

RECEIVED BY OSTI FEB 12 1986

**THE OBSERVATION AND PHENOMENOLOGY OF GLUEBALLS\*****S.J. LINDENBAUM****Brookhaven National Laboratory, Upton, New York 11973****and****City College of New York, New York, New York 10031**

BNL--37412

DE86 006243

**Invited Lecture presented at the  
Summer Workshop in High Energy Physics and Cosmology  
International Centre for Theoretical Physics  
Trieste, Italy  
7-14 July 1985**

**DISCLAIMER**

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

**MASTER**

\* This research was supported by the U.S. Department of Energy under Contract Nos. DE-AC02-76CH00016 (BNL) and DE-AC02-83ER40107 (CCNY).

# THE OBSERVATION AND PHENOMENOLOGY OF GLUEBALLS\*

S.J. Lindenbaum

Brookhaven National Laboratory and City College of New York  
Dept. of Physics, Bldg. 510A      Dept. of Physics  
Upton, New York 11973 U.S.A.      138th St. and Convent Ave.  
New York, New York 10031

## INTRODUCTION

The standard model explains the electro-weak interaction by  $SU(2)_L \times U(1)$ . The critical features of the electro-weak interaction at presently attainable energies have been successfully quantitatively described by this standard electro-weak model.<sup>1)</sup>

The hadronic interactions which have been traditionally theoretically intractable, are explained in the standard model by QCD.<sup>2)</sup> The heart of QCD is the locally gauge invariant non-abelian group  $SU(3)_{\text{color}}$  which has eight colored massless gauge bosons, which self-interact. Their interaction exhibits asymptotic freedom and is described by a running coupling constant. The experimental values of  $\Lambda$  are of the order of 100 MeV. Thus the asymptotic freedom is quite "precious" even at moderate  $q^2$  and energies. Color confinement and the running coupling constant make the existence of multi-gluon resonances or glueballs<sup>3)</sup> inescapable. This has been quantitatively demonstrated by Lattice Gauge calculations.<sup>4)</sup> In fact in a pure Yang Mills Theory<sup>5)</sup> of  $SU(3)_{\text{color}}$ , glueballs would be the only hadrons in the world. The addition to the theory of quarks interacting via gluons to yield QCD should in no way remove the glueballs.

Thus in the soft QCD sector where the quark-gluon and gluon-gluon coupling constants become strong, the fact that the Particle Data Group tables contain hundreds of quark-built meson and baryon states and there is no glueball section is a serious missing link in QCD. Thus, in spite of the fact that perturbative QCD has had many successes which include quantitative ones to ~ 10-20%. The glueball missing link must be found if the theory is to survive.

---

\* This research was supported by the U.S. Department of Energy under Contract Nos. DE-AC02-76CH00016 (BNL) and DE-AC02-83ER40107 (CCNY).

Evidence has been presented for glueball candidates.<sup>6-9,16,17)</sup> However, in only one case the BNL/CCNY  $1^{G}J^{PC} = 0^{+}2^{++}$  states,  $g_T(2050)$ ,  $g_T(2300)$  and  $g_T(2350)$ , the glueball resonance hypothesis (i.e. these states are produced by 1-3 glueballs) has been shown to well explain the very striking and unusual characteristics of the data naturally within the context of QCD; whereas the published alternative explanations have been shown to be both incorrect and ruled out by the data.<sup>10-12)</sup> Thus we have very probably discovered glueballs and found the missing link in QCD.

In the workshop lecture on which this paper is based, the experimental evidence and the relevant phenomenology of glueballs was reviewed.

#### SEARCHING FOR GLUEBALLS

I. USE AN OZI<sup>13,6-8)</sup> SUPPRESSED CHANNEL WITH VARIABLE MASS SUCH AS THE REACTION  $\pi^-p \rightarrow \phi n$ . The breakdown of the OZI suppression signals a glueball. The OZI suppression is a filter which allows resonating gluons or glueballs to pass, while strongly rejecting conventional quark-built hadronic states.<sup>†</sup> Therefore as has been previously concluded, the BNL/CCNY<sup>7-8,10-12,14)</sup>  $g_T(2050)$ ,  $g_T(2300)$  and  $g_T(2350)$  are produced by glueballs.

II. LOOK IN A CHANNEL ENRICHED IN GLUONS SUCH AS THE RADIATIVE DECAY OF THE  $J/\psi$  AND SEARCH FOR NEW PHENOMENA such as the  $\iota(1440)$ ,<sup>9a)</sup> the  $\theta(1640)$ ,<sup>9b)</sup> and the  $\zeta(2220)$ .<sup>15)</sup>

III. Pattern recognition of a decuplet - a  $q\bar{q}$  nonet + glueball + decuplet with characteristic splitting and mixing. The  $g_8(1240)$ <sup>16)</sup> and the  $G(1590)$ <sup>17)††</sup> are examples.

IV. Double Pomeron exchange.<sup>7)</sup>

V.  $\phi\phi$  inclusive.<sup>18)</sup>

Method III, pattern recognition of a decuplet has glueball candidates<sup>16-17)</sup> which are relatively weak and inconclusive.

---

† Provided  $\phi\phi$  system  $J \geq 1$  so that vacuum mixing is neglectable, otherwise this vacuum mixing could possibly lead to violations of Zweig suppression, since it can lead to large departures from ideal mixing.

Method IV, double Pomeron exchange has no stand-alone glueball candidates but some indications for them.<sup>7)</sup>

Method V, the  $\phi\phi$  inclusive experiment of Booth et al.,<sup>18)</sup> shows consistency with the BNL/CCNY  $J^{PC} = 2^{++}$  resonances in moments activity, and fitting the shape of the correlated  $\phi\phi$  spectrum, requires two resonances consistent with the BNL/CCNY states.

It is generally agreed that the most prominent glueball candidates are the BNL/CCNY  $g_T$ ,  $g_{T'}$ , and  $g_{T''}$  found in the (OZI forbidden) reaction  $\pi^-p \rightarrow \phi\phi n$ , and the SLAC  $J/\psi$  radiative decay candidates (i.e. the  $\iota$  and the  $\theta$ ).

In the case of the  $\iota$  and the  $\theta$ , plausible alternative explanations<sup>19)</sup> other than the glueball resonance hypothesis have been proposed and published and have not been refuted. Thus these candidates are inconclusive.

However in the case of the BNL/CCNY  $g_T$ ,  $g_{T'}$  and  $g_{T''}$ , We have been able to refute<sup>10-12)</sup> published alternatives. I will show that these alternatives are either incorrect, or do not fit the data, or both.

Thus in my opinion the glueball resonance explanation is the only viable one (which fits the data) of those proposed and published after several years of effort by various authors. Hence I conclude the  $g_T$ ,  $g_{T'}$ , and  $g_{T''}$  are produced (to a very high probability) by glueball(s) and will spend the major portion of this paper discussing them.

#### THE OZI RULE<sup>23)</sup>

In the u,d,s quark system it has been well established experimentally and via phenomenological analyses, that  $q\bar{q}$  meson nonets exist for  $J^{PC} = 0^{-+}$ ,  $1^{--}$ ,  $2^{++}$  and  $3^{--}$ . Except for the  $0^{-+}$  nonet, all those with  $J \geq 1$  are nearly ideally mixed.<sup>24)</sup> Ideal mixing is characterized by the requirement that the singlet state be composed of an  $s\bar{s}$  pair exclusively and that the singlet of the octet states contains no strange quarks. An ideally mixed nonet is conveniently representable by Zweig's Quark Line Diagrams.

---

††. The  $G(1590)$  and the  $S^*(1750)$  may be the same particle.

The disconnected Zweig diagrams for decay and the production of the  $\phi$  and  $f'$  are shown in Figs. 1 and 2. The connected Zweig diagrams in QCD (Fig. 3) are characterized by a continuous flow of color carried by the quark lines, which allows a series of single gluon exchanges which involve strong collective soft glue effects to create or annihilate  $q\bar{q}$  pairs relatively easily, and thus gives us the relatively highly probable Zweig-allowed decay and production processes. However, when the diagram became disconnected (Figs. 1 and 2) the  $s\bar{s}$  pair in the  $\phi$  has to be annihilated or created by at least three hard gluons to conserve all quantum numbers including color and by at least two hard gluons in the case of the  $f'$ . Asymptotic freedom strongly decouples hard glue from quarks. This has been observed to occur at relatively moderate gluon energies such as those involved in the three-gluon decay of the  $\phi$  and thus is often referred to as "precocious" asymptotic freedom. The resultant relatively weak coupling constants of the hard gluons naturally explains the observed OZI suppression factors  $\sim 100$  for both  $\phi$  and  $f'$  decay and production, and the even larger OZI suppression in the decay of the  $J/\psi$  and  $T$ .

In meson states built from  $q\bar{q}$  pairs, departures from ideal mixing can only be expected to occur when flavor changing diagrams which convert  $s\bar{s}$  quark pairs into  $u\bar{u}$  or  $d\bar{d}$  quark pairs or vice versa have the connecting gluons relatively strongly coupled.

The relatively heavy  $s\bar{s}$  pairs and precocious asymptotic freedom can explain why this does not happen in  $J \geq 1$  nonets, assuming there is no flavor mixing mechanism. However if there is a flavor mixing mechanism as for example vacuum effects (i.e. instantons, etc.) which as Novikov et al.<sup>25)</sup> have pointed out are expected to be important for the  $J^{PC} = 0^{-+}$  nonet, we can get a badly mixed nonet. Novikov et al.<sup>25)</sup> estimate vacuum effects are important for  $J = 0$  but are unimportant for  $J \geq 1$ . This is certainly consistent with the experimental results which show all established nonets with  $J \geq 1$  are nearly ideally mixed whereas the well-established  $J^{PC} = 0^{-+}$  nonet is far from ideally mixed.

In QCD there is only one other basic mixing mechanism, namely the presence of glueballs with the same quantum numbers near enough to the nonet singlet masses and with the appropriate width to effectively mix with the singlets. This glueball mixing mechanism could destroy ideal

**DISCONNECTED**  
FORBIDDEN (SUPPRESSED)

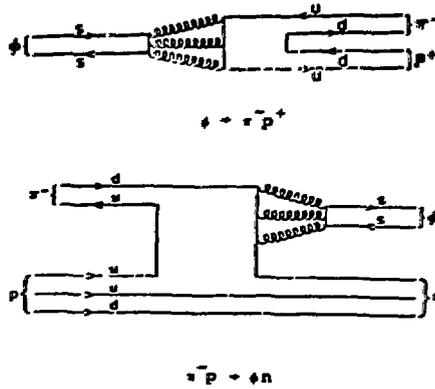


Figure 1

Zweig disconnected diagrams (suppressed reaction) for the u,d,s, quark system. The helixes represent gluons bridging the disconnection for the decay and production of the  $\phi$ .

**DISCONNECTED**  
FORBIDDEN (SUPPRESSED)

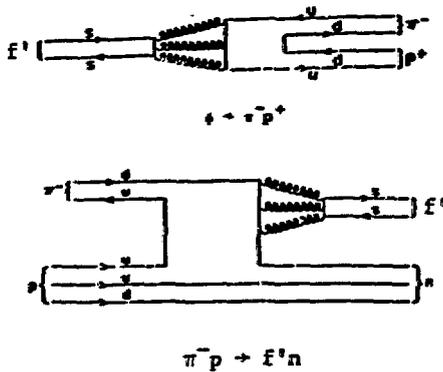


Figure 2

Zweig disconnected diagram (suppressed reaction) for the u, d, s quark system for the decay and production of the  $f'$ .

CONNECTED  
ALLOWED PROCESS

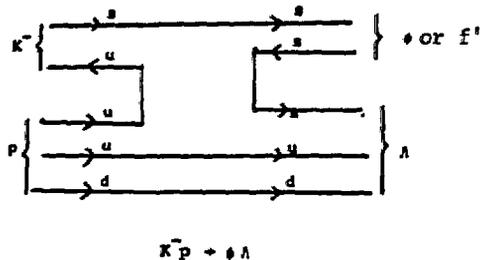
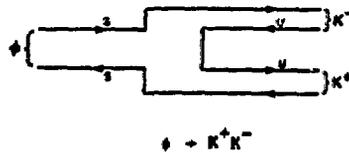


Figure 3

Zweig connected (allowed reaction) diagrams for the u,d,s quark system.

mixing and lead to a badly mixed  $J \geq 1$  nonet. Furthermore a glueball resonance is expected to be a collection of strongly coupled gluons since the gluons can easily split. As they move apart and become softer, their couplings increase (i.e. infrared slavery). Thus glueballs like other hadronic resonance are expected to be relatively strongly coupled.

It is a well-known experimental fact that in all Zweig disconnected diagrams in the  $\phi$ ,  $f'$ ,  $J/\psi$  and T systems the OZI rule appears universal as illustrated in Figs. 1 through 5. Furthermore the OZI suppression in the  $J/\psi$  and T systems is much larger than that for the  $\phi$  and  $f'$ . With u,d,s,c quarks we expect sixteen-plets and with u, d, s, c, b quarks twenty-five plets. If  $J \geq 1$  and there are no glueballs with the same quantum numbers near enough to the singlets, with the right width to cause appreciable mixing, these higher multiplets should be nearly ideally mixed and the u,d,s nonets contained within them will also be nearly ideally mixed.

