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AND HEAVY FERMION SUPERCONDUCTIVITY

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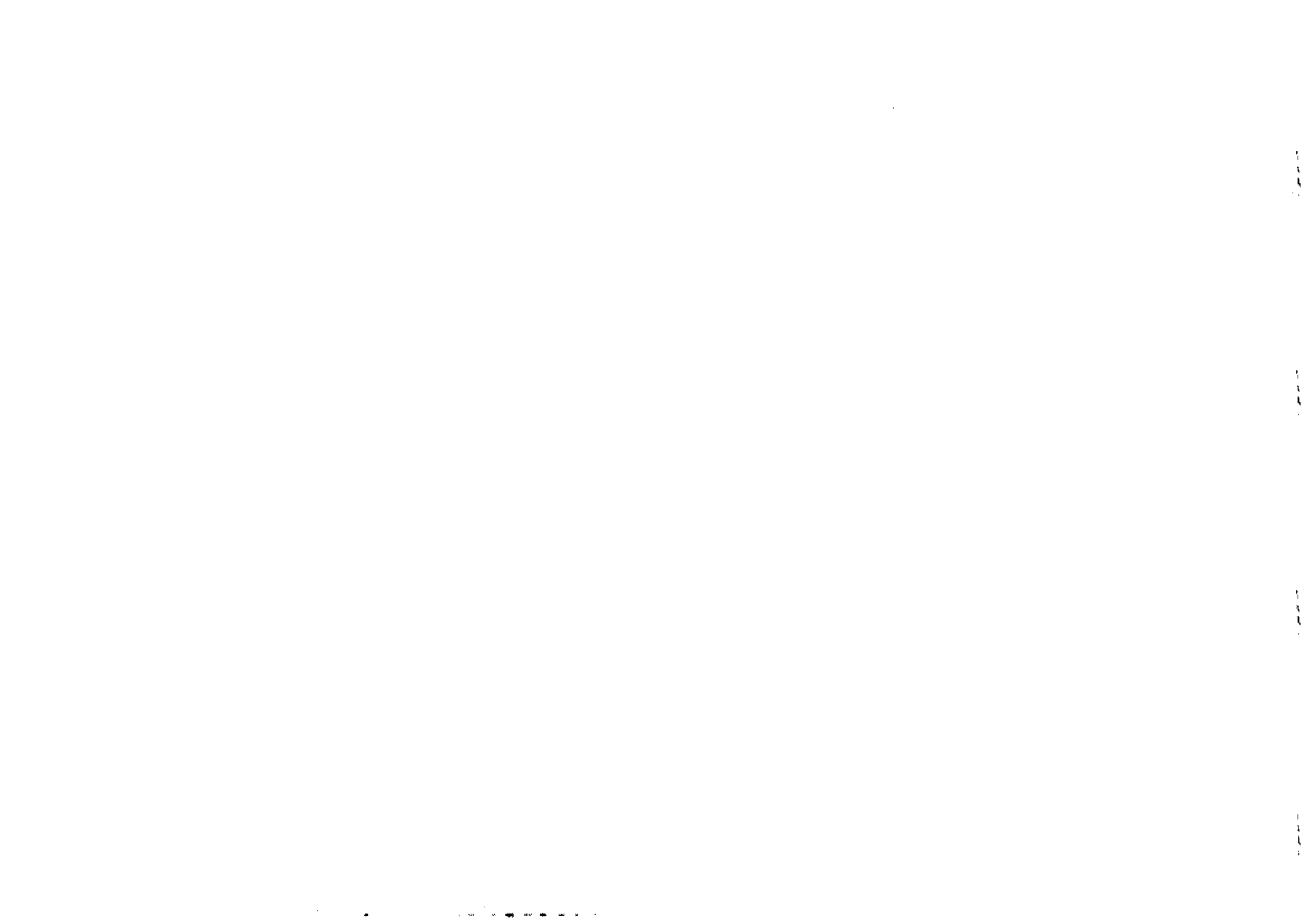


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RKKY INTERACTION IN MIXED VALENCE SYSTEM  
AND HEAVY FERMION SUPERCONDUCTIVITY \*

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ABSTRACT

The 1-D RKKY interaction of mixed valence system is given by using the thermodynamic perturbation theory. The numerical comparisons of 1-D and 3-D RKKY interaction between systems with localized magnetic moments of mixed valence and non-mixed valence show that the former is much stronger than the latter. From some analyses we propose that the heavy Fermion superconductivity comes from the RKKY interaction between two local  $f$  electrons which hop off the impurity site to become two continuum electrons. The source of the two impurity electrons hopping is the Coulomb interaction. It is also emphasized that the RKKY interaction does not disappear for the Kondo lattice, when the temperature is less than the Kondo temperature.

