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SOME PROBLEMS OF DYNAMICAL SYSTEMS ON THREE DIMENSIONAL MANIFOLDS

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ABSTRACT

It is important to study the dynamical systems on 3 dimensional manifolds, its importance is showing up in its close relation with our life. Because of the complication of topological structure of Dynamical systems on 3-dimensional manifolds, generally speaking, the search for 3-dynamical systems is not easier than 2-dynamical systems. This paper is a summary of the partial result of dynamical systems on 3-dimensional manifolds.

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1. SOME BASIC CONCEPTS OF DYNAMICAL SYSTEMS [1)-4)]

Let $\mathbb{R}$ be a complete metric space and $I = (-\infty, \infty)$.

Let a mapping $f: \mathbb{R} \times I \to \mathbb{R}$. Satisfy the following conditions.

(i) $\forall p \in \mathbb{R}$, $f(p, 0) = p$;

(ii) $f \in C^0$;

(iii) $\forall p \in \mathbb{R}$, $t_1, t_2 \in I$ have

$$f(f(p, t_1), t_2) = f(p, t_1 + t_2)$$

then f is called a topological dynamical systems on $\mathbb{R}$ or a continuous flows on $\mathbb{R}$.

if we replace $\mathbb{R}$ by n dimensional differential manifolds M^n and replace (ii) above by the condition $f \in C^r$ $(r > 1)$, then we call f a C^r differential dynamical systems or a C^r flows on M^n.

Let f be a topological dynamical system on $\mathbb{R}$.

<u>Definition</u>: For some point $q (\in \mathbb{R})$ if there exist $t_n \to +\infty$ $(t_n \to -\infty)$ so that as $n \to \infty$ they have

$$f(p, t_n) \longrightarrow q$$

then we call q a ω-(α-) limiting point of the trajectory $f(p,t)$ and mark all ω-(α-) limiting point of the trajectory $f(p,t)$ $\Omega_p (A_p)$

It is obvious that the limiting set $\Omega_p (A_p)$ of the trajectory $f(p,t)$ is closely invariant and connective.

<u>Definition</u>: Let $f(p,t)$ be a trejectory of f

i) If $\Omega_p (A_p) = \phi$, we call $f(p,t)$ is a positive- far out (negative-far out) trajectory of f;

ii) If $\Omega_p (A_p) \neq \phi$ and $\Omega_p (A_p) \cap f(p,t) = \phi$,we call $f(p,t)$ a positive (negative) asymptotic trajectory of f;

iii) If $\Omega_p (A_p) \neq \phi$ and $\Omega_p (A_p) \cap f(p,t) \neq \phi$, we call $f(p,t)$ a positive (negative) P-stable trajectory of f.

The closure of all positive or negative P-stable trajectories of f is called the centre of f, and we mark the centre of f as M.

Definition: Let A be an invariably close set of f and there is no any invariant close pure subset of A for f, then we call A a minimal set of f.

We can show that the minimal set of f is closure of some P-stable trajectory f(p,t) and $\forall q \in \overline{f(p,I)}$ there exists

$$\overline{f(q,I)} = \overline{f(p,I)}$$

2. THE CENTRE AND THE ORDER OF THE CENTRE SEQUENCE

Let f be a topological dynamical system on R.

Let point $p \in R$, p be called a wondering point of f, if there is neighborhood U of p and T > 0, so that, as t > T, we have,

$$f(U,t) \cap U = \emptyset$$

The set of all wandering point on R for f is marked W, obviously, W is invariably open set. Let $\Omega = R/W$. Ω be called non wandering set of f on R. It is easy to show Ω is invariably close set of f.

Let Q be a invariably close set of R under f. Let the wandering set of Q for f be W_Q and the set that its number is not limiting point of any trajectories in Q be $\check{W}_Q$.

