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DIFFRACTION OF LOVE WAVES BY TWO PARALLEL PERFECTLY WEAK HALF PLANES *

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ABSTRACT

We consider the diffraction of Love waves by two parallel perfectly weak half planes in a layer overlying a half space. The problem is formulated in terms of the Wiener-Hopf equations in the transformed plane. The transmitted waves are then calculated using the Wiener-Hopf procedure and inverse transforms.

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1. INTRODUCTION

The diffraction of seismic waves by structural discontinuities is of considerable importance in seismology because of the existence of such discontinuities in the Earth's crust. Exact analytical solutions of these problems are difficult to obtain even for simple geometries. de Hoop (1958) presented a method based upon the Wiener - Hopf technique for the solution of body waves by a single perfectly rigid or perfectly weak half plane. Kazi (1975) considered the diffraction of Love-waves by perfectly rigid and perfectly weak half planes lying in a surface layer overlying a half space. Recently, Asghar and Zaman (1986) have considered the diffraction of Love waves by taking the rigid barrier to be of finite extension.

In this paper, we set up and solve the problem of diffraction of Love waves normally incident on two parallel perfectly weak (crack) half planes lying in a surface layer and parallel to the interface between the layer and the half space. The problem is formulated in terms of the two Wiener-Hopf equations and can be solved by the technique introduced by Jones (1952). The weak screens separate the layer into three loosely coupled layers. The transmitted waves in these three regions have been calculated analytically. As expected on physical grounds, it has been shown that the transmitted wave in each region satisfies the dispersion relation of the Love waves travelling in a layer of uniform thickness under similar boundary condition.

2. FORMULATION OF THE PROBLEMS

We consider the diffraction of Love waves by two parallel weak half planes (cracks) lying in a layer of uniform thickness H over an elastic half space. The half space has a rigidity μ_1 and shear wave velocity β_1 and the layer has rigidity μ_2 and the shear wave velocity β_2. The coordinate system is chosen in such a way that the interface between the half space and the layered medium coincides with the xy plane, the z axis is directed into the half space and the two semi-infinite planes occupy $z = -h_1$, $x \leq 0$ and $z = -h_2$, $x \leq 0$. The free surface is $z = -H$. The geometry of the problem is shown in Fig. 1.

The incident Love waves of the N^{th} mode have displacements:

$$v_1^{inc} = A\cos(\sigma_{2N}H)\exp\left\{-\sigma_{1N}z - iK_{1N}(x-x_0)\right\}, \quad z\geq 0$$

$$v_2^{inc} = A\cos\{(z+H)\sigma_{2N}\}\exp\left\{-iK_{1N}(x-x_0)\right\}, \quad 0\geq z\geq -H \tag{1}$$

where

$$\sigma_{1N} = (K_{1N}^2-k_1^2)^{\frac{1}{2}}, \quad \sigma_{2N} = (k_2^2-K_{1N}^2)^{\frac{1}{2}}, \quad k_1 = \frac{\omega}{\beta_1}, \quad k_2 = \frac{\omega}{\beta_2} \tag{2}$$

and K_{1N} is the N^{th} root of the Love wave dispersion equation

$$\tan\left\{\sqrt{k_2^2-K^2}\,H\right\} = \nu\frac{\sqrt{K^2-k_1^2}}{\sqrt{k_2^2-K^2}}, \quad \nu = \frac{\mu_1}{\mu_2}, \tag{3}$$

corresponding to the layer thickness H. Moreover, $K_{1N} = \frac{\omega}{C_{1N}}$, where C_{1N} is the phase velocity of the Love waves of the N^{th} mode.

Let the total displacement field due to the presence of perfectly weak screens be written as

$$v^{tot} = \begin{cases} v_1^{inc} + v_1, & z\geq 0, \quad \infty > x > -\infty \\ v_2^{inc} + v_2, & 0\geq z\geq -h_1, \quad \infty > x > -\infty \\ v_2^{inc} + v_3, & -h_1\geq z\geq -h_2, \quad \infty > x > -\infty \\ v_2^{inc} + v_4, & -h_2\geq z\geq -H, \quad \infty > x > -\infty \end{cases} \tag{4}$$

The geometry of the problem leads to the following boundary conditions:

a) At $z = 0$, $\infty > x > -\infty$

$$\left.\begin{aligned} v_1 &= v_2 \\ \nu\frac{\partial v_1}{\partial z} &= \frac{\partial v_2}{\partial z} \end{aligned}\right\} \tag{5a}$$

b) For $x \leq 0$.

$$\left.\begin{aligned} &\text{At } z = -h_1+0, && \frac{\partial v_2}{\partial z} \\ &\text{At } z = -h_1-0, \; z = -h_2+0, && \frac{\partial v_3}{\partial z} \\ &\text{At } z = -h_2-0, && \frac{\partial v_4}{\partial z} \end{aligned}\right\} = -\frac{\partial v_2^{inc}}{\partial z} = A\sigma_{2N}\sin(\sigma_{2N}\delta_i)\exp\{-iK_{1N}(x-x_0)\},$$

$$\delta_i = H-h_i; \; i=1,2. \tag{5b}$$

c) At $z = -H$, $-\infty < x < \infty$

$$\frac{\partial v_4}{\partial z} = 0 \tag{5c}$$

d) At $z = -h_1$, $x \geq 0$

$$v_2 = v_3, \quad \frac{\partial v_2}{\partial z} = \frac{\partial v_3}{\partial z} \tag{5d}$$

e) At $z = -h_2$, $x \geq 0$

$$v_3 = v_4, \quad \frac{\partial v_3}{\partial z} = \frac{\partial v_4}{\partial z}. \tag{5e}$$

The displacements satisfy the differential equations

$$\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial z^2} = \frac{1}{\beta_1}\frac{\partial^2 v_1}{\partial t^2} \tag{6}$$

$$\frac{\partial^2 v_j}{\partial x^2} + \frac{\partial^2 v_j}{\partial z^2} = \frac{1}{\beta_2}\frac{\partial^2 v_j}{\partial t^2} \quad (j=2,3,4) \tag{7}$$

The differential equations (6) and (7) can be transformed into

$$\frac{\partial^2 \upsilon_1}{\partial x^2} + \frac{\partial^2 \upsilon_1}{\partial z^2} + k_1^2 \upsilon_1 = 0 \tag{8}$$

$$\frac{\partial^2 \upsilon_j}{\partial x^2} + \frac{\partial^2 \upsilon_j}{\partial z^2} + k_2^2 \upsilon_j = 0 , \quad j = 2, 3, 4, \tag{9}$$

where $k_i = \frac{\omega}{\beta_i}$, $i = 1,2$; $|k_1| < |k_2|$ and the time dependence is taken as $e^{i\omega t}$ and will be suppressed throughout. The differential equations (8) and (9) together with the boundary conditions (5a) through (5e) constitute the boundary value problem.

