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ABSTRACT

As was first pointed out by Witten, large number of colours (N_c) leads to a simplification in the theory of baryon masses in that the quarks may be assumed to move in a mean field which can be found self-consistently. The interquark potential in such a description can be borrowed form the meson sector phenomenology in the absence of an accurate evaluation of it from large N_c quantum chromodynamics (QCD). We have carried out this program with such a potential due to Richardson, used often by workers in the meson sector. This potential has the advantage of incorporating the two main features of QCD, namely confinement and asymptotic freedom. In view of the small number of parameters involved, the results agree surprisingly well with experiment for the case of three identical quarks.

1. INTRODUCTION

It is now generally believed that QCD is the correct theory of strong interactions. In spite of great progress in the understanding of such a theory, it is as yet difficult to compute the mass spectrum of the observed hadrons from QCD, and approximations have to be made. Of course we have in mind continuum QCD calculations: it is quite conceivable that the steadily improving lattice calculations will yield the correct baryon properties. An attempt is made in this paper to calculate baryon properties, keeping as close to QCD as present day techniques permit.

Although QCD is a theory for which the number of colors N_c is equal to three only, the existence of mesons as a large number of narrow resonances and properties like Zweig's rule can be understood readily when one is willing to think about the whole class of QCD-like theories of arbitrary N_c. This was first suggested by t'Hooft [1], almost as soon as QCD was formulated, and became an extremely powerful tool for understanding baryons after the work of Witten [2]. To leading order, QCD is equivalent to a classical field theory with a coupling constant proportional to $1/N_c$ for any N_c. It is virtually impossible, however, to discover or prove this equivalence if one restricts oneself to the $N_c = 3$ case. This $1/N_c$ expansion does give, to leading order, most of the qualitative features expected of low-energy QCD, such as confinement [2] and chiral symmetry breaking [3]. For mesons it leads to a Bethe-Salpeter equation with a $q\bar{q}$ potential, while for baryons it yields a Hartree-Fock-like equation. The alternative for baryons is to solve classical meson

Lagrangians, as in the Skyrme model [4], for instance, where the valence quark degrees of freedom are completely suppressed. But it is difficult to deduce these meson field models from QCD without encountering terms of sixth or higher order[5] in the isospin current of the pions: one is then faced with the problem of justifying the fourth order truncation of the Skyrme model. Another difficulty connected with this model is the fact that important low-mass mesons, like the ρ-meson, are difficult to incorporate into the model[6]. At this stage, therefore, relativistic HF with a potential looks like a very promising alternative for describing baryons[7] the potential being chosen so as to fit the meson sector properties. Summing all planar gluon diagrams of QCD would yield this potential unambiguously. Since this is unfortunately beyond the present-day capabilities, the best alternative is to borrow a potential devised for meson sector calculations. As it turns out, the situation is quite satisfactory here, for one has the Richardson potential[8], which in spite of its simplicity is able to provide many of the major features of the QCD interaction with only one free parameter. It incorporates the correct renormalization group behaviour at high q^2. At small q^2 it has the correct q^{-4} behaviour predicted by the Schwinger-Dyson equations of QCD, as shown by Mandelstam and Baker et al.[9], and pointed out from different considerations by Nair and Rosenzweig[10]. Note that although this q^{-4} behaviour is not directly based on a $1/N_c$ expansion, the fact that one has confinement to leading order in $1/N_c$ implies that the $q\bar{q}$ potential, in such an expansion, should have a q^{-4} behaviour for low q^2. The Richardson potential fits the charmonium data, and additional cross-checks are

possible on its only free parameter Λ, which is related to the string tension and hence to other parameters like the critical temperature T_c through lattice gauge calculations, as discussed recently by Bailin et al [11].

In the spirit of remaining close to the first principles of QCD, we have not, in our calculation, allowed the quark mass m_q, to be a free parameter; we have rather fixed it at the current algebra value[12]. With m_s = 150 MeV for the strange quark, we find very good agreement with the Ω^- energy for a reasonable value of the parameter Λ in Richardson's potential. There is some uncertainty in the current quark mass values. Since, however, the kinetic energy decreases as the quark mass increases, the result in fact is not very sensitive to small changes in the value of m_q. On the other hand, if one chooses m_q of the order of 10 MeV which is appropriate for u, d quarks, there are problems of confinement connected with the Lorentz vector nature of the effective potential as developed in sec. II. These problems can however be alleviated by splitting the confining part of the interaction into a scalar and a vector part, and this prescription allows one to deal with light quark systems quite successfully.

In Sec. II we derive the relativistic HF equations and we describe the method that allows to subtract the spurious center-of-mass kinetic energy. Throughout the paper we limit our considerations, for the sake of simplicity, to systems formed of three quarks of a single flavor in the same orbital wave function. Numerical results are presented in Sec. III. In particular, we study the confining properties of the various potentials.