DISCONNECTED

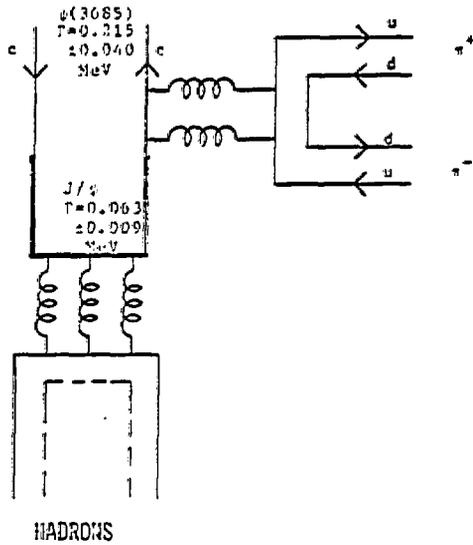
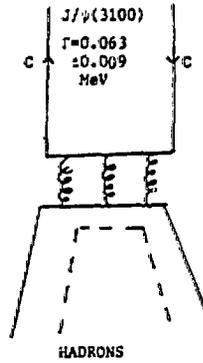


Figure 4  
Zweig disconnected diagrams in the  $J/\psi$  and excited  $\psi$  states.

DISCONNECTED ZWEIG DIAGRAMS IN  $T$  SYSTEM

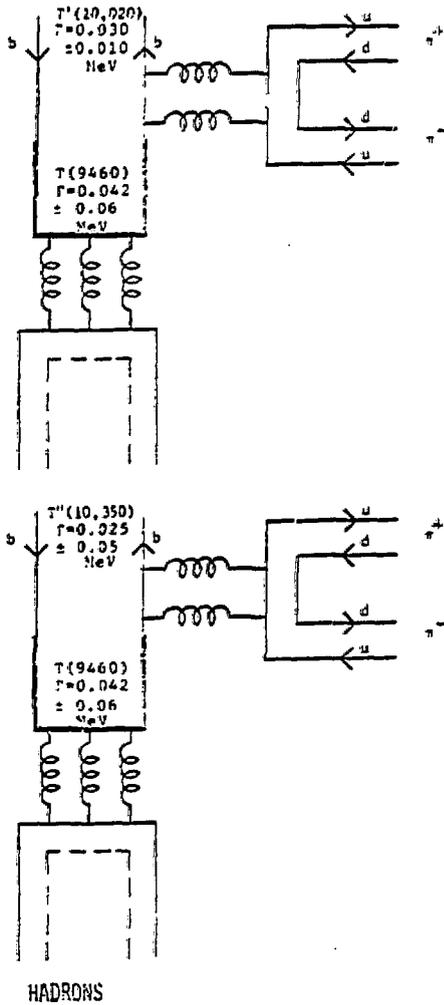


Figure 5  
Zweig disconnected diagrams in the  $T$  system.

As previously discussed,<sup>14,8,26)</sup> the OZI rule appears on paper to be defeatable by two-step processes, each of which are OZI allowed.

For example:

In decay,

- 1)  $\phi \rightarrow K^+K^- \rightarrow \rho\pi$  or  $3\pi$
- 2)  $f' \rightarrow K^+K^- \rightarrow \pi\pi$
- 3)  $f' \rightarrow \eta\eta \rightarrow \pi^+\pi^-$

In production,

- 4)  $\pi^-p \rightarrow K^+K^-n \rightarrow \phi n$
- 5)  $\pi^-p \rightarrow K^+K^-n \rightarrow f'n$
- 6)  $\pi^-p \rightarrow n\eta n \rightarrow f'n$
- 7)  $\pi^-p \rightarrow \phi K^+K^-n \rightarrow \phi\eta n^\dagger$

Reactions 1-6 are experimentally determined to be OZI suppressed clearly demonstrating that such two-step processes do not break the OZI suppression. One should note that in reactions 2, 3, 5 and 6 the intermediate step is considerably above threshold whereas in reactions 1 and 4 are just above threshold. In order to properly consider such two-step processes, the QCD dynamics, the overall quantum numbers of the system, and cancellations between all possible intermediate steps should be taken into account. The reason they probably are suppressed is partially due to the fact that hadronization in the first step takes place at the outer regions of the confinement region where the first  $q\bar{q}$  pair is far apart (and probably moving away from each other) and the coupling is strong. Then for the two-step process to occur (except in reaction 3), a  $q\bar{q}$  pair of quarks have to return to short distances where annihilation takes place and then annihilate. This is expected to be discriminated against dynamically and in fact the  $K^+$  and  $K^-$  or other particles formed in the intermediate step may already be color singlets (i.e. have their own bags) thus further inhibiting the subsequent needed annihilation of the  $q\bar{q}$  pair for the two step process to break the OZI suppression.

In the case of reaction 3, each  $\eta$  can in principle be created via its  $s\bar{s}$  component and then by vacuum mixing transform to  $u\bar{u}$  and  $d\bar{d}$

---

<sup>†</sup> The OZI suppression is expected if the  $\phi\phi$  system has  $J \geq 1$ . For  $J = 0$ , there is the possibility of vacuum mixing.

components and then by a quark rearrangement interaction  $\pi^+$  and  $\pi^-$ . This would partially break the OZI suppression with a process analogous to that proposed by Donoghue<sup>22)</sup> to break the OZI suppression in  $\pi^-p \rightarrow \phi n$ . The fact that the coupling of the  $f'$  to  $\pi^+\pi^-$  has experimentally been observed to be small in both decay and production demonstrations that such processes are unlikely enough so that they do not materially affect the OZI suppression. In the context of QCD the only way you can break the OZI suppression in the  $(u,d,s)$   $q\bar{q}$  quark states is by changing the near ideal mixing observed in the nonets for  $J \geq 1$  by a strong enough flavor changing mixing mechanism which converts  $u\bar{u}$  or  $d\bar{d} \leftrightarrow s\bar{s}$ . Vacuum effects<sup>25)</sup> can do this for  $J = 0$  (e.g., the  $J^P = 0^-$  nonet). However, they are not expected to and do not appear to affect  $J \geq 1$  nonets, and thus would not be expected to appreciably affect the  $J^{PC} = 2^{++} \phi\phi$  states observed in the BNL/CCNY experiment. The only other known basic flavor-changing mechanism is a glueball.

Figures 6, 7, and 8 are the Zweig Quark Line Diagrams for the three reactions studied by the BNL/CCNY group. Figure 9 shows a scatter plot of  $K^+K^-$  masses from the BNL/CCNY experiment which used the BNL MPS II. We see the general  $\approx$  uniform background from the reaction a)  $\pi^-p \rightarrow K^+K^-K^+K^-n$  which is OZI allowed, and the two  $\phi$  bands representing b)  $\pi^-p \rightarrow \phi K^+K^-n$  which is also OZI allowed. Where the two bands cross we have the Zweig forbidden reaction  $\pi^-p \rightarrow \phi n$ . Although there are two  $\phi$  mesons instead of one, one would expect this reaction to be more or less as forbidden as  $\pi^-p \rightarrow \phi n$  provided the  $\phi\phi$  system does not have the quantum number  $J = 0$  so that the vacuum can mix flavors.

The black spot where the two  $\phi$  bands cross shows an obviously more or less complete breakdown of the Zweig suppression. This has been quantitatively shown to be so in these reactions<sup>26)</sup> and also by comparing  $K^-$  induced  $\phi$  and  $\phi\phi$  production.<sup>27)</sup>

The black spot when corrected for double counting and resolution is  $\approx 1,000$  times the density of reaction (a) and  $\approx 50$  times the density of reaction (b). If one projects out the  $\phi$  bands, even with rather wide cuts  $\pm 14$  MeV, there is a huge  $\phi\phi$  signal which is  $\approx 10$  times greater than the background from reaction (b). The recoil neutron signal is also very clean,  $\approx 97\%$  neutron (see Figs. 10a and 10b).

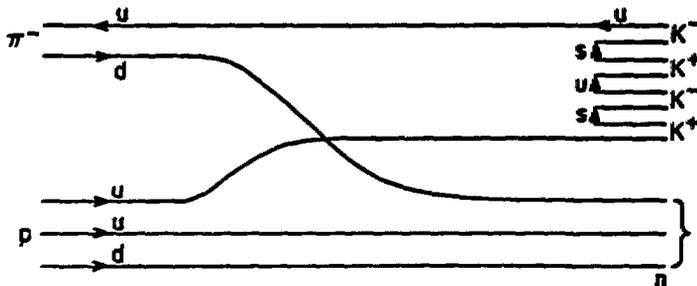


Figure 6  
 The Zweig quark line diagram for the reaction  $\pi^- p \rightarrow K^- K^+ K^- K^+ n$ , which is connected and OZI allowed.

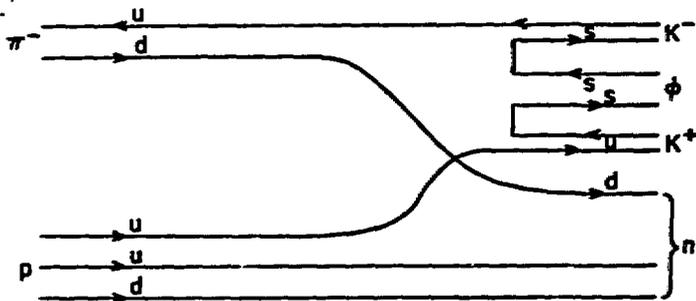


Figure 7  
 The Zweig quark line diagram for the reaction  $\pi^- p \rightarrow K^- \phi K^+ n$ , which is connected and OZI allowed.

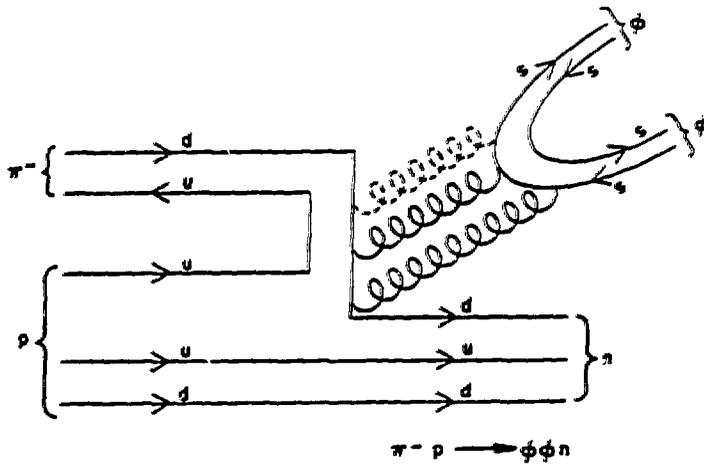


Figure 8a

The Zweig quark line diagram for the reaction  $\pi^- p \rightarrow \phi\phi n$  which is disconnected (i.e. a double hairpin diagram) and is OZI forbidden. Two or three gluons are shown connecting the disconnected parts of the diagram depending upon the quantum numbers of the  $\phi\phi$  system. For the  $8_T$ 's,  $J^{PC} = 2^{++}$ , and only two gluons are required. From the data analysis they come from the annihilation of the incident  $\pi^-$  and a  $\pi^+$  exchanged between the lower and the upper parts of the diagram.

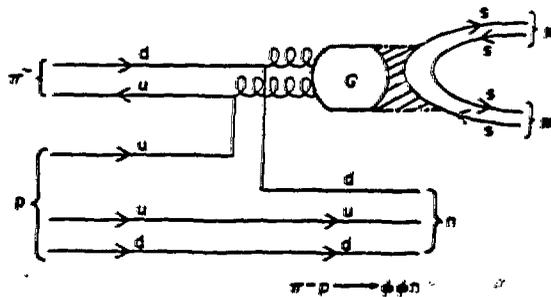


Figure 8b

The  $J^{PC} = 2^{++}$  glueball intermediate state in  $\pi^- p \rightarrow \phi\phi n$ . The dash-dot lines with crosshatch lines region indicates that we don't know details of the glueball hadronization into  $\phi\phi$ .

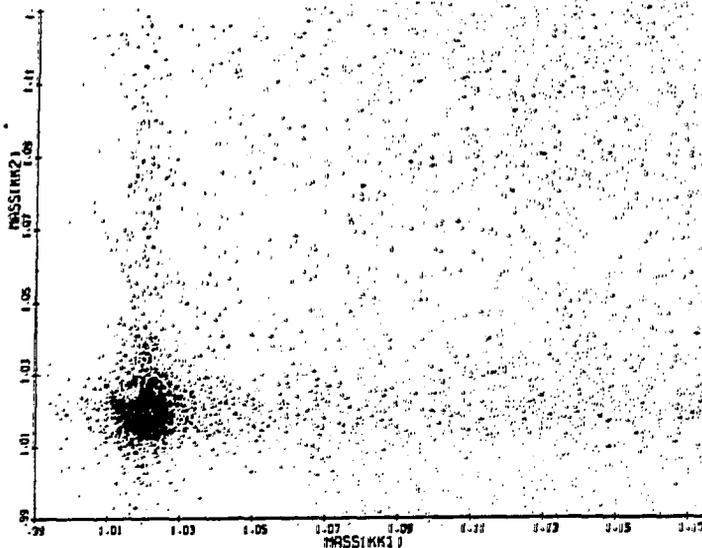


Figure 9:  
 Scatter plot of  $K^+K^-$  effective mass for each pair of  $K^+K^-$  masses. Clear bands of  $\phi(1020)$  are seen with enormous enhancement (black spot) where they overlap (i.e.  $\phi\phi$ ) showing essentially complete breakdown of OZI suppression.

Figure 10c shows the acceptance corrected  $\phi\phi$  mass spectrum in the ten mass bins which were used for the partial wave analysis.

All waves with  $J = 0 - 4$ ,  $L = 0 - 3$ ,  $S = 0 - 2$ ,  $P = \pm$  and  $\eta$  (exchange naturality)  $= \pm$  were allowed in the partial wave analysis. Thus 52 waves were considered. The incident  $\pi^-$  lab momentum vector and the lab momentum vectors of the four kaons completely specified an event. The Gottfried-Jackson frame angles  $\beta$ (polar) and  $\gamma$ (azimuthal) are shown in Fig. 11a. These and the polar angles  $(\theta_1, \theta_2)$  of the  $K^+$  decay in the  $\phi$  rest systems relative to the  $\phi$  direction and the azimuthal angles  $\alpha_1$  and  $\alpha_2$  of the  $K^+$  decay direction in the  $\phi_1, \phi_2$  rest systems (see Fig. 11b) were used to specify an event.