3. THE WIENER-HOPF EQUATIONS

Following a notation similar to that of Noble (1958), define

$$V(\alpha, z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \upsilon(x, z)\, e^{i\alpha x}\, dx$$

$$= V_+(\alpha, z) + V_-(\alpha, z), \tag{10}$$

where

$$V_+(\alpha, z) = \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} \upsilon(x, z)\, e^{i\alpha x}\, dx$$

$$V_-(\alpha, z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} \upsilon(x, z)\, e^{i\alpha x}\, dx .$$

If $|V| \leqslant A \exp(\eta_- x)$ as $x \to \infty$ and $|V| \leqslant B \exp(\eta_+ x)$ as $x \to -\infty$ then $V_+(\alpha,z)$ is analytic for $\eta > \eta_-$ and $V_-(\alpha,z)$ is analytic for $\eta < \eta_+$. Thus $V(\alpha,z)$ is analytic in the strip $\eta_+ > \eta > \eta_-$, where $\eta_+ > \eta_-$. It seems reasonable to expect $\eta_+ = \mathrm{Im}(k_1)$, $\eta_- = -\mathrm{Im}(k_1)$ and $\eta = \mathrm{Im}(\alpha)$ (Noble, 1958). Transforming the differential equations (8) and (9) ($j = 2$) by (10), we obtain

$$\frac{d^2}{dz^2} V_1(\alpha, z) - \sigma_1^2 V_1(\alpha, z) = 0 , \quad \sigma_1 = (\alpha^2 - k_1^2)^{\frac{1}{2}}, \ Re\ \sigma_1 \geq 0 \tag{11}$$

$$\frac{d^2}{dz^2} V_2(\alpha, z) - \sigma_2^2 V_2(\alpha, z) = 0 , \quad \sigma_2^2 = (\alpha^2 - k_2^2)^{\frac{1}{2}}, \ Re\ \sigma_2 \geq 0 . \tag{12}$$

The solution of Eq. (11) satisfying $V_1 = 0$ as $|z| \to \infty$ is

$$V_1(\alpha, z) = V_{1+}(\alpha, z) + V_{1-}(\alpha, z) = B(\alpha) \exp(-\sigma_1 z), \tag{13}$$

and of Eq. (12) is

$$V_2(\alpha, z) = V_{2+}(\alpha, z) + V_{2-}(\alpha, z) = C(\alpha) \exp(-\sigma_2 z) + D(\alpha) \exp(\sigma_2 z). \tag{14}$$

Using the boundary conditions (5a) in Eqs. (13) and (14), we get

$$C(\alpha) = \frac{\sigma_2 + \nu \sigma_1}{2\sigma_2} B(\alpha), \quad D(\alpha) = \frac{\sigma_2 - \nu \sigma_1}{2\sigma_2} B(\alpha). \tag{15}$$

Eq. (14) can thus be written as

$$V_{2+}(\alpha, z) + V_{2-}(\alpha, z) = \frac{1}{\sigma_2} \left[\sigma_2 \cosh \sigma_2 z - \nu \sigma_1 \sinh \sigma_2 (z) \right] B(\alpha). \tag{16}$$

Similarly, the solutions of (9) for $j = 3$ and 4, after taking its transforms, are given by

$$V_{3+}(\alpha, z) + V_{3-}(\alpha, z) = E(\alpha) \exp(-\sigma_2 z) + F(\alpha) \exp(\sigma_2 z), \tag{17}$$

and

$$V_{4+}(\alpha, z) + V_{4-}(\alpha, z) = 2 \exp(\sigma_2 H) \cosh \sigma_2 (z + H)\, G(\alpha). \tag{18}$$

In writing the equation (18), we have used the boundary condition (5c).

We now attempt to eliminate the constants $B(\alpha)$, $E(\alpha)$, $G(\alpha)$ and $F(\alpha)$ from the previous equations. First of all, we differentiate Eq. (16) with respect to z and put $z = -h_1$, to obtain

$$B(\alpha) = -\frac{V'_{2+}(\alpha,-h_1) + V'_{2-}(\alpha,-h_1)}{\sigma_2 \sinh \sigma_2 h_1 + \nu\sigma_1 \cosh \sigma_2 h_1} . \tag{19}$$

Similar manipulations lead to

$$E(\alpha) = \frac{\left[V'_{3+}(\alpha,-h_1) + V'_{3-}(\alpha,-h_1)\right] \exp(-\sigma_2 h_2) - \left[V'_{3+}(\alpha,-h_2) + V'_{3-}(\alpha,-h_2)\right] \exp(-\sigma_2 h_1)}{2\,\sigma_2 \sinh \sigma_2 (h_2-h_1)} \tag{20}$$

$$F(\alpha) = \frac{\left[V'_{3+}(\alpha,-h_1) + V'_{3-}(\alpha,-h_1)\right] \exp(\sigma_2 h_2) - \left[V'_{3+}(\alpha,-h_2) + V'_{3-}(\alpha,-h_2)\right] \exp(\sigma_2 h_1)}{2\,\sigma_2 \sinh \sigma_2 (h_2-h_1)} \tag{21}$$

and

$$G(\alpha) = \frac{V'_{4+}(\alpha,-h_2) + V'_{4-}(\alpha,-h_2)}{2\sigma_2 \exp(\sigma_2 H) \sinh \sigma_2 \delta_2} , \quad \delta_2 = H - h_2. \tag{22}$$