II. GENERAL FORMALISM

The action for a system of interacting quarks and gluons can be written as

$$S = S_{gluon} + \int [\bar{q}(i\not{\partial} - m)q + j^a_\mu A^{a\mu}]\, d^4x, \tag{2.1}$$

where S_{gluon} denotes, collectively, the action of the free gluonic fields $A^{a\mu}$, the gauge fixing terms and the action of the ghost fields, while j^a_μ is the quark current

$$j^a_\mu = \frac{g}{2}\, \bar{q}\, \gamma_\mu \lambda^a q\ , \tag{2.2}$$

where the $\lambda^{a'}$s are the Gell-Mann matrices. The connected Green functions of gluons are generated by the functional

$$e^{iW(j)} = \int e^{i[S_{gluon} + \int j^a_\mu A^{a\mu} d^4x]}\, d\mu[A], \tag{2.3}$$

where the volume element in path space, $d\mu\,[A]$, includes the ghost fields. The full generating functional is given by

$$Z = \int e^{iS}\, d\mu[A]\, [d\bar{q}\, dq]\ . \tag{2.4}$$

Using (2.3), one can integrate formally over the gluons and the ghosts, and write

$$\begin{aligned} Z &= \int e^{i[\int \bar{q}(i\not{\partial} - m)q\, d^4x \; + W(j)]}\, [d\bar{q}\, dq] \\ &\equiv \int e^{i S_{eff}}\, [d\bar{q}\, dq], \end{aligned} \tag{2.5}$$

thus obtaining an effective action for the quarks.

$$\begin{aligned} S_{eff} = \int d^4x \{ &\bar{q}(i\not{\partial} - m)q - \frac{1}{2!}\int j^{a\mu}(x)\, V^{ab}_{\mu\nu}(x,y)\, j^{b\nu}(y)\, d^4x\, d^4y \\ &- \frac{1}{3!}\int\int j^{a\mu}(x)\, j^{b\nu}(y)\, j^{c\rho}(z)\, V^{abc}_{\mu\nu\rho}(x,y,z)\, d^4x\, d^4y\, d^4z + \cdots \}, \end{aligned} \tag{2.6}$$

where the V's are connected Green functions. Now, eq. (2.6) is an infinite expansion and it is absolutely essential to have a truncation scheme if we want to extract meaningful numbers. For N_c-quark systems like baryons, the $1/N_c$ expansion provides such a scheme. This can be seen by going back to the formalism of canonical quantization and considering S_{eff} as a function of the field operators q and $\bar{q}$. For a N_c-body baryon, the expectation value of the N' currents ($N' > N_c$) that are contracted with $V^{a_1 \ldots\ldots a_{N'}}_{\mu_1 \ldots\ldots \mu_{N'}}$ involves

$$\langle N_c\ q\text{'s}|\underbrace{\bar{q}q\ \ldots\ \bar{q}q}_{N'\ \text{times}}|N_c\, q\text{'s}\rangle \sim \psi^*_{N_c}(1, \ldots, N_c)\langle 0|\underbrace{\bar{q}q\ \ldots\ \bar{q}q}_{(N'-N_c)\,\text{times}}|0\rangle\psi_{N_c}(1,\ldots,N_c). \tag{2.7}$$

Now, the factor $\langle 0|\,\bar{q}q \ldots \bar{q}q|0\rangle$ corresponds to the production of virtual $q\bar{q}$ pairs and quark loops. It is suppressed by $1/N_c$ and all terms involving more than N_c currents can be dropped in a similar manner.

In spite of this restriction, one cannot actually compute even the two-point Green function $V^{ab}_{\mu\nu}$ to all orders (Fig. 1). Moreover, a fully relativistic treatment of the two-body problem keeping all components of $V^{ab}_{\mu\nu}$ (1,2) is prevented by the standard relative-time difficulties in the Bethe Salpeter equation. We therefore go to the static limit in our calculation and use for V_{00} (1,2) the Richardson potential, the simplest interpolation incorporating both asymptotic freedom and confinement.

With such a static limit for all Green functions in (2.6), it is easy to write down a Schrödinger equation for the subspace of N_c-quark states:

$$\Big[\sum_{i=1}^{N_c} (\underset{\sim}{\alpha}_i \cdot \underset{\sim}{p}_i + m_i \beta) + \frac{1}{4} \sum_{i<j} \lambda(i).\lambda(j)\, V(\underset{\sim}{x}_i - \underset{\sim}{x}_j) + \cdots$$

$$+ 3, \ldots, N_c\text{-body-terms} \Big] \Psi(\underset{\sim}{x}_1, \ldots, \underset{\sim}{x}_{N_c}) = E \Psi(\underset{\sim}{x}_1, \ldots, \underset{\sim}{x}_{N_c}). \quad (2.8)$$

Little is known about the interactions involving more than two quarks, except that they should be all of the same order of magnitude as far as the $1/N_c$ expansion is concerned. Other considerations however may reduce their importance. For 3-quark systems with $N_c = 3$, the diagram shown in Fig. 2a gives no contribution [13], because the expectation value of $f^{abc} \lambda^a \lambda^b \lambda^c$ between color singlets vanishes. Some higher order diagrams (Fig. 2b) can easily be shown[14] to vanish in the same manner. In view of the present lack of knowledge on the remaining non-vanishing 3-body diagrams, we will drop them in our computational work. When dealing with more than three colours, we will do the same for all Green functions involving more than two quarks. Our Hamiltonian will thus be reduced to

$$H = \sum_{i=1}^{N} (\underset{\sim}{\alpha}_i \cdot \underset{\sim}{p}_i + m_i \beta) + \frac{1}{4} \sum_{i<j} \lambda(i).\lambda(j)\, V(r_{ij}), \quad (2.9)$$

with the Richardson potential for $\frac{1}{4}\ \lambda(i).\ \lambda(j)\ V(r_{ij})$.

