IC/85/228

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

1985 MIRAMARE-TRIESTE

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International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization
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ABSTRACT

A multiparticle theory of the Integral Quantum Hall Effect (IQHE) was constructed operating with pairs wave function as an order parameter. The IQHE is described with bosonic macroscopic states while the fractional QHE with fermionic ones. The calculation of the critical current and Hall conductivity temperature dependence is presented.

MIRAMARE – TRIESTE
November 1985

* To be submitted for publication.

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1. INTRODUCTION

Remarkable progress has been achieved recently in the theory of Integral Quantum Hall Effect [1]-[3] (IQHE). In an early stage the Laughlin theorem [4] explains the exact integral quantization of the resistance $R = \frac{h}{e^2} \cdot \frac{1}{\nu}$, $\nu = 1,2,3...$ The role of the weak disorder was discussed by Halperin [5] in terms of the Laughlin theorem. A more consistent theory taking into account the localization was basically constructed by Levine, Libby and Pruisken [6],[7]. These authors discovered an important third order term in the field theory of localization appearing in magnetic field. Taking into account the topological contributions arising from different configurations conductivity trajectories were constructed in the plane σ_{xx}, σ_{xy} with exact quantized values [8],[9]. The theory was extended recently for the Fractional Quantum Hall Effect [10] (FQHE). Though the theory is in rather good shape it does not account for the critical currents observed [11],[12], [13] and the a.c. field frequency dependence [14]-[16] is unclear. Another theory [17] describing FQHE was developed by introducing multi-electron wave functions as order parameters for each fractional quantum state [18] of the 2-D electronic liquid. A field theory accounting for the disorder and localization was constructed [19] recently extending the multi-electron scheme to the critical region of strong disorder.

Here we discuss the IQHE in relation to this multi-electron order parameter theory. In such a framework the IQHE is realized by carriers pairing and the two particles wave function plays the role of order parameter very similar to BCS theory. The fractional quantum states then are fermionic while the integer filling corresponds to a bosonic type of macroscopic quantum states. This marginal difference of the FQHE and IQHE accounts for ω-dependence and critical current

2. EXCITATIONS SPECTRUM, ORDER PARAMETER

Here will be discussed only the lowest Landau subbands corresponding to the spin-up and spin-down states of $n = 0$ inversion layer. According to a theorem which was proved [17],[20] in magnetic field there is a specific degeneracy of the single particle states persistent to any degree and type of disorder. Let the total number of states N_g defined as ϕ/ϕ_0, where $\phi = BS$ is the magnetic flux and $\phi_0 = h/ec$ is the flux quantum, be of the type

$$N_g = k.N$$

k being a small integer. Then the states of number N_g form a band which remains k-times degernerate, whatever potential is applied to the system with accuracy $\sqrt{N}$. In the FQHE the odd numbers k play a role [17]-[20], which leads to fermionic macroscopic states.

Here even numbers k will be considered or more precisely k = 2 as the simplest case. In this case the total number of states N_g is

$$N_g = 2N$$

and all of them are twice degenerate. Converting the initial set of eigenstates to equivalent 2-dimensional $\vec{k}$ representation [20] one sees that the states with momentum $\vec{k}$ and $-\vec{k}$ are degenerate having the same energy $\varepsilon(k)$. Since the single particle potential fails to lift this degeneracy the interaction should be introduced in order to remove it. As is well known from the BCS theory the model Hamiltonian

$$H = -|g| \int \bar{\Psi}_\alpha \bar{\Psi}_\beta \Psi_\beta \Psi_\alpha \tag{1}$$

provides effective scattering between the k and -k states and lead to electron pairing. From the single particle states $\psi_k(\tau)$ the symmetric and antisymmetric two particle states can be constructed as follows

$$\Psi_k^{\pm}(1,2) = \frac{1}{\sqrt{2}}\left[\Psi_k(1)\Psi_{-k}(2) \pm \Psi_k(2)\Psi_{-k}(1)\right] \tag{2}$$

These two particle states are non-degenerate since the spin is fixed by the magnetic field and only the set $\psi_k^-(1,2)$ can be realized in the lowest $\sigma = \frac{1}{2}$ Landau subband. Thus the two-particle states $\psi_k^-(1,2)$ and $\psi_k^+(1,2)$ are separated by the spin gap

$$2\Delta_0 = g\mu_B B \tag{3}$$

Therefore in the vicinity of the integral filling number $\nu = 1$ the double degeneracy is removed by pairing-the electrons free the states $\psi_k^-(1,2)$. These states are of number N and contain two electrons each so that the total number of electrons is equal to the N_g - the Landau level degeneracy. In other words the even number k = 2 corresponds to integral filling number $\nu = 1$ and the spin-gap is the one that separates the nearest electronic states.