In the PWA, the standard LBL/SLAC isobar model program was used but modified so that the spectator particle was replaced by the second  $\phi$ . Bose statistics was satisfied by the requirements  $L + S$  must be even for the  $\phi\phi$  system. Because of the narrowness of the  $\phi$  (less than the experimental resolution  $\Gamma_{\phi\phi} \approx 8$  MeV), the partial wave analysis is

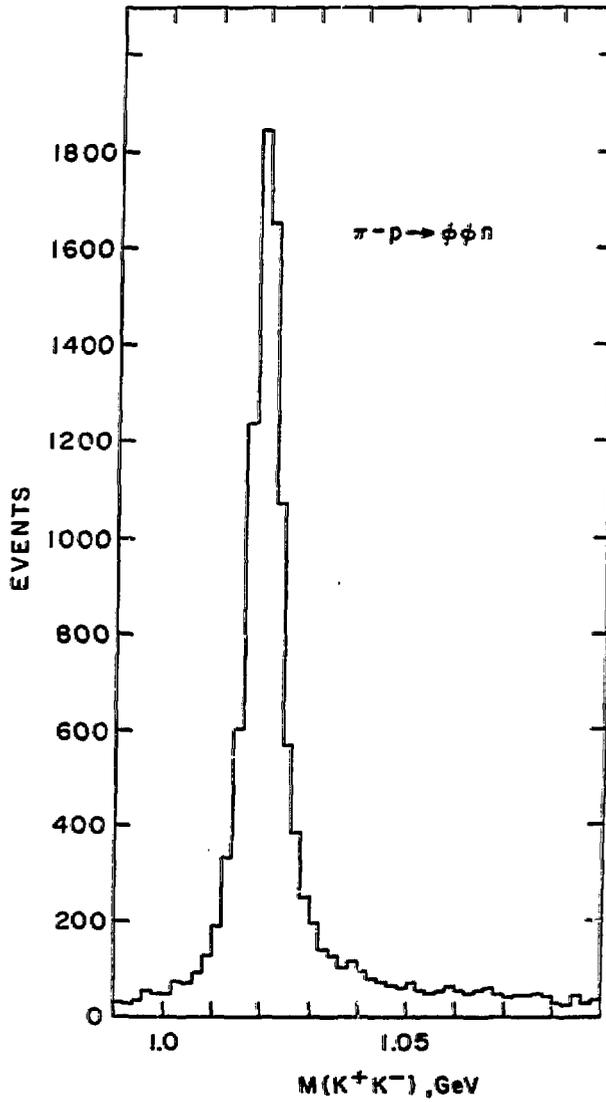


Figure 10a  
The effective mass of each  $K^+K^-$  pair for which the other pair was in the  $\phi$  mass band.

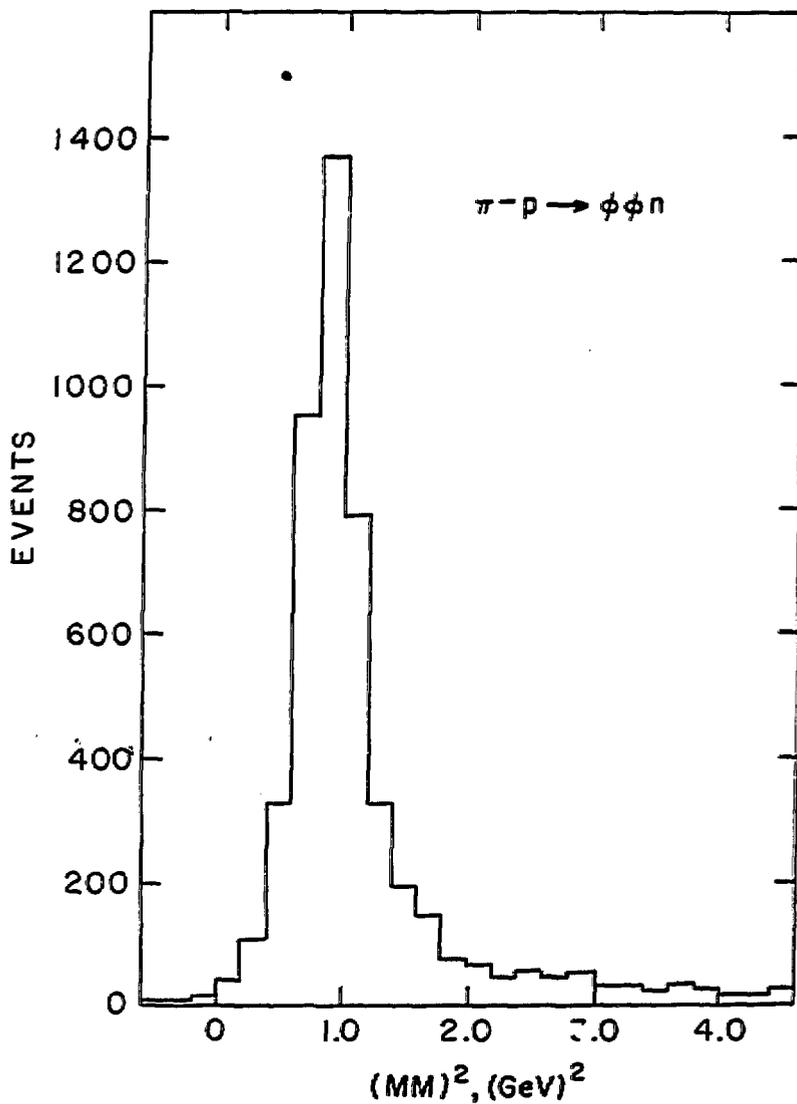


Figure 10b  
The missing mass squared for the neutral system recoiling from the  $\phi\phi$ .

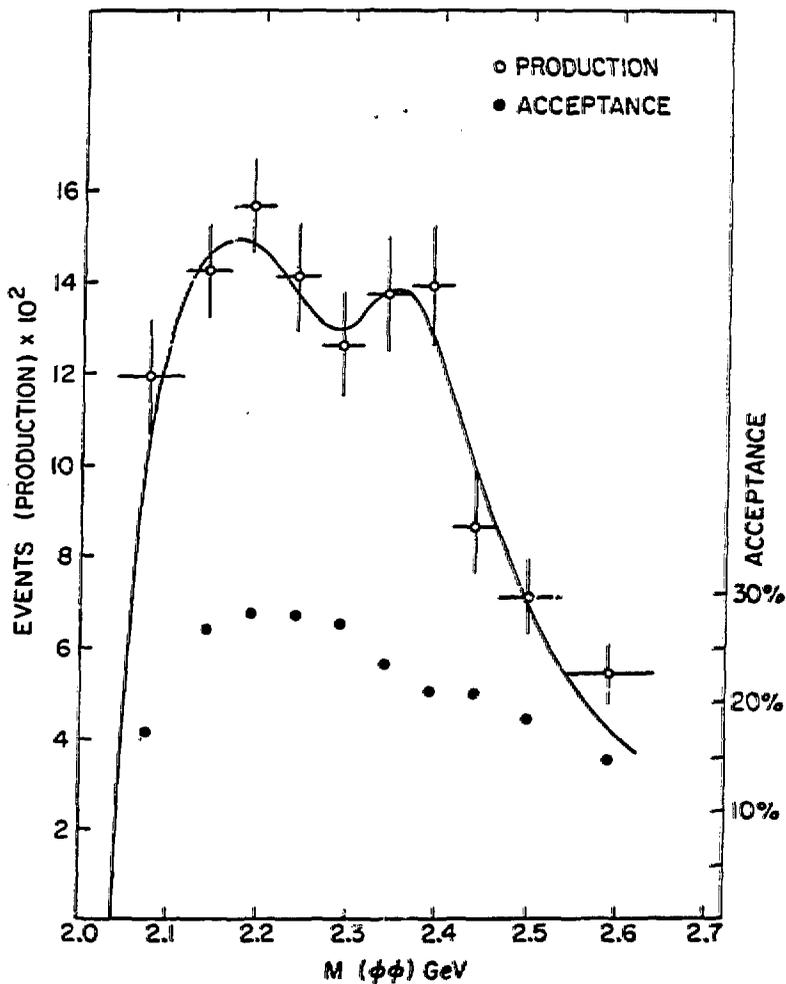
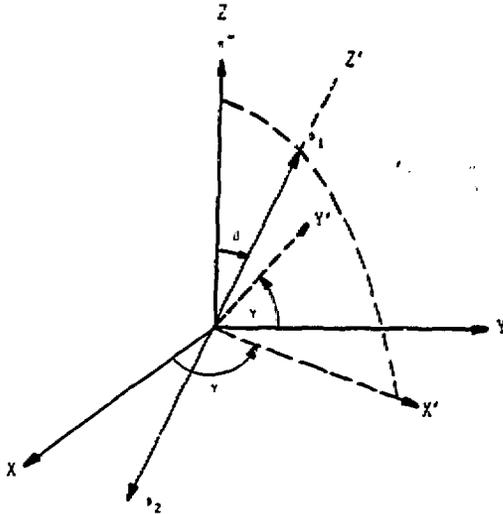


Figure 10c

The  $\phi\phi$  mass spectrum corrected for acceptance. The solid line is the fit to the data with the three resonant states to be described later. The points at the bottom of the diagram are the acceptance for each mass bin to be read with the scale at the right.

G.J. FRAME

$$\begin{aligned} Z &= \text{e}^- \text{ BEAM} \\ \hat{Y} &= \hat{P} \times \hat{N} \\ \hat{X} &= \hat{Y} \times \hat{Z} \end{aligned}$$



$\phi_1$  AND  $\phi_2$  LIE IN  $(Z, X')$  PLANE

Figure 11a: The Gottfried-Jackson frame with polar angle  $\beta$  and azimuthal angle  $\gamma$ .

REST FRAME OF  $\phi_1$

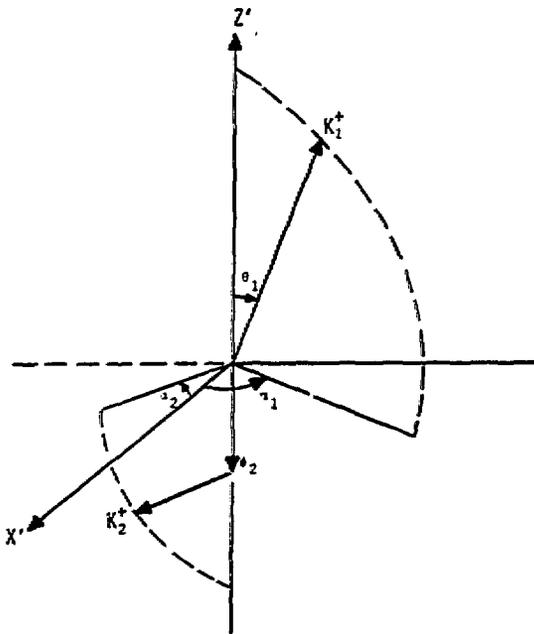


Figure 11b: The  $\phi_1$  rest frame with the polar angle  $\theta_1$  of the decay  $K_1^+$  (relative to  $\phi$  direction) and the azimuthal angle  $\alpha_1$  of the decay  $K_1^+$ .

model independent and only depends on the angular characteristics of the partial waves. Due to the spins of the two narrow decaying  $\phi$ 's, this partial wave analysis is much more powerful and selective than any other previously employed.<sup>†</sup> Thus, although we have allowed all waves up to  $L = 3$  and  $J = 4$  (i.e. 52 waves), we find a unique solution (Figs. 12a and 12b) which consists of only three  $J^{PC} = 2^{++}$  waves, an S-wave with  $S = 2$ , a D-wave with  $S = 2$ , and a D-wave with  $S = 0$ , which is a good fit. All three waves have  $J_Z = 0$  in the Gottfried-Jackson frame and the exchange naturality =  $(-1)$ , the characteristics of pion exchange. The observed  $(d\sigma/dt')_{\phi\phi} = e^{(9.4 \pm 0.7)t'}$  for  $t' < 0.3$ , the low  $t'$ -region which contains most of the data, is shown in Fig. 13. The only charged particle exchange which will give this is pion exchange. The best two-wave fit is  $\sim 30 \sigma$  away. Our selected three-wave fit is  $\sim 15 \sigma$  better than the next best three-wave fit.

The few percent background was estimated to be almost entirely composed of the reaction  $\pi^- p \rightarrow \phi K^+ K^- n$ . The wide cuts that were used allowed the background to be  $\sim 13\%$  of the  $\phi\phi n$  events. This ensured that no biases in  $\phi$  selection were introduced and also allowed the possibility of finding a coherent wave in this background to serve as a reference for our phase motion. However, the partial wave analysis of this background in the region where the  $K^+ K^-$  mass was slightly larger than that of the  $\phi$  revealed that  $\sim 65\%$  of it was structureless and incoherent. Only approximately 7% of the background was  $2^{++}$  which had an  $\sim 0$  amplitude in the threshold region and peaked at  $\sim 2.4$  GeV (see Fig. 14). Thus the  $\phi K^+ K^-$  background reaction was totally different than the  $2^{++}$  observed in the  $\phi\phi$  system. There was  $\sim 28\%$  of  $1^{--}$  background which are expected quantum numbers for a  $\phi K^+ K^-$  system where all particles are in an S-wave with respect to each other and the production is via  $\pi$ -exchange. Thus in the  $\phi K^+ K^-$  background we are dominated by the structureless incoherent background one would expect to be the result of the addition of the many possible partial waves. The  $2^{++}$  wave intensity is a small fraction of the total.

The partial wave amplitudes and phase behavior of the  $\phi\phi$  system (shown in Figs. 12a and 12b) clearly suggest that these three waves are produced by resonances. A two-pole K-matrix fit which allows all

<sup>†</sup> See Refs. 8 and 14 for a discussion of selectivity.