Let us come now to the HF approximation. In order to avoid the complexities of open-shell and multi-configuration HF, we will limit ourselves in this paper to the simplest case, in which N_c quarks all have the same flavor and occupy the same state $\phi_{jm}(\underset{\sim}{r})$. In the formalism of second quantization, this Slater determinant is written as

$$\Psi = \prod_{c=1}^{N_c} a^{+}_{fcjm} |0\rangle, \quad (2.10)$$

where a^{+}_{fcjm} creates a quark of colour c and flavor f in state (jm). The state ϕ_{jm} is chosen so as to minimize the average value of the energy,

$$E = \langle \Psi | H | \Psi \rangle . \quad (2.11)$$

In order to compute this quantity, one needs the matrix element

$$\langle \text{color singlet} \,|\, \lambda(i).\lambda(j)/4 \,|\, \text{color singlet} \rangle = -\frac{N_c+1}{2N_c} . \quad (2.12)$$

One then obtains readily

$$E = N_c \int \phi^{+}_{jm}(\underset{\sim}{r})\ t\ \phi_{jm}(\underset{\sim}{r})\, d\underset{\sim}{r}$$

$$- \frac{(N_c^2-1)}{4} \int \phi^{+}_{jm}(\underset{\sim}{r}_1)\, \phi^{+}_{jm}(\underset{\sim}{r}_2)\, V(r_{12})\, \phi_{jm}(\underset{\sim}{r}_1)\, \phi_{jm}(\underset{\sim}{r}_2)\, d\underset{\sim}{r}_1\, d\underset{\sim}{r}_2 , \quad (2.13)$$

where

$$t \equiv \underset{\sim}{\alpha} . \underset{\sim}{p} + \beta m . \quad (2.14)$$

Varying this expression with respect to ϕ_{jm}, subject to the constraint

$$\int \phi^{+}_{j'm'}(\underset{\sim}{r})\ \phi_{jm}(\underset{\sim}{r})\ d\underset{\sim}{r} = \delta_{jj'}\,\delta_{mm'}, \tag{2.15}$$

yields the HF equation

$$\left[t - \frac{N_c^2-1}{2N_c}\int \phi^{+}_{jm}(\underset{\sim}{r}')\,V(\underset{\sim}{r}-\underset{\sim}{r}')\,\phi_{jm}(\underset{\sim}{r}')\,d\underset{\sim}{r}'\right]\phi_{jm}(\underset{\sim}{r}) = \epsilon\,\phi_{jm}(\underset{\sim}{r}), \tag{2.16}$$

where ϵ is the single-particle energy. One gets a single equation, instead of a set of coupled equations, since all particles are in the same orbital ϕ_{jm}. For the same reason, there is no exchange matrix element for V and the single-particle self-consistent potential

$$w_{av}(\underset{\sim}{r}) = -\frac{N_c^2-1}{2N_c}\int \phi^{+}_{jm}(\underset{\sim}{r}')\,V(\underset{\sim}{r}-\underset{\sim}{r}')\,\phi_{jm}(\underset{\sim}{r}')\,d\underset{\sim}{r}' \tag{2.17}$$

is local. This self-consistent potential obviously behaves like the time component of a vector, and not as a mass term. The total energy E_{HF} is related to the single-quark energy ϵ by the usual relation

$$E_{HF} = N\left[\epsilon - \tfrac{1}{2}\langle\phi_{jm}|\,w_{av}\,|\phi_{jm}\rangle\right]. \tag{2.18}$$

For quarks in the lowest ($1s\tfrac{1}{2}$) orbital, one may write

$$\phi_{1s\frac{1}{2}m}(\underset{\sim}{r}) = \sqrt{\frac{1}{4\pi}}\begin{pmatrix} i\,G(r)\,\chi_m \\ \underset{\sim}{\sigma}\cdot\hat{\underset{\sim}{r}}\,F(r)\,\chi_m\end{pmatrix}, \tag{2.19}$$

where χ_m is a Pauli spinor, and eq. (2.16) yields the system of coupled equations,

$$\frac{dG}{dr} - (m - w_{av} + \epsilon)\,F = 0 \tag{2.20a}$$

$$\frac{dF}{dr} + \frac{2}{r}F + (\epsilon - w_{av} - m)\,G = 0, \tag{2.20b}$$

where

$$w_{av} \equiv -\frac{N_c^2-1}{2N_c}\int\left[G^2(r') + F^2(r')\right]V_0(r,r')\,r'^2dr', \tag{2.21}$$

$V_0(r,r')$ being the $\ell = 0$ part of $V(\underset{\sim}{r}-\underset{\sim}{r}')$:

$$V_0(r,r') = \frac{1}{2}\int_0^{\pi} V(\underset{\sim}{r}-\underset{\sim}{r}')\,\sin\theta\,d\theta. \tag{2.22}$$

Quite obviously, the single-particle energy ϵ does not depend on N_c when w_{av} is itself independent of this parameter. In the limit $N_c \to \infty$, this condition is seen to be realized if V is proportional to $1/N_c$, in accordance with the philosophy of the $1/N_c$ expansion [2)].

It may be noticed, in eqns. (2.20), that the single-particle potential (2.21) is added to the energy term. This will be shown later to lead to too large a radius for the confined system. On the other hand one expects a potential which will behave like a mass term if the explicit chiral symmetry breaking is introduced as a constraint in the action as done for example by Simić [15]. In fact in recent work with the Richardson potential in the meson sector, Crater and Van Alstine [16] have broken up the confining part of the potential into an equal vector and scalar part and put the terms appropriately with the energy and the mass respectively. We have tried both approaches i) treating the entire potential like an energy term and ii) splitting the confining part into half energy-like and half mass-like terms and the results will be discussed in the following section.