Using Eqs. (19), (20), (21) and (22), Eqs. (16), (17) and (18) can be written as

$$V_{2+}(\alpha,z) + V_{2-}(\alpha,z) = -\frac{1}{\sigma_2} \cdot \frac{\sigma_2 \cosh \sigma_2 z - \nu\sigma_1 \sinh \sigma_2 z}{\sigma_2 \sinh \sigma_2 h_1 + \nu\sigma_1 \cosh \sigma_2 h_1} \left[V'_{2+}(\alpha,-h_1) + V'_{2-}(\alpha,-h_1)\right] \tag{23}$$

$$V_{3+}(\alpha,z) + V_{3-}(\alpha,z) = \left\{ \left[V'_{3+}(\alpha,-h_1) + V'_{3-}(\alpha,-h_1)\right] \cosh \sigma_2 (z+h_2) - \left[V'_{3+}(\alpha,-h_2) + V'_{3-}(\alpha,-h_2)\right] \cosh \sigma_2 (z+h_1) \right\} \div \left\{ \sigma_2 \sinh \sigma_2 (h_2-h_1) \right\} , \tag{24}$$

$$V_{4+}(\alpha,z) + V_{4-}(\alpha,z) = \frac{1}{\sigma_2} \frac{\cosh \sigma_2 (z+H)}{\sinh \sigma_2 \delta_2} \left[V'_{4+}(\alpha,-h_2) + V'_{4-}(\alpha,-h_2)\right]. \tag{25}$$

Now, putting $z = -h_1$ in (23) and (24) gives

$$V_{2+}(\alpha,-h_1) + V_{2-}(\alpha,-h_1) = -\frac{1}{\sigma_2} \frac{\sigma_2 \cosh \sigma_2 h_1 + \nu\sigma_1 \sinh \sigma_2 h_1}{\sigma_2 \sinh \sigma_2 h_1 + \nu\sigma_1 \cosh \sigma_2 h_1} \left[V'_{2+}(\alpha,-h_1) + V'_{2-}(\alpha,-h_1)\right] \tag{26}$$

and

$$V_{3+}(\alpha,-h_1) + V_{3-}(\alpha,-h_1) = \frac{1}{\sigma_2} \coth \sigma_2 (h_2-h_1) \left[V'_{3+}(\alpha,-h_1) + V'_{3-}(\alpha,-h_1)\right] - \frac{\left[V'_{3+}(\alpha,-h_2) + V'_{3-}(\alpha,-h_2)\right]}{\sigma_2 \sinh \sigma_2 (h_2-h_1)} . \tag{27}$$

Also, Eqs. (24) and (25) for $z = -h_2$ can be written as

$$V_{3+}(\alpha,-h_2) + V_{3-}(\alpha,-h_2) = \frac{\left[V'_{3+}(\alpha,-h_1) + V'_{3-}(\alpha,-h_1)\right]}{\sigma_2 \sinh \sigma_2 h} - \frac{\coth(\sigma_2 h)}{\sigma_2} \times \left[V'_{3+}(\alpha,-h_2) + V'_{3-}(\alpha,-h_2)\right] , \tag{28}$$

where $h = h_2 - h_1$ and

$$\left[V_{4+}(\alpha,-h_2) + V_{4-}(\alpha,-h_2)\right] = \frac{\coth(\sigma_2 \delta_2)}{\sigma_2} \left[V'_{4+}(\alpha,-h_2) + V'_{4-}(\alpha,-h_2)\right]. \tag{29}$$

Transforming the boundary conditions (5b), we get

$$\left.\begin{aligned} V'_{2-}(\alpha,-h_1) &= V'_{3-}(\alpha,-h_1) \\ V'_{4-}(\alpha,-h_1) &= V'_{3-}(\alpha,-h_1) \end{aligned}\right\} = -\frac{iA\,\sigma_{2N} \sin(\sigma_{2N}\delta_i) \exp(iK_{1N}x_0)}{\sqrt{2\pi}\,(\alpha-K_{1N})} . \tag{30}$$

Applying the transforms to (5d) and (5e) yields

$$V_{2+}(\alpha,-h_1) = V_{3+}(\alpha,-h_1) \quad ; \quad V_{4+}(\alpha,-h_2) = V_{3+}(\alpha,-h_2)$$

$$V'_{2+}(\alpha,-h_1) = V'_{3+}(\alpha,-h_1) \quad ; \quad V'_{4+}(\alpha,-h_2) = V'_{3+}(\alpha,-h_2). \tag{31}$$

Eqs. (27) and (28) together with Eqs. (30) and (31) give

$$V_{2+}(\alpha,-h_1) + V_{2-}(\alpha,-h_1) + V_{3-}(\alpha,-h_1) - V_{2-}(\alpha,-h_1) = \frac{\coth\sigma_2 h}{\sigma_2}\left[V'_{2+}(\alpha,-h_1) + V'_{2-}(\alpha,-h_1)\right]$$

$$- \frac{\left[V'_{4+}(\alpha,-h_2) + V'_{4-}(\alpha,-h_2)\right]}{\sigma_2\sinh\sigma_2 h}, \tag{32}$$

and

$$V_{4+}(\alpha,-h_2) + V_{4-}(\alpha,-h_2) + V_{3-}(\alpha,-h_2) - V_{4-}(\alpha,-h_2) = \frac{\left[V'_{2+}(\alpha,-h_1) + V'_{2-}(\alpha,-h_1)\right]}{\sigma_2\sinh\sigma_2 h}$$

$$- \frac{\coth(\sigma_2 h)}{\sigma_2}\left[V'_{4+}(\alpha,-h_2) + V'_{4-}(\alpha,-h_2)\right], \tag{33}$$

where $h = h_2 - h_1$.

Using Eqs. (26) and (29) in Eqs. (32) and (33) and simplifying, we have

$$\frac{1}{\sigma_2^2}\,\frac{\sigma_2 h}{\sinh\sigma_2 h}\,\frac{I(\alpha,h_2)}{I(\alpha,h_1)}\left[V'_{2+}(\alpha,-h_1) + V'_{2-}(\alpha,-h_1)\right] - \frac{1}{\sigma_2^2}\,\frac{\sigma_2 h}{\sinh\sigma_2 h}\left[V'_{4+}(\alpha,-h_2) + V'_{4-}(\alpha,-h_2)\right]$$

$$= h\left[V_{3-}(\alpha,-h_1) - V_{2-}(\alpha,-h_1)\right] \tag{34}$$

and

$$\frac{1}{\sigma_2^2}\cdot\frac{\sigma_2 h}{\sinh\sigma_2 h}\left[V'_{2+}(\alpha,-h_1) + V'_{2-}(\alpha,-h_1)\right] - \frac{1}{\sigma_2^2}\,\frac{\sigma_2 h}{\sinh\sigma_2 h}\cdot\frac{\sinh\sigma_2\delta_1}{\sigma_2\delta_1}\cdot\frac{\sigma_2\delta_2}{\sinh\sigma_2\delta_2}\,\frac{\delta_1}{\delta_2}\times$$

$$\times\left[V'_{4+}(\alpha,-h_2) + V'_{4-}(\alpha,-h_2)\right] = \left[V_{3-}(\alpha,-h_2) - V_{4-}(\alpha,-h_2)\right], \tag{35}$$

where

$$I(\alpha,h_i) = \sigma_2\sinh\sigma_2 h_i + \nu\sigma_1\cosh\sigma_2 h_i \tag{36}$$

and $h = h_2 - h_1$, $\delta_1 = H - h_1$, $\delta_2 = H - h_2$. Eqs. (34) and (35) are the required Wiener equations and can be solved by the usual Wiener-Hopf procedure.