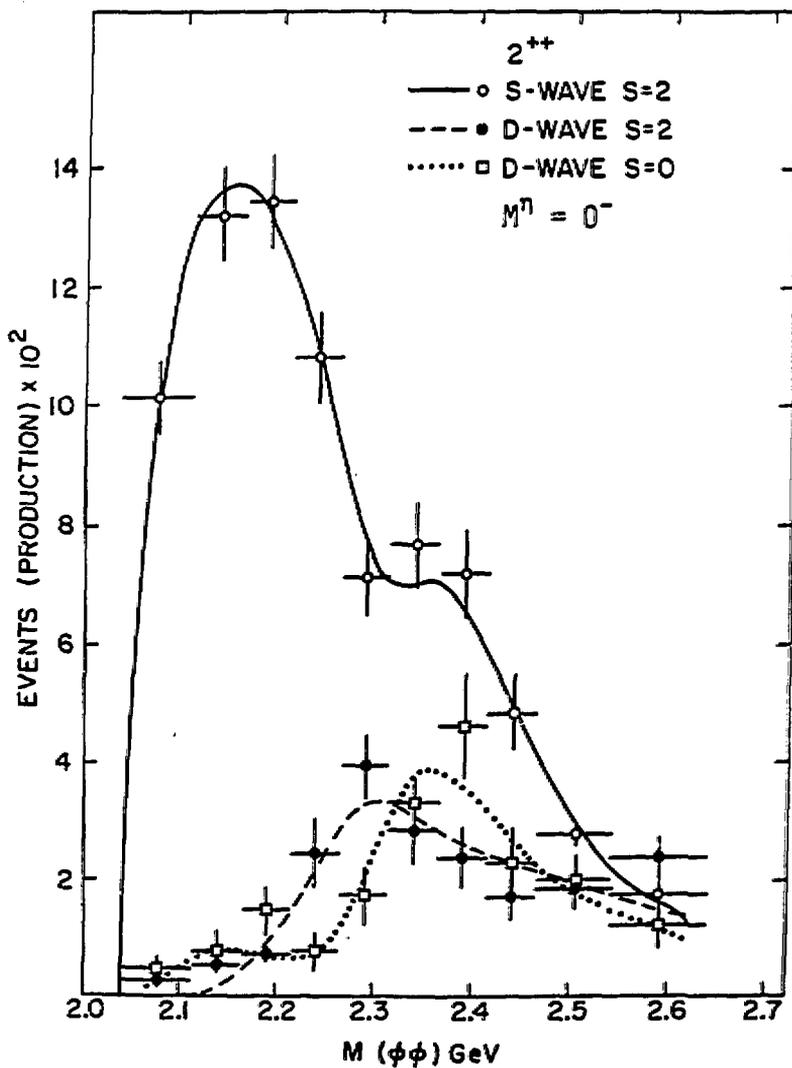


Figure 12a: The three  $J^{PC} = 2^{++}$  partial waves at production in 50 MeV mass bins (except at ends).  $J_z = 0$  in the Gottfried-Jackson frame and the exchange naturality is (-) corresponding to pion exchange for all three waves. The smooth curves are derived from a three-pole K-matrix fit.

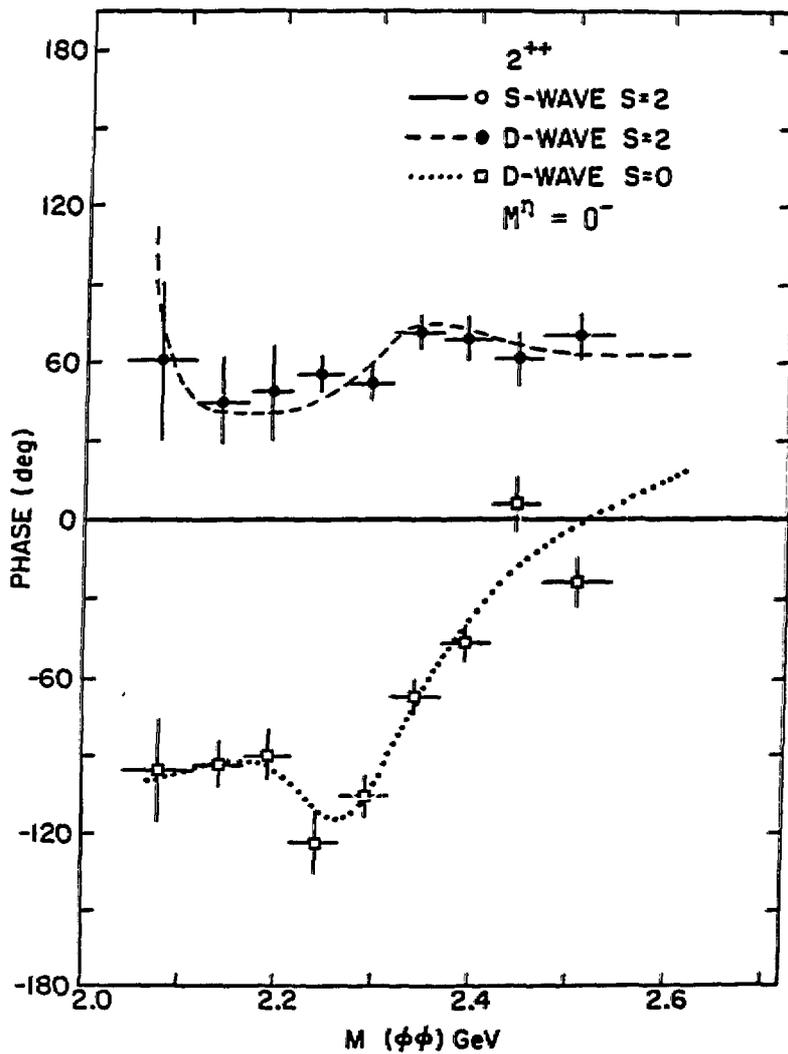


Figure 12b: D-S phase difference from the partial wave analysis vs.  $\phi\phi$  mass. The smooth curves are derived from a three-pole K-matrix fit.

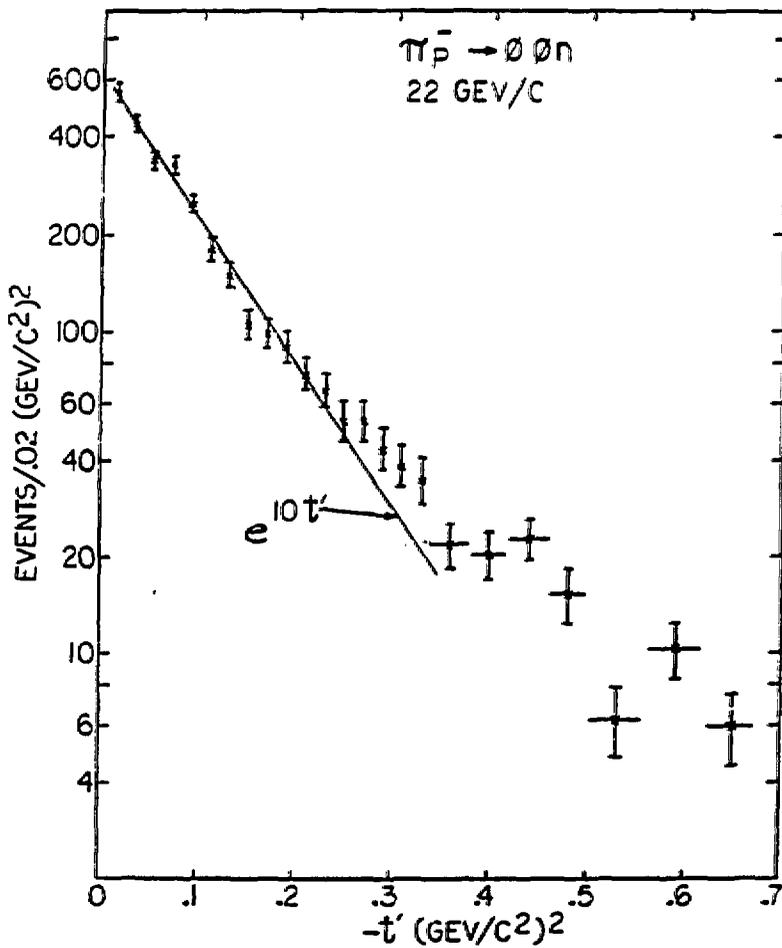


Figure 13:  $d\sigma/dt$  vs.  $t'$ .  
 For  $t' < 0.3$  which contains most of the data.  $d\sigma/dt' = e^{(9.4 \pm 0.7)t'}$  which is characteristic of pion exchange.

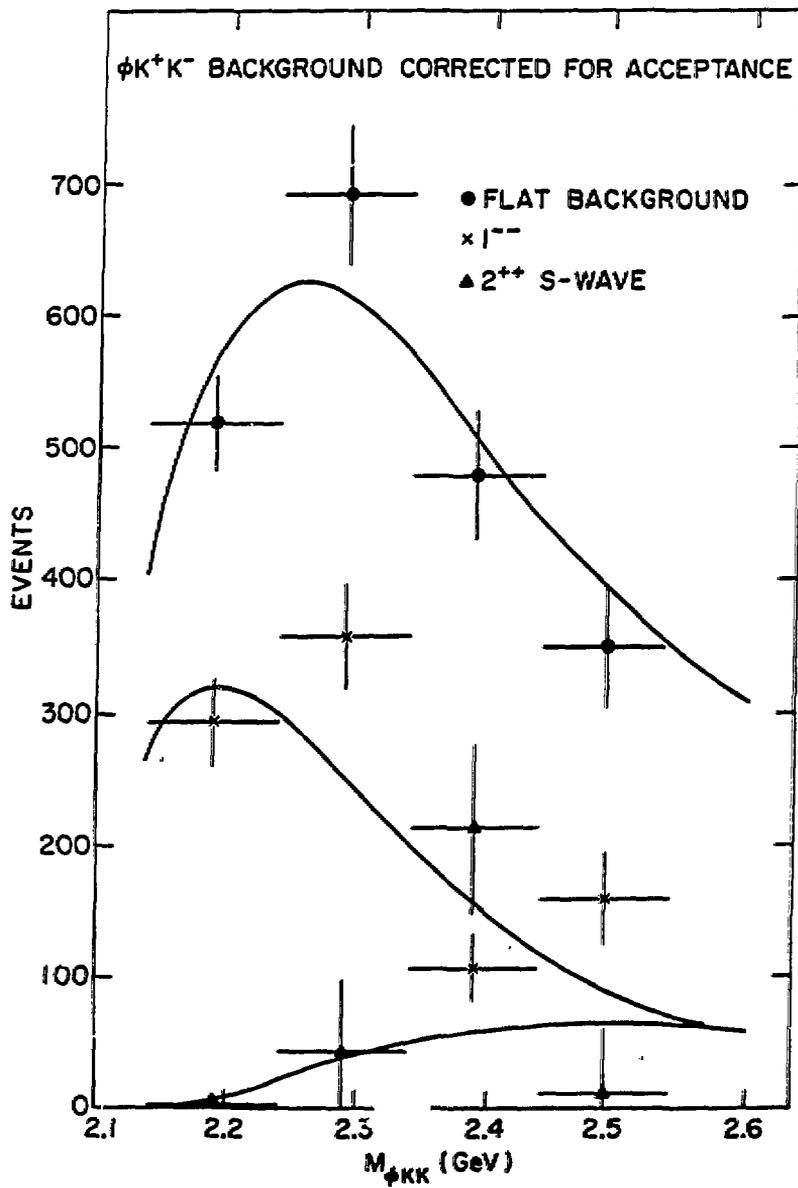


Figure 14: The partial wave intensities in the background reaction  $\pi^- p + \phi K^+ K^- n$ .

three observed waves to mix in each pole was  $\approx 15^\circ$  away from a good fit.

A three-pole K-matrix fit gave a good fit. The Argand diagram for this fit is shown in Fig. 15. A four-pole K-matrix fit did not lead to any further improvement. The three-pole K-matrix fit was used to fit the data which was contained in 90 angular variable bins for each of the ten mass bins used, thus yielding a total of 900 data bins. The fit to these 900 data bins was  $\lesssim$  a one  $\sigma$  fit. The resonance parameters of this fit are given in Table I.

Due to the small background and the fact that the background is mostly incoherent, the S-wave which dominates the  $g_T(2050)$  must be used as a phase reference. The phase difference of this and the other two D-waves precisely match the 3-pole K-matrix fit and thus clearly demonstrate that all three states have the pole behavior which is the best and only critical definition of a resonance.

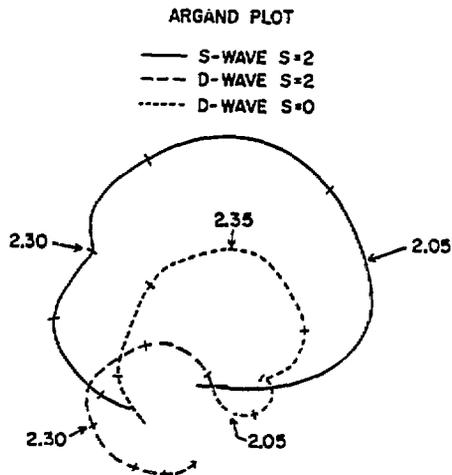
Attributing the production of these states to 1-3 primary  $J^{PC} = 2^{++}$  glueballs has explained all their features in a clear-cut and simple manner.<sup>7,8,14)</sup>

It should be noted that the mixing of waves is substantial in these three  $J^{PC} = 2^{++}$  states and the exact wave content and parameters of each resonance or K-matrix pole is therefore somewhat sensitive to details and somewhat uncertain. However from the glueball physics point of view we are at present mostly interested in the quantum numbers and general characteristics of the parameters of the resonant states and not very concerned about their exact values and wave contents.