It is well known that HF solutions violate the translation invariance of the underlying Hamiltonian (2.9), since they are formed of single-particle wave functions derived from an average potential w_{av} that is not translation-invariant. As a consequence, the center-of-mass momentum is not well defined in HF solutions and this entails a spurious contribution from the center-of-mass kinetic energy to the total energy. Since the relative importance of this spurious effect increases as the number of particles decreases, it is important that it should be corrected for systems formed of few quarks. This can be done by extending to relativistic HF the Peierls-Yoccoz procedure of nuclear physics.

Let $\Psi(\underset{\sim}{x})$ be the HF wave function in configuration space, where $\underset{\sim}{x}$ denotes in a collective way the quark coordinates. It is possible to expand $\Psi(\underset{\sim}{x})$ on the basis of eigenvectors of the center-of-mass momentum

$$\Psi(\underset{\sim}{x}) = \int e^{i\underset{\sim}{k}\cdot\underset{\sim}{R}} \, \psi_{\underset{\sim}{k}}^{int}(\underset{\sim}{x}-\underset{\sim}{R})\, d\underset{\sim}{k}\,, \tag{2.23}$$

where $\underset{\sim}{R}$ is the center-of-mass coordinate and $\psi_{\underset{\sim}{k}}^{int}(\underset{\sim}{x}-\underset{\sim}{R})$ describes the internal motion of the system. Of course, this internal wave

function will be different for every value of $\underset{\sim}{k}$, since there is no factorization of the center-of-mass motion. The Peierls-Yoccoz method allows to project out of $\Psi(\underset{\sim}{x})$ a state $\Psi_{\underset{\sim}{k}}^{PY}$ of definite total momentum $\underset{\sim}{k}$:

$$\Psi_{\underset{\sim}{k}}^{PY}(\underset{\sim}{x}) = \frac{1}{(2\pi)^3}\int e^{i\underset{\sim}{k}\cdot\underset{\sim}{a}}\, \Psi(\underset{\sim}{x}-\underset{\sim}{a})\, d\underset{\sim}{a}$$

$$= e^{i\underset{\sim}{k}\cdot\underset{\sim}{R}}\, \psi_{\underset{\sim}{k}}^{int}(\underset{\sim}{x}-\underset{\sim}{R})\,, \tag{2.24}$$

where $\psi(\underset{\sim}{x}-\underset{\sim}{a})$ is the HF wave function translated in such a way that it is centered at point $\underset{\sim}{a}$.

Once this projection is performed, it is quite easy to compute the expectation value of any operator $F(\underset{\sim}{P})$ depending on the total momentum $\underset{\sim}{P}$, when the system is in the state $\Psi(\underset{\sim}{x})$,

$$\langle \Psi(\underset{\sim}{x}) | F(\underset{\sim}{P}) | \Psi(\underset{\sim}{x})\rangle = \int \langle \Psi_{\underset{\sim}{k}'}^{PY}(\underset{\sim}{x}) | F(\underset{\sim}{P}) | \Psi_{\underset{\sim}{k}}^{PY}\rangle\, d\underset{\sim}{k}\, d\underset{\sim}{k}'$$

$$= \int F(\underset{\sim}{k}) \langle \Psi_{\underset{\sim}{k}'}^{PY} | \Psi_{\underset{\sim}{k}}^{PY}\rangle\, d\underset{\sim}{k}\, d\underset{\sim}{k}' \;. \tag{2.25}$$

Now, it is trivial to show that

$$\langle \Psi_{\underset{\sim}{k}'}^{PY} | \Psi_{\underset{\sim}{k}}^{PY}\rangle = \delta(\underset{\sim}{k}-\underset{\sim}{k}')\, I(\underset{\sim}{k})\,, \tag{2.26}$$

where

$$I(\underset{\sim}{k}) \equiv \frac{1}{(2\pi)^3}\int e^{i\underset{\sim}{k}\cdot\underset{\sim}{\alpha}}\, N(\underset{\sim}{\alpha})\, d\underset{\sim}{\alpha} \tag{2.27}$$

is the Fourier transform of the overlap kernel

$$N(\underset{\sim}{a}-\underset{\sim}{a}') \equiv \langle \Psi(\underset{\sim}{x}-\underset{\sim}{a}') \mid \Psi(\underset{\sim}{x}-\underset{\sim}{a}) \rangle \quad . \qquad (2.28)$$

Substituting (2.26) into (2.25), one gets

$$\langle \Psi(\underset{\sim}{x}) \mid F(\underset{\sim}{p}) \mid \Psi(\underset{\sim}{x}) \rangle = \int F(\underset{\sim}{k})\, I(\underset{\sim}{k})\, d\underset{\sim}{k} \quad . \qquad (2.29)$$

One is now left with the task of computing $I(\underset{\sim}{k})$. The single-quark overlap kernel for wave functions (2.19) is

$$m(\underset{\sim}{\alpha}) = \langle \phi_{1S_{1/2}m}(\underset{\sim}{r}+\underset{\sim}{\alpha}/2) \mid \phi_{1S_{1/2}m}(\underset{\sim}{r}-\underset{\sim}{\alpha}/2) \rangle$$

$$= \frac{1}{4\pi} \int \left[G(r_+)\,G(r_-) + F(r_+)\,F(r_-)\; \underset{\sim}{\sigma}\cdot\hat{\underset{\sim}{r}}_+\; \underset{\sim}{\sigma}\cdot\hat{\underset{\sim}{r}}_- \right] d\underset{\sim}{r} \,, \qquad (2.30)$$

where

$$\underset{\sim}{r}_\pm \equiv \underset{\sim}{r} \pm \underset{\sim}{\alpha}/2 \quad . \qquad (2.31)$$