Note that

$$(Q)_A = Q / W_Q$$

$$(Q)_B = \overline{Q / \check{W}_Q}$$

$$(Q)_C = Q / \tilde{W}_Q$$

We define respectively

$$\begin{cases} M^{(1)} = (\mathbb{R})_A & \\ M^{(\alpha)} = (M^{(\alpha-1)})_A & \text{for a first class number } \alpha \\ M^{(\alpha)} = \bigcap_{\beta<\alpha} M^{(\beta)} & \text{for the second class number } \alpha \end{cases}$$

$$\begin{cases} \tilde{M}^{(1)} = (\mathbb{R})_B & \\ \tilde{M}^{(\alpha)} = (\tilde{M}^{(\alpha-1)})_B & \text{for a first class number } \alpha \\ \tilde{M}^{(\alpha)} = \bigcap_{\beta<\alpha} \tilde{M}^{(\beta)} & \text{for the second class number } \alpha \end{cases}$$

and

$$\begin{cases} \hat{M}^{(1)} = (\mathbb{R})_C & \\ \hat{M}^{(\alpha)} = (\hat{M}^{(\alpha-1)})_C & \text{for a first class number } \alpha \\ \hat{M}^{(\alpha)} = \bigcap_{\beta<\alpha} \hat{M}^{(\beta)} & \text{for the second class number } \alpha \end{cases}$$

Consequently we obtain three transfinite sequence

$$M^{(1)}, M^{(2)}, \cdots, M^{(n)}, \cdots, M^{(\omega)}, \cdots, M^{(\beta)}, \cdots,$$

$$\tilde{M}^{(1)}, \tilde{M}^{(2)}, \cdots, \tilde{M}^{(n)}, \cdots, \tilde{M}^{(\omega)}, \cdots, \tilde{M}^{(\beta)}, \cdots,$$

$$\hat{M}^{(1)}, \hat{M}^{(2)}, \cdots, \hat{M}^{(n)}, \cdots, \hat{M}^{(\omega)}, \cdots, \hat{M}^{(\beta)}, \cdots.$$

and there hold

$$M^{(1)} \supset M^{(2)} \supset \cdots \supset M^{(\omega)} \supset \cdots \supset M^{(\beta)} \supset \cdots,$$

$$\tilde{M}^{(1)} \supset \tilde{M}^{(2)} \supset \cdots \supset \tilde{M}^{(\omega)} \supset \cdots \supset \tilde{M}^{(\beta)} \supset \cdots,$$

$$\hat{M}^{(1)} \supset \hat{M}^{(2)} \supset \cdots \supset \hat{M}^{(\omega)} \supset \cdots \supset \hat{M}^{(\beta)} \supset \cdots.$$

obviously, in these sequences $M^{(\beta)}$, $\tilde{M}^{(\beta)}$, $\hat{M}^{(\beta)}$ are invariant and $M^{(\beta)}$ $\tilde{M}^{(\beta)}$ is close.

From Baire theorem, there are respectively $\alpha_1, \alpha_2, \alpha_3$ so that starting from α_1, α_2 and α_3, we have

$$M^{(\alpha_1)} = M^{(\alpha_1+1)} = M^{(\alpha_1+2)} = \cdots,$$

$$\tilde{M}^{(\alpha_2)} = \tilde{M}^{(\alpha_2+1)} = \tilde{M}^{(\alpha_2+2)} = \cdots,$$

$$\hat{M}^{(\alpha_3)} = \hat{M}^{(\alpha_3+1)} = \hat{M}^{(\alpha_3+2)} = \cdots.$$

We can show

$$M^{(\alpha_1)} = \overset{\vee}{M}^{(\alpha_2)} = \overline{\hat{M}^{(\alpha_3)}} = M$$

i.e. $M^{(\alpha_1)}$, $M^{(\alpha_2)}$, $M^{(\alpha_3)}$ respectively is centre M of f.

We call α_1 the first order of teh centre M for f, denote by β_A i.e. $\beta_A = \alpha_1$.

We call α_2 the second order of the centre M for f denote by β_B, i.e. $\beta_B = \alpha_2$.

We define the third class order of the centre M for f

$$\beta_c = \min \{ \alpha_3 \mid \overline{\hat{M}^{(\alpha_3)}} = M \} .$$

It is easy to show

$$\beta_A \geqslant \beta_B \geqslant \beta_c . \qquad \text{(see 5) - 12))}$$

Let M^2 be 2-dimensional compact manifolds and topological dynamical systems f_2 be defined on M^2. For the order of centre sequence of f_2. A.P. Maĭep[8], Schwartz Thomas[15] get the following theorem

Theorem 1 For any topological dynamical systems f_2 on M^2, inequalities

$$\beta_A \leq 3 , \qquad \beta_c \leq 2 .$$

hold.