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1. INTRODUCTION

In a series of papers we have emphasized the importance of the RKKY interaction for resistivity <sup>1)</sup>, superconductivity enhancement <sup>2)</sup>, superconductivity reentrant <sup>3)</sup>, resistivity maximum <sup>4)</sup>, thermodynamic properties <sup>5)</sup> in systems with local magnetic moment. The form of 1-D RKKY interaction was obtained in Ref. 6. For the mixed valence system it is much more difficult to treat than for the non-mixed valence system with local magnetic moment. The RKKY interaction of 3-D for mixed valence was obtained in Refs. 7, 8 and 9. In this paper we will derive the form of 1-D RKKY interaction by using the thermodynamic perturbation theory <sup>10)</sup>. Then we make numerical comparisons of RKKY interaction between systems with local magnetic moments of mixed valence and non-mixed valence. We find the former is much larger than the latter. The superconductivity caused by the RKKY interaction has been proposed in Ref. 11. There is also a review paper on the effect of RKKY interaction <sup>12)</sup>. The mechanism of superconductivity which is proposed in this paper is RKKY mediated interaction between two local  $f$  electrons which hop off the impurity site to become two continuum electrons. The source of the two impurity electrons hopping is the Coulomb interaction. In the last part of this paper some discussions are made. Especially, we will argue that the RKKY interaction does not disappear for the Kondo lattice, when the temperature is less than the Kondo temperature.

2. RKKY INTERACTION IN MIXED VALENCE SYSTEM

At first we derive the form of 1-D RKKY interaction. The Hamiltonian to be used is  $U \rightarrow \infty$  limit of the degeneracy generalized Anderson lattice Hamiltonian

$$H = \sum_{\mathbf{R}m} \epsilon_{\mathbf{R}m} d_{\mathbf{R}m}^{\dagger} d_{\mathbf{R}m} + \sum_{m,\alpha} \left\{ E_m f_{\alpha m}^{\dagger} f_{\alpha m} + \frac{U}{2} \sum_n \left[ f_{\alpha m}^{\dagger} f_{\alpha n}^{\dagger} f_{\alpha n} f_{\alpha m} \right] \right\} + \sum_{\alpha} \frac{1}{N_s} \left( V_{\mathbf{R}m} e^{i\mathbf{R} \cdot \mathbf{r}_{\alpha}} d_{\mathbf{R}m}^{\dagger} f_{\alpha m} + \text{h.c.} \right) \quad (1)$$

in which the hybridization (last term in Eq.(1)) is treated as a perturbation.  $N_s$  denotes the number of sites labelled by  $\alpha$ ,  $m$  and  $n$  are angular momentum quantum numbers, their number being  $N = 2J + 1$ . The large  $N$  limit has to be performed with the restriction  $NV^2 = \text{constant}$ . We see from Fig. 1 that since the band electrons running between the sites conserve angular momentum, any contribution in which three or more sites are connected, is of higher order (e.g.  $N^{-2}$ ). We obtain the RKKY interaction diagrammatic rules from Ref. 8:

$$RKKY = -[\beta(1+S_1^z)(1+S_2^z)]^{-1} \sum_m \sum_{\omega_n} [G_{T_0}(R_1-R_2, i\omega_n) \frac{P_0}{2\pi i} \int dz \frac{\exp(-\beta z)}{(z+i\omega_n-E_m)(z-\Sigma(z))}]^2 \quad (2)$$

The diagrams which were summed up are displayed in Fig. 1. The Green-function for the band is

$$G_{T_m}(R_1-R_2, i\omega_n) = \frac{1}{N_s} \sum_k |V_k|^2 \frac{\exp[ik(R_1-R_2)]}{i\omega_n - \epsilon_{km}} \quad (3)$$

By using the residue theorem we get

$$G_m(R, i\omega_n) = -i\pi N(\omega) V^2 \exp k_F R (i \text{Sign} \omega_n - |n + \frac{1}{2}| \frac{\pi T}{E_F}) \cdot \text{Sign} \omega_n \quad (4)$$

where  $R = |R_1 - R_2|$ . We next evaluate the contour integral in Eq.(2) by the pole approximation at  $z = E_0$ .

