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ABSTRACT

The $K^0 - \bar{K}^0$ mixing amplitude is calculated without using the standard zero external momentum approximation. The resulting corrections $\left(\sim \frac{m_k^2}{m_c^2} \ln \frac{m_c^2}{m_s^2}\right)$ are numerically significant for the real part of the amplitude. In the imaginary part of the amplitude the effects of similar corrections are less important. Implications for Δm_k and ϵ are discussed.

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The box diagram amplitude for $K^0 - \bar{K}^0$ mixing was first computed by Gaillard and Lee [1] who used the zero external momentum approximation. Subsequently it was pointed out [2] in a four quark model that corrections arising due to non-vanishing external momenta are numerically significant $\left(\sim \frac{m_k^2}{m_c^2} \ln \frac{m_c^2}{m_s^2}\right)$. The purpose of this note is to generalize the results of Ref.2 to a six quark model so that the analysis of CP violation may also be included. This is particularly important since several authors [3] have already observed that the Kobayashi-Maskawa (KM) model of CP violation based on the standard $SU(2)_L \otimes U(1)$ model may not be compatible with the observed value of the ε parameter. Since numerical factors play an important role in this analysis and the above corrections are apparently significant, their effects should be carefully examined [4].

In Ref.2 the approximation that M_W is much larger compared to all other relevant masses in the amplitude was used. This approximation, however, is not appropriate when the top quark is included in the analysis. We have therefore, used the exact formula [5] for the integrals that appear in the box diagram amplitude. In Ref. 5 the integrals were evaluated numerically in the context of $B^0 - \bar{B}^0$ systems. For the $K^0 - \bar{K}^0$ system they may be written in a compact form under reasonable approximations ($m_u, m_d \ll m_s \ll m_c \ll m_t, m_W$). We thus obtain (neglecting three momenta of the external quarks)

$$M_{12}^{Box} = \langle K^0 | H_{eff} | \bar{K}^0 \rangle$$
$$= \frac{G_F^2 M_W^2}{8\pi^2} \left[\lambda_c^2 \eta_1 (\mathcal{M}_1 B_1 + \mathcal{M}_2 \mathcal{C}_1) + \lambda_t^2 \eta_2 (\mathcal{M}_1 B_2 + \mathcal{M}_2 \mathcal{C}_2) + 2\lambda_c \lambda_t (\mathcal{M}_1 B_3 + \mathcal{M}_2 \mathcal{C}_3) \right], \quad (1.a)$$

where $\eta_1 = 0.85$, $\eta_2 = 0.6$, $\eta_3 = 0.4$ are QCD correction factors [6)]

$$\lambda_q = K^*_{qd} K_{qs} \qquad (1.b)$$

K_{ij}'s are elements of the KM matrix [7)] $\mathcal{M}_1$ and $\mathcal{M}_2$ are given by

$$\mathcal{M}_1 \equiv \langle K^0 | (\bar{d}_L \gamma_\mu s_L)^2 | \bar{K}^0 \rangle$$

$$= \frac{4}{3} f_K^2 m_K \mathcal{B} , \qquad (1.c)$$

$$\mathcal{M}_2 \equiv \langle K^0 | (\bar{d}_R s_L)^2 | \bar{K}^0 \rangle$$

$$= - 5/6 \; f_K^2 m_K (m_K/m_s)^2 \mathcal{B} \qquad (1.d)$$

where $\mathcal{B}$ is the well known bag parameter [3)]. It should be emphasized that the scalar and pseudo scalar operators (Eq.1.d) arise by contracting external four momenta in the amplitude with relevant spinor bilinears and then by using the Dirac equation. The matrix elements of these operators in fact lead to the most significant corrections due to external momenta. The functions B_i and C_i (i = 1, 2, 3) are given by

$$B_1 = \frac{a_c}{2} \left[1 + \frac{A}{3a_c} \ln (a_c/A) \right] \qquad (2.a)$$

$$B_2 = \frac{a_t}{2} \left[\frac{1}{1-a_t} - \frac{7a_t - a_t^2}{4(1-a_t^2)} - \frac{3a_t^2 - a_t^3}{2(1-a_t)^3} \ln a_t \right] \qquad (2.b)$$

$$B_3 = \frac{a_c}{2} \left[\ln (a_t/a_c) + \frac{2 - 9a_t}{4(1-a_t)} - \frac{3a_t^2 \ln a_t}{4(1-a_t)^2} \right] \qquad (2.c)$$

