BNL 32230
Conf-820941--10

Why QCD Lattice Theory is Important to Spin Physicists*

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*Summary of a talk presented at the 5th International Symposium on High Energy Spin Physics, BNL - Sept. 16-22, 1982.

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The lattice formulation of a quantum field theory allows calculations in the regime of strong coupling, by expansion techniques, and for intermediate coupling, by Monte Carlo simulations. These computations are especially valuable in the case of Quantum Chromodynamics (QCD), where several of the most important problems are not amenable to a perturbative analysis. Monte Carlo simulations, in particular, have recently emerged as a very powerful tool and have been used to evaluate a variety of important physical quantities, such as the string tension, the deconfinement temperature, the scale of the interquark potential, glueball masses and masses in the quark model spectrum[1]. Monte Carlo calculations are based on an algorithm which generates a large number of configurations distributed according to the proper quantum measure. Quantum expectation values are then approximated by averages taken over this large set of configurations. In principle (given an infinite amount of computer time and infinite memory) every observable could be evaluated by Monte Carlo simulations: in practice one has to contend with computational constraints which limit the space-time size of the lattice and the attainable accuracy (because of statistical fluctuations, inherent in the method).

If we consider those problems of strong interactions where spin plays an important role, it is unlikely, for the moment at least, that the lattice formulation may be of relevance where the phenomena being investigated invove propagations over extended domains of space-time: thus, for instance, the limitations mentioned above make it impossible to perform a meaningful simulation of a scattering experiment on the lattice. But we are at the stage where Monte Carlo calculations begin to provide relevant information on spectroscopic properties related to spin.

One of the topics which has been considered in this Symposium and which is the focus of theoretical and experimental attention concerns the spectroscopy of glueballs, i.e., of those hadronic particles which are expected to be mainly excitations of the gluonic field and which ought to ssurvive, as pure gauge quantum states, also in the limit where the quarks are decoupled from the theory.

Monte Carlo estimates of particle masses proceed through the numerical determination of the Green's function G(t) for a product of operators which create and subsequently annihilate, after a time t, states with definite quantum numbers: $G(t) = \langle \bar{\mathcal{O}}(t)\, \mathcal{O}(0) \rangle$. In Euclidean space-time each intermediate physical state of energy E_n contributes to G(t) a term behaving as $\exp\{-E_n t\}$. From the analysis of the rate of decay in time of the Green's functions one obtains estimates for the lowest masses in the spectrum. This method has been applied to the study of the glueball spectrum. The result $m_g \approx 2\sqrt{\kappa}$ (κ being the string tension), with a possible error of 15 to 20%, for the mass of the lowest $J^{PC} = 0^{++}$ state has been found rather consistently[2]. This number, which places the ground state glueball at $\approx$ 0.9 GeV, agrees also with Monte Carlo results obtained following different techniques and with the results of strong coupling expansions. More recently the method outlined above has been extended to an analysis of the states of higher angular momenta. Masses of the order of 2 m_g or higher have been found[2]. These last results although they may agree with some experimental observations are, in my opinion, only preliminary and may be affected by a substantial margin or error. Yet they are interesting and point to the fact that future, more refined Monte Carlo simulations can provide quite useful information on the spectroscopy of gluonic matter.

Extending Monte Carlo simulations to systems with fermionic degrees of freedom invovles rather severe computational problems. The calculations become more manageable in an approximation where, while a few fermions (quarks) are allowed to propagate, the effects of pair creation and annihilation (i.e., from closed quark loops) are neglected. This scheme of approximation has been successfully applied to numerical evaluations of the masses in the quark model spectrum[3]. The only free parameters of the calculation are the quark masses: the numerical results indicate that if the quark masses are set to zero, the masses of the lowest pseudoscalar bosons also go to zero, thus pointing to a dynamical realization of chiral symmetry, in agreement with theoretical prejudices. The masses of the other states in the spectrum are stable and tend to finite limits as the quark masses tend to zero. The experimental masses of the lowest pseudoscalar mesons can be used to determine the quark masses: all other masses can then be expressed in terms of the string tension, which sets the scale for all physical quantities. Equivalently, one can use the mass of one of the particles to set the scale, expressing all other masses and the string tension in terms of that. Typical results, using for instance m_π to determine the average mass of the light quarks and m_ρ to set the scale, give $m_u + m_d/2 \approx 6$ MeV, $\sqrt{\kappa} \approx 450$ MeV, $m_\delta \approx 950$ MeV, $m_{A_1} \approx 1100$ MeV, $m_p \approx 1000$ MeV, $m_\Delta \approx 1300$ MeV, with errors ranging from 15 to 20%.[3]

Adding to the Lagrangian terms which affect the propagation of the particles via a coupling to spin degrees of freedom one can estimate the magnitude of spin related observables. Thus, by evaluating the change induced in the Green's functions by the action of an external magnetic field, two groups have been able to calculate magnetic moments.[4-5] The results of the two independent investigations are consistent and give

$$\mu_p = 2.7 \quad (\exp 2.79),$$

$$\mu_N = -1.6 \ (\exp -1.91),$$

$\mu_\Omega = -1.7$ (exp ?) with errors on the order of 25%.

These results, as well as the other results mentioned in this very concise exposition, demonstrate the power of the lattice formulation of QCD and of the Monte Carlo technique. The great advantage of this calculation procedure is that the results derive entirely from first principles: they do not depend on any parameters other than those present at the Lagrangian level (and not even on an arbitrary coupling constant, since this is traded off for a scale, via dimensional transmutation). The limitations come from the approximate, numerical nature of the results, as well as from the finite and rather contained extent of the lattices one can practically consider. These limitations certainly prevent the direct calculation of several quantities which would be of interest for the physics of spin; yet there remains a variety of spin related observables, about which the lattice formulation can provide relevant and useful information.

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