Now,

$$\underset{\sim}{\sigma}\cdot\hat{\underset{\sim}{r}}_+\; \underset{\sim}{\sigma}\cdot\hat{\underset{\sim}{r}}_- = \frac{(r^2-\alpha^2/4) + \underset{\sim}{\sigma}\cdot\underset{\sim}{r}\times\underset{\sim}{\alpha}}{r_+\, r_-} \,,$$

and the term in $\underset{\sim}{\sigma}\cdot\underset{\sim}{r}\times\underset{\sim}{\alpha}$ brings no contribution to (2.30) since it yields an integrand that is odd in $\underset{\sim}{r}$. One is thus left with

$$m(\underset{\sim}{\alpha}) = m(\alpha)$$

$$= \frac{1}{2} \int_0^\infty r^2 dr \int_{-1}^{1} du \left[G(r_+)\,G(r_-) + \frac{F(r_+)\,F(r_-)}{r_+ r_-} \left(r^2 - \frac{\alpha^2}{4}\right) \right] , \qquad (2.32)$$

where u is the cosine of the angle between $\underset{\sim}{r}$ and $\underset{\sim}{\alpha}$.

The overlap for N_c quarks in the same orbital is simply

$$N(\alpha) = [m(\alpha)]^{N_c} \qquad (2.33)$$

and

$$I(\underset{\sim}{k}) = I(k)$$

$$= \frac{1}{2\pi^2 k} \int_0^\infty \alpha \sin k\alpha \, [m(\alpha)]^{N_c} \, d\alpha \quad . \qquad (2.34)$$

Taking for $F(\underset{\sim}{p})$ the center-of-mass kinetic energy operator,

$$T_{cm} = F(\underset{\sim}{P}) = \sqrt{M^2 + \underset{\sim}{P}^2} - M \,, \qquad (2.35)$$

where M is the mass of the system, one gets finally

$$\langle \Psi \mid T_{cm} \mid \Psi \rangle = \frac{2}{\pi} \int_0^\infty k\alpha \left[\sqrt{M^2+k^2} - M \right] \sin(k\alpha)\, [m(\alpha)]^{N_c}\, dk\, d\alpha \quad . \qquad (2.36)$$

The calculation of this quantity thus involves a 4-fold integral. Particular attention has to be paid to the integration over k, which was handled through Filon's method.

Although the Peierls-Yoccoz method enables one to subtract the average value of the center-of-mass kinetic energy, it does obviously not eliminate all effects connected with spurious center-of-mass motion: in particular, it does not change anything to the fact that the HF wave function is a linear combination or internal wave functions, eq. (2.23), each of which yields a different average value of the internal energy.

III NUMERICAL RESULTS

Taking for granted the legitimacy of describing hadrons by an effective Hamiltonian with two-body interactions, there remains the question of the validity of the HF approximation. We obviously have no exact relativistic solutions for the 3-body problem to compare our HF solutions with, and the best we can do is to perform such a calculation in a situation where relativistic effects are presumably small. We took as a reference Richard's calculation[17] of the Ω^- mass, using the Martin [18] potential in the hyperspherical formalism. Since the strange quark is given a mass of 518 MeV in this calculation, one might expect the situation to be non-relativistic. Richard found a mass of 1617 MeV before including spin-dependent corrections. With the same potential and quark mass, we obtained E_{HF} = 1821 MeV. The average value of the center of mass was computed as described in Section II. Since such a procedure removes center-of-mass spuriosity in an approximate manner only, we did not try to reach self-consistency between the mass used in computing the correction and the corrected mass. We thus used the experimental value of the Ω^- mass in eq. (2.36), finding $< T_{CM} >$ = 237 MeV. The corrected mass thus went down to 1584 MeV. Although this is reasonably close

to Richard's value, it turns out to be difficult to draw any definite conclusion from this agreement, since the wave functions, somewhat to our surprise, turned out to have relatively large small components (Fig. 3), and thus not to be non-relativistic. This raises, among other, the question of the appropriateness of the Martin potential for our calculation, since it was devised for non-relativistic systems.

Such a question does not arise, of course, for Richardson's potential,

$$V(r) = \frac{\lambda(1)\cdot\lambda(2)}{4}\,\frac{6\pi}{33-2N_f}\,\Lambda\left(\Lambda r - \frac{f(\Lambda r)}{\Lambda r}\right), \tag{3.1}$$

where N_f is the number of flavors, taken to be three, and

$$f(t) \equiv 1 - 4\int_1^\infty \frac{dq}{q}\,\frac{e^{-qt}}{[\ln(q^2-1)]^2+\pi^2} \tag{3.2}$$

We have computed the Ω^- mass with this potential, which contains only one free parameter, the scale factor Λ. Since our calculation is relativistic, we need not work with an unrealistically large quark mass and we can take the current algebra value [12], m_s = 150 MeV. The results for the case where the entire potential is treated like the time-part of a vector (eqs. (2.20a) and (2.20b) are given in Table 1 for several values of Λ. They are rather sensitive the value of this parameter. In view of uncertainties in the value of the quark mass, we have checked how sensitive the HF energy is to m_s. One sees on Table 2 that doubling the mass from 150 to 300 MeV changes E_{HF} by less than 5% for Λ = 400 MeV and 10% for Λ = 300 MeV.

This relative lack of sensitivity is due to the fact that the increase in m_s is compensated, to a large extent, by a decrease of the kinetic energy. However, as shall be seen later, a decrease of the quark mass enhances the small component of the wave function and leads to confinement problems.

