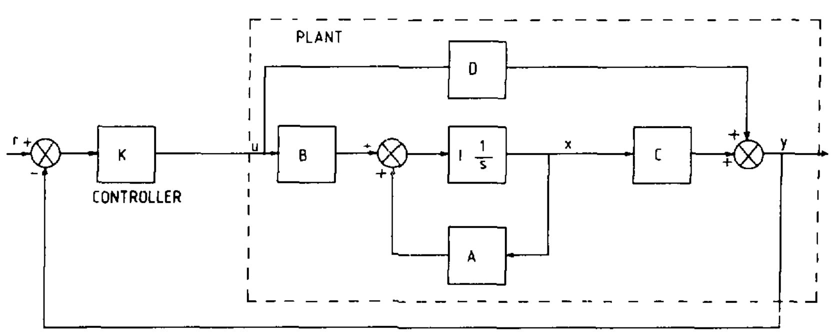

Figure 1. Output-Feedback Controller on a Multivariable Plant

- real matrices,
- polynomial matrices,
- rational polynomial matrices,
- frequency responses, and
- time histories.

It contains the following types of modules:

- utilities such as LOAD, EDIT, OUT, PLOT, BODE and NYQ,
- math library to perform standard matrix operations such as ADD, SUBtract, MULtiply, INVerse, TRAnspose and EIGenvalues.
- high-level modules for

  --order reduction
  --modal control
  --pole shifting
  --optimal control
  --Kalman filtering
  --state space to Laplace and frequency domain
  --inverse Nyquist array methods
  --simulation

The structure of the program is such that more modules may be added as required. Also, the program is designed to be user-friendly; extensive help messages are available at every stage in the design.

The user interacts with the design at various points by supplying data or choosing methods. He can also iterate by repeating a design with modified parameters if the results of the previous iteration were not satisfactory.

NUCLEAR STEAM GENERATOR

The objective is to design a control system for the nuclear steam generator (NSG) described in Appendix A. It consists of a 15th order model (n=15) having two inputs:

- steam throttle valve lift, and
- feedwater flow,

and two outputs:

- steam pressure, and
- downcomer level.

The open-loop frequency responses from inputs to outputs are shown in Figure 2. Step responses are shown in Figure 3. Note that the initial response to the step input is often in the opposite direction to the longer term effect. This non-minimum phase characteristic together with the fact that all four responses are significant, leads to the problems commonly associated with steam generator control.

The eigenvalues of this system are given in Table 1; the system is open-loop unstable as the first eigenvalue is positive. Note that there is one uncontrollable mode in the system associated with the primary water inlet plenum temperature and one unobservable mode associated with the primary outlet plenum temperature. Elimination of these two variables leads to a 13th order system which is fully controllable and observable.

TABLE 1

STEAM GENERATOR EIGENVALUES

Number	Eigenvalues	Frequency (Hz)	Damping
1	+2.47E-5		
2	-5.18E-2		
3	-0.239		
4	-0.331		
5	-0.886+j0.255	0.147	0.961
6	-0.886-j0.255		
7	-1.30		
8	-1.43*		
9	-1.43*		
10	-3.20		
11	-3.29		
12	-5.84		
13	-5.98+j0.0670	0.952	~1
14	-5.98-j0.0670		
15	-6.30		

*Associated with unobservable or uncontrollable modes.