#### GLUEBALL MASSES

The constituent (i.e. gluon has effective mass) gluon models would predict three low lying  $J^{PC} = 2^{++}$  glueballs. The mass estimates from the lattice gauge groups cover the range  $\approx 1.7$  to 2.5 GeV for  $J^{PC} = 2^{++}$  glueballs.<sup>4,28-29)</sup> with which we are clearly consistent.

T.D. Lee has analytically calculated  $J = 2$  glueballs in the strong coupling limit<sup>30)</sup> and obtains three glueball states which correspond to our three states. His strong coupling calculation gives the mass differences between these three states in terms of two



Argand plot from K-matrix fit.

parameters, one being essentially the effective strength of the coupling and the other a mass scale parameter. In order to try to adjust his strong coupling calculation to the real world of intermediate coupling we took the mass of the  $0^{++}$  glueball as  $\approx 1$  GeV from the Lattice Gauge calculations, and fit our three masses with the other parameter and found a reasonable fit.

Many years ago I employed a similar procedure in the case of the Pauli-Dancoff strong coupling calculations of the nucleon isobars. In that case when I put in the known  $f^2$  and a reasonable value for the cut-off, the strong coupling calculation results gave reasonable agreement with the experimental observations on nucleon isobars.

#### GLUEBALL WIDTH

In hadrons, the hadronization process consists of creation of one or more  $\bar{q}q$  pairs. This must occur near the outer region of confinement involving strongly interacting soft glue, including collective interactions, if we are to have resonances decay with typical hadronic widths ( $\Gamma_{\text{hadronic}} \sim 100$  to several hundred MeV). It appears that hadronic widths are more or less independent of the number of particles in the dominant decay mode.

Table 1

Resonance parameters of the three-pole K-matrix fit

$M_1 = 2.050^{+.090}_{-.050}$	$\Gamma_1 = .200^{+.160}_{-.050}$	$\approx 50\%^{+10\%}_{-10\%}$	data
S-wave, S = 2	$\approx 98\%^{+02\%}_{-70\%}$	coupling sign (+) defined	
D-wave, S = 2	$\approx 0\% + 50\%$	coupling sign (-)	
D-wave, S = 0	$\approx 2\%^{+25\%}_{-02\%}$	coupling sign (-)	
$M_2 = 2.300^{+.020}_{-.100}$	$\Gamma_2 = .200^{+.060}_{-.050}$	$\approx 20\%^{+20\%}_{-10\%}$	data
S-wave, S = 2	$\approx 30\%^{+20\%}_{-20\%}$	coupling sign (+)	
D-wave, S = 2	$\approx 50\%^{+20\%}_{-10\%}$	coupling sign (+)	
D-wave, S = 0	$\approx 20\%^{+20\%}_{-20\%}$	coupling sign (+)	
$M_3 = 2.350^{+.020}_{-.030}$	$\Gamma_3 = .270^{+.090}_{-.130}$	$\approx 30\%^{+10\%}_{-20\%}$	data
S-wave, S = 2	$\approx 40\%^{+10\%}_{-20\%}$	coupling sign (+)	
D-wave, S = 2	$\approx 05\%^{+15\%}_{-05\%}$	coupling sign (-)	
D-wave, S = 0	$\approx 55\%^{+20\%}_{-15\%}$	coupling sign (+)	

For example the  $\rho(770) \rightarrow \pi\pi$  requires production of one quark pair. The width of the  $\rho(770)$  is  $\Gamma_\rho = 154 \pm 5$  MeV. The  $\rho'(1600) \rightarrow 4\pi$  requires the production of three quark pairs.<sup>†</sup> Yet  $\Gamma_{\rho'}$  =  $300 \pm 100$  MeV. Hence even though production of two-three additional quark pairs is required the  $\Gamma_{\text{hadronic}}$  actually increases. This example clearly shows that hadronization easily occurs via collective soft glue effects and this is the basis of typical hadronic widths.

A glueball is a resonating multi-gluon system. The glue-gluon coupling is stronger than the quark-gluon coupling and thus it would be

---

<sup>†</sup> Even assuming  $\rho'(1600) \rightarrow \rho\pi\pi$  requires initial production of two additional  $q\bar{q}$  pairs.

expected, via gluon splittings before the final hadronization, to have a similar hadronization process to a  $q\bar{q}$  hadron. Therefore a glueball would be expected to have typical hadronic widths. This is clearly to be expected for ordinary (non-exotic)  $J^{PC}$  states. In the case of exotic  $J^{PC}$  states, this argument may not hold since no one yet knows what suppresses the unobserved exotic sector. Therefore Meshkov's oddballs<sup>19)</sup> may be narrow.

The observed characteristics of the reaction  $\pi^-p \rightarrow \phi\phi n$  are very unusual and striking in the following respects:

1. The expected OZI suppression is completely broken in a very unusual manner, since virtually all of the cross section is composed of three resonant  $\phi\phi$  states, the  $g_T(2050)$ , the  $g_T'(2300)$  and the  $g_T''(2350)$ , all with the same quantum numbers  $I^{GJPC} = 0^{++2^{++}}$ . In contrast, hadronic reactions in other channels show no such selective quantum numbers or resonance phenomena. For example the highest statistics<sup>31)</sup> experiment on  $\pi^-p \rightarrow K_S^0 K_S^0 n$ , which also exhibits  $\pi$ -exchange characteristics, has a slowly varying non-resonant  $2^{++}$  amplitude behavior with essentially no phase motion. The  $0^{++}$  and  $4^{++}$  amplitudes are also both populated in contrast to  $\pi^-p \rightarrow \phi\phi n$  where only  $2^{++}$  is populated. A second three times higher statistics experiment<sup>42)</sup> gives similar results.

2. The OZI allowed background reaction  $\pi^-p \rightarrow \phi K^+ K^- n$  which is unexpectedly only a few percent of the OZI forbidden  $\pi^-p \rightarrow \phi\phi n$  has been partial wave analyzed in the  $K^+ K^-$  mass region just slightly heavier than the  $\phi$  mass and  $\approx 65\%$  of the reaction consists of a structureless, incoherent, flat in all angular distributions, background which is expected for the superposition of many waves. Approximately 28% of the reaction has  $J^{PC} = 1^{--}$  which are expected quantum numbers for a  $\phi K^+ K^-$  system where all particles are in a relative s-wave. Thus this is a kinematic effect. Only  $\approx 7\%$  of the cross section is a  $2^{++}$  S-wave\* with an amplitude near zero at the threshold and peaking at  $\approx 2.4$  GeV, and exhibiting no evidence of resonant behavior. Thus it is entirely different than the  $\phi\phi$   $2^{++}$  amplitudes. This reaction has about the same threshold and similar kinematics to the  $\pi^-p \rightarrow \phi\phi n$

---

\* The  $K^+ K^-$  system and the  $\phi$  are in a relative S-wave. The  $K^+ K^-$  pair are in a relative P-wave.

reaction, thus threshold effects would be quite similar in the two reactions and it is clear that the striking characteristics of the  $\phi\phi$  data cannot be attributed to such a naive mechanism.

The above characteristics of the data can be very well explained naturally within the context of QCD, if one assumes 1-3 primary glueballs with  $J^{PC} = 2^{++}$  produce these states.<sup>7, 8, 10)</sup>

One or two primary  $J^{PC} = 2^{++}$  glueballs could break the OZI suppression and mix with nearby  $q\bar{q}$  states with the same quantum numbers and similar masses. However the simplest explanation is that we have a triplet of  $J^{PC} = 2^{++}$  glueballs. These would be in the right mass range predicted from Lattice Gauge calculations<sup>4, 28-29)</sup> and would fit the prediction of three distinct masses made by T.D. Lee.<sup>30)</sup> As is earlier described, we can adapt his calculation to fit the splitting.

#### CONCLUSION ON $\pi^-p \rightarrow \phi\phi n$

To prove that these states are produced by glueballs, as in all proofs one requires the appropriate input axioms.

##### If one assumes as input axioms:

1. QCD is correct;
2. The OZI rule is universal for weakly coupled glue in Zweig disconnected diagrams where the disconnection is due to the introduction of new flavors of quarks. All we need under Axiom 2 is the more restrictive statement that the OZI rule is operative in the reaction  $\pi^-p \rightarrow \phi\phi n$  in the absence of a resonating glue system (i.e. a glueball) for  $J \geq 1^{**}$  for the  $\phi\phi$  system (i.e. disconnected system).

Then the  $g_T$ ,  $g_{T'}$ , and  $g_{T''}$  states we observe must represent the discovery of 1-3 glueballs.

Note that axiom (2) allows only resonating glue (i.e. glueballs) to break the Zweig suppression. However, one or two

---

\*\* To avoid the possibility of vacuum mixing mechanisms affecting the OZI rule. It should also be noted that when a vacuum mixing effect is present such as in the  $O^-$  nonet  $\eta$  and  $\eta'$  you cannot draw a unique Zweig diagram due to the mixed nature of the  $q\bar{q}$  states. Therefore you do not actually have Zweig disconnected diagrams thus the statement in the first sentence under 2 is technically correct but may lead to

primary glueball(s) could break down the OZI suppression and possibly mix with two quark or other possible states.

These axioms strikingly agree with the data in the  $\phi$ ,  $f'$ ,  $J/\psi$  and  $T$  systems, and merely represent modern QCD practice. Thus the glueball resonance hypothesis naturally explains the data within the context of QCD.

#### ALTERNATIVE EXPLANATIONS AND CRITICISMS

Let us now examine alternative explanations and criticisms of the glueball resonance explanation which have been published. Recent differences<sup>32)</sup> regarding the degree of OZI forbiddance of the reaction  $\pi^-p \rightarrow \phi\phi n$  observed by BNL/CUNY have been resolved<sup>33)</sup> and it was concluded that these resonances would be OZI forbidden if they were of the  $q\bar{q}$  type and therefore they constituted strong evidence for glueball(s).

Gomm<sup>20)</sup> has argued that two gluon exchanges are not intrinsically OZI suppressed in the case of tensor mesons ( $J^{PC} = 2^{++}$ ) since the hadron wave function is antisymmetric in space. Thus the quark and anti-quark are not likely to annihilate into gluons. This argument is self inconsistent. When an  $s\bar{s}$  pair annihilates into 2 gluons, precocious asymptotic freedom will lead to an OZI suppression. If the antisymmetric wave function inhibits the  $s\bar{s}$  annihilation, the observed overall suppression would merely be increased. Thus the observed breakdown of the OZI suppression in the  $J^{PC} = 2^{++}$   $g_T$ ,  $g_T'$ , and  $g_T''$  states would be even more difficult to explain. The production ( $\pi^-p \rightarrow f'n$ ) and decay of the  $J^{PC} = 2^{++}$   $f'$  (a two-gluon exchange) clearly exhibits an OZI-like suppression<sup>34)</sup> of the same order as in the equivalent  $\phi$  interactions. This clearly demonstrates that both the production and decay of  $J^{PC} = 2^{++}$  mesons are as inhibited as those of  $J^{PC} = 1^{--}$  mesons.

As one measure of the breakdown of the OZI suppression we have used the ratio:

$$a) \quad \frac{\sigma(\pi^-p \rightarrow \phi\phi n)}{\sigma(K^-p \rightarrow \phi\phi\Lambda)} = \frac{1}{3}$$

since the numerator is OZI forbidden while the denominator is OZI allowed. Reference 20 states that this ratio is comparable to that of similar reactions, e.g.

b) 
$$\sigma(\psi \rightarrow \phi\pi\pi)/\sigma(\psi \rightarrow \omega\pi\pi) = 0.21 \pm 0.10$$

and this is used to imply two-gluon exchanges are not suppressed. In Ref. 20 it is apparently not realized that both of the reactions in b) proceed via three-gluon exchange, and are both OZI forbidden and thus have no relation to the previous ratio a) where the numerator is OZI forbidden and the denominator is OZI allowed. Since gluons are flavor blind, it is not unexpected that the  $\phi\pi\pi$  and  $\omega\pi\pi$  final states in radiative  $J/\psi$  decay (reaction b) should have the same order of magnitude cross section which is what is observed.

The fact that the final state quark lines in the numerator involves a disconnected Zweig diagram while that in the denominator does not, is not simply interpretable since the  $c\bar{c}$  annihilation creates three hard gluons and these three hard gluons which are " flavor blind then produce the final quark states. The final state hadronization process of course always involves numerous soft gluons which can take care of color conservation among the new particles produced in the final state. This paper contains many other examples of confusion and erroneous statements. Reference 20 estimates the relative importance of quark-antiquark annihilation into three versus two gluons by comparing the annihilation diagrams (Fig. 1 of Ref. 20) which split  $\omega$  and  $\rho$  and  $\pi$  and  $\eta$ , and concludes there is in general no suppression of disconnected diagrams if only two gluons need to be exchanged. This comparison in Ref. 20 is used as a general measure of two-gluon versus three-gluon exchanges, but is not realistic and very unreliable since the  $0^-$  nonet is badly mixed by vacuum effects (instantons, etc.) whereas  $J \geq 1$  nonets appear (as expected) to be relatively uninfluenced by vacuum effects<sup>25)</sup> and are observed to be " ideally mixed.

Ref. 20 then tries to explain the BNL/CCNY observations via kinematical effects which lead to mass peaks. There is no explanation in that kinematical approach for the breakdown of the OZI suppression, the selection of one set of quantum numbers for the three partial wave resonances which exhibit the classic phase as well as amplitude behavior generated by poles.

---

† The measured cross section is the sum of  $\phi\phi\Lambda$  and  $\phi\phi\Sigma^0$ . We have divided by a factor of 2 to obtain  $\phi\phi\Lambda$ .