The single-quark wave functions obtained with the Martin and the Richardson (Λ = 300 MeV) potentials are compared in Fig. 3 and appear to be less confined in the second case. As can be expected, the Richardson wave functions are less confined for Λ = 300 MeV than for Λ = 400 MeV (Fig. 4), and more relativistic for m_s = 150 MeV than for m_s = 500 MeV (Fig. 5).

It can be seen on Fig. 5 that as the quark mass is decreased the wave function, particularly the small component, is enhanced at large radius. The average radius of about 1 fm given in Table 1 is already large. If one were to calculate the radius of systems like the nucleon or the delta isobar one would have to use current quark masses of the order of 10 MeV. With a Λ = 300 MeV and m_q = 10 MeV the Richardson potential yields wave functions leading to an average radius of about 1.7 fm. Clearly this is unrealistic and the purely vector potential is unable to confine massless quarks. As we have indicated before one expects the spontaneous breakdown of chiral symmetry to generate a mass-like or scalar potential and one can simulate this by dividing the linear part of the Richarson potential in eqn. (3.1) into two equal parts, as done by Crater and Van Alstine[16], $S = V = \frac{8\pi\Lambda^2 r}{54}$, the vector linear part in the average potential

w_{av} (eqn. 2.21) being half of what it was, while the contribution from the second term of the Richardson potential remains the same. The S – term now gives a v_{av} which is found through eqs. (2.21) and (2.22), with V replaced by S, so that the equations (2.20) are now changed to

$$\frac{dG}{dr} - (m + v_{av} - w_{av} + \epsilon)F = 0 \qquad (3.3a)$$

$$\frac{dF}{dr} + \frac{2}{r}F + (\epsilon - w_{av} - m - v_{av})G = 0. \qquad (3.3b)$$

It should be pointed out that an equal scalar and vector linear confining potential does not contribute any spin-orbit splitting to the p-states, for instance, in agreement with phenomenology[19].

For m_q = 10 MeV and Λ = 300 MeV, the radius of the system now becomes 1.1 fm, which is more reasonable. The results for various values of Λ are given in Table 3. For the Ω^- system as shown in Table 4 the mass is 1588 MeV for Λ = 400 MeV in very good agreement with experiment. The magnetic moment of the Ω^-, $\mu(\Omega^-)$, is not experimentally known. However, in large N_c expansion it is related to the $\mu(\Lambda^0)$, as shown by Karl and Paton[20],

$$\mu(\Omega^-) = 3\,\mu(\Lambda^0) \,. \qquad (3.4)$$

Since $\mu(\Lambda^0)$ is known to be $-.6138 \pm .0047$ nuclear magneton[21] the agreement between the $\mu(\Omega^-)$ in Table 4 for Λ = 400 MeV and eq. (3.4) is very good. In Table 3 we give the calculated magnetic moment of the nucleon-like system assuming it is a proton composed of u u d. Whereas we neglected the difference between the u and d flavors in the Hartree-Fock calculation and assumed a shell of identical quarks, we put appropriate charges on these quarks in the magnetic calculation. In the absence of strong spin-dependent forces, this may be justified. Again, the magnetic moments in Table 3 are close to the experimental proton magnetic moment. The mass of the nucleon-like system is close to the mean of the nucleon and the delta isobar for Λ = 300 MeV. But we feel less sure about the results of Table 3 since in the case of light quarks there are: i) greater uncertainties in the masses, ii) mass difference between the two flavours and, iii) effects from the open shell character of the wave function. We prefer the value of Λ = 400 MeV since this is in accord with lattice string tension estimates and meson sector results.

IV CONCLUSION

In conclusion we are of the opinion that the Richardson potential with Λ = 400 MeV can explain baryon properties as well as meson properties provided the confinement part is split into equal scalar and vector parts. For a purely vector potential there are problems with confinement. It is remarkable that a large N_c theory with practically no free parameters can achieve such good results.

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Table I. Hartree-Fock energy E_{HF} and centre-of-mass kinetic energy $<T_{cm}>$ for a time-like vector average potential derived from the Richardson two body interaction for the Ω^- with m_s = 150 MeV. All quantities except the average radius $<r^2>^{1/2}$ (fm) are in MeV. E_o is the single particle energy and $\Delta\epsilon$ is the nodal excitation energy when the Hartree-Fock field is kept unchanged.

Λ	E_{HF}	$<T_{cm}>$	$E_{HF} - <T_{cm}>$	E_o	$\Delta\epsilon$	$(<r^2>)^{1/2}$
400	2247	197	2050	1081	334	.92
350	1950	155	1795	916	294	.99
325	1831	140	1691	860	281	1.08
300	1690	122	1568	781	263	1.13

Table 2. Dependency of the Hartree-Fock energy $E_{HF}(m_s)$ on the strange quark mass, m_s. All quantities are in MeV.

Λ	E_{HF} (150)	E_{HF} (300)	E_{HF} (500)
400	2247	2320	2605
300	1690	1833	2198

Table 3. Hartree-Fock energy for half-vector, half-scalar confining potential for nucleon-like systems with m_q = 10 MeV. The notation is as in Table 1, μ is the magnetic moment in nuclear magnetons.

Λ	E_{HF}	$< T_{cm} >$	$E_{HF} - < T_{cm} >$	$< r^2 >^{1/2}$	ε_o	$\Delta\varepsilon$	μ
400	1523	154	1369	0.96	647	282	2.48
350	1373	124	1249	1.03	562	245	2.48
300	1212	87	1125	1.15	481	189	2.38

Table 4. Same as in Table 3 but for quark mass m_s = 150 MeV appropriate for the Ω^-.