But, for any topological dynamical systems f_3 on 3-dimensional manifolds M^3, particularly on R^3 (3-dimensional Euclidean space) we propose the following theorem

Theorem 2 [14] Let α, β and ν be any given finit or transfinit numbers with $\alpha \geqslant \beta \geqslant \nu > 2$. Then we can construct the system of ordinary differential equations

$$\frac{dx_i}{dt} = X_i (x_1 , x_2 , x_3) , \quad i = 1, 2, 3$$

on R^3, where X_i satisfy the Lipschitz condition and this system just has

$$\beta_A = \alpha , \qquad \beta_B = \beta , \qquad \beta_c = \nu .$$

From this theorem, it is easily shown that the above theorem is hold for any topological dynamical systems on M^3.

We got this theorem in 1963. It is the author's graduation thesis published in 1980. D.A. Neumann[16] got also a similar result in 1978. In these two papers the methods used to construct systems are different.

From the above result, we can see that topological construction of 3-dynamical systems is more complicated than 2-dynamical systems.

3. THE EXISTENCE OF PERIODIC SOLUTIONS

As one knows, in dynamical systems on the plane, determination of the existence on periodic solutions is mainly based on the Poincarè-Bendixson theorem.

This theorem can extend to dynamical systems on general 2-manifolds.

Theorem 3 [24] Let M^2 be a compact 2-manifolds and c' dynamical systems M_t^2 be defined on M^2. Let the region D of M_t^2 have $\ell (>1)$ boundaries that consist of ℓ mutually non-intersection simple curves and suppose its direction of M_t^2 on every boundary is agrreable. M_t^2 having no singularity in D. Then there are close trajectories about every boundary of D.

For 3-c' dynamical systems M_t^3 is there similar Poincaré-Bendixon theorem? Seifert has presented a famous conjecture as follows

Conjecture [17] "It is unknown if every continuous vector field of the three dimensional sphere S^3 contains a closed integral curve".

The positive assertion of this statement has subsequently become known is the Seifert conjecture, a term which was popularized by F.W.Wilson Jr.[18] Hsin Chu[19] and C.C. Pugh[20].

The statement of the Seifert conjecture with C^r v ctor field replacing continuous vector fields will be called the C^r Seifert conjecture.

In 1974 an important counter example to the C' Seifert conjecture was constructed by P. Schweitzer [22].

P. Schweitzer constructed C' flow in a solid torus that every trajectories through 2-torus T^2 is attracted to a singular minimal set on T^2, the singular minimal set being due to A. Denjoy.

From A.J.Schwartz' theorem*) there do not any C^r $(r > 1)$ vector fields on 2-manifolds such that they have singular minimal set. As a result to solve the c^r $(r > 1)$ Seifert conjecture, one needs to find an other method so as to distinguish it from Schweitzer method.

Recently J. Harrison [25] declared that the C^2 Seifert conjecture has been solved. But for $r \geq 3$, the c^r Seifert conjecture is still open.

4. THE MINIMAL SET

The topological construction of the minimal set of 2-dynamical system is not clear enough. And in this way the topological construction of the minimal set of 3-dynamical systems is more ambiguous. To show this we attempt to discuss the well-known Gottschalk's conjecture. It was presented by Gottschalk in 1968 [28].

Conjecture (Gottschalk) The 3-sphere is not minimal under any continuous flows.

The conjecture is obviously false for the 1-sphere and true for 2-spheres and even dimensional spheres [29].

We have study this conjecture and answered it by a positive solution.

First, we discuss the conjecture for dynamical systems on R^3 and get

Theorem 4 [30] The R^3 is not minimal under any continuous flows.

Next, we discuss the conjecture for dynamical systems on S^3 and give the following

Theorem 5 [30] The 3-sphere S^3 is not minimal under any continuous flows.

To prove these theorems the main method is to find their global cross section of R^3, S^3 under f; respectively. We show the existence of their global cross section on R^3 or S^3. From this it is easily derived a contradiction with our presumption that R^3 or S^3 is minimal set under f.

5. ASYMPTOTIC TRAJECTORY SYSTEMS

Asymptotic trajectory system f on a complete metric space R is the dynamical system f on R that all trajectories are asymptotic.

V.V.Nemytskii [31] posed a question that whether there are asymptotic trajectory systems on complete metric space. Z.F. Zheng [32] has constructed the asymptotic trajectory system in Genytob systems. This is the earliest answer to the Nemytskii's problem. We have also construct the asymptotic trajectories system on partial complete subspace in R^3 [14]. Then J.H. Mai gave the asymptotic trajectory system on infinit dimensional Herbert spaces.