$$C_1 = - \frac{A}{3} \ln (a_c/A) \qquad (2.d)$$

$$C_2 = A \left[-\frac{1}{3} \ln (a_t/A) + \frac{20a_t - 83a_t^2 + 42a_t^3 - 5a_t^4}{36(1-a_t)^4} + \frac{a_t^2 \ln a_t}{3} \left(\frac{1}{4} + \frac{1}{(1-a_t)^2} - \frac{6a_t + 4a_t^2 - 2a_t^3}{(1-a_t)^5} \right) \right] \qquad (2.e)$$

$$C_3 = - \frac{A}{3} \ln (a_c/A) - \frac{5A}{18} \qquad (2.f)$$

where $a_q = (m_q/M_W)^2$ and $A = (m_s/M_W)^2$. The $O(a_t, a_t^2 ....)$ corrections though numerically significant in principle, do not affect Re M_{12} (see below). Their effect should be included in the computation of ϵ. The O(A) corrections are of the same order of magnitude as the corresponding terms in Ref.2. The numerical coefficients obtained in the present analysis are somewhat different. It should be noted that the B_i's are practically the same as the corresponding quantities obtained in the zero external momentum approximation. The correction $(O(m_s^2/m_c^2))$ is numerically insignificant. The significant corrections arise due to the C_i's. Using the unitarity of the KM matrix (i.e. $\lambda_u + \lambda_c + \lambda_t = 0$) and the fact that in a convenient parametrization $\mathrm{Im}\lambda_u = 0$, we finally obtain (from the real part of the amplitude) the box diagram contribution to the $K_L - K_S$ mass differences:

$$\Delta m_K^{Box} = \frac{G_F^2 m_c^2}{6\pi^2} s_1^2 c_1^2 f_K^2 \mathcal{B} \eta_1 m_K \left(1 + \frac{5}{12}\frac{m_K^2}{m_c^2} \ln \frac{m_c^2}{m_s^2}\right) \tag{3}$$

The second term in the parenthesis gives the correction to the zero external momentum approximation. In writing Eq.(3) we have used the well-known result [8] that in view of the unexpectedly large B lifetime [9] and the bound $R = \Gamma(b \to u\ell\nu)/\Gamma(b \to c\ell\nu) < 0.4$ [10] one obtains $(\mathrm{Re}\lambda_c)^2 \gg (\mathrm{Re}\lambda_c\, \mathrm{Re}\lambda_t) \gg (\mathrm{Re}\lambda_t)^2, (\mathrm{Im}\lambda_c)^2$. Using the current quark masses [11] $m_c \simeq 1{\cdot}2$ GeV and $m_s \simeq 0{\cdot}15$ GeV and the well known values for the other parameters we obtain

$$\Delta m_k^{BOX} = (0.17)\ B\ (1 + 0.31)\ 10^{-14}\mathrm{GeV} \tag{4}$$

The theoretical estimates of the $\mathcal{B}$ parameter are infested with the usual uncertainties of strong interaction physics. The popular vacuum saturation approximation [1] gives $\mathcal{B} \approx 1$ while estimates from MIT bag model [13] $(\mathcal{B} \approx 0{\cdot}4)$, current algebra [14] $(\mathcal{B} \approx 0{\cdot}33)$ and chiral perturbation theory [15] $(\mathcal{B} \approx 0{\cdot}33)$ lead to much smaller values. It is, however, interesting to note that even with the optimistic choice $\mathcal{B} = 1$, Δm_k^{BOX} in the zero external momentum approximation is somewhat smaller than the experimental value $\Delta m_k = 3.5 \times 10^{-15}$ GeV and that the correction due to the Kaon mass ($\approx$31%) shifts it in the right direction [16]. Unfortunately, the comparison of Δm_k^{BOX} with the experimental result is by no means straightforward. In addition to the already quoted uncertainties in the $\mathcal{B}$ parameter, contributions from low mass intermediate states complicate the situation [17]. Introducing an independent parameter D to represent the dispersive contributions to Δm_k one usually writes

$$\Delta m_k = \Delta m_k^{BOX} + D\Delta m_k \tag{5}$$

Using Eq.(4) it is easy to see that the consistency of the standard model requires $D = 0.4(\mathcal{B} = 1)$, $0.8(\mathcal{B} = \frac{1}{3})$ etc. This should be compared with the recent theoretical estimates i) $-0.7 < D < 3$ [18], ii) 0.67 ± 0.17 [19], iii) $D \approx 46\%$ (considering contributions form 2π intermediate states only) [20]. As is well known the uncertainties in the theoretical estimates are too large to permit any definite conclusion regarding the status of the standard model vis-a-vis Δm_k.