The treatment naively focuses on giving possible qualitative and incomplete arguments for the kinematical generation of two peaks (one an S-wave and one a D-wave) in the  $\phi\phi$  mass spectrum and ignores the fact that we have detailed partial wave amplitude and phase behavior for three peaks (one S- and two D-waves), and three resonances which is not explained. The treatment is incorrect in many other respects. For example, in generating the mass values of the S and D mass peaks, Ref. 20 assumes that the two  $\phi$  masses are each produced by two hard gluons which are approximately colinear and argues that the production process should be similar to that where "hard gluons decay into lepton pairs with  $\langle P_{\perp} \rangle = 0.6$  GeV and that higher transverse momenta are strongly suppressed." This is then used to estimate the values of the mass peaks, this type of process would not explain the forward peripheral nature of the  $\phi\phi$  system observed by BNL/CCNY where  $(d\sigma/dt')_{\phi\phi} = e^{(9.4 \pm 0.7)t'}$ , and in fact would lead to much higher  $t'$ -values for the  $\phi\phi$  system than observed. The Figure caption of Fig. 3 (Ref. 20) and the entire Fig. 4 are very misleading. The quark lines in Fig. 4 (showing  $K^{\bar{p}} \rightarrow \phi\phi\Lambda$ ) are correct but showing two hard gluons each producing a  $\phi$  would only contribute a very small part of the cross section. The quark line diagram of Fig. 4 is a classic Zweig connected diagram which would proceed by the single gluon and collective soft glue exchanges at a rate which would overwhelm the two hard gluon process Gomm illustrates.

Karl et al.<sup>21)</sup> attempts to explain the  $\pi^{\bar{p}} \rightarrow \phi\phi n$  data by a semi-classical time sequential pair creation model. One should note that a proper quantum mechanical treatment could easily wash out the mass peaks obtained with this method (although a lump at threshold might remain).

The unexpected selection of only  $J^P = 2^+$  in the  $\phi\phi$  system is attributed in Ref. 21 to "At these energies the annihilation is dominated by  $u\bar{u}$  pairs with  $J_z = \pm 1$  which can only have angular momentum  $2^+$  or larger." As we can see from Fig. 8 (and the PWA), in the exclusive process we are observing  $\pi^{\bar{p}} \rightarrow \phi\phi n$ , we are annihilating a  $\pi^-$  and a  $\pi^+$  (i.e.  $\pi$  exchange) thus there is no inherent net  $J_z$  in the annihilation system which is ( $u\bar{d} = \pi^+$  annihilating  $u\bar{d} = \pi^-$ ). This is borne out clearly by our partial wave analysis which selects three waves all with  $J^{PC} = 2^{++}$ ,  $J_z = 0$  (in the Gottfried-Jackson frame)

and naturality (-) [see Fig. 12a]. These are the characteristics of  $\pi$ -exchange. Furthermore the peripheral nature of our reaction  $d\sigma/dt'(\pi^-p \rightarrow \phi\phi n) = e(9.4 \pm 0.7)t'$  for  $t' < 0.3$  is clearly indicative of  $\pi$ -exchange. This information was clearly stated in our paper<sup>8</sup> that Ref. 21 quoted as a basis for their work. Hence the Ref. 21 mechanism for selecting  $J^P = 2^+$  is clearly ruled out by the characteristics and analysis of the experiment. It should be noted that  $J_z = \pm 1$  is rejected by 27 $\sigma$  in our partial wave analysis clearly indicating that the proposal of Ref. 21 is ruled out.

The mechanism of Ref. 21 could not explain the breakdown of the OZI suppression and the clear-cut resonant phase and amplitude behavior of our data. If such effects<sup>21)</sup> were to occur at all they would occur at the level of the OZI suppression, not the much higher cross section level corresponding to the breakdown of the OZI suppression. Furthermore, crude qualitative treatments which do not attempt to explain our detailed quantitative data and PWA are not a satisfactory explanation.

Donoghue<sup>22)</sup> in his Yukon Conference paper and in his summary talk at the Maryland Conference, and also his Bari talk proposed to explain our data (at least the first one or two partial waves) by kinematical hard gluon production which falls off rapidly combined with threshold effects. Among the obvious deficiencies of these arguments are 1) There is no mechanism for selecting  $J^{PC} = 2^{++}$  only. For example, an S-wave  $\phi\phi$  system\* could have  $J^{PC} = 0^{++}$  and  $2^{++}$ , and a D-wave  $\phi\phi$  system could have  $J^{PC} = 0^{++}, 2^{++}, 4^{++}$ . Ref. 22 does not explain the complete lack of those other than  $J^P = 2^+$  waves in our data. Furthermore ad hoc assumptions and free-hand drawings as Donoghue made<sup>11)</sup> are not a satisfactory explanation of our quantitative detailed partial wave analysis and fitting of the data.

2) Donoghue grants that the third PWA peak phase motion is clear enough so that it should be considered a resonance, however he questions the resonance status of the first and possibly the second. Although we do use the S-wave as a phase reference, the phase differences between it and the second partial wave (D-wave with  $S = 2$ )

---

\* All  $J^{P+}$  quantum numbers are possible for the  $\phi\phi$  system. We have made the further restriction of  $\pi$ -exchange.

are precisely what one needs to explain the data. Secondly, if you accept the first D-wave dominated pole as a resonance (as Donoghue did),<sup>11,22b)</sup> the S-wave must have resonance phase behavior also to reproduce the observed D-S behavior.

The precise phase differences given in Fig. 12b would be very difficult (if at all possible) to generate by a non-resonant mechanism.

3) Donoghue, as stated in his Maryland Conference summary talk,<sup>22)</sup> concluded that the observed break of the OZI suppression must occur due to unitarity by the two-step real process 1)  $\pi^- p \rightarrow \eta\pi \rightarrow \phi\phi$ . He states both of these steps can proceed by a simple quark interchange. Thus  $\pi^- p \rightarrow \eta\eta n \rightarrow \phi\phi n$  would not be OZI suppressed and proceed quite strongly. He does not take into account QCD dynamics, cancellations, and the overall quantum numbers of the system in his incorrectly applied unitarity argument. If we consider this two-step process, the  $u\bar{u}$  and  $d\bar{d}$  quarks in each  $\eta$  have to be in a  $0^{-+}$  system for the vacuum mixing which is responsible for the transition  $u\bar{u}$  (or  $d\bar{d}$ ) to  $s\bar{s}$  to occur. Then in order for the two  $\eta$  mesons in the intermediate step to become two  $\phi$  mesons, each  $\eta$  must then change to a  $1^{-}$  system. This requires complex dynamical interactions which we expect would be strongly suppressed and could not account for the more-or-less complete breakdown of the OZI suppression, and its selectivity. Even if we adopt his naive view of the two-step process, there is no reason the  $\phi\phi$  system would select only  $J^{PC} = 2^{++}$ .  $J^{PC} = 0^{++}$  and  $4^{++}$  would also occur and they do not. Furthermore this two-step process could proceed by Zweig allowed single and multiple soft glue exchanges and thus his mechanism for creating dynamical peaks would evaporate.

There are other real two-step processes, such as 1)  $\pi^- p \rightarrow \eta\eta n \rightarrow f'n$ , 2)  $f' \rightarrow \eta\eta \rightarrow \pi\pi$ , 3)  $\pi^- p \rightarrow K^+K^- n \rightarrow \phi n$ , and 4)  $\pi^- p \rightarrow K^+K^- n \rightarrow f'n$ , which do not break the OZI suppression. All of these two-step processes can occur as real processes, Reactions 1, 2 and 4 being considerably above threshold. The reason (except in case 2) they probably are suppressed is at least partially due to the fact that hadronization in the first step takes place at the outer regions of the confinement region where the first  $q\bar{q}$  pair is far apart and the coupling is strong. Then for the two-step process to occur, a  $q\bar{q}$  pair

of quarks have to return to short distances where annihilation takes place and then annihilate. This is probably discriminated against dynamically and in fact the  $K^+$  and  $K^-$  or other particles formed in the intermediate step may already be color singlets (i.e. have their own bags) thus further inhibiting the subsequent needed annihilation of the  $q\bar{q}$  pair for the two step process to break the OZI suppression. In the case of Reaction 3, each  $n$  can in principle be created via its  $s\bar{s}$  component and then by vacuum mixing transform to  $u\bar{u}$  and  $d\bar{d}$  components and by quark rearrangement interactions  $\pi^+$  and  $\pi^-$ . The fact that the coupling of the  $f'$  to  $\pi^+\pi^-$  has experimentally been observed to be small in both production and decay demonstrates that such processes are unlikely enough so that they do not materially affect the OZI suppression.

In the context of QCD the only way you can appreciably break the OZI suppression in the (u,d,s)  $q\bar{q}$  quark states is by changing the near ideal mixing observed in the nonets for  $J \geq 1$  by a flavor changing mixing mechanism which converts  $u\bar{u}$  or  $d\bar{d}$  to  $s\bar{s}$ . Vacuum effects<sup>25)</sup> can do this for  $J = 0$  (the  $J^P = 0^-$  nonet). However, they are not expected to and do not appear to affect  $J \geq 1$  nonets. The only other known basic flavor changing mechanism is a glueball. Clearly Donoghue's proposed mechanism would not do this for a  $J^{PC} = 2^{++} \phi\phi$  state.

In any event there is no experimental evidence for two-step OZI allowed processes significantly breaking OZI suppression. However, Donoghue, following his own philosophy, as expressed in his summary talk<sup>22b)</sup> and transparencies,<sup>11)</sup> naively invents a model for OZI and other suppression which is totally in disagreement with many known experimental facts, and in conflict with QCD. In this model, quark exchange dominates, and each  $q\bar{q}$  pair produced or destroyed is simply counted and gives a factor  $\epsilon \ll 1$ . He then uses this model (in many cases calculated incorrectly) to explain various ratios. One obvious example why the model is unrealistic is that it is well established that the  $\rho'(1600)$ , for which the predominant decay mode is  $4\pi$  (which requires the creation of three additional  $q\bar{q}$  pairs) would have a suppression of  $\epsilon^2$  compared to the  $\rho(770)$ . Even if one takes account of the fact that the decay mode is probably  $\rho\pi\pi$  then the  $\rho'$  would still be suppressed by a factor  $\epsilon \ll 1$  compared to the  $\rho$ . Thus its

decay width would be much smaller than the  $\rho$  decay width, whereas in fact the  $\rho'$  is  $\approx$  twice as wide as the  $\rho$ .

The ratio

$$B \frac{(\rho'(1600) + 2\pi)}{(\rho'(1600) + 4\pi)} = \frac{23 \pm 7}{60 \pm 7} \approx 0.38$$

According to Donoghue's model, this ratio should be  $1/\epsilon^2$  (possibly  $1/\epsilon$  for  $\rho\pi\pi$ ) where  $\epsilon \ll 1$  and thus should be extremely large.

The fact that most hadronic widths are of the same order more-or-less independent of the number of quark pairs in the dominant decay mode shows the naivety of this model. Making jets would certainly be very difficult with Donoghue's model.

What Donoghue seems to have overlooked is that creation of additional  $q\bar{q}$  pairs (of the u,d type) by hadronization where there is no disconnected Zweig diagram, seems experimentally to cost you a factor near  $1^\dagger$  whereas creation or annihilation of a new type of  $q\bar{q}$  pair in a disconnected Zweig diagram costs you a big factor (i.e.  $\epsilon \ll 1$ ). This is explainable by general characteristics of QCD because in the hadronization corresponding to a connected Zweig diagram, collective soft glue effects can easily create additional  $q\bar{q}$  pairs, whereas precocious asymptotic freedom in QCD gives hard gluons a small coupling constant and leads naturally to the OZI rule.

Let us for the sake of argument grant Donoghue his desired breakdown of the OZI suppression. If that were to occur, he would be dealing with an OZI allowed process which can proceed via a series of soft single gluon exchanges and soft multi-gluon collective effects. Thus his hard gluon mechanism (combined with a threshold effect) for generating the mass peaks does not occur. Secondly, there would be no mechanism for selecting  $J^{PC} = 2^{++}$  only.

Furthermore, we have analyzed the OZI allowed background process  $\pi^- p \rightarrow \phi K^+ K^- n$  in the  $K^+ K^-$  mass region just above the  $\phi$  mass and found that  $\approx 65\%$  is incoherent (i.e., flat structureless background)  $\approx 28\%$  is a  $1^-$  wave and only  $\approx 7\%$  is a broad  $J^{PC} = 2^{++}$  wave which is absent at threshold and has a broad peak at about 2.4 GeV. Since the  $K^+ K^-$  pair have almost the  $\phi$  mass, this process should exhibit

---

$\dagger$  There is some phenomenological penalty for creating the first  $s\bar{s}$  pair even in a Zweig connected diagram.

threshold effects similar to any which occur in  $\phi\phi$  if the OZI suppression is broken. Due to the vast differences in the  $\phi K^+ K^- n$  and the  $\phi\phi n$  data threshold enhancement arguments to explain  $\phi\phi n$  are not plausible.

In summary, threshold enhancement effects would not select one  $J^P$ , would not break the OZI suppression and would not give the characteristics exhibited by our data.

To summarize, the alternative explanations to our conclusion that the  $g_T$ ,  $g_{T'}$ , and  $g_{T''}$  are resonances have been treated qualitatively, crudely, incompletely and incorrectly. They have not been seriously compared by the authors to the BNL/CCNY data which is quantitative, detailed and has considerable statistics. They have not explained the breakdown of the OZI suppression accompanied by the selection of virtually only three  $J^{PC} = 2^{++}$  partial wave amplitudes which exhibit pole behavior. As we have pointed out, these explanations have serious errors and deficiencies and definitely do not explain the critical features of the BNL/CCNY observations as Refs. 8 and 10 do.

Thus the  $g_T$ ,  $g_{T'}$ , and  $g_{T''}$  are three resonances with  $I^G J^{PC} = 0^+ 2^{++}$  which break down the OZI suppression and contain practically all of the reaction cross section in their mass region. These facts can be well explained by the conclusion that they are produced by glueball(s),<sup>8,10)</sup> whereas the alternative explanations<sup>20-22)</sup> discussed and similar ones have been shown<sup>11-12)</sup> to be incorrect and do not fit the experimental facts.