From the Schwartz-Thomas theorem [15] in §2, it is easy to show that there are no any asymptotic trajectory systems on 2 compact complete manifolds. But for the whole R^3, are there an asymptotic trajectory system? For this we shall give the following negative answer.

Theorem 6 [34] There are not any asymptotic trajectory systems on R^3.

To prove this theorem, using the method of canonical regions, we gave the concept of trajectory bunch, analyse its topological construction and further study its limiting property of the trajectory bunch. At last we claim the correctness of this theorem.

6. STRUCTURALLY STABLETY

We denote a compact n-dimensional Reimann smooth manifold $(n \geq 2)$ by M^n, and we define a C' vector field X, from this we get C' dynamical systems f on M^n. let $X = X(M^n)$ denote the set of all C' vector fields on M^n. In X we can define C^0-module and C'-module, and so X is a linear space with modules C^0 and C'.

Definition [4] Let Z_1, Z_2 be two C' vector fields on M_1^n, M_2^n respectively. Z_1 and Z_2 is topologically equivalent, if and only if there is a homeomorphism h: $M_1^n \to M_2^n$, so that h maps every trajectory of Z_1 on M_1^n onto one of Z_2 on M_2^n under preservation of their direction of every trajectory mark $Z_1 \sim Z_2$.

Definition [4] let Z be C' vector field on M^n, Z be C' structurally stable if and only if there are C' neighborhood U of Z in $X(M^n)$, so that $\forall Y \in U$, we have $Y \sim Z$.

In this area, the mainly problem is to look for characterizations of structurally stability on C' dynamical systems. To study this problem is the chief contents in Differential Dynamical Systems Theory.

For C' dynamical systems on 2-manifolds, they have been solved early by Peixoto [35].

For C' dynamical systems on n(3) -manifolds they are problem of most difficulty.

Up to now, the main resolt is due to Liao S.T. [37)-43)], Smale [36)]-Smale and Palis [*1)] Robbin [46)] Robinson, [47)] Pugh-Shub [*2)] Maré [*3)] Pugh [45)] etc.

There exists the following famous conjecture (posed by Smale [36)] primarily for discrete dynamical systems generated by diffeomorphism on M^n) of structurally stable on C' dynamical systems on M^n.

Conjecture A necessary and sufficient condition for $S \in X(M^n)$ ($Diff(M^n)$) to be structurally stable is that S satisfies Axiom A and the strong transversalbity condition.

This famous conjecture is open for many year... Robbin-Robinson solved its sufficient part [46)-47)]. For the necessary part of this conjecture, as far as we know the best work is achieved by Liao S.T. For n=2, 3 he has solved this conjecture for its diffeomorphism 's form and for n=3,4 he has solved this conjecture on its flow's form under the condition that s is not singularity.

These results are as following

Theorem 7 [40)] (Liao S.T.) Let $f \in Diff'(M^2)$; then a necessary and sufficient condition for f to be structurally stable is that f satisfies Axiom A and the strong transversality condition

Theorem 8 (Liao S.T.) Let $f \in Diff'(M^3)$; then a necessary and sufficient condition for f to be structurally stable is that f satisfies Axiom A and the strong transversality condition.

Theorem 9 [40)] (Liao S.T.) if $S \in X(M^3)$ has no singularities, then a necessary and sufficient condition for S to be structurally stable is that S satisfies Axiom A and the strong transversality condition.

7. TOPOLOGICAL CLASSIFICATION

In § 6 we gave the definition of topological equivalent for two vector fields $Z_1 \in X(M_1^n)$ and $Z_2 \in X(M_2^n)$. With this definition, we can classify in $X(M^n)$ under the above mentioned sense of topological equivalence. Here is the problem now to describe their characterizations of any topological classification in $x(M^n)$.

In case n=2 L. Markus [48)] M.M.Peixoto [49)] D.A. Neumann [50),51)] and the author [52)] study this problem successively.

Theorem 10 [52)] Let $M_t^{(1)}$, $M_t^{(2)}$ be two continuous flows with isolated critical point on 2-manifolds M^2. Then $M_t^{(1)}$ and $M_t^{(2)}$ are topologically equivalent if and only if they are decomposable into generalized canonical regions by the same fashion i.e;, they have the same number and type of generalized canonical regions and the over-lapping boundaries of the generalized canonical regions on $M_t^{(1)}$ and the corrsponding ones on M_t^2 are of the same type.