We do not discuss here the origin of the electron-electron attraction and restrict ourselves to the model (1). The nearest even number of K = 4 will be discussed elsewhere. Here we only mention that it corresponds to $\nu = \frac{1}{2}$ and the spin-gap (3) is no longer present.

The BCS model (1) in the presence of spin gap leads to triplet pairing, but only one component of the triplet is important - the antisymmetric two-particles function $\psi^-(r_1,r_2)$. With this simplification the BCS equations are the same with one single difference - the energy Δ_0 enters the BCS equations as an addition to the chemical potential μ. This difference is important, however, because the cut-off on both sides of the Fermi surface $\varepsilon_{(K)} = \mu$ is symmetric $(\mu \pm \omega_0)$. Solving the BCS equations one has a single particle excitation spectrum with a gap Δ_{tot}

$$\Delta_{tot} = \sqrt{\Delta_0^2 + \Delta^2} \tag{4}$$

Here the order parameter Δ as found from the BCS equation at T = 0 is

$$\Delta = \frac{|\Delta_0^2 - \omega_0^2|}{\sqrt{\Delta_0 \omega_0}} \sqrt{2\left(\frac{1}{\eta_c} - \frac{1}{\eta}\right)} \qquad \eta > \eta_c \tag{5}$$

The BCS parameter is $\eta = |g|\rho_F$ and the critical value η_c is defined as

$$\frac{2}{\eta_c} = \ln\left|\frac{\Delta_0 + \omega_0}{\Delta_0 - \omega_0}\right| \tag{6}$$

Below η_c no pairing exists and the order parameter Δ vanishes, Therefore the condition

$$\eta_c < |g|\rho_F \tag{7}$$

is vital for IQHE to appear. It is reasonable to assume the cut-off energy ω_0 to be a linear function of the magnetic field B

$$\omega_0 = \bar{\omega}\left(\frac{B}{B_0}\right)$$

In such a case the value n_c is a constant and the condition (7) defines a lower bound for the interaction strength g_c. In the case of V_g = const (magnetic field control) the density of states ρ_F is roughly proportional to the magnetic field. Therefore the condition (7) is easily reversed in strong magnetic field. One can conclude that the relative strength of the interaction $|g_c|$ creating a new phase with pairs concentration $n_c \neq 0$ is lower at high magnetic fields.

For $T \neq 0$ the gap vanishes at temperature T_c defined by the condition $\Delta(T_c) = 0$ with solution

$$T_c = \frac{\Delta_0}{\ln\left(\frac{2\eta}{\eta - \eta_c}\right)} \qquad (8)$$

Due to the magnetic field dependence of n, T_c is not exactly a linear function of B but is very close to it. In the vicinity of T_c the temperature dependence of Δ is determined by the mean field exponent (1/2)

$$\Delta \sim \sqrt{T_c - T} \qquad (9)$$

3. CRITICAL CURRENT

Once the BCS order parameter Δ is introduced, the Ginsburg-Landau free energy is derived in the usual form

$$2f = -\alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left|\left(-i\hbar\nabla + \frac{e^*}{c}\vec{A}\right)\psi\right|^2 \qquad (10)$$

where $\psi(\tau) = \Delta(\tau)$ is the two particles wave function. From (10) the Ginsburg-Landau equation is found by minimizing f in respect to ψ and the current density has the usual form

$$j(r) = -\frac{i\hbar}{2m}\left(\psi^*\nabla\psi - c.c.\right) \qquad (11)$$

This equation will be employed here in order to calculate the critical current destructing the ordered state with macroscopic wave function $\psi(\tau)$. One can write the longitudinal current j_x in the form valid only in the vicinity of

the transition to normal state

$$j_x = e n_0 V_d \qquad (12)$$

Here n_0 is electrons concentration. Let us consider an infinite sample so that the order parameter $\psi(\tau)$ is of the type

$$|\psi| = const \; , \quad \psi = c\, e^{i\phi(r)} \qquad (13)$$

In order to determine the phase $\phi(\tau)$ let us transform ψ to the reference frame moving with velocity V_d parallel to the applied electric field ε. In this frame the longitudinal current (12) vanishes and the phase $\phi(\tau)$ of $\psi(\tau)$ is fixed by the condition

$$(\nabla\phi)_x = 0 \qquad (14)$$

Returning to the actual reference frame by Galilean transformation of the type

$$\psi_{J_x} = \psi_{J_x=0}\, e^{i\frac{m\vec{V}_d\cdot\vec{r}}{\hbar}} \qquad (15)$$

one reaches the following expression for the free energy:

$$2f = -\alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{m^* V_d^2}{2}|\psi|^2 \qquad (16)$$