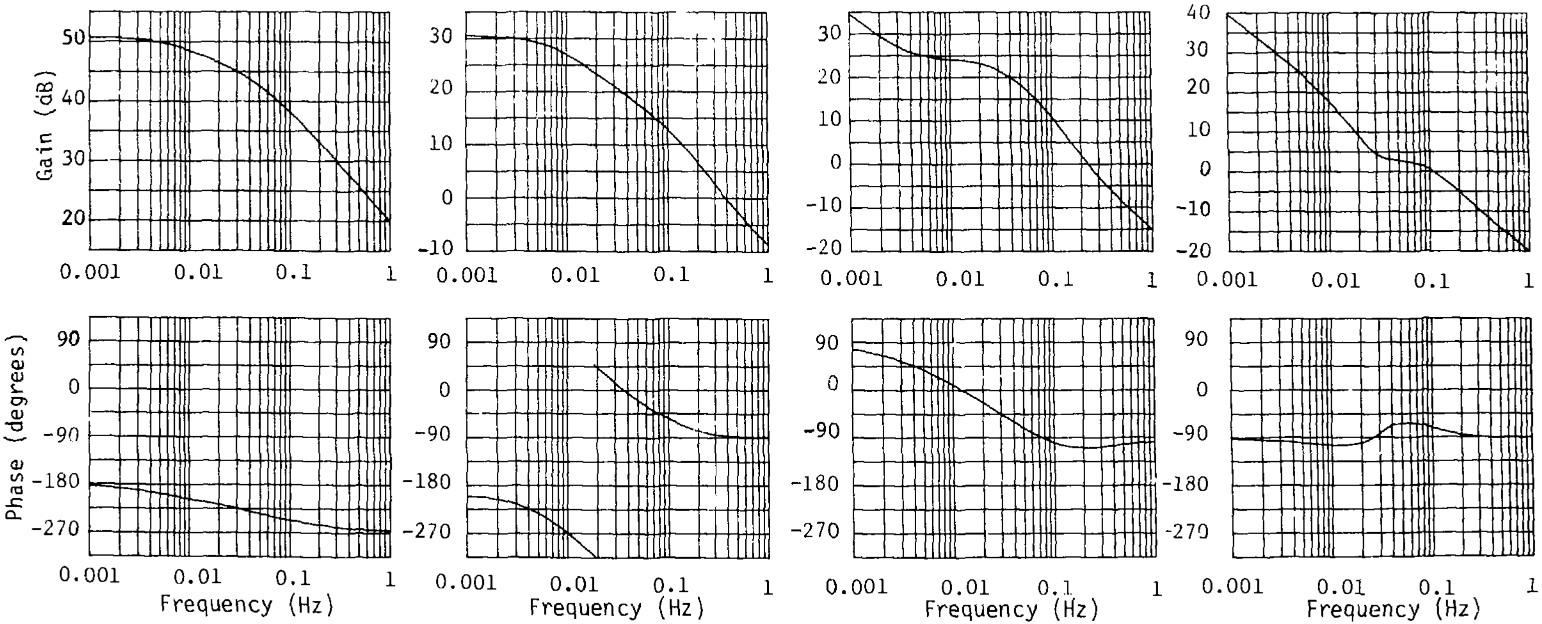

Figure 2. Open-Loop Frequency Response of the Nuclear Steam Generator

(a) $G_{11} = \dfrac{\text{Steam Pressure}}{\text{Steam Valve Lift}}$ (b) $G_{12} = \dfrac{\text{Steam Pressure}}{\text{Feedwater Flow}}$ (c) $G_{21} = \dfrac{\text{Downcomer Level}}{\text{Steam Valve Lift}}$ (d) $G_{22} = \dfrac{\text{Downcomer Level}}{\text{Feedwater Flow}}$

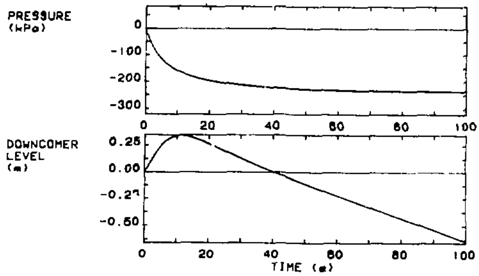

Figure 3a. Steam Generator Open-Loop Response to a Step Change of 10% in Steam Valve Lift

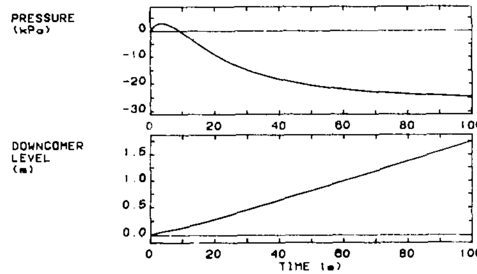

Figure 3b Steam Generator Open-Loop Response to a Step Change of 10% in Feedwater Flow

DESIGN METHODS

Modal Control

The principle of modal control is that the matrix K can be factored into 3 parts as

$$K = K_1 K_d K_2 \qquad (8)$$

such that

- K_2 converts the output y into modes
- K_d is a diagonal matrix of modal gains to shift the eigen-values of the modes = diag(k_i)
- K_1 converts the resulting modal inputs into actual inputs.

Ideally, we would wish

$$U^T B K_1 = \begin{bmatrix} I_p \\ 0 \end{bmatrix} \qquad (9)$$

and

$$K_2 C V = [I_p, 0] \qquad (10)$$

These cannot in general be achieved exactly but suitable approximations are available (1), the simplest of which is

$$K_1 = (U_p^T B)^{-1} \tag{11}$$

$$K_2 = (CV_p)^{-1} \tag{12}$$

where

V_p are the first p columns of V, and

U_p^T consists of the first p rows of U^T and is suitable for m=p, as in our case.