#### GLUEBALL CANDIDATES FROM THE $J/\psi$ RADIATIVE DECAY

The radiative decay of the  $J/\psi$  is thought to occur in leading order via the usual three gluons emitted in the annihilation of the  $c\bar{c}$  pair where one is replaced by a photon. Thus it has been argued<sup>35</sup> that the two-gluon system could recoil from the photon and preferentially form a glueball. The first and most discussed glueball candidate of this type is the iota (1440)<sup>3a)</sup> since it is a  $J^{PC} = 0^{-+}$  state seen in place of the hadronic  $E(1420)$  with  $J^{PC} = 1^{++}$ . The status of the iota (1440) with  $J^{PC} = 0^{-+}$ ,  $M = 1440^{+20}_{-15}$  and  $\Gamma = 55^{+28}_{-30}$  as of July 1982 was reviewed in the Paris Conference.<sup>3b)</sup> Some concern was expressed that the ITNEP<sup>25</sup> calculations on instanton effects would move a  $0^{-+}$  glueball up to 2.0-2.5 GeV mass region. The

alternative that the iota (1440) is a radial excitation etc. rather than a glueball has also been discussed.<sup>19)</sup> The question of whether the iota is really different from the E seen in earlier hadronic production experiments<sup>37)</sup> has also been raised.

The recent E/iota results in hadronic interactions also appear to be experiment dependent. The Dionisi et al.<sup>38)</sup> and Armstrong et al.<sup>39)</sup> results on the centrally produced D and E from  $\pi^+p$  and  $pp$  interactions at 85 GeV/c find the conventional  $J^{PC} = 1^{++}$  "hadronic E".

However a recent experiment<sup>40)</sup> in the reaction  $\pi^-p + K^+K_S^0\pi^-n$  at 8 GeV/c found, from a Dalitz plot analysis, that although the E region has a large  $1^{++}$  (S-wave mostly  $K^*K$ ) its intensity continues to rise through the E region and thus does not exhibit behavior typical of a resonant state.

This data show an E(1420) resonance in the  $J^{PC} = 0^{++}$  wave amplitude coupling mostly to  $\delta^*$  and some  $K^*K$ . They concluded this is not inconsistent with the analysis of Baillon et al.<sup>37)</sup> Thus the status of the iota is still unclear.

Another glueball candidate of this type is the  $\theta$ (1700).<sup>9b)</sup>  $J^{PC} = 2^{++}$  was favored originally with a 95% C.L. but (I understand that recently this has been improved). The resonance parameters were  $M \approx 1700 \pm 50$ .  $\Gamma \approx 160 \pm 50$ . New data in the radiative  $J/\psi$  decay were recently reported by the Mark III collaboration.<sup>15)</sup> They observed the iota in the  $K^+K^-\pi^0$  and also  $K_S^0K_S^0\pi^0$  mode. The Breit-Wigner fit parameters determined were  $M = 1.46 \pm 0.01$  GeV and  $\Gamma = 0.097 \pm 0.025$  GeV. In the case of the  $\theta$ , the Breit-Wigner parameters were determined as  $M = 1.719 \pm 0.006$  GeV.  $\Gamma = 0.117 \pm 0.023$  GeV. The iota and  $\theta$  situation did not appear to change substantially from the earlier review.<sup>36)</sup> The essentially new development was the evidence for a new narrow structure [ $\xi$ (2200)].<sup>15)</sup> Reference 41 does not find evidence for the  $\xi$ (2200).

A recent review of the glueball candidates by Fishbane and Meshkov<sup>19b)</sup> concluded that the iota and  $\theta$  were probably not glueballs but they considered alternative explanations in which they could be.

Let us consider why the BNL/CCNY  $\phi\phi$  states have not been seen in the radiative decay of the  $J/\psi$ . The new MK III results observe  $J/\psi \rightarrow \gamma\phi\phi$ .<sup>15)</sup> Their detection efficiency for  $\phi\phi$  is very low in the mass region of the  $g_T(2050)$ ,  $g_T(2300)$  and  $g_T(2350)$ . Thus they find only

- 10 events in this mass region. However if one corrects their  $\phi\phi$  mass spectrum for the detection efficiency it is not inconsistent with the shape of the mass spectrum seen by BNL/CCNY. However one should note we are comparing ~ 4,000 observed events to - 10. It appears that the MK III can only observe strong signal, narrow, high mass  $\phi\phi$  states such as the decay of the  $\eta_c$ , and thus is not likely to be able to observe the BNL/CCNY states.

The DM2 group<sup>41)</sup> has reported at the Bari Conference ~ 50  $\Upsilon\phi\phi$  events in the mass region of the BNL/CCNY experiment. At present due to the limited statistics they are unable to say whether this signal is related to the resonant structures (i.e., the  $g_T, g_{T'}, g_{T''}$ ) observed by BNL/CCNY.

#### WHY HAVE THE $g_T$ 's NOT BEEN SEEN IN OTHER CHANNELS?

One can also raise the question why some other decay mode of the  $g_T, g_{T'}$  and  $g_{T''}$  have not been seen in other hadronic production experiments or in particular in the radiative decay of the  $J/\psi$  since this is considered to be a gluon enriched channel.

First it should be noted that in a related experiment  $\pi^-Be \rightarrow \phi\phi$  inclusive<sup>18</sup> the data are found to be consistent with the  $g_{T'}$  and  $g_{T''}$  and needs two Breit Wigner resonances to explain the results. This channel would only be expected to be partially Zweig suppressed since the Zweig suppression would not apply if a  $K\bar{K}_2^0$  or  $K(\Lambda)$  pair were created.

All other hadronic production experiments involve OZI-allowed channels therefore one would expect the  $g_T$ 's to be submerged in the many other hadronic states one could expect. Thus their detection would likely require very large statistics and even then it might be quite difficult to separate these from the other hadronic states.

Figure 16 shows the results of the analysis of a 23 GeV/c  $\pi^-p \rightarrow K_S^0 K_S^0 n$  experiment.<sup>31)</sup> In the mass region of the  $g_T$ 's, the  $J^{PC} = 2^{++}$  amplitude behavior is smooth and structureless and shows no phase motion. Furthermore, the  $0^{++}$  and  $4^{++}$  states are also populated unlike the selection of only  $J^{PC} = 2^{++}$  in  $\pi^-p \rightarrow \phi\phi n$ . This experiment has had its statistics raised by a factor of ~ 3 recently<sup>42)</sup> and the results are the same. This is what I would expect when the effects of

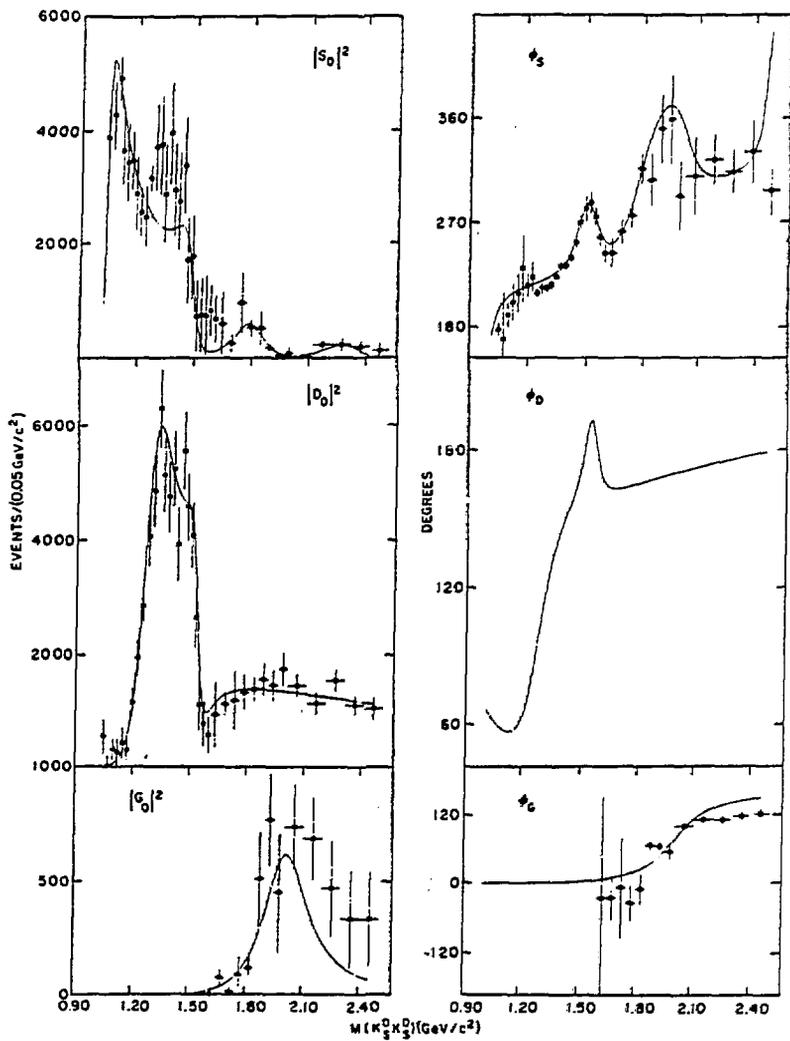


Figure 16: The square of the moduli of the  $S_0$ ,  $D_0$ , and  $G_0$  amplitudes together with their absolute phases from the best fit as functions of  $K_0^0 K_0^0$  effective mass for  $t' < 0.1 (\text{GeV}/c)^2$ . The solid curves are the results of our preferred mass-dependent fit in the same  $t'$  interval (Ref. 31).

the OZI suppression filter action are eliminated as they are in this reaction.

As to why the  $g_T$ 's have not yet been seen in the radiative  $J/\psi$  decay I would suggest the following:

We argue the Zweig suppression in our channel (with a pure glue intermediate state) should filter out other hadronic states and give a highly enriched sample of glueballs. What we found in the data is certainly consistent with this, namely we find three new states with the same quantum numbers and nothing else accompanied by very little background.

In the  $J/\psi$  radiative decay  $\approx 90\%$  of the observed states are known conventional ones and thus it is at most an inefficient filter for glueballs. If it really were almost completely glueball dominated, then we could have the reaction  $J/\psi + \gamma$  [glueball(s)] dominating. Since I would expect glueballs to be relatively strongly coupled (note perturbation theory arguments do not apply to resonant states of glue or anything else) then the width of the  $J/\psi$  would be expected to be broadened. The reason is that if the glueball is strongly coupled, the only suppressant left is the weak coupling of the  $\gamma$ . Thus widths of as much as a few MeV instead of .06 MeV might be expected for the  $J/\psi$ . Hence the fact that the  $J/\psi$  is so narrow implies to me that its radiative decay is not dominated by glueballs, which is consistent with the experimental observations. Furthermore perturbation theory gives the experimental ratio of the radiative decay to the total decay width. If glueballs were strongly coupled in the radiative decay channel I would expect the percentage of radiative decay to be higher than that predicted by perturbation theory where the two gluons are weakly coupled. Sinha<sup>44</sup> concludes that  $2^{++}$  glueballs are weakly coupled to the radiative  $J/\psi$  decay.

When one considers the very limited statistics gathered in the  $J/\psi$  radiative decay channels combined with the inefficient glueball filter nature of this channel, I am not surprised that the  $g_T$ 's have not yet been seen in it.

In regard to the  $g_T$ 's, it is also worth noting that Chanowitz and Sharpe<sup>43)</sup> have concluded that strange quarks may well be favored in glueball decay and in particular in the  $\phi\phi$  S-wave.\*

Furthermore I would like to point out that except for color, the quantum numbers of a gluon and a  $\phi$  are the same. Thus one can imagine that gluons would like to go into  $\phi$  mesons just like photons like to go into vector mesons (i.e. similar to VDM). Of course the color must be changed to a singlet but such color rearrangements might perhaps easily be accomplished by soft gluon exchanges in the final hadronization. Thus this may also be another reason why the  $g_T$ 's if they are glueballs are only seen in the  $\phi\phi$  decay mode. In this regard if sufficient statistics are gathered in  $J/\psi \rightarrow \gamma\phi\phi$  some evidence for the  $g_T$ ,  $g_{T'}$ , and  $g_{T''}$  states may be seen.

#### CONCLUSIONS ON THE STATUS OF GLUEBALL STATES

One can prove the  $g_T$ ,  $g_{T'}$ , and  $g_{T''}$  are glueballs with the appropriate input axioms. Then as we concluded previously the  $g_T(2050)$ ,  $g_{T'}(2300)$  and  $g_{T''}(2350)$  are produced by 1-3 primary  $J^{PC} = 2^{++}$  glueballs, if you assume as input axioms:

1. QCD is correct.
2. The OZI rule is universal for weakly coupled glue in disconnected Zweig diagrams where the disconnection is due to the creation or annihilation of new flavor(s) of quark(s), and  $J \geq 1$  for the disconnected system (to avoid possible vacuum mixing effects).

We have previously stated that the BNL/CCNY  $g_T(2050)$ ,  $g_{T'}(2300)$  and  $g_{T''}(2350)$  are naturally explained within the context of QCD by concluding they are produced by 1-3 primary glueballs. One or two broad primary glueballs could in principle break down the OZI suppression and mix with one or two quark states which accidentally have the same quantum numbers and nearly the same mass. However the simplest explanation of the rather unusual characteristics of our data is that we have found a triplet of  $J^{PC} = 2^{++}$  glueball states.

---

\* They also have meikton states breaking the OZI suppression and possibly being associated with our states as well as glueballs. However this argument depends on bag calculations and the dynamical mechanisms are not clear.

Alternatives to the Glueball resonance explanation have been discussed earlier and found to be incorrect or do not fit the data or both.