4. DETERMINATION OF THE WIENER-HOPF SOLUTION

Sato (1961) has fully described the factorization of $\frac{\sin\sigma_2 h}{\sigma_2 h}$ and $\frac{I(\alpha,h_2)}{I(\alpha,h_1)}$, a brief account of which is given in Appx.A. Eqs. (A4), (A5) and (A7) are

$$\frac{\sinh\sigma_2 h}{\sigma_2 h} = H_+(\alpha)\cdot H_-(\alpha) \tag{37a}$$

$$\frac{I(\alpha,h_2)}{I(\alpha,h_1)} = T_+(\alpha)\cdot T_-(\alpha) \tag{37b}$$

$$\frac{\sinh\sigma_2\delta_1}{\sigma_2\delta_1}\cdot\frac{\sigma_2\delta_2}{\sinh\sigma_2\delta_2} = Y_+(\alpha)\cdot Y_-(\alpha). \tag{37c}$$

By making use of Eqs. (37), Eqs. (34) and (35) can be rewritten as

$$\frac{1}{(\alpha^2-k_2^2)}\cdot\frac{T_+(\alpha)\,T_-(\alpha)}{H_+(\alpha)\,H_-(\alpha)}\left[V'_{2+}(\alpha,-h_1) + V'_{2-}(\alpha,-h_1)\right] - \left[V'_{4+}(\alpha,-h_2) + V'_{4-}(\alpha,-h_2)\right]$$

$$= h\left[V_{3-}(\alpha,-h_1) - V_{4-}(\alpha,-h_1)\right] \tag{38}$$

and

$$\frac{\left[V'_{2+}(\alpha,-h_1) + V'_{2-}(\alpha,-h_1)\right]}{(\alpha^2-k_2^2)\,H_+(\alpha)\,H_-(\alpha)} - \frac{Y_+(\alpha)\,Y_-(\alpha)\,\delta}{(\alpha^2-k_2^2)\,H_+(\alpha)\,H_-(\alpha)}\left[V'_{4+}(\alpha,-h_2) + V'_{4-}(\alpha,-h_2)\right]$$

$$= h\left[V_{3-}(\alpha,-h_2) - V_{4-}(\alpha,-h_2)\right], \tag{39}$$

where $\delta = \frac{\delta_1}{\delta_2}$.

Eq. (38) gives after rearrangement

$$\frac{T_+(\alpha)}{(\alpha+k_2)}\frac{V_2'(\alpha,-h_1)}{H(\alpha)} - \frac{V_{4+}'(\alpha,-h_2)}{(\alpha+k_2)H_+(\alpha)T_-(\alpha)} - \frac{V_{4-}'(\alpha,-h_2)}{(\alpha+k_2)H_+(\alpha)T_-(\alpha)}$$

$$= h\frac{H_-(\alpha)(\alpha-k_2)}{T_-(\alpha)}\left[V_{3-}(\alpha,-h_1) - V_{2-}(\alpha,-h_1)\right]. \tag{40}$$

Using the splitting technique of Noble (1958), we can write

$$\frac{V_{4+}'(\alpha,-h_2)}{(\alpha+k_2)H_+(\alpha)T_-(\alpha)} = N_+(\alpha) + N_-(\alpha) \tag{41}$$

and

$$\frac{1}{(\alpha+k_2)H_+(\alpha)T_-(\alpha)} = P_+(\alpha) + P_-(\alpha). \tag{42}$$

Using (30), (41) and (42), Eq. (40) can be written in the form

$$\frac{T_+(\alpha)V_2'(\alpha,h_1)}{(\alpha+k_2)H_+(\alpha)} + \frac{T_+(K_{IN})}{(K_{IN}+k_2)H_+(K_{IN})}\frac{iA}{\sqrt{2\pi}}\frac{\sigma_{2N}\sin\sigma_{2N}\delta_1}{(\alpha-K_{IN})}\exp(iK_{IN}x_0) +$$

$$\frac{iA}{\sqrt{2\pi}}\frac{\sigma_{2N}\sin\sigma_{2N}\delta_2}{(\alpha-K_{IN})}\exp(iK_{IN}x)\left\{P_+(\alpha) - P_+(K_{IN})\right\} - N_+(\alpha) =$$

$$N_-(\alpha) - \frac{iA}{\sqrt{2\pi}}\frac{\sigma_{2N}\sin\sigma_{2N}\delta_2}{(\alpha-K_{IN})}\exp(iK_{IN}x_0)\left\{P_-(\alpha) + P_+(K_{IN})\right\} +$$

$$\frac{T_+(K_{IN})}{(K_{IN}+k_2)H_+(K_{IN})}\frac{iA}{\sqrt{2\pi}}\frac{\sigma_{2N}\sin\sigma_{2N}\delta_1}{(\alpha-K_{IN})}\exp(iK_{IN}x_0). \tag{43}$$