A more robust method is based on pseudo-inverses

$$K_1 = (B^T B)^{-1} B^T V_p \tag{13}$$

$$K_2 = U_p^T C^T (CC^T)^{-1} \tag{14}$$

but was not found to work as well in our NSG case.

With K_1 and K_2 defined by equations (11) and (12), trial and error was used to adjust K_d until the first two eigenvalues for the closed-loop were shifted as given in Table 2. The arrays K_1, K_d, K_2 and K are given below:

$$K_1 = \begin{bmatrix} -0.644 & 0.229 \\ 1.867 & 0.123 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} -4.61E-2 & 0.732 \\ -0.432 & -2.60E-2 \end{bmatrix}$$

$$K_d = 0.02\ I$$

$$K = \begin{bmatrix} -1.38E-3 & -9.55E-3 \\ -2.78E-3 & 2.73E-2 \end{bmatrix}$$

The gain k_1 is as large as possible (within a factor of 2) without causing complex closed-loop poles to appear. The system is relatively insensitive to gain

k_2. *Increases in k_2 cause the first eigenvalue (the one nominally controlled* by k_1) to decrease while increasing only slightly the second eigenvalue. The value $k_2=k_1$ was selected as a simple compromise.

Closed-loop simulation of this system for initial offsets of the output variables is shown in Figure 4.

TABLE 2

CLOSED-LOOP POLES USING MODAL CONTROLLER

Number	Eigenvalue	Frequency	Damping
1	-3.21E-2		
2	-5.64E-2		
3	-0.182		
4	-0.416		
5	-0.897+j0.256	0.148	0.962
6	-0.897-j0.256		
7	-1.30		
8	-3.20		
9	-3.29		
10	-5.84		
11	-5.99+j6.96E-2	0.953	~1
12	-5.99-j6.96E-2		
13	-6.30		

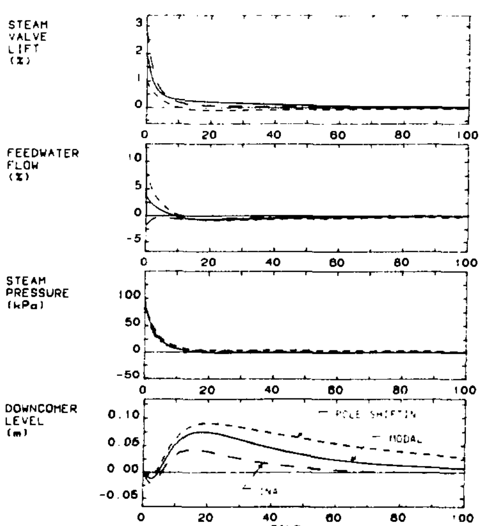

Figure 4a Closed-Loop Response to Initial Offset of Steam Pressure

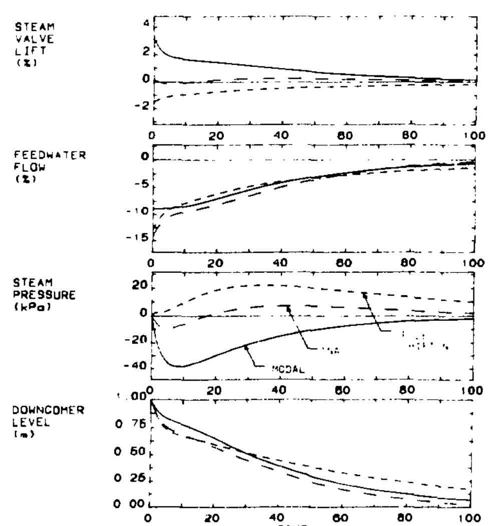

Figure 4b Closed-Loop Response to Initial Offset of Downcomer Level

Pole Shifting

Pole shifting is based on the principle that a cyclic system, which is completely controllable and observable, can be reduced to a single-input single-output system. In practice, this means that the controller can be factored into 2 parts

$$K = qk^* \tag{15}$$

The vector q may be chosen either arbitrarily or to approximately satisfy (2)

$$V^{-1} Bq = W \tag{16}$$

where

$$W = \begin{bmatrix} W_1 \\ \\ W_2 \end{bmatrix}$$ is a vector with no zero elements,

W_1 = vector of dimension p corresponding to shifted poles,

W_2 = vector of dimension n-p corresponding to non-shifted poles.

Normally, if each element of W_2 is chosen to be less than every element of W_1, then the n-p remaining poles will remain stable.

