

ON THE EVALUATION OF SEMICLASSICAL NUCLEAR MANY-PARTICLE
MANY-HOLE LEVEL DENSITIES

A.H. Blin - Laboratoire de Physique Nucléaire, Centre d'Etudes Nucléaires
de Grenoble, F-38041 Grenoble Cedex, France

R.W. Hasse^x - Institut Laue-Langevin, F-38042 Grenoble Cedex, France

B. Hiller, P. Schuck and C. Yannouleas - Institut des Sciences Nucléaires,
F-38026 Grenoble Cedex, France

ABSTRACT

An exact general scheme is described to calculate the m -particle n -hole fermion level densities for an arbitrary single particle Hamiltonian taking into account the Pauli exclusion principle. This technique is applied to obtain level densities of the three dimensional isotropic harmonic oscillator semiclassically in the Thomas-Fermi approach. In addition, we study the l -particle l -hole level density of the Woods-Saxon potential. For the harmonic oscillator we