We note that the left-hand side of Eq. (43) is analytic in the domain $\mathrm{Im}(\alpha + k_1) > 0$ and the right-hand side is analytic in the domain $\mathrm{Im}(\alpha - k_1) < 0$. Therefore, both the expressions define an entire function because of the strip common to both the domains. If both sides tend to zero as $\alpha \to \infty$ in appropriate half planes, then the entire function can be shown to be zero using Liouville's theorem. In order to ensure the uniqueness of the solution we have to specify

the edge conditions. We may assume the edge conditions $\frac{\partial v_2}{\partial z} \to$ constant $x^{-1/2}$ as $x \to 0_+$ on $z = -h_1$ and $z = -h_2$ (cf. Kazi, 1975). Hence $|V_{2+}'(\alpha, -h)| <$ constant $x|\alpha|^{-1/2}$ and both sides of Eq. (43) can be taken to be identically zero. Thus, equating the left-hand side of (39) to zero, gives

$$V_2'(\alpha,-h_1) = \frac{(\alpha+k_2)H_+(\alpha)}{T_+(\alpha)}\left[N_+(\alpha) - \frac{iA}{\sqrt{2\pi}}\frac{\sigma_{2N}\exp(iK_{IN}x_0)}{(\alpha-K_{IN})}\left\{\frac{T_+(K_{IN})\sin\sigma_{2N}\delta_1}{(K_{IN}+k_2)H_+(K_{IN})} - \right.\right.$$

$$\left.\left. - \sin\sigma_{2N}\delta_2\left(P_+(\alpha) - P_+(K_{IN})\right)\right\}\right]. \tag{44}$$

Placing Eq. (44) in Eq. (23), we obtain

$$V_2(\alpha,z) = -\frac{\sigma_2\cosh\sigma_2 z - \nu\sigma_1\sinh\sigma_2 z}{\sigma_2\sinh\sigma_2 h_1 + \nu\sigma_1\cosh\sigma_2 h_1}\frac{H_+(\alpha)}{T_+(\alpha)}\left(\frac{\alpha+k_2}{\alpha-k_2}\right)^{\frac{1}{2}}\left[N_+(\alpha) - \frac{iA}{\sqrt{2\pi}}\frac{\sigma_{2N}}{(\alpha-K_{IN})}\times\right.$$

$$\left.\exp(iK_{IN}x_0)\left\{\frac{T_+(K_{IN})\sin\sigma_{2N}\delta_1}{(K_{IN}+k_2)H_+(K_{IN})} - \sin\sigma_{2N}\delta_2\left[P_+(\alpha) - P_+(K_{IN})\right]\right\}\right]. \tag{45}$$

Similarly, starting from Eq. (39), we arrive at the following equation:

$$V_4'(\alpha,-h_2) = \frac{(\alpha+k_2)H_+(\alpha)}{Y_+(\alpha)}\left[Q_+(\alpha) - \frac{iA}{\sqrt{2\pi}}\frac{\sigma_{2N}\exp(iK_{IN}x_0)}{(\alpha-K_{IN})}\left\{\frac{Y_+(K_{IN})\quad\sin\sigma_{2N}\delta}{(K_{IN}+k_2)H_+(K_{IN})}\right.\right.$$

$$\left.\left. - \sin\sigma_{2N}\delta - \sin\sigma_{2N}\delta_1\left[W_+(\alpha) - W_+(K_{IN})\right]\right\}\right], \tag{46}$$

where we have used the following additive decompositions

$$\frac{V_{2+}'(\alpha,-h_1)}{\delta(\alpha+k_2)H_+(\alpha)Y_-(\alpha)} = Q_+(\alpha) + Q_-(\alpha) \tag{47}$$

and

$$\frac{1}{(\alpha+k_2)H_+(\alpha)Y_-(\alpha)} = W_+(\alpha) + W_-(\alpha). \tag{48}$$

In order to obtain $V_4(\alpha,z)$, we make use of Eq. (46) in Eq. (25). This gives

$$V_4(\alpha,z) = \frac{\cosh\sigma_2(z+H)}{\sinh\sigma_2\delta_2}\,\frac{H_+(\alpha)}{Y_+(\alpha)}\left(\frac{\alpha+k}{\alpha-k}\right)^{\frac{1}{2}}\left[O_+(\alpha) - \frac{iA}{\sqrt{2\pi}}\,\frac{\sigma_{2N}\exp(iK_{IN}x_0)}{(\alpha-K_{IN})}\right.$$

$$\left.\times\left\{\frac{Y_+(K_{IN})\sin\sigma_{2N}\delta_2}{(K_{IN}+k_2)H_+(K_{IN})} - \sin\sigma_{2N}\delta_1\left[W_+(\alpha)-W_+(K_{IN})\right]\right\}\right]. \tag{49}$$

Similar manipulations yield $V_3(\alpha,z)$ as

$$V_3(\alpha,z) = \frac{\cosh\sigma_2(z+H)}{\sinh\sigma_2 H}\cdot\frac{H_+(\alpha)}{T_+(\alpha)}\left(\frac{\alpha+k_2}{\alpha-k_2}\right)^{\frac{1}{2}}\left[N_+(\alpha) - \frac{iA}{\sqrt{2\pi}}\,\frac{\sigma_{2N}\exp(iK_{IN}x_0)}{(\alpha-K_{IN})}\times\right.$$

$$\left.\times\left\{\frac{T_+(K_{IN})\sin\sigma_{2N}\delta_1}{(K_{IN}+k_2)H_+(K_{IN})} - \sin\sigma_{2N}\delta_2\left[P_+(\alpha)-P_+(K_{IN})\right]\right\}\right] -$$

$$-\frac{\cosh\sigma_2(z+h_1)}{\sinh\sigma_2 h}\,\frac{H_+(\alpha)}{Y_+(\alpha)}\left(\frac{\alpha+k_2}{\alpha-k_2}\right)^{\frac{1}{2}}\left[O_+(\alpha) - \frac{iA}{\sqrt{2\pi}}\,\frac{\sigma_{2N}\exp(iK_{IN}x_0)}{(\alpha-K_{IN})}\times\right.$$

$$\left.\left\{\frac{Y_+(K_{IN})\sin\sigma_{2N}\delta_2}{(K_{IN}+k_2)H_+(K_{IN})} - \sin\sigma_{2N}\delta_1\left[W_+(\alpha)-W_+(K_{IN})\right]\right\}\right]. \tag{50}$$

Eqs. (45), (49) and (50) give the transmitted waves in the three regions of interest in the transformed α plane. The explicit representations of $N_\pm(\alpha)$, $P_\pm(\alpha)$, $O_\pm(\alpha)$ and $W_\pm(\alpha)$ are given in Appx.B.