In case n=3, the result on this problem is not many, as we can see. Recently author gave the following theorem [53)].

Theorem 11 [53)] Let two continuous flows $M_t^{(1)}$, $M_t^{(2)}$ be defined on 3-manifolds M^3 and have only isolated critical point and their separatries be denoted by $S^{(1)}$, $S^{(2)}$ respectively. If there are homeomorphism $h : S^{(1)} \to S^{(2)}$, so that we may take every separatrix onto one of $S^{(2)}$ under direction of trajectory preservation and if the canonical regions of every separatrix V_1 in $S^{(1)}$ and the corresponding canonical regions at the separatrix $h(V_1) = V_2$ are the same then $M_t^{(1)}$, $M_t^{(2)}$ are topological equivalent.

In comparison of the above theorem on M^2 with the above theorem on M^3, the condition described topological equivalence is not so clear as the above theorem on M^2. For example in the latter theorem we did not give the conditions to describe topological equivalence on set of all their separatrixes. Because of the complexition of 3 dynamical systems. We think, it is not easy to give these conditions.

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REFERENCES

1) V.V. Nemytskii and V.V. Stepanov; "Qualitative theory of differential equations", Princeton University Press, 1960.

2) N.P. Bhatia and G. Szego; "Stability theory of dynamical systems", Springer, berlin 1970.

3) W.A. Gottschalk and G.A. Hedland; "Topological dynamic", A.M.S. Collog pub. 36 (1965).

4) J.Palis and W. de Melo, "geometric Theory of Dynamical Syatems An Introduction. Springer verlag (1982).

5) G.D. Birkhoff; Uber gewisse Zentralbewegungen dynamischer Systeme, Gott Nachr.(1926) p. 81-92.

6) G.D. Birkhoff, Some unsolved problems of theoretical dynamics, Science 94(1941) p. 598-600.

7) G.D. Birkhoff and P.A. Smith, Structure analysis of surface transformations. J.Math.Pures.Appl. 7(1928) p. 345-379.

8) A.P. Maйep, MaT c 12 54(1943) p. 71-84.

9) A.P. Maйep, RAH LV 6 (1947) p. 477-479.

10) A.P. Maйep, RAH LV 7 (1947) p. 583-585.

11) A.P. Maйep, RAH X 8 (1948) p. 1393-1396.

12) A.P. Maйep, MaT c 26(68)(1950) p. 265-290.

13) Z.X. Dong, On a problem of Ordinal Number of Central Trajectories, Acta Scientiaruma Naturalium Universitatis Pekinensis, 3(1980) p.11-21.

14) Z.X. Dong, Dynamical Systems with Limiting Trajectories. Acta Mathematica Sinica 5(25)(1982) p. 295-602.

15) A.J. Schwartz and E.S. Thomas, The depth of the center of 2-manifolds Global Anal. proc. Symp. pure Math. 14 Amer.Math.Soc. providence Rhode Island 1970 p. 253-264.

16) D.A. Neumann , Central Sequences in Dynamical systems, J.Math.Vol. 100 N°1 (1978) p. 1-18.

17) H. Seifert, Closed Intergral curves in 3-space and isotopic two-dimensional deformations PAMS V.I. (1950) p. 287-302.

18) F.W. Wilson Jr. , ON the minimal sets of non-singular vector fields. Ann of Math. V 84(1966) p. 529-536.