The vector k* is calculated as the solution of (2)

$$k^*\widetilde{C}_K = \Lambda = (\Delta_1, \Delta_2, \ldots, \Delta_p) \tag{17}$$

where

$\widetilde{C}$ = companion form of C

$\Delta_i = \lambda_i^n - a_n\lambda_i^{n-1} - a_{n-1}\lambda_i^{n-2} - \ldots\ldots - a_2\lambda_i - a_1$

λ_i = preassigned eigenvalues

Applying this method to the steam generator with the two assigned poles at -0.016 and -0.070 gives the matrices

$$k^* = [-4.681E-3 \ , \ 6.964E-4]$$

$$q = \begin{bmatrix} 1 \\ 8.436 \end{bmatrix}$$

and the resulting controller

$$K = \begin{bmatrix} -4.68E-3 & 6.96E-4 \\ -3.95E-2 & 5.88E-3 \end{bmatrix}$$

Closed-loop simulation of this system for initial offsets of the output variables are shown in Figure 4.

Optimal Control

Optimal control is designed to minimize a quadratic cost function

$$J = \int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)]dt \qquad (18)$$

However, the controller produced requires that all states be fed back, and in practice this is not possible. A Kalman filter can be designed to estimate all states given the outputs only. The Kalman filter has the structure shown in Figure 5.

The details of the calculation of the controller matrix K and the Kalman filter gain matrix H are given in (3).

The Kalman filter design is based on a model of the plant. In this case, we chose a reduced, 6th order model which was computed using the MVPACK module RED (4). The 6th order model was checked against the 15th order model and found to be identical for practical purposes. We also chose noise covariance matrices representing independent Gaussian noise of

$$\Xi = \mathrm{diag}(10^3,\ 10^3,\ 10^6,\ 10^6,\ 10^6,\ 10^6)$$

$$\Theta = \mathrm{diag}(1,1)$$

The Kalman filter gain is

$$H = \begin{bmatrix} 2.09 & 180.6 \\ 53.8 & 2.09 \\ 850.2 & 49.0 \\ -13.6 & -996.4 \\ 289.5 & 8.57 \\ 797.2 & -0.739 \end{bmatrix}$$

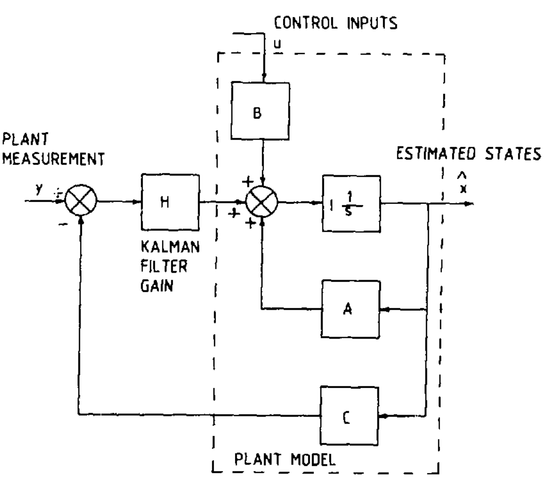

Figure 5. Kalman Filter

The cost matrices selected are a reasonable compromise between the dynamics of the states and the control actions. The chosen cost matrices are

$$Q = \mathrm{diag}(6,\ 0.4,\ 0.04,\ 0.04,\ 0.04,\ 0.04)$$
$$R = \mathrm{diag}(10^4,\ 10^4)$$

and the resulting state-feedback matrix is

$$K = \begin{bmatrix} -9.72\times10^{-4} & -3.43\times10^{-3} & -3.82\times10^{-3} & 0.211 & -8.92\times10^{-3} & -2.22\times10^{-3} \\ 2.12\times10^{-2} & 2.97\times10^{-4} & -6.44\times10^{-4} & -1.024 & 2.81\times10^{-2} & -1.36\times10^{-4} \end{bmatrix}$$

When this controller is applied to the full model, the first 8 poles are -1.63×10^{-2}, -7.06×10^{-2}, -0.251, $-0.363\pm j0.044$, $-0.902\pm j0.250$, -1.30. The closed-loop response of this system is given in Figure 6. The response depends on whether the Kalman filter has the correct initial conditions or not.

Inverse Nyquist Array Method

The inverse Nyquist array (INA) method is somewhat different from the other methods described above as the controller matrix is chosen directly by the designer using an iterative approach rather than being the result of a calculation. The inverse Nyquist array is a graphical tool that aids the designer in his choice. Another major difference is that the INA method works in the frequency domain with the input-output transfer matrix rather than in state space.