In the imaginary part of the amplitude the contribution proportional to m_c^2 ceases to be the most important piece and the contributions due to external momenta turns out to be insignificant in comparison to the dominant m_t^2 contribution as is seen from the following expression:

$$\begin{aligned}\mathrm{Im}\, M_{12}^{box} = \frac{G_F^2}{3\pi^2} f_K^2 m_K \mathcal{B}\, \mathrm{Im}\lambda_t \Big[&\mathrm{Re}\lambda_c (\eta_3 B_3 - \eta_1 B_1) \\ &+ \mathrm{Re}\lambda_t (\eta_2 B_2 - \eta_3 B_3) \\ &+ \frac{5}{24}\mathrm{Re}\lambda_c (\eta_3 - \eta_1)\frac{m_K^2}{m_c^2} \ln\frac{m_c^2}{m_s^2} \Big]\end{aligned} \tag{6}$$

where we have retained only the most significant correction due to the Kaon mass. Using the values of $\mathrm{Re}\lambda_t$ permitted by b decay data, $m_t \simeq 40$ GeV and already quoted values of the other parameters it is easy to see that the Kaon mass correction is indeed insignificant. Nevertheless, an interesting contribution to the observed CP violating parameter ϵ parameter may still arise as follows. The ϵ parameter is given by

$$\epsilon = \frac{1}{\sqrt{2}} \frac{e^{i\pi/4}}{\Delta m_K} \left(\mathrm{Im}\, M_{12} + 2\xi\, \mathrm{Re}\, M_{12} \right) \ldots \quad (7)$$

where $\xi = \mathrm{Im}\, A_0/\mathrm{Re}\, A_0$ with $\mathrm{Amp}(K^0 \to (\pi\pi)_I) = A_I e^{i\delta_I}$. Since the 2π intermediate state is expected to dominate the non-perturbative long range contributions to M_{12} and by definition [21] $\mathrm{Im}\, M_{12}^{(2\pi)} = -2\xi\, \mathrm{Re}\, M_{12}^{2\pi}$, where the superscript refers to the 2π intermediate state contribution, one obtains the much more convenient expression

$$\epsilon = \frac{1}{\sqrt{2}} \frac{e^{i\pi/4}}{\Delta m_K} \left(\mathrm{Im}\, M_{12}^{\mathrm{Box}} + 2\xi\, \mathrm{Re}\, M_{12}^{\mathrm{Box}} \right) \ldots \quad (8)$$

At the moment the uncertainties (theoretical and/or experimental) in the $\mathcal{B}$ parameter, m_t, $\mathrm{Re}\lambda_t$, $\mathrm{Im}\lambda_t$, ξ etc. are too large to draw any firm conclusion regarding the compatibility of the theoretical estimate of Eq. (8) with experimental data. Some interesting features of the Kaon mass corrections to the second term of Eq. (8) are, however, clear. The parameter ξ has been theoretically estimated by several authors [22]. Although the magnitude of ξ involves considerable uncertainties its sign is found to be negative. The enhancement of $\mathrm{Re}M_{12}^{\mathrm{BOX}}$ through the Kaon mass correction as discussed above, therefore, effectively, reduces the contributions of $\mathrm{Im}M_{12}^{\mathrm{BOX}}$ to ϵ. It has already been observed that if the current algebraic estimate $\mathcal{B} \approx 0.33$ is taken seriously then $\mathrm{Im}M_{12}^{\mathrm{BOX}}$ in Eq. (8) may not fit the experimental value of ϵ for $30 < m_t < 50$, the range suggested by recent UA1 results [23]. The effective reduction of $\mathrm{Im}M_{12}^{\mathrm{BOX}}$ as discussed above will worsen the fit. For example, Buras et al. (Ref. 3) have shown that (see Fig. 4 and Fig. 5) inclusion of the ξ term in Eq. (8) increases the minimum m_t required to fit ϵ by a small amount. The inclusion of the Kaon mass correction which enhances $\mathrm{Re}M_{12}^{\mathrm{BOX}}$ will therefore, further increase $(m_t)_{\min}$. Such an effect could indeed be important when m_t is measured with sufficient accuracy.

The importance of the Kaon mass correction to the standard model estimates of Δm_K and ϵ may not be fully appreciated at first sight. This is essentially due to the presence of other larger uncertainties both theoretical and experimental in the said estimates. Significant improvements in the experimental data on τ_B, R and m_t and further sharpening of the

theoretical tools for analyzing D, $\mathcal{B}$, etc. will hopefully pin down the standard model estimates of Δm_K and ϵ to reasonable accuracy. The numerically significant contribution due to the Kaon mass correction will then play a significant role in confronting the standard model with experimental results.