$$RKKY_{1-D} = -\pi N V^4 N^2(0) T \left( \frac{\pi T_A}{\pi T_A + N\Delta} \right)^2 \sum_{\omega_n} \frac{\exp 2k_F R (i \text{Sign} \omega_n - |n + \frac{1}{2}| \frac{\pi T}{E_F})}{(i\omega_n - T_A)^2} \quad (5)$$

where  $\Delta = \pi N(E_F) V^2$ ,  $T_A = E_m - E_0$ . We then perform the sum on  $\omega_n$ .

$$RKKY_{1-D} = \frac{N V^4 N^2(0)}{2T} \left( \frac{\pi T_A}{\pi T_A + N\Delta} \right)^2 \varphi(R) \quad (6)$$

Assume that  $T \ll E_F$ ,  $E_m - E_0 \gg T$ , then the  $\varphi(R)$  is

$$\varphi(R) = -4\pi^2 \left( \frac{T}{T_A} \right)^2 e^{-\frac{\pi T k_F R}{E_F}} \text{Re} \left\{ e^{i(k_F R + \frac{2\pi T}{T_A})} \sum_{n=0}^{\infty} e^{-\pi n \left( \frac{2T k_F R}{E_F} - i \frac{4T}{T_A} \right)} \right\} \quad (7)$$

If  $2\pi T/(E_m - E_0) \ll 2k_F R \ll E_F/(E_m - E_0)$ , then

$$\varphi(R) = \frac{\pi T}{T_A} \sin 2k_F R$$

$$RKKY_{1-D} = N V^4 N^2(E_F) \frac{\pi}{2} \left( \frac{\pi T_A}{\pi T_A + N\Delta} \right)^2 \frac{1}{T_A} \sin 2k_F R \quad (8)$$

If  $E_F/(E_m - E_0) \ll 2k_F R \ll E_F/T$ , then

$$\varphi(R) = -\frac{4\pi E_F T}{T_A^2} \frac{\cos 2k_F R}{2k_F R}$$

$$RKKY_{1-D} = -\frac{2\pi N V^4 N^2(0) E_F}{T_A^2} \left( \frac{\pi T_A}{\pi T_A + N\Delta} \right)^2 \frac{\cos 2k_F R}{2k_F R} \quad (9)$$

If  $2k_F R \gg E_F/T$ , then

$$\varphi(R) = -\frac{4\pi^2}{T_A^2} e^{-\frac{\pi T k_F R}{E_F}} \cos 2k_F R$$

$$RKKY_{1-D} = -\frac{2\pi^2 N V^4 N^2(0) T}{T_A^2} \left( \frac{\pi T_A}{\pi T_A + N\Delta} \right)^2 e^{-\frac{\pi T k_F R}{E_F}} \cos 2k_F R \quad (10)$$

The form of 1-D non-mixed valence RKKY is <sup>6)</sup>

$$RKKY_{1-D \text{ non}} = -\frac{J^2}{16 E_F} \left[ \frac{\cos 2k_F R}{2k_F R} + \frac{\sin 2k_F R}{(2k_F R)^2} \right] \quad (11)$$

To get the numerical comparison between forms of 1-D RKKY we assume  $J = 0.1$  ev,  $E_F = 5$  ev, the degeneracy of f level  $N$  is 6,  $T_A = 500$ , then from Schrieffer-Wolff transformation  $V \sim 0.05$  ev. So, if  $2 < 2k_F R < 20$ , then from Eq. (8) the 1-D RKKY is about  $0.63 \sin 2k_F R (K)$ . From Eq.(11) the non-mixed valence 1-D RKKY is about  $0.29 (\cos 2k_F R + (\sin 2k_F R)/5)$ . In average there is

$$\frac{\text{maximum value of 1-D RKKY for mixed valence}}{\text{maximum value of 1-D RKKY for non-mixed valence}} \sim 5 \quad (12)$$

The 3-D RKKY interaction of mixed valence was given in Ref. 8.