5. THE TRANSMITTED WAVES

We determine the transmitted waves in the three regions that are formed by the half planes in the layer. This can be done by taking inverse transforms. We do this for each region separately.

a) The region $-h_1 \leq z \leq 0$; $x < 0$:

The Fourier inversion formula gives

$$v_2(x,z) = \frac{1}{\sqrt{2\pi}}\int_{c-i\infty}^{c+i\infty} \exp(-i\alpha x)\,V_2(\alpha,z)\,d\alpha, \tag{51}$$

where $V_2(\alpha,z)$ is given by (45) and $\mathrm{Im}(k_1) > c > -\mathrm{Im}(k_1)$. For $x < 0$; $-h_1 \leq z \leq 0$, we can close the contour in the upper half plane. Lapwood,(1949) showed that the contributions to the surface waves come through the poles. The branch point contributions give rise to body waves which are of no interest to us for the present study. The integrand in Eq. (51) has simple poles at $\alpha = K_{IN}$ and at the zeros of $I(\alpha,h_1)$ located in the upper half plane. Using the relations (2) and (3), the contribution $v_{2,1}$ from the pole at $\alpha = K_{IN}$ can be written as

$$v_{2,1}(x,z) = -A\exp\{-iK_{IN}(x-x_0)\}\cos\sigma_{2N}(z+H), \tag{52}$$

which cancels exactly the incident Love wave as is to be expected. Let K_{2m} ($m = 1,2,3,\ldots$) denote the zeros of $I(\alpha,h_1)$ in the upper half plane. Then using

$$\sigma_{1m} = (K_{2m}^2 - k_1^2)^{\frac{1}{2}},\quad \sigma_{2m} = (k_2^2 - K_{2m}^2)^{\frac{1}{2}},$$

we can write

$$v_{2,2} = \sqrt{2\pi}\,i\sum_{m=1}^{\infty}\frac{\cos\sigma_{2m}(z+H)\,K_{2m}}{\sigma_{2m}h_1\sin\sigma_{2m}h_1}\cos(\sigma_{2m}h_1)\left\{\frac{c_{2m}}{U_{2m}}-1\right\}\exp(-iK_{2m}x)$$

$$\times\left[\frac{K_{2m}+k_2}{K_{2m}-k_2}\right]^{\frac{1}{2}}\frac{H_+(K_{2m})}{T_+(K_{2m})}\left[N_+(K_{2m}) - \frac{iA}{\sqrt{2\pi}}\,\frac{\sigma_{2N}\exp(iK_{IN}x_0)}{(K_{2m}-K_{IN})}\times\right.$$

$$\left.\left\{\frac{T_+(K_{IN})\sin\sigma_{2N}\delta_1}{H_+(K_{IN})(K_{IN}+k_2)} - \sin\sigma_{2N}\delta_2\left[P_+(K_{2m})-P_+(K_{IN})\right]\right\}\right]. \tag{53}$$

In (53), we have used

$$\left.\frac{dI(\alpha,h_1)}{d\alpha}\right|_{\alpha=K_{2m}} = \frac{\sigma_{2m}^2 h_1}{K_{2m}\cos(\sigma_{2m}h_1)\left[\frac{c_{2m}}{U_{2m}}-1\right]}, \tag{54}$$

where $c_{2m} = \frac{\omega}{K_{2m}}$ is the phase velocity of the Love type waves of the m^{th} mode. It may be noted that (53) requires the relations $I(K_2m) = 0$ which is equivalent to

$$\tan \sigma_{2m} h_1 = \frac{\nu \, \sigma_{1m}}{\sigma_{2m}} . \tag{55}$$

Since (55) is the relation for the propagation of Love type waves in layered structure consisting of a semi-infinite solid of rigidity μ_1 covered by a surface layer of uniform thickness h_1 and rigidity μ_2, $v_{2,2}$ is therefore a Love wave propagating in the geometry as shown in Fig. 1.

b) The region $-h_2 \leqslant z \leqslant -h$; $x < 0$.

The transmitted wave $v_3(x,z)$ in this region is determined by applying inversion formula to Eq. (50). The contour of integration in (46) is closed in the upper half plane. The integrand has simple poles at $\alpha = K_{IN}$ and at the zeros of $\sin h^2\sigma_2 h$ lying in the upper half plane. The contribution $v_{3,1}(x,z)$ arising from the pole at $\alpha = K_{IN}$ cancels the incident wave v_2^{inc} in this region. The contribution from the poles at $\alpha = ip_n$, where $p_n = \left[\frac{n^2\pi^2}{h^2} - k^2\right]^{1/2}$, denoted by $v_{3,2}$, is given by

$$v_{3,2} = \sqrt{2\pi}\, i \sum_{n=1}^{\infty} \exp(p_n x) \left[\frac{ip_n + k_2}{ip_n - k_2}\right]^{\frac{1}{2}} H_+(ip_n) \Bigg[\frac{\cos \frac{n\pi}{h}(z+H)}{T_+(ip_n)} \Big[N_+(ip_n)$$

$$- \frac{iA}{\sqrt{2\pi}} \frac{\sigma_{2N} \exp(iK_{IN}x_0)}{ip_n - K_{IN}} \left\{ \frac{T_+(K_{IN}) \sin(\sigma_{2N}\delta_1)}{(K_{IN}+k_2) H_+(K_{IN})} - \sin\sigma_{2N}\delta_2 \left(P_+(ip_n) - P_+(K_{IN})\right)\right\}\Big]$$

$$- \frac{\cos \frac{n\pi}{h}(z+h_1)}{Y_+(ip_n)} \Bigg[Q_+ - \frac{iA}{\sqrt{2\pi}} \frac{\sigma_{2N} \exp(iK_{IN}x_0)}{(ip_n - K_{IN})} \times \left\{ \frac{Y_+(K_{IN}) \sin(\sigma_{2N}\delta_2)}{(K_{IN}+k_2) H_+(K_{IN})} \right.$$

$$\left. - \sin(\sigma_{2N}\delta)\left(W_+(ip_n) - W_+(K_{IN})\right)\right\}\Bigg]\Bigg]^{*} \tag{56}$$

Note that in deriving Eq. (56), we have used the relation $\sin h\sigma_2 h = 0$, which is the dispersion relation for waves in an infinite strip of uniform thickness δ and rigidity μ_2 with weak upper and lower surfaces at $z = -h_2$ and $z = -h_1$ respectively.

c) The region $-H \leqslant z \leqslant -h_2$; $x < 0$.