Λ	E_{HF}	$< T_{cm} >$	$E_{HF} - < T_{cm} >$	$< r^2 >^{1/2}$ fm	ε_o	$\Delta\varepsilon$	μ
400	1778	190	1588	.75	652	345	- 1.81
350	1598	150	1448	.83	578	304	- 2.19
300	1419	115	1304	.93	503	264	- 2.00

FIGURE CAPTIONS

Figure 1. Some of the terms appearing in the two-point Green function for the gluons. As discussed in the text, the second diagram is suppressed by $1/N_c$ and is dropped.

Figure 2. Vanishing 3-body diagrams.

Figure 3. Large (upper half of the figure) and small (lower half) components of the wave functions for the potentials of Richardson (Λ = 300 MeV) and Martin.

Figure 4. Large (upper half of the figure) and small (lower half) components of the wave functions for two different values of the parameter Λ in the Richardson potential.

Figure 5. Large (upper half of the figure) and small (lower half) components of the wave functions for two different values of the quark mass and the Richardson potential (Λ = 300 MeV).

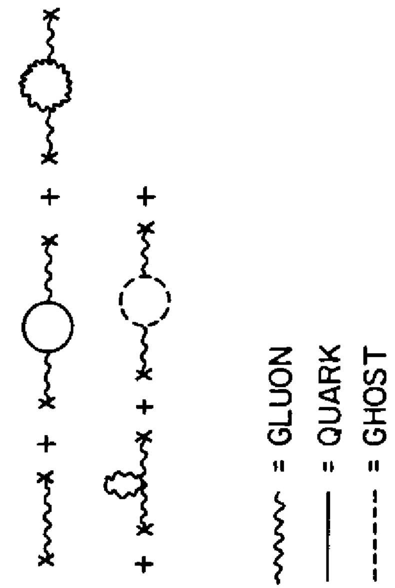

Fig. 1

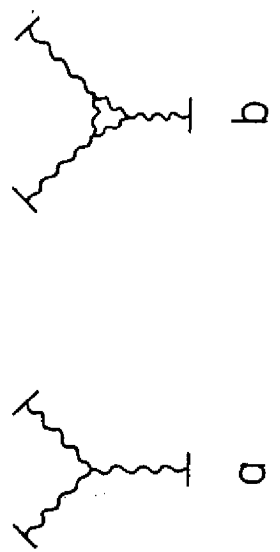
b
a

Fig. 2

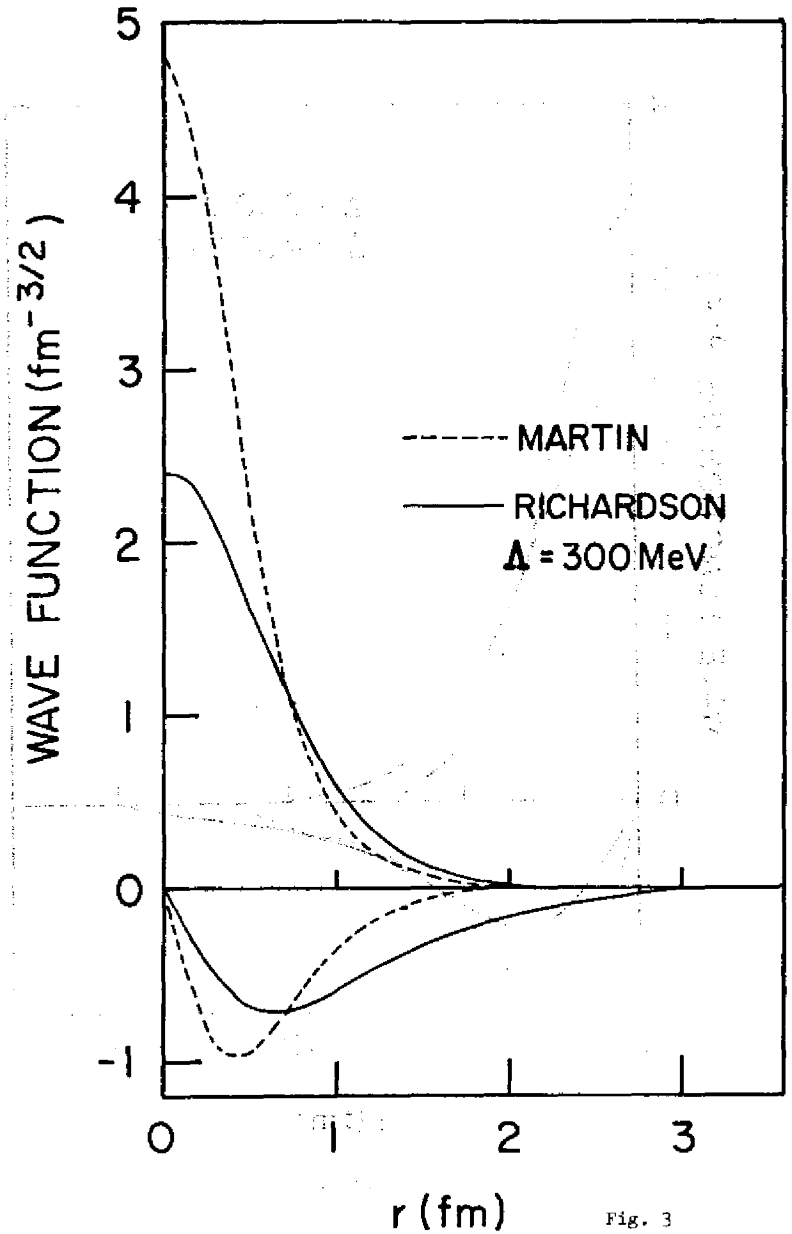
WAVE FUNCTION (fm$^{-3/2}$)
MARTIN
RICHARDSON
Λ = 300 MeV
r (fm)
0
1
2
3
4
5
-1