Suppose that we factor the controller into 3 portions, L, F, and K_1 where F is a diagonal matrix.

$$\text{By definition, } Q(s) = LG(s)K_1 \qquad (19)$$

Then the closed-loop transfer function is given by

$$H(s) = [I + Q(s)F]^{-1}\, Q(s) \qquad (20)$$

where we have, for convenience, placed some blocks in the feedback path as shown in Figure 7.

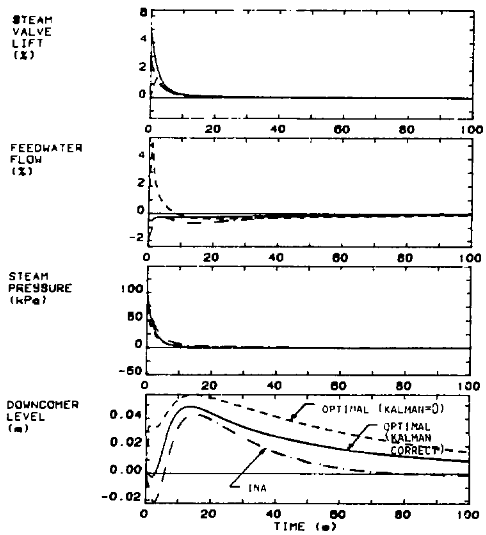

Figure 6a. Closed-Loop Response to Initial Offset of Steam Pressure

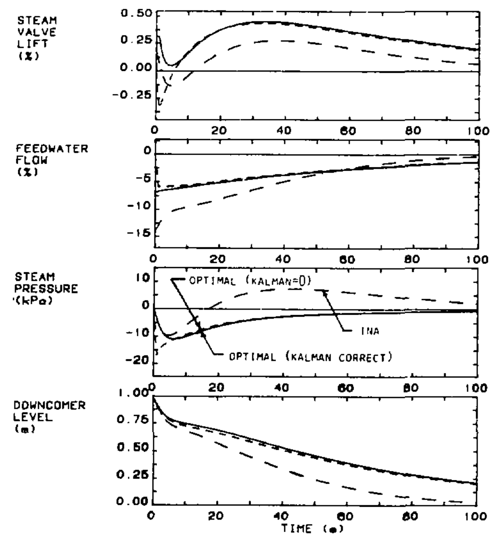

Figure 6b. Closed-Loop Response to Initial Offset of Downcomer Level

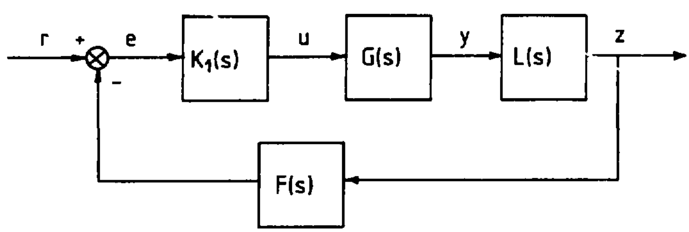

Figure 7. General INA Design Model

Rewriting Eq. 20 in terms of inverses gives

$$\hat{H}(s) = F + \hat{Q}(s) \tag{21}$$

where we use the notation

$$\hat{H}(s) = H(s)^{-1} \text{ and } \hat{Q}(s) = Q(s)^{-1}$$

The system stability can be examined by plotting the determinants $|\hat{Q}(s)|$ and $|\hat{H}(s)|$ and counting their encirclements of the origin which we will denote by N_1 and N_2, respectively. The system is stable if (5)

$$N_1 - N_2 = p_o \tag{22}$$

where p_o = number of open-loop poles in the right half plane.

This criterion is a modified version of the standard Nyquist criterion, but is not useful in general. However, if the system is diagonally dominant, then the number of encirclements is equal to the sum of the encirclements by the individual diagonal elements.

Thus N_1 and N_2 can be determined for a diagonally dominant system by examing the individual elements $\hat{q}_{ii}$ and $\hat{h}_{ii}$. Note also that $\hat{h}_{ii} = f_i + \hat{q}_{ii}$ and thus both N_1 and N_2 can be obtained from the same curve by shifting the critical point to $-f_i$ from the origin.

Diagonal (row) dominance can be examined by plotting the Gershgorin circles which have radii

$$d_i(s) = \sum_{j \neq i} |\hat{q}_{ij}(s)| \tag{23}$$

Provided these circles do not encircle the origin, the matrix $\hat{Q}(s)$ is diagonally dominant. These circles define a band about the $\hat{q}_{ii}$ curves. The Gershgorin circles are dependent on the open-loop system only.