To find the transmitted waves in this range, we apply inversion formula to Eq. (49). Closing the contour of integrations in (49) in the upper half plane, we find that the integrand has simple poles at $\alpha = K_{IN}$ and at zeros of $\sin h\sigma_2\delta_2$ that lie in the upper half plane. The contribution, $v_{4,1}$, from $\alpha = K_{IN}$ exactly cancels the incident wave v_2^{inc} in this region. The poles arising from the zeros of $\sin h\sigma_2\delta_2$ are $\alpha = ip_n' = \left\{k_2^2 - \frac{n^2\pi^2}{\delta^2}\right\}^{1/2}$, $n = 1,2,3,\ldots$

These poles give rise to the contribution

$$v_{4,2} = \sqrt{2\pi}\, i \sum_{n=0}^{\infty} \exp(p_n' x) \cos \frac{n\pi}{\delta_2}(z+H) \left(\frac{ip_n' + k_2}{ip_n' - k_2}\right)^{\frac{1}{2}} \frac{H_+(ip_n')}{Y_+(ip_n')} \times$$

$$\left[Q_+(ip_n') - \frac{iA}{\sqrt{2\pi}} \frac{\sigma_{2N} \exp(iK_{IN}x_0)}{(ip_n' - K_{IN})} \left\{ \frac{Y_+(K_{IN}) \sin \sigma_{2N}\delta_2}{(K_{IN}+k_2) H_+(K_{IN})} - \sin\sigma_{2N}\delta_1 \left\{W_+(ip_n') - W_+(K_{IN})\right\}\right\}\right].$$

The dispersion equation satisfied by $v_{4,2}$ corresponds to the relation $\sin h\sigma_2\delta_2 = 0$, that is, $\alpha = \left[k_2^2 - \frac{n^2\pi^2}{\delta^2}\right]^{1/2}$, which is the dispersion relation for waves in an infinite strip of uniform thickness δ_2 and rigidity μ_2 with free upper surface and free/weak lower surface respectively.

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APPENDIX A

The factorization of the functions, involved in Eq. (34) and (35), has been fully described by Sato, 1961. We only quote the results here.

a) Let us write

$$\frac{\sinh \sigma_2 \delta_2}{\sigma_2 \delta} = \prod_{n=1}^{\infty} \left\{ \bar{p}_n^2 \bar{\delta}_n^{-2} + \alpha^2 \bar{\delta}_n^{-2} \right\} = \bar{H}(\alpha) \tag{A1}$$

where

$$\bar{p}_n \bar{\delta}_n = \left[1 - k_2^2 \bar{\delta}_n^2\right]^{\frac{1}{2}} = i\left(k_2^2 \bar{\delta}_n^{-2} - 1\right), \quad \bar{\delta}_n = \frac{\delta_n}{n\pi} \tag{A2}$$

Also $\bar{H}(\alpha) = \bar{H}_+ \cdot \bar{H}_-(\alpha)$, where

$$H_\pm(\alpha) = \prod_{n=1}^{\infty} \left\{ \bar{p}_n \bar{\delta}_n \mp i \alpha \bar{\delta}_n \right\} \exp\left\{ \mp i \alpha \bar{\delta}_n + \bar{\chi}(\alpha) \right\}. \tag{A3}$$

If $\bar{\chi}(\alpha) = -\left[i\alpha \frac{\delta}{\pi}\right]\left[1 - c - \log \frac{\alpha\delta}{\pi}\right] + \frac{\alpha\delta}{2}$, then $\bar{H}_\pm(\alpha) \sim |\alpha|^{-1/2}$ as $|\alpha| \to \infty$ in appropriate half planes. Hence

$$\frac{\sinh \sigma_2 \delta}{\sigma_2 \delta} = \bar{H}_+(\alpha) \cdot \bar{H}_-(\alpha). \tag{A4}$$

b) If $\pm\hat{K}_{1m}$ and $\pm\hat{K}_{2m}$ $(m = 1,2,\ldots)$ denote the zeros of $I(\alpha,h_2)$ and $I(\alpha,h_1)$ respectively, we can write

$$\frac{I(\alpha,h_2)}{I(\alpha,h_1)} = \prod_{m=1}^{\infty} \left(\frac{\alpha^2 - \hat{K}_{1m}^2}{\alpha^2 - \hat{K}_{2m}^2} \right) \frac{G_1(\alpha)}{G_2(\alpha)} .$$

where

$$G_1(\alpha) = I(\alpha,h_2) \Big/ \prod_{m=1}^{\infty} (\alpha^2 - \hat{K}_{1m}^2),$$

$$G_2(\alpha) = I(\alpha,h_1) \Big/ \prod_{m=1}^{\infty} (\alpha^2 - \hat{K}_{2m}^2),$$

and $G_{1,2}(\alpha)$ has no zeros.

Let

$$Q(\alpha) = \frac{G_1(\alpha)}{G_2(\alpha)} = Q_+(\alpha) \cdot Q_-(\alpha),$$

then

$$\log Q_+(\alpha) = \frac{1}{2\pi i} \int_\Gamma \log \frac{Q_+(\omega)}{(\omega - \alpha)} d\omega \quad ; \quad Q_-(\alpha) = Q_+(-\alpha),$$

$$\frac{I(\alpha,h_2)}{I(\alpha,h_1)} = \prod_{m=1}^{\infty} \left(\frac{\alpha^2 - \hat{K}_{1m}^2}{\alpha^2 - \hat{K}_{2m}^2} \right) Q_+(\alpha) \cdot Q_-(\alpha) = T_+(\alpha)\, T_-(\alpha), \tag{A5}$$

where

$$T_\pm(\alpha) = Q_\pm(\alpha) \prod_{m=1}^{\infty} \left(\frac{\alpha + \hat{K}_{1m}}{\alpha + \hat{K}_{2m}} \right). \tag{A6}$$

c) The parallel calculations from (A1) to (A4) for $\frac{\sin h\sigma_2\delta_1}{\sigma_2\delta_1}$ and $\frac{\sin h\sigma_2\delta_2}{\sigma_2\delta_2}$ lead to

$$\frac{\sinh \sigma_2 \delta_1}{\sigma_2 \delta_1} \cdot \frac{\sigma_2 \delta_2}{\sinh \sigma_2 \delta_2} = Y_+(\alpha) \cdot Y_-(\alpha), \tag{A7}$$

where

$$\frac{\sinh \sigma_2 \delta_1}{\sigma_2 \delta_1} = Y_{1+}(\alpha) \cdot Y_{1-}(\alpha)$$