For  $T_A \gg T$ ,  $2T/(E_m - E_0) \ll 2k_F R \ll E_F/(E_m - E_0)$ ,

$$RKKY_{3-D} = -2N\pi N^2(E_F) V^4 \frac{\sin 2k_F R}{(2k_F R)^3} \frac{\pi^2 T_A}{(\pi T_A + N\Delta)^2} \quad (13)$$

For  $T_A \gg T$ ,  $E_F/(E_m - E_0) \ll 2k_F R \ll E_F/T$ ,

$$RKKY_{3-D} = 8NE_F \pi N^2(E_F) V^4 \frac{\cos 2k_F R}{(2k_F R)^3} \frac{\pi^2}{(\pi T_A + N\Delta)^2} \quad (14)$$

For  $T_A \gg T$ ,  $2k_F R \gg E_F/T$ ,

$$RKKY_{3-D} = \frac{8\pi^2 N V^4 N^2(E_F) T}{T_A^2} \left( \frac{\pi T_A}{\pi T_A + N\Delta} \right)^2 e^{-\frac{\pi T k_F R}{E_F}} \frac{\cos 2k_F R}{(2k_F R)^3} \quad (15)$$

The form of 3-D non-mixed valence RKKY is

$$RKKY_{3-D \text{ non}} = \frac{9\pi}{8E_F} J^2 \left( \frac{\cos 2k_F R}{(2k_F R)^3} - \frac{\sin 2k_F R}{(2k_F R)^4} \right) \quad (16)$$

If the value of  $2k_F R$  is the area of 2--20, then the two kinds of RKKY interaction are nearly equal in the case of 3-D.

Let us discuss another limit:  $(E_m - E_0)/k_B T \ll 1$ , i.e. the  $E_m \sim E_0$ , for example  $E_m - E_0 = T_A \sim 1$  K. In this condition the renormalization factor  $(1 - \partial \sum (E_0)/\partial E_0)$  is not easy to treat, so we do not consider the renormalization in the following formulae of mixed valence RKKY temporarily. The  $\varphi(R)$  in Eq.(6) is

$$\varphi(R) = \sum_{n=0}^{\infty} \frac{1}{(n + \frac{1}{2})^2} e^{-\frac{n\pi 2k_F R T}{E_F} - \frac{\pi k_F R T}{E_F}} \cos 2k_F R \quad (17)$$

If  $2k_F R \ll E_F/T$ , then  $\varphi(R) = (\pi^2/2) \cos 2k_F R$ . Inserting this  $\varphi(R)$  into Eq.(6), we obtain that 1-D RKKY interaction in mixed valence system is

$$RKKY_{1-D} = \frac{N V^4 N^2(E_F)}{T} \left( \frac{\pi}{2} \right)^2 \cos 2k_F R \quad (18)$$

If  $2k_F R \gg E_F/T$ , then  $\varphi(R) = 4 \exp(-k_F R \pi T/E_F) \cos 2k_F R$ . Inserting this  $\varphi(R)$  into Eq.(6), we find that the 1-D RKKY interaction is

$$RKKY_{1-D} = \frac{2N V^4 N^2(E_F)}{R_B T} e^{-\frac{\pi k_F R T}{E_F}} \cos 2k_F R \quad (19)$$

In the case of  $(E_m - E_0) \ll T$ , the 3-D RKKY interaction of mixed valence system is given by Ref.(8) as

$$RKKY_{3-D} = -N\pi^2 N^2(E_F) V^4 \frac{1}{T} \frac{\cos 2k_F R}{(2k_F R)^2} \quad (20)$$

for  $2k_F R \ll E_F/T$ .

$$RKKY_{3-D} = -8N \cdot N^2(E_F) V^4 \frac{1}{T} e^{-\frac{\pi k_F R T}{E_F}} \frac{\cos 2k_F R}{(2k_F R)^2} \quad (21)$$

for  $2k_F R \gg E_F/T$ .

Assuming that  $T = 10$  K, then from the Eq.(18) and Eq.(11) we can get the following numerical estimation:

$$\frac{\text{maximum value of 1-D RKKY for mixed valence}}{\text{maximum value of 1-D RKKY for non-mixed valence}} \sim 10 \quad (22a)$$

From Eq.(16) and Eq.(20) we can get the following numerical estimation:

$$\frac{\text{maximum value of 3-D RKKY for mixed valence}}{\text{maximum value of 3-D RKKY for non-mixed valence}} \sim 12 \quad (22b)$$

The reason why the RKKY interaction in mixed valence system is much stronger than the one in non-mixed valence system is that in the mixed valence case the mediated conduction electron becomes local  $f$  electron on one site since the hybridization and the  $f$  electron previously localized on the site becomes conduction electron which carries and transmits more information to another site than in the case of non-mixed valence.