By minimization of f the modulus $|\psi|$ is now found to be

$$|\psi|^2 = |\psi_0|^2\left(1 - \frac{mV_d^2}{2\alpha}\right) = |\psi_0|^2\left(1 - \frac{J_x^2}{J_c^2}\right) \qquad (17)$$

From (17) is seen that the order parameter vanishes for $J_x = J_c$ and the ordered state is destroyed. This result is very important. The system is in the regime of fixed temperature, magnetic field and gate voltage. In these conditions the same sample displays quantized behaviour (below J_c) and normal one (above J_c). The working explanation [3] is that the current J produces Joule heating of the sample at the contacts. This explanation is unsatisfactory since the change of the current by less than 1% results in a jump of the potential

V_x by a factor of 10^4. This certainly is not typical for thermal effects, not to mention that the high resistive state $\sigma_{xx} = 0$ is maintained again when the current J_x is reduced back to small values. Another important observation is that the normalized critical current j^{cr}/j^{cr}_{max} depends on the magnetic field in the same way as the normalized activation energy found by Ebert et al.[11] (Fig. 1). Our expression for the critical current J_c entering (17) is the following

$$J_c = e|\Psi_0| n_0 \sqrt{\frac{2\beta}{m^*}} \tag{18}$$

It is therefore clear that the critical current is proportional to the gap in the spectrum $\Delta = |\psi_0|$. It depends on the magnetic field and in the vicinity of the transition close to $\nu = \nu_c$ this dependence is of the mean field type

$$|\Psi_0| \sim \sqrt{|\nu - \nu_c|} \tag{19}$$

Here ν_c is the value of the magnetic field for which the order parameter vanishes

$$\Delta(\nu_c) = 0$$

The critical current depends also on the temperature T through $\Delta(\tau)$ according to (g) but this dependence is not an exponential one as seen from the data of Ebert et al. displayed in Fig. 2. The broad band nonthermal noise observed by Cage et al. [12] and the current enhancement of the plateau region in a.c. conditions observed by Pepper et al. [3] will be discussed elsewhere. Next we calculate the temperature dependence of the Hall current.

4. HALL CURRENT

In this section the temperature is assumed to be much lower than the order parameter $|\psi_0| >> T$. In these conditions for $\nu = \nu_c$ the system is in coherent state and the normal electrons concentration $\tilde{n}$ is exponentially small

$$\tilde{n} \sim e^{-\frac{\Delta_{tot}}{T}}$$

In order to calculate the Hall current from the current definition (11) it is necessary [17]-[20] to determine the phase $\phi(\tau)$ of the order parameter since its magnitude is found from the free energy (10) minimization. For this reason we perform a Galilean transformation to the reference frame, where the electric field ε is zero. In this frame the total current is also zero and in this way the phase $\phi(\tau)$ is fixed by the condition

$$\left(-i\hbar\vec{\nabla} + \frac{e^*}{c}\vec{A}\right)\phi(x) = 0 \tag{20}$$

In the laboratory reference frame the macroscopic wave function is ψ_ε

$$\Psi_\varepsilon = \Psi_{\varepsilon=0}\; e^{\frac{i m^* \vec{v}\cdot\vec{r}}{\hbar}} \quad , \quad |v| = c\,\frac{\varepsilon}{B} \tag{21}$$

Substituting this solution into the current definition one has exact quantization of the Hall current at $T = 0$

$$j_y = e^* |\Psi_0|^2 c\,\frac{\mathcal{E}_x}{B} \tag{22}$$

In this equation $|\psi_0|^2$ represent the concentration of the coherent electrons n_c which coincide with the total number of electrons in the Landau subband at $T = 0$. At finite temperatures, however, a fraction $\tilde{n}$ of the electrons are thermally excited and the Hall conductivity σ_{xy} takes the form

$$\sigma_{xy} = \frac{e^2}{h}\,\nu_c(T) \quad , \quad \nu_c(T) = 1 - \tilde{\nu}(T) \tag{23}$$

The filling number $\tilde{\nu}(T)$ correspond to the thermally excited electrons concentration $\tilde{n}(T)$. It depends exponentially on the temperature ($T << |\psi_0|$)

$$\tilde{\nu}(T) \sim e^{-\frac{\Delta_{tot}}{2T}} \tag{24}$$

The experimental observations of Paalanen et al. [22] demonstrate activated behaviour for the $\rho_{xy} - \frac{h}{e^2}\frac{1}{n}$ and ρ_{xx} at integral fillings ($\nu = 8$) and as well at fractional filling numbers in low mobility samples. Similar observations were reported for fractional filling numbers by Ebert et al. [23] for very high mobility samples both for GaAs/$Al_xGa_{1-x}As$. In fact deviations from activated behaviour were seen [22],[24]. In respect to this situation it may be stressed that the expression (22) temperature dependence is determined by the order parameters temperature dependence, which is exponential only at very low temperatures. In writing (22) the effect of disorder is not included. It is however included in the field theory already constructed [20].