For our steam generator problem, over the frequency range 0.001 to 1 Hz, most INA plots had amplitudes less than 1. To obtain all curves >1, the following scaling was performed (see Table 3):

ROW S,1,100	scale row 1 by 100
ROW S,2,10	scale row 2 by 10

Row scaling does not affect the diagonal (row) dominance. The resulting INA plot with Gershgorin circles is shown in Figure 8a.

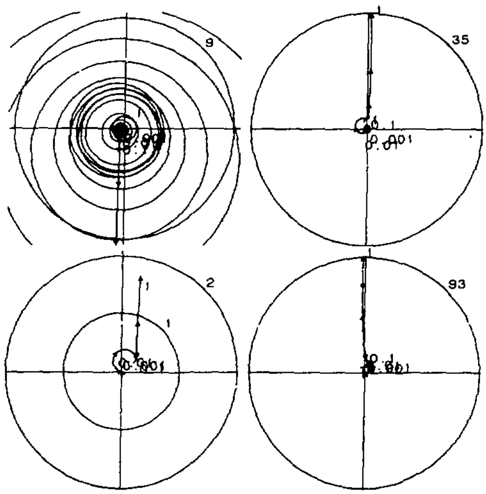

Figure 8a. INA Plot
after Initial Scaling

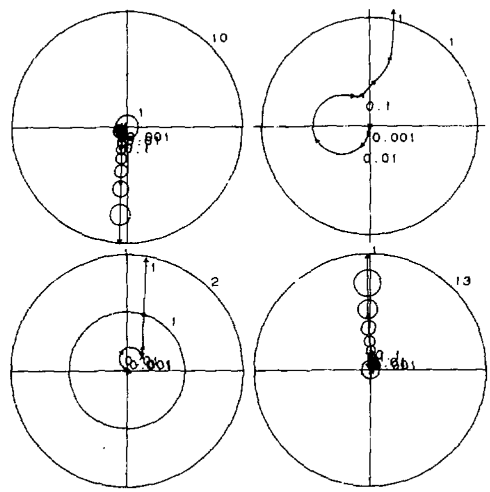

Figure 8b. INA Plot
after Achieving Dominance

TABLE 3

OPERATIONS TO ACHIEVE DIAGONAL DOMINANCE

MVPACK Command	Description
ROW*P,i,j	Permutation of rows i and j
ROW*S,i,α	Scaling row i with a constant, α
ROW*A,i,j,α	Adding α times row j to row i
ROW*C,i	Application of Rosenbrock's hill-climbing algorithm on row i
ROW*NAME,I	Premultiply Q by matrix NAME (invert NAME if I specified)
QZE	Apply Q(0) to Q

*Similar commands for column operations are also available.

Note that the system is not diagonally dominant as, in general, $\hat{q}_{12} > \hat{q}_{11}$; the Gershgorin circles of element (1,1) encircle the origin.

Diagonal dominance can be achieved by trial and error using the methods shown in Table 3. Most of these are simple row and column operations. The Rosenbrock hillclimb is an algorithm for minimizing the dominance ratio

$$r_i(s) = \frac{d_i(s)}{|\hat{q}_{ii}(s)|} \tag{24}$$

where $d_i(s)$ is given by Eq. 23, by performing a set of ROW A,i,j,α operations.

QZE is used to diagonalize $\hat{Q}(s)$ as $s \to 0$, by using $Q(s)$ as $s \to 0$ for either $\hat{K}$ or $\hat{L}$. It ensures diagonal dominance at low frequencies but, in general, has not been found useful.

To achieve diagonal dominance, we performed, by intuition, the following sequence

COL S,2,0.14	scale column 2 by 0.14
ROW C,1	apply hillclimbing to row 1
ROW C,2	apply hillclimbing to row 2

The resulting INA plot is shown in Figure 8b. Note that the system is now diagonally dominant. The values of K and L are given by

$$K = \begin{bmatrix} 0.01 & 0.00293 \\ 0.00316 & 0.101 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 \\ 0 & 7.14 \end{bmatrix}$$

The final task is to choose the value of $F = diag(f_i)$ which will give the desired response. The problem is thus in the same form as that of modal control (Section 5.1) where

- K_2 is equivalent to L
- K_1 is equivalent to K_1
- K_d is equivalent to F

The gains of F were selected to be as high as possible without causing a damping ratio to be less than 0.8. The gains found were

$$F = diag(-0.2,\ 0.06)$$

and thus the total controller gain is given by

$$K = \begin{bmatrix} -2.0E-3 & -5.859E-4 \\ 1.356E-3 & 4.325E-2 \end{bmatrix}$$

The response to initial offsets in the output variables is shown in both Figures 4 and 6.