$$\frac{\sinh \sigma_2 \delta_2}{\sigma_2 \delta_2} = Y_{2+}(\alpha) \cdot Y_{2-}(\alpha)$$

and

$$Y_\pm(\alpha) = \frac{Y_{1\pm}(\alpha)}{Y_{2\pm}(\alpha)} .$$

APPENDIX B

Following the general decomposition theorem (Noble, 1958), the explicit representations of $N_\pm(\alpha)$, $P_\pm(\alpha)$, $O_\pm(\alpha)$ and $W_\pm(\alpha)$ are given by

$$N_\pm(\alpha) = \pm\frac{1}{2\pi i}\int_{\substack{ic-\infty \\ id-\infty}}^{\substack{id+\infty \\ ic+\infty}} \frac{V'_{4+}(\xi,-k_2)\,d\xi}{(\xi+k_2)H_+(\xi)T_-(\xi)(\xi-\alpha)} \tag{B1}$$

$$P_\pm(\alpha) = \pm\frac{1}{2\pi i}\int_{\substack{ic-\infty \\ id-\infty}}^{\substack{id+\infty \\ ic+\infty}} \frac{d\xi}{(\xi+k_2)H_+(\xi)T_-(\xi)(\xi-\alpha)} \tag{B2}$$

$$O_\pm(\alpha) = \pm\frac{1}{2\pi i}\int_{\substack{ic-\infty \\ id-\infty}}^{\substack{id+\infty \\ ic+\infty}} \frac{V'_{2}(\xi,-k_2)\,d\xi}{\delta(\xi+k_2)H_+(\xi)Y_-(\xi)(\xi-\alpha)} \tag{B3}$$

$$W_\pm(\alpha) = \pm\frac{1}{2\pi i}\int_{\substack{ic-\infty \\ id-\infty}}^{\substack{id+\infty \\ ic+\infty}} \frac{d\xi}{\delta(\xi+k_2)H_+(\xi)Y_-(\xi)(\xi-\alpha)}, \tag{B4}$$

where $-\mathrm{Im}(k_1) < c < \mathrm{Im}(\alpha) < d < \mathrm{Im}(k_1)$.

The integrals in (B1), (B2), (B3) and (B4) can be calculated by the contour integration method. Let us consider

$$N_+(\alpha) = \frac{1}{2\pi i}\int_{ic-\infty}^{ic+\infty} \frac{V'_{4+}(\xi)\,d\xi}{(\xi+k_2)H_+(\xi)(\xi-\alpha)T_-(\xi)} \tag{B5}$$

For α lying in $(-\eta, \eta)$, we can write $\alpha = -\eta\cos\theta$, where $|\theta| \leq \pi$. Thus, Eq. (B5) can be written as

$$N_+(-\eta\cos\theta) = \frac{1}{2\pi i}\int_{ic-\infty}^{ic+\infty} \frac{V'_{4+}(\xi,-k_2)}{(\xi+k_2)H_+(\xi)T_-(\xi)(\xi+\eta\cos\theta)}, \tag{B6}$$

where $T_-(\xi) = Q_-(\xi)\,\pi_{m=1}^{\infty}\left[\frac{\xi-\hat{K}_{1m}}{\xi-\hat{K}_{2m}}\right]$, and $Q_-(\xi)$ has no zeros. Putting the value of $T_-(\xi)$ in (B6), we obtain

$$N_+(-\eta\cos\theta) = \frac{1}{2\pi i}\int_{ic-\infty}^{ic+\infty} \frac{V'_{4+}(\xi,-k_2)\prod_{m=1}^{\infty}(\xi-\hat{K}_{2m})}{(\xi+k_2)H_+(\xi)Q_-(\xi)\prod_{m=1}^{\infty}(\xi-\hat{K}_{1m})(\xi+\eta\cos\theta)} \tag{B7}$$

Closing the line of integration by a semi-circle in the upper half plane the poles captured are $\xi = -\eta\cos\theta$, $\xi = \hat{K}_{1m}$ $(m = 1,2,\ldots)$. Thus

$$N_+(-\eta\cos\theta) = \sum_{l=1}^{\infty} \frac{V'_{4+}(K_{1l}-k_2)}{(\hat{K}_{1l}+k_2)H_+(\hat{K}_{1l})Q_+(\hat{K}_{1l})}\cdot\frac{\prod_{m=1}^{\infty}(\hat{K}_{1l}-\hat{K}_{2m})}{\prod_{m=1}^{\infty}{}'(\hat{K}_{1l}-\hat{K}_{1m})} + \frac{V'_{4+}(-\eta\cos\theta,-k_2)}{(k_2-\eta\cos\theta)H_+(-\eta\cos\theta)T_-(-\eta\cos\theta)} \tag{B8}$$

The other integrals can be evaluated similarly.

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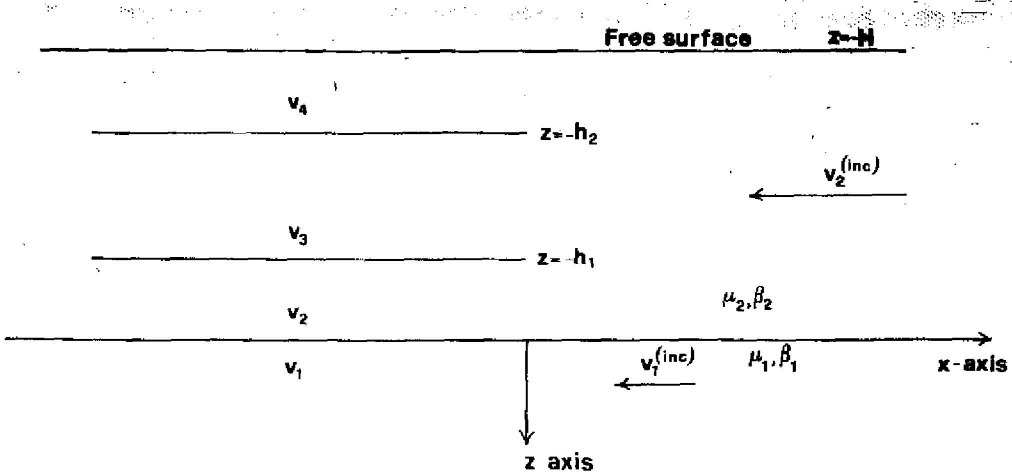

Fig. 1