3. EFFECTIVE ATTRACTIVE INTERACTION AND MODEL OF HEAVY FERMION SUPERCONDUCTIVITY

At first we write the general Coulomb interaction for a system which contains continuum electrons and local electrons. Impurity atom is located at the position  $\vec{R}_n$  and has localized electron orbital  $\phi_L(\vec{r} - \vec{R}_n)$ , when the electron is on the sites. Otherwise the electron is in a continuum state  $k$  with wave function  $\phi_k(\vec{r})$  and  $\epsilon_k$ . The wave function may be considered as plane waves or alternatively as Bloch functions of the crystal. A generalized state function is the summation over all possible states,

$$\Psi(\vec{r}, \sigma) = \sum_{R, S} \phi_R(\vec{r}) X_{S, \sigma} C_{R, S} + \sum_{S, n} \phi_L(\vec{r} - \vec{R}_n) X_{S, \sigma} a_{n, S} \quad (23)$$

where the  $X$  are the spin wave functions, which denote spin up  $X_1$  or down  $X_2$ . The interesting magnetic phenomena comes from the terms involving electron-electron interaction, thus we consider

$$V = \frac{1}{2} \int d\vec{r}_1 d\vec{r}_2 \sum_{\sigma_1, \sigma_2} \Psi^+(\vec{r}_1, \sigma_1) \Psi^+(\vec{r}_2, \sigma_2) \frac{e^2}{|\vec{r}_1 - \vec{r}_2|} \Psi(\vec{r}_2, \sigma_2) \Psi(\vec{r}_1, \sigma_1) \quad (24)$$

inserting Eq.(23) into Eq.(24), we get sixteen terms.

$$\begin{aligned} V = \frac{1}{2} \int d\vec{r}_1 d\vec{r}_2 \sum_{\sigma_1, \sigma_2} & \left\{ \sum_{R_1, S_1} \phi_{R_1}^*(\vec{r}_1) X_{S_1, \sigma_1}^+ C_{R_1, S_1}^+ \sum_{R_2, S_2} \phi_{R_2}^*(\vec{r}_2) X_{S_2, \sigma_2}^+ C_{R_2, S_2}^+ \frac{e^2}{|\vec{r}_1 - \vec{r}_2|} \right. \\ & \sum_{R_3, S_3} \phi_{R_3}(\vec{r}_2) X_{S_3, \sigma_2} C_{R_3, S_3} \sum_{R_4, S_4} \phi_{R_4}(\vec{r}_1) X_{S_4, \sigma_1} C_{R_4, S_4} \\ & + \sum_{R_1, S_1} \phi_{R_1}^*(\vec{r}_1) X_{S_1, \sigma_1}^+ C_{R_1, S_1}^+ \sum_{R_2, S_2} \phi_{R_2}^*(\vec{r}_2) X_{S_2, \sigma_2}^+ C_{R_2, S_2}^+ \frac{e^2}{|\vec{r}_1 - \vec{r}_2|} \\ & \sum_{R_3, S_3} \phi_{R_3}(\vec{r}_1) X_{S_3, \sigma_1} C_{R_3, S_3} \sum_{R_4, S_4} \phi_L(\vec{r}_2 - \vec{R}_n) X_{S_4, \sigma_2} a_{n, S_4} \\ & \left. + \sum_{R_1, S_1} \phi_{R_1}^*(\vec{r}_1) X_{S_1, \sigma_1}^+ C_{R_1, S_1}^+ \sum_{R_2, S_2} \phi_{R_2}^*(\vec{r}_2) X_{S_2, \sigma_2}^+ C_{R_2, S_2}^+ \frac{e^2}{|\vec{r}_1 - \vec{r}_2|} \right. \end{aligned}$$