Other Methods

Other, more elaborate, design methods are available in MVPACK but were not investigated as part of this report, namely,

- design and analysis of a dynamic compensator,
- design of an integral controller,
- pole shifting with constrained output feedback.

Steady-State Control

The controllers designed in the previous sections were all tested by considering a relaxation from an initial offset. In many cases, the response to changes in the setpoint are of interest. The steady-state behaviour of the system is given by:

$$y = H_0 r \qquad (25)$$

where H_0 = closed loop, steady-state gain matrix. Ideally H_0 should be the identity matrix, but in practice it is considerably different; changes in the variables do not approach the changes in the setpoints.

This could be corrected by

- integral action (a feature available in MVPACK which is not discussed in this report), and/or
- by pre-multiplying by H_0^{-1}.

In the latter case, the system is represented as shown in Figure 9.

SUMMARY AND CONCLUSIONS

The paper describes the use of four basic controller design methods: modal, pole shifting, optimal, and inverse Nyquist array (INA), to design controllers for a nuclear steam generator. The results from all four methods are presented in a manner which is directly comparable. A brief comparison of some peak values is presented in Table 4. It appears from Table 4 that the INA method gives the best controller, in that the peak crosstalk is less. Optimal control, however, is a close second and could be improved further by a different choice of cost parameters. However, this conclusion is very subjective and depends strongly on the criteria of interest.

MVPACK was used as a tool in all the calculations and plotting shown in this paper. It is very convenient to use and offers extensive "help" facilities to the novice user. By having all the design methods on a common database and programming system, their intercomparison was simplified. The INA example illustrates the convenience with which one can go from state-space to frequency response and then back to time responses. It is a very versatile package.

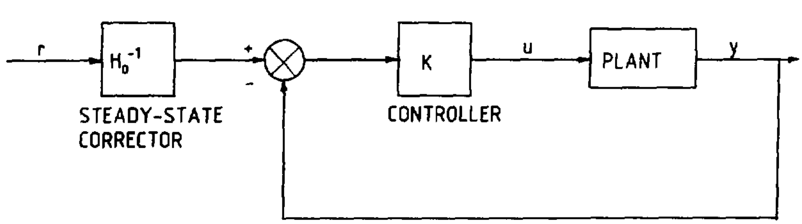

Figure 9. Correction for Steady-State Errors

TABLE 4

SUMMARY OF PERFORMANCE PARAMETERS

Initial Offset	Design Method	Peak Steam Valve Lift %	Peak Feedwater Flow %	Peak Crosstalk
Steam Pressure	Modal	2.0	4.0	75 mm
	Pole Shift	1.0	8.0	91 mm
	Optimal (1)	1.4	6.4	58 mm
	Optimal (2)	5.0	0.7	49 mm
	INA	2.9	1.8	43 mm
Downcomer Level	Modal	3.1	9.0	38.3 kPa
	Pole Shifting	1.5	13.0	22.2 kPa
	Optimal (1)	0.4	5.9	16.8 kPa
	Optimal (2)	0.5	7.0	10.9 kPa
	INA	0.3	14.2	9.7 kPa

(1) Kalman filter initialized to zero.

(2) Kalman filter initialized correctly.

REFERENCES

1. P.D. McMorran and T.A. Cole, "Multivariable Control in Nuclear Power Stations: Modal Control", Atomic Energy of Canada Limited, report AECL-6690, 1979 December.

2. S. Mensah, P.D. McMorran, M. Polis and W. Paskievici, "Multivariable Controller for Nuclear Plants Based on Pole Shifting", Atomic Energy of Canada Limited, report AECL-7249, 1981 February.

3. M. Parent and P.D. McMorran, "Multivariable Control in Nuclear Power Stations: Optimal Control", Atomic Energy of Canada Limited, report AECL-7244, 1982 November.

4. M. Parent and P.D. McMorran, "Multivariable Control in Nuclear Power Stations: Order Reduction", Atomic Energy of Canada Limited, report AECL-7245, 1982 December.

5. N. Roy, S. Mensah and J. Boisvert, "Design of a Multivariable Controller for a CANDU 600 MWe Nuclear Power Plant Using the INA Method", Atomic Energy of Canada Limited, report AECL-8342, 1984 April.