19) H.Chu, A remark on the Seifert conjecture, Topology Vol.9(1970) p. 275-281
20) C.C. Pugh, The closing Lemma, Am.J.Math. Vol.59 (1967) p. 956-1009.
21) P. Schweitzer, Compact leaves of foliations. Proc.Intl.Cong. of Math. Vancouver 1974, Vol.1 p.543-546.
22) P. Schweitzer, Counterexamples to the Seifert conjecture and opening close leaves of foliations. Ann. of Math. Vol.100 (1974) p. 386-400.
23) A. Denjoy, Sur les courbes définies par les équations différentielles à la surface du tore, J.Math. Pures et Appl. Vol. 11 (2932) P. 333-375.
24) Z.X. Dong, Generalized Poincarè-Bendixson Theorem on 2 dimensional manifolds Advances in Mathematics 3 (12) (1983) p. 222-225.
25) J.Harrison, A C^2 Counterexample to the Seifert Conjecture, preprint
26) J. Harrison and J.A. Yorke, Flows on S^3 and R^3 without periodic orbits, Lecture Notes in Mathematics Vol. 100, Springer Verlag 401-407.
27) R. Ellis, Lectures on Topological Dynamics W.A. Benjamin, New York, 1969.
28)J. Auslander and W.H. Gottschalk (editors) Topological Dynamics, Benjamin New York, 1968.
29) O. Hàjek, Non-minimality of 3-manifolds, Lecture Notes in Mathematics 318 Springer verlag p. 140-142.
30) Z.X. Dong, On Gottschalk Conjecture (to appear)
31) V.V. Nemytskii, Topological problems of the theory of dynamical systems. Amer.Math.Soc.Transl. 103 (1954).
32) Z.F. Zhang, T.G. Ding and W.Z. Huang, Answers to some questions on Topological Dynamical systems posed by V.V. Nemytskii and the others, KEXUE TONGBAO (1980).
33) Z.F. Zhang, T.G. Ding and W. Z. Huang , Some problems on Topological Dynamical Systems, kexue TUNGBAO (1980)
34) Z.X. Dong, Existence of the Asymptotic Trajectories systems on R^3 (to appear).
35) M. Peixoto, Structural stability on two-dimensional manifolds, Topology 1 (1962) p. 101-120.
36) S. Smale, Differentiable dynamical systems, Bull. AMS 73(1967) 747-817.

37) S.T. Liao, Certain ergodic properties of a differential system on a compact differentiable manifolds, Acta Scientiarum Naturalium Universitatis Pekinensis 9 (1963) N° 3 241-265; 309-324.
38) S.T. Liao, Standard Systems of differential equations, Acta Math. Sinica,17 (1974) 100-109, 175-196, 270-295.
39) S.T. Liao, An extension of the C^1 closing lemma, Acta Scientiarum Naturalium Universitatis Pekinensis 1979, N°3 1 - 41.
40) S.T. LIao, On the stability Conjecture, Chinese Annals of Math. 1 (1980) 9 - 29.
41) S.T. Liao, Obstruction sets and strong transversality. Acta Math. Sinica 19 (1976) 203-209.
42) S.T. Liao, Obstruction sets (I) Acta Math. Sinica 23(1980) 411-453, (II) Acta Scientiarums Naturalium Universitatis Pekinensis 1981 N°2 1 - 36.
43) S.T. Liao, Standard systems of Differential Equations and Obstruction sets with Applications to Structural Stability problems. Report on DD4 (1983).
44) Z. Nietecki, Differentiable dynamics 1971.
45) C. Pugh, An improved closing lemma and a general density theorem, Amer. J. Math. 89 (1967) 1010-1021.
46) J. Robbin, A structural stability theorem, Annals of math. 94 (1971) 447-493.
47) C. Robinson, Structural Stability for C^1 flows, Worwick 1974, Lecture Notes 468, Springer Verlag.
48) L. Markus, Global structure of ordinary differential equations in the plane, trans.Amer.Math.Soc. 76 (1954) 127-148.
49) M.M. Peixoto, On the classification of flows on 2-manifolds Dynamical Systems. Academic Press, New York 1973.
50) D.A. Neumann and T.O'brien, Global structure of continuous flows on 2-manifolds, J.Diff.Eqs. 22 (1976) 89-110.

51) D.A. Neumann, Classification of continuous flows on 2-manifolds, Proc. Amer.Math.Soc. 48 (1975) 73-81.

52) Z.X. Dong, On the classification of Dynamical systems on 2-manifolds The Report on DD4 (1983) (to appear).

53) Z.X. Dong, On the classification of Dynamical systems on 3-manifolds (to appear)

*) A.J. Schwartz, A generalization of A Poicarè-Bendixson Theorem to Closed Two-Dimensional Manifolds.

*1) J.Palis ans S.Smale, Structural Stability theorems, Proc.Symp. Pure Math AMS 14(1970) 223-231.

*2) C. Pugh and M. Shub, The   stability theorems for flows; Invetiones Math. 11 (1980) 150-158.

*3) H. Manè, An ergodic closing Lemma, Annals of Math. 116 (1982) 503-540.