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REFERENCES AND FOOTNOTES

1) M.K. Gaillard and B.W. Lee, Phys. Rev. D10, 897 (1974).

2) A. Datta and K. Neogi, Phys. Rev. D20, 2441 (1979).

3) P.H. Ginsparg, S.L. Glashow and M.B. Wise, Phys. Rev. Lett. 50, 1415 (1983); A.J. Buras, W. Slominski and H. Steger, Nucl. Phys. B238, 529 (1984).

4) A recent analysis of $D^0 - \bar{D}^0$ mixing reveals that the inclusion of external momenta dramatically suppresses the amplitude. See A. Datta and D. Kumbhakar, Z. Phys. C27, 515 (1985).

5) A.J. Buras, W. Slominski and H. Steger, Nucl. Phys. B245, 369 (1984).

6) F.J. Gilman and M.B. Wise, Phys. Rev. D27, 1128 (1983).

7) M. Kobayashi and K. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

8) E.A. Paschos, B. Stech and U. Turke, Phys. Lett. 128B, 240 (1983); K. Kleinknecht and B. Renk, Phys. Lett. 130B, 459 (1983); L.-L. Chau and W. Keung, Phys. Rev. D29, 592 (1984). See also Ref. 3.

9) N.S. Lockyer et. al., Phys. Rev. Lett. 51, 1316 (1983); E. Fernandez et al., Phys. Rev. Lett. 51, 1022 (1983); For a review, see for example, R. Klanner, XXIIth Int. Conf. on High Energy Physics (Leipzig, July 1984).

10) C. Klopfenstein et al., Phys. Lett. 130B, 444 (1983); A. Chen et al., Phys. Rev. Lett. 52, 1084 (1984).

11) For the heavy flavours (c, b, t) one cannot use current algebra to compute quark mass ratios even in principle. However, it is believed (see, for example, Ref. 12) that the current algebraic masses are somewhat smaller than the constituent masses. For the light quarks (u, d, s) the difference between the two masses are estimated to be 350 MeV. Since this difference is beleived to arise due to chiral symmetry breaking in QCD and is, therefore, expected to be flavour independent, one estimates the current algebraic mass of the charmed quark to be $m_c \approx 1.2$ GeV from the "observed" (constituent) mass of 1.5 GeV [12].

12) S. Weinberg, in A Festschrift for I.I. Rabi, Ed. L. Motz (New York Academy of Science, New York 1977) p. 185.

13) R.E. Shrock and S.B. Triman, Phys. Rev. D19, 2148 (1979); For a different point of view, see P. Colić et al., Nucl. Phys. B, 221, 141 (1983).

14) J.F. Donoghue, E. Golowich and B.R. Holstein, Phys. Lett. 119B, 412 (1982).

15) A. Pich and E. de Rafael, to appear in Phys. Lett. B (1985); For a different point of view see J. Bijnens, H. Sonada and M. Wise, Phys. Rev. Lett. 53, 2367 (1984).

16) It should be noted that with $m_c = 1.5$ GeV, $B \approx 1$ and without the QCD factor one obtains $\Delta m_k^{BOX} = 0.34 \times 10^{-14}$ GeV which is the celebrated result of Ref. 1. However, since it is generally believed that the current algebraic mass should be used in the charm quark propagator the choice $m_c \approx 1.5$ GeV is questionable.

17) L. Wolfenstein, Nucl. Phys. B160, 501 (1979); C.T. Hill, Phys. Lett. 97B, 275 (1980).

18) I.I. Bigi and A. Sanda, Phys. Lett. 148B, 205 (1985).

19) J.F. Donoghue, B. Holstein and E. Golowich, Phys. Lett. 135B, 481 (1984); J.F. Donoghue and B. Holstein, Phys. Rev. D29, 2088 (1984).

20) M.R. Pennington, Phys. Lett. 153B, 439 (1985).

21) See for example, the second paper of Ref. 19.

22) F.J. Gilman and M. Wise, Phys. Lett. 83B, 83 (1979). B. Guberina and R.D. Peccei, Nucl. Phys. B163, 289 (1980); R.D.C. Miller and B.H.J. McKellar, Aust. J. Phys. 35, 235 (1982).

23) C. Rubbia, Dortmund Neutrino Conference (1984).