$$\begin{aligned} & \sum_{R_3, S_3} \phi_L(\vec{r}_1 - \vec{R}_n) X_{S_3, \sigma_1} a_{n, S_3} \sum_{R_4, S_4} \phi_{R_4}(\vec{r}_2) X_{S_4, \sigma_2} C_{R_4, S_4} \\ & + \sum_{R_1, S_1} \sum_{R_2, S_2} \sum_{R_3, S_3} \sum_{R_4, S_4} \dots + \sum_{R_1, S_1} \sum_{R_2, S_2} \sum_{R_3, S_3} \sum_{R_4, S_4} \dots \\ & + \sum_{R_1, S_1} \sum_{R_2, S_2} \sum_{R_3, S_3} \sum_{R_4, S_4} \dots + \sum_{R_1, S_1} \sum_{R_2, S_2} \sum_{R_3, S_3} \sum_{R_4, S_4} \dots \\ & + \sum_{R_1, S_1} \sum_{R_2, S_2} \sum_{R_3, S_3} \sum_{R_4, S_4} \dots + \sum_{R_1, S_1} \sum_{R_2, S_2} \sum_{R_3, S_3} \sum_{R_4, S_4} \dots \\ & + \sum_{R_1, S_1} \sum_{R_2, S_2} \sum_{R_3, S_3} \sum_{R_4, S_4} \dots + \sum_{R_1, S_1} \sum_{R_2, S_2} \sum_{R_3, S_3} \sum_{R_4, S_4} \dots \\ & + \sum_{R_1, S_1} \sum_{R_2, S_2} \sum_{R_3, S_3} \sum_{R_4, S_4} \dots + \sum_{R_1, S_1} \sum_{R_2, S_2} \sum_{R_3, S_3} \sum_{R_4, S_4} \dots \\ & + \sum_{R_1, S_1} \sum_{R_2, S_2} \sum_{R_3, S_3} \sum_{R_4, S_4} \dots + \sum_{R_1, S_1} \sum_{R_2, S_2} \sum_{R_3, S_3} \sum_{R_4, S_4} \dots \\ & + \sum_{R_1, S_1} \sum_{R_2, S_2} \sum_{R_3, S_3} \sum_{R_4, S_4} \dots \end{aligned} \quad (25)$$

They correspond to the following Feynman diagrams and can be separated into five types (see Fig. 1). The physical process of type D.1 of Coulomb interaction between double site synchronism mixed valence electrons is that two impurity electrons hop off the impurity site to become two continuum electrons and vice versa. The two impurity electrons can be simply called hopping electrons.

The estimations in Sec. 2 tell us that the RKKY interaction of mixed valence system is not small. The RKKY interactions are attractive potential for some interval of values of  $2k_F R$ . By the famous Cooper argument, if there are pure attractive forces between two electrons above Fermi sea, then the ground state will be reconstructed and the Cooper pair will be formed. The convenient

method to get the critical temperature of superconductivity  $T_c$  is to determine the  $T > 0$  K retardation K matrix  $K^R(\omega)$ . From the pole at  $\omega = 0$  of  $K^R(\omega)$  the  $T_c$  can be determined. The expression of  $K^R(\omega)$  is

$$K^R(\omega) = V_{ef} u_p \left\{ 1 + N(E_F) V_{ef} \int_{-\omega_c}^{\omega_c} d\xi \frac{\tanh(\beta\xi/2)}{2\xi - \omega} + i N(E_F) V_{ef} \pi \tanh \frac{\beta\omega}{4} \right\}^{-1} \quad (26)$$

where the  $u_p$  is cut off factor of energy, when  $-\omega_c > \xi > \omega_c$ ,  $u_p = 1$ , otherwise  $u_p = 0$ . The  $p$  in Eq.(26) denotes main value integral.  $\omega_c$  is cut off energy, and is about several hundred degrees up or down the Fermi surface. When  $T \ll \omega_c$ , the approximation expression of  $K^R(\omega = 0)$  is

$$K^R(0) \approx V_{ef} u_p \left\{ 1 + N(E_F) V_{ef} \ln \frac{\omega_c}{2T} \right\}^{-1} \quad (27)$$