The  $\text{iota}(1440)$  and the  $\theta(1700)$  observed in  $J/\psi$  radiative decay are glueball candidates, the pros and cons of which have been discussed briefly here and more extensively in the references cited. Other glueball candidates<sup>16-17)</sup> are relatively weak ones, and recent glueball searches<sup>7)</sup> have not yet led to definite candidates.

## REFERENCES

1. A. Salam, Elementary Particle Theory, Nobel Symposium, Ed. N. Svartholm (Wiley Interscience 1968); S. Glashow, J. Iliopoulos, L. Maiani, Phys. Rev. D 2, 1285 (1970); S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).
2. H. Fritzsch and M. Gell-Mann, XVI Int. Conf. on High Energy Physics, Chicago-Batavia, 1972, Vol. 2, pp. 135; H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. 47B, 365 (1973); S. Weinberg, Phys. Rev. Lett. 31, 49 (1973); S. Weinberg, Phys. Rev. D 8, 4482 (1973); D.J. Gross and F. Wilczek, ibid, 3633 (1973).
3. a) Fritzsch and Minkowski, Nuovo Cimento 30A, 393 (1975).  
 b) R.P. Freund and Y. Nambu, Phys. Rev. Lett. 34, 1645 (1975).  
 c) R. Jaffee and K. Johnson, Phys. Lett. 60B, 201 (1976).  
 d) Kogut, Sinclair and Susskind, Nucl. Phys. B114, 199 (1975).  
 e) D. Robson, Nucl. Phys. B130, 328 (1977). F) J. Bjorken, SLAC Pub. 2372.
4. a) C. Michael and I.. Teasdale, Glueballs From Asymmetric Lattices, Liverpool Univ. Preprint LTH-127, March 1985.  
 b) Ph. de Forcrand, G. Schierholz, H. Schneider and M. Teper, Phys. Lett. 152B, 107 (1985).  
 c) S.W. Otto and P. Stolorz, Phys. Lett. 151B, 428 (1985).  
 d) G. Schierholz and M. Teper, Phys. Lett. 136B, 64 (1984).  
 e) Berud Berg, The Spectrum in Lattice Gauge Theories, DESY Preprint 84-012, February 1984.
5. C.N. Yang and R.L. Mills, Phys. Rev. 96, 191 (1954).
6. S.J. Lindenbaum, C. Chan, A. Etkin, K.J. Foley, M.A. Kramer, R.S. Longacre, W.A. Love, T.W. Morris, E.D. Platner, V.A. Polychronakos, A.C. Saulys, Y. Teramoto, C.D. Wheeler. A New Higher Statistics Study of  $\pi^+p \rightarrow \pi^+n$  and Evidence for Glueballs. Proc. 21st Intern. Conf. on High Energy Physics, Paris, France, 26-31 July 1982, Journal de Physique 43, P. Petitau and M. Porneuf, Editors (Les Editions de Physique, Les Ulis, France), pp. C3-87 - C3-88; A. Etkin, K.J. Foley, R.S. Longacre, W.A. Love, T.W. Morris, E.D. Platner, V.A. Polychronakos, A.C. Saulys, C.D. Wheeler, C.S. Chan, M.A. Kramer, Y. Teramoto, S.J. Lindenbaum. The Reaction  $\pi^+p \rightarrow \pi^+n$  and Evidence for Glueballs. Phys. Rev. Lett. 49, 1620-1623 (1982).
7. S.J. Lindenbaum. Hadronic Production of Glueballs. Proc. 1983 Intern. Europhysics Conf. on High Energy Physics, Brighton, U.K., July 20-27, 1983, J. Guy and C. Costain, Editors (Rutherford Appleton Laboratory), p. 351-360.
8. S.J. Lindenbaum. Production of Glueballs. Comments on Nuclear and Particle Physics 13, #6, 285-311 (1984).
9. a) Edwards et al., Phys. Rev. Lett. 49, 259 (1982);  
 b) Edwards et al., Phys. Rev. Lett. 48, 458 (1982).

REFERENCES (continued)

10. Lindenbaum, S.J. The Glueballs of QCD and Beyond. Invited Lecture. Proc. 22nd Course of the International School of Subnuclear Physics on "Quarks, Leptons and their Constituents," Erice, Trapani-Sicily, Italy, 5-15 August 1984, (to be published).
11. Lindenbaum, S.J. and Longacre, R.S. The Glueball Resonance and Alternative Explanations of the Reaction  $\pi^+p + \pi^+n$ . Hadron Spectroscopy - 1985 (International Conference, Univ. of Maryland), S. Oneda, Editor, AIP Conf. Proc. 132, p. 51-66 (American Institute of Physics, New York, 1985).
12. Lindenbaum, S.J. and Longacre, R.S. The Glueball Resonance and Alternative Explanations of the Reaction  $\pi^+p + \pi^+n$ . Phys. Lett. B (in press).
13. A. Etkin, K.J. Foley, J.H. Goldman, W.A. Love, T.W. Morris, S. Ozaki, E.D. Platner, A.C. Saulys, C.D. Wheeler, E.H. Willen, S.J. Lindenbaum, M.A. Kramer, U. Mallik, Phys. Rev. Lett. 40, 422-425 (1978); Phys. Rev. Lett. 41, 784-787 (1978).
14. S.J. Lindenbaum, Status of the Glueballs. Invited Lecture. Proc. of the 21st Course of the International School of Subnuclear Physics on "How Far we are From the Electroweak Interactions and the Other Gauge Forces", Erice, Trapani-Sicily, 3-14 August, 1983 (to be published).
15. a) K. Einsweiler. Proc. 1983 Intern. Europhysics Conf. on High Energy Physics, Brighton, U.K., July 20-27, 1983, J. Guy and C. Costain, Editors (Rutherford Appleton Laboratory), p. 348-350;  
 b) D. Hitlin, Radiative Decays and Glueball Searches. Proc. of the 1983 Int. Symposium on Lepton and Photon Interactions at High Energies, Cornell University, August 4-9, 1983, David G. Cassel and David L. Kreinick, Editors, pp. 746-778.  
 c) C. Heusch, Proc. 22nd Course of the International School of Subnuclear Physics on "Quarks, Leptons and their Constituents," Erice, Trapani-Sicily, Italy, 5-15 August 1984, (to be published).
16. A. Etkin, K.J. Foley, R.S. Longacre, W.A. Love, T.W. Morris, S. Ozaki, E.D. Platner, V.A. Polychronakos, A.C. Saulys, Y. Teramoto, C.D. Wheeler, E.H. Willen, K.W. Lai, S.J. Lindenbaum, M.A. Kramer, U. Mallik, W.A. Mann, R. Merenyi, J. Marraffino, C.E. Roos, M.S. Webster, Phys. Rev. D 25, 2446 (1982).
17. Binon et al., Il Nuovo Cimento 78A, 313 (1983).
18. a) Booth et al., Angular Correlations in the  $\pi\pi$  System. Proc. of the XXII Intern. Conf. on High Energy Physics, Leipzig, July 1984.  
 b) Booth et al., A High Statistics Study of the  $\pi\pi$  Mass Spectrum. Proc. of the XXII Intern. Conf. on High Energy Physics, Leipzig, July 1984. See paper by Chung, et al.

(continued)

REFERENCES (continued)

18. c) Booth et al., A High Statistics Study of the  $\phi\phi$  Mass Spectrum, to be published in Zeitschrift der Physik.
19. a) S. Meshkov, Proc. of the Seventh Intern. Conf. on Experimental Meson Spectroscopy, April 14-16, 1983, Brookhaven National Laboratory, S.J. Lindenbaum, Editor, AIP Conf. Proc. No. 113, p. 125-156.  
b) P.M. Fishbane and S. Meshkov, Comments on Nuclear and Particle Physics 13, 325 (1984).
20. H. Gomm, Phys. Rev. D30, 1120 (1984).
21. G. Karl, W. Roberts and N. Zagury, Phys. Lett. 149B (1984) 403; G. Karl, Hadron Spectroscopy - 1985 (International Conference, Univ. of Maryland, S. Oneda, Editor, AIP Conf. Proc. 132, p. 73 (American Institute of Physics, New York, 1985).
22. a) J. Donoghue, The Status of Unusual Meson Candidates, Proc. of the Yukon Advanced study Institute: The Quark Structure of Matter, August 11-27, 1984, (to be published); UMHEP-209.  
b) J. Donoghue, Theory Summary, Hadron Spectroscopy - 1985 (International Conference, Univ. of Maryland, S. Oneda, Editor, AIP Conf. Proc. 132, p. 460 (American Institute of Physics, New York, 1985).
23. a) S. Okubo, Phys. Lett. 5, 165 (1963); Phys. Rev. D16, 2336 (1977).  
b) G. Zweig, CERN REPORTS TH401 and 412 (1964).  
c) J. Iizuba, Prog. Theor. Physics, Suppl. 37-38, 21 (1966); J. Iizuba, K. Okuda and O. Shito, Prog. Theor. Phys. 35, 1061 (1966).
24. Particle Data Group, Review of Particle Properties, Review of Modern Physics 56, No. 2, Part II (April 1984).
25. V.A. Novikov et al., Nucl. Phys. B191, 301 (1981).
26. a) S.J. Lindenbaum, Hadronic Physics of  $q\bar{q}$  Light Quark Mesons, Quark Molecules and Glueballs. Proc. of the XVIII Course of the International School of Subnuclear Physics, July 31-August 11, 1980, Erice, Trapani, Italy, Subnuclear Series, Vol. 18, "High Energy Limit", Ed. A. Zichichi, pp. 509-562; b) S.J. Lindenbaum, Il Nuovo Cimento, 65A, 222-238 (1981).
27. S.J. Lindenbaum. The Discovery of Glueballs. Surveys in High Energy Physics, Vol. 4, 69-126, John M. Charap, Editor (Harvard Academic Publishers, London, 1983).
28. a) B. Berg and A. Billoire, Nucl. Phys. B221, 109 (1983); B226, 405 (1983).  
b) M. Teper, Proc. 1983 Intern. Europhysics Conf. on High Energy Physics, Brighton, U.K., July 20-27, 1983, J. Guy and C. Costain, Editors (Rutherford Appleton Laboratory); Preprint LAPP-TH-91 (1983).

REFERENCES (continued)

29. See Ref. 19 for a fuller discussion of this topic.
30. T.D. Lee. Time as a dynamical variable CU-TP-266; Talk at Shelter Island II Conf., June 2, 1983; also, Proc. of the 21st Course of the International School of Subnuclear Physics on "How Far we are From the Electronuclear Interactions and the Other Gauge Forces", Erice, Trapani-Sicily, 3-14 August 1983, to be published.
31. Etkin, A., Foley, K.J., Longacre, R.S., Love, W.A., Morris, T.W., Ozaki, S., Platner, E.D., Polychronakos, V.A., Saulys, A.C., Teramoto, Y., Wheeler, C.D., Willen, E.H., Lai, K.W., Lindenbaum, S.J., Kramer, M.A., Mallik, U., Mann, W.A., Merenyi, R., Marraffino, J., Roos, C.E., Webster, M.S. Amplitude Analysis of the  $K^0 K^0$  System Produced in the Reaction  $\pi^+ p \rightarrow K^0 K^0 n$  at 23 GeV/c. *Phys. Rev. D* 25, 1786-1802 (1982).
32. a) H.J. Lipkin, *Phys. Lett.* 124B (1983) 509; *Nucl. Phys.* B224, 147 (1984).  
b) S.J. Lindenbaum, *Phys. Lett.* 131B (1983) 221; also Ref. 11.
33. S.J. Lindenbaum and H.J. Lipkin, *Phys. Lett.* 149B (1984) 407.
34. A.J. Pawlicki *et al.*, *Phys. Rev.* D15, 3196 (1977); Particle Data Group, *Reviews of Modern Physics* 56, No. 2, Part II, April 1984 which lists  $f' \rightarrow 2\pi$  as possibly seen, thus the suppression is clearly large.
35. M. Chanowitz, *Phys. Rev. Lett.* 46, 981 (1981).
36. E. Bloom. Proc. 21st Intern. Conf. on High Energy Physics, Paris, France, 26-31 July 1982, *Journal de Physique* 43, P. Petiau and M. Porneuf, Editors (Les Editions de Physique, Les Ulis, France), pp. C3-407.
37. P. Baillon. Resonance in the  $KK\pi$  System Below 1.6 GeV/c<sup>2</sup>. Experimental Meson Spectroscopy -- 1983 (Seventh International Conference, Brookhaven), S.J. Lindenbaum, Editor, AIP Conf. Proc. No. 113, p. 78-106 (American Institute of Physics, 1984).
38. Dionisi *et al.*, *Nucl. Phys.* B169, 1 (1980).
39. T.A. Armstrong ( $\pi^+/p$ ) $p \rightarrow \pi^+/p K\bar{K}\pi$  at 85 GeV/c, CERN/EP 84-88; also submitted to XXII Intern. Intern. Conf. on High Energy Physics, Leipzig, July 17-25, 1984.
40. S.U. Chung *et al.*, *Phys. Rev. Lett.* 55, 779 (1985).
41. B. Jean Marie, DM2 Results on Hadronic and Radiative  $J/\psi$  Decays, Proc. of the Intern. Europhysics Conf. on High Energy Physics, Bari, Italy, 18-24 July 1985 (to be published); Preprint, LAL 85-27, July 1985. This paper contains other results relevant to the *iora*, <sup>9</sup>.

REFERENCES (continued)

42. Brandeis/BNL/CCNY/Duke/Notre Dame Collaboration, private communication.
43. M.A. Beg. Dynamical Symmetry Breaking and Hypercolor. Proc. XXth Intern. Conf. on High Energy Physics, July 1980, Madison, Wisconsin, pp. 489-492 (AIP Conf. Proceedings No. 68, Part I).
44. R. Sinha, Isoscalar Meson Mixing and Glueballs, Preprint (1985).