APPENDIX A

STEAM GENERATOR MODEL (1)

State Variables

1.	T_{pi}	Primary Water Inlet Plenum Temperature
2.	T_{p1}	First Primary Water Lump Temperature
3.	T_{p2}	Second Primary Water Lump Temperature
4.	T_{p3}	Third Primary Water Lump Temperature
5.	T_{p4}	Fourth Primary Water Lump Temperature
6.	T_{po}	Primary Outlet Plenum Temperature
7.	T_{m1}	First Tube Metal Lump Temperature
8.	T_{m2}	Second Tube Metal Lump Temperature
9.	T_{m3}	Third Tube Metal Lump Temperature
10.	T_{m4}	Fourth Tube Metal Lump Temperature
11.	L_d	Downcomer Level
12.	L_{s1}	Subcooled Length
13.	P_s	Steam Pressure
14.	x_e	Boiling Section Exit Quality
15.	T_d	Downcomer Temperature

Control Variables

1. $\frac{\partial C_L}{C_L}$ — Steam Valve Lift

2. $\frac{\partial W_{Fi}}{W_{Fi}}$ — Feedwater Flow

Output Variables

P_s and L_d.

STEAM GENERATOR MATRICES

MATRIX A

1	-1.431	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	
2	4.945	-5.593	0	0	0	0	0.6486	0
	0	0	0	-6.001	0	0	0	
3	0	0.7607	-1.409	0	0	0	4.7474E-02	0.5924
	-5.6193E-02	4.7474E-02	-7.0209E-02	1.970	-2.6219E-02	-0.1448	0.2225	
4	0	0	0.7607	-1.409	0	0	0	0
	0.6486	0	0	0.5554	0	0	0	
5	0	0	0	4.945	-5.593	0	-2.0482E-02	2.4243E-02
	2.4243-02	0.6281	3.0290E-02	-2.969	1.1312E-02	6.2463E-02	-9.5977E-02	
6	0	0	0	0	1.431	-1.431	0	0
	0	0	0	0	0	0	0	
7	0	2.406	0	0	0	0	-3.788	-9.8553E-02
	-9.8553E-02	8.3261E-02	-0.1231	1.890	6.0216E-02	-0.2539	1.123	
8	0	0	2.406	0	0	0	1.2807E-02	-5.415
	-1.5159E-02	1.2807E-02	-1.8940E-02	0.2906	0.4271	-3.9056E-02	6.0012E-02	
9	0	0	0	2.406	0	0	-2.8646E-03	3.3907E-03
	-5.397	-2.8646E-03	4.2365E-03	-6.5011E-02	0.4358	8.7362E-03	-1.3424E-02	
10	0	0	0	0	2.406	0	-1.8622E-02	2.2042E-02
	2.2042E-02	-3.890	2.7539E-02	-0.4226	0.1165	5.6790E-02	0.6452	
11	0	0	0	0	0	0	1.9680E-05	2.6471E-02
	2.6471E-02	1.9680E-05	-4.9864E-02	-1.2821E-02	-5.6503E-03	-15.88	9.2223E-05	
12	0	0	0	0	0	0	-4.0438E-02	4.7864E-02
	4.7864E-02	-4.0438E-02	5.9803E-02	-0.9177	2.2333E-02	0.1233	-0.1895	

STEAM GENERATOR MATRICES

MATRIX A (cont'd)

13	0	0	0	0	0	0	9.7633E-04	1.313
	1.313	9.7633E-04	-5.6933E-04	-2.427	-0.4447	-0.3819	4.5751E-03	
14	0	0	0	0	0	0	-1.1336E-03	2.3680E-03
	2.3680E-03	-1.1336E-03	6.9899E-04	-2.6930E-02	3.6157E-04	-0.3084	-5.3120E-03	
15	0	0	0	0	0		7.9461E-06	1.0688E-02
	1.0688E-02	7.9461E-06	3.0538E-03	-2.1967E-02	7.1755E-03	-5.837	-8.2840E-02	

MATRIX B

1	0	0
2	0	0
3	2.444	0.2151
4	0	0
5	-1.055	-9.2801E-02
6	0	0
7	4.287	0.3772
8	0.6594	5.8026E-02
9	-0.1475	-1.2979E-02
10	-0.9587	-8.4373E-02
11	1.060	0.6264
12	-2.082	-0.1832
13	-63.28	2.317
14	-4.8407E-02	-5.4998E-03
15	0.4279	-1.106

MATRIX C

1	0	0	0	0	0	0	0	0
	0	0	0	0	1	0	0	
2	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	

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