When  $V_{ef} < 0$ , from  $K_0^{(R)}(0) = \infty$  we get

$$T_c = \frac{\omega_c}{2} \cdot e^{-\frac{1}{N(E_F)|V_{ef}|}} \quad (28)$$

Assume our system is 3-D,  $T_A = 100$  K,  $k_F = 1.7 \text{ \AA}^{-1}$ ,  $R = 4.1 \text{ \AA}$ ,  $E_F = 0.5$  eV,  $N(E_F) = 2$  states/ev. atom. spin,  $N = 6$ ,  $\omega_c = 500$  K, then  $2k_F R = 13.94$ ,  $2T/T_A \ll 2k_F R \ll E_F/T_A$  and  $T_A \gg T$ , so we can use Eq.(13) to get the RKKY interaction. It is easy to get the critical temperature  $T_c$  from Eq.(28) and the value of RKKY interaction. The  $T_c$  in this case is about 0.5 K. As we know, the experimental value of  $T_c$  of  $\text{CeCu}_2\text{Si}_2$  is also about 0.5 K. So, the above estimation shows that the RKKY interaction is really large enough to provide an attractive potential between two ions of Ce. Our model of heavy Fermion superconductivity is the interaction between two local  $f$  electrons which hop off the impurity site to become two continuum electrons. The diagrams of double site synchronism mixed valence in Fig. 1 clearly show this process. The source of the two impurity electrons hopping is the Coulomb interaction between  $s$  (or  $d$ ) and  $f$  electrons. The high density of state, i.e. large effective mass may be naturally interpreted by the above mechanism, because the Cooper pair is formed from  $f$  electrons.

#### 4. DISCUSSION

In this section we shall give some discussion on four questions. For the case of non-mixed valence magnetic impurities, for example  $\text{Fe}_{0.05}\text{TaS}_2$ , we had proposed a RKKY superconductivity mechanism also. The differences between models for the cases of non-mixed valence and mixed valence are that the Cooper pair of the latter is formed from two electrons on the two sites with mixed valence ions, but the Cooper pair of the former is formed from two electrons which scatter on the different sites with non-mixed valence ions. In the former case the two electrons scattering with a magnetic ion pair can also experience the RKKY interaction between the magnetic ions, so in some cases they may become a Cooper pair. But there is big differences for the two cases. As we know from Refs. 1 and 2, there is always a factor in the expression of self-energy  $\sum (i\omega_n + 0 + i0^+)$ ,

$$I = \sum_{\mathbf{R}} e^{i\mathbf{R} \cdot (\vec{R}_i - \vec{R}_j)} \frac{1}{i0^+ - (\hbar^2 \mathbf{R}^2 / (2m) - \mu)} = \frac{2m\pi^2}{\hbar^2 |\vec{R}_i - \vec{R}_j|} (-\cos R_F |\vec{R}_i - \vec{R}_j| - i 3 \sin R_F |\vec{R}_i - \vec{R}_j|) \quad (29)$$

In Eq.(29) the imaginary and real parts do not always have the same sign. The  $T_c$  formula is given in Refs. 2 and 3 as

$$\ln \frac{T_c'}{T_c} = \Psi\left(\frac{1}{2}\right) - \Psi\left[\frac{1}{2} + C_1(-I_m \sum_{\mathbf{A} \neq \mathbf{R}_i, \mathbf{R}_j} (i\omega_n \rightarrow i0^+)) + C_2(-I_m \sum (i\omega_n \rightarrow i0^+))\right] \quad (30)$$

where  $\Psi$  is the digamma function,  $T_c'$  is critical temperature including the influences of imaginary parts of self-energy and  $T_c$  is critical temperature without the influences of the latter. From Eq.(30) we can see that  $T_c'$  is not always larger than the  $T_c$ , because of the change of sign of the imaginary part of the self energy. Especially, the former process mentioned at the beginning of this section corresponds to a process in the eighth-order approximation of the Anderson Hamiltonian, and the latter only in the fourth. So, the contribution of the former process to superconductivity is less than the contribution of the latter, i.e. the latter process is much more important for superconductivity than the former. 1-D RKKY interaction, in general, is much larger than the RKKY of the 3-D because in the latter expression there is

a factor  $1/(2k_F R)^n$  which is much smaller than 1, for example it is about  $1/14^2$  for  $CeCu_2Si_2$ . Someone may think that there is no RKKY interaction when  $T < T_K$ . There are two reasons to think that the RKKY interaction is still important. The first reason is that  $T_K$  itself is decreased by the RKKY interaction<sup>15),16)</sup>. The second reason is that although under the condition  $T < T_K$  there is a formation of a bound state whose magnetic moment is cancelled by the electron cloud surrounding the ion in average, the magnetic ion can still feel RKKY interactions with the other magnetic ion because of the loose structure of the electron cloud.

In principle, the Cooper pair mediated by the RKKY interaction can be a spin-singlet state or a spin-triplet state. The decisive condition for the appearance of superconductivity is purely that the attractive force is not zero. This means the RKKY attractive potential should overcome Coulomb repulsive potential.

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FIGURE CAPTION

Fig. 1 Feynman diagrams of general Coulomb interaction for the system including continuum and local electrons. The Arabic numbers correspond to the order of terms in Eq.(25).