5. CONCLUSIONS

Summarizing the results of the present work we should mention that the IQHE was included in the multiparticle theory of the FQHE [17]. As a result the IQHE is described by bosonic states with pairs wave function as order parameter while the FQHE is due to fermionic states of odd number particles. The order parameter permits calculation of the critical current in the FQHE (18) and the Hall conductivity temperature dependence (22)-(24). In this framework the theory can account for the a.c.field frequency dependence and the size effects [25]. Together with the field theory [20] accounting for the disorder the multiparticle description represents a possible complete scheme of the Hall effect.

ACKNOWLEDGMENTS

The author is indebted to Professor M.P. Tosi for critical reading of the manuscript. He would also like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

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FIGURE CAPTIONS

Fig. 1 Normalized critical current (crosses) and activation energy (circles) magnetic field dependence from the data of Ebert et al. [11].

Fig. 2 Current-voltage characteristics for different temperatures T_L from the data of Ebert et al. [11].

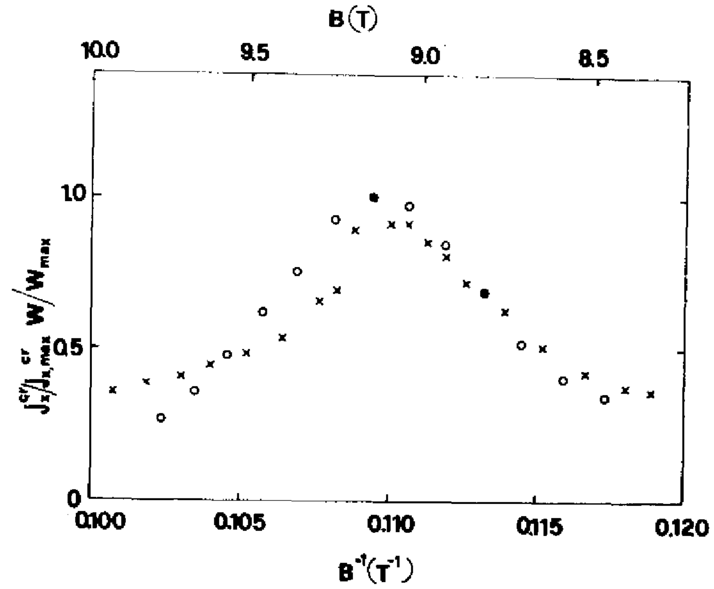
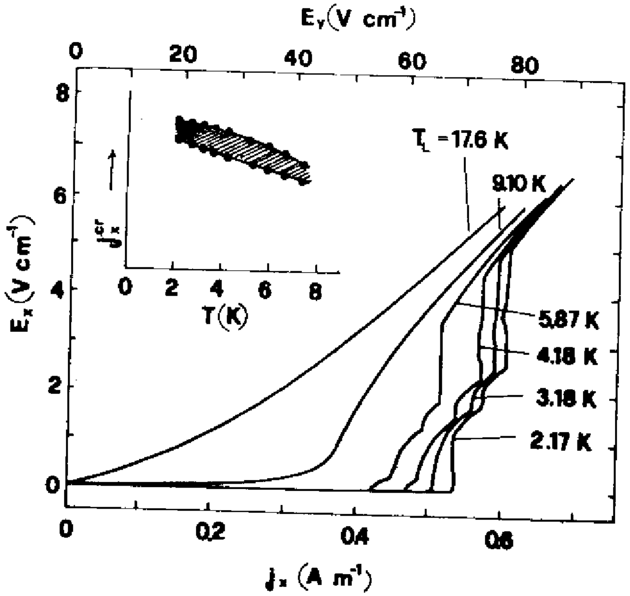


Fig.1



E_Y (V cm^{-1})
0
20
40
60
80
T_L = 17.6 K
9.10 K
5.87 K
4.18 K
3.18 K
2.17 K
E_x (V cm^{-1})
8
6
4
2
0
0
0.2
0.4
0.6
j_x (A m^{-1})
j_x^{cr}
T(K)

Fig.2