Fig. 3

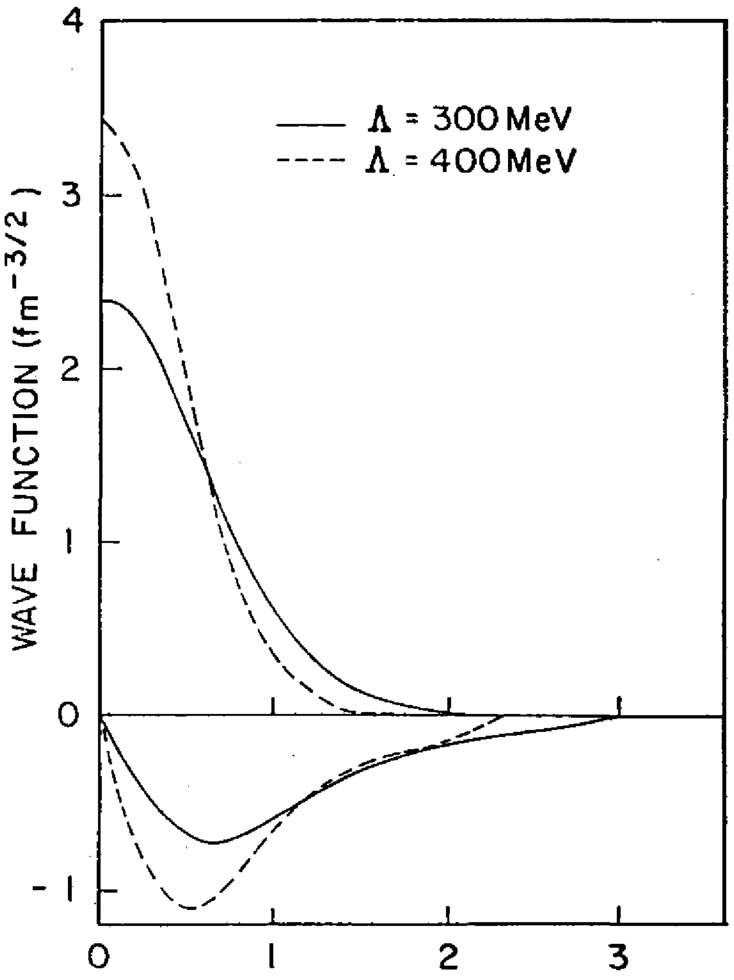
WAVE FUNCTION (fm$^{-3/2}$)
Λ = 300 MeV
Λ = 400 MeV
r (fm)

Fig. 4

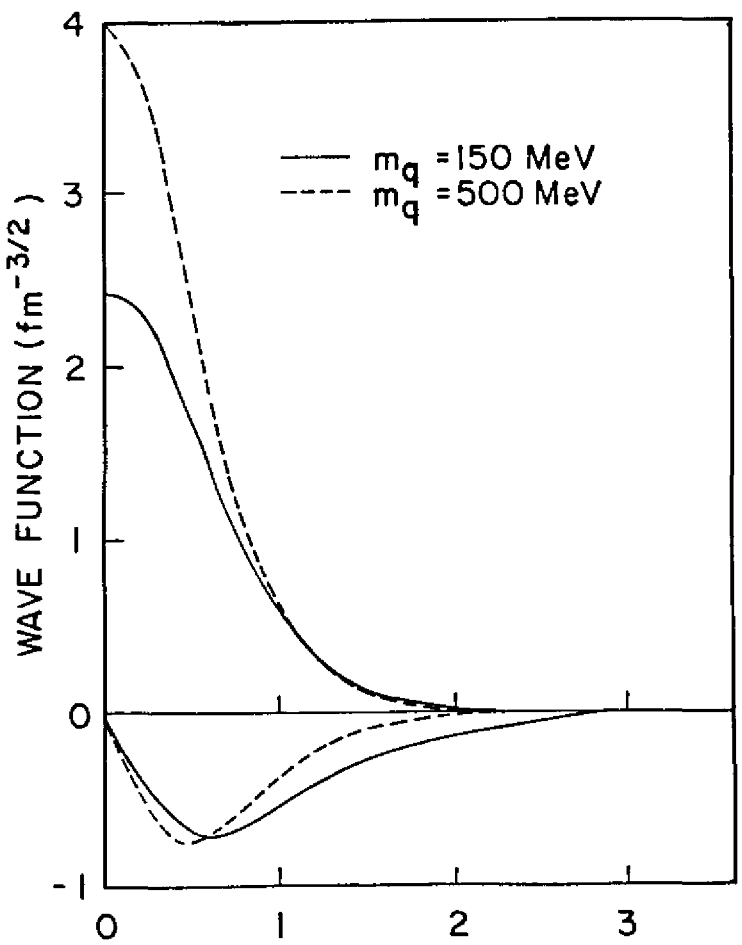
WAVE FUNCTION (fm$^{-3/2}$)
m_q = 150 MeV
m_q = 500 MeV
r (fm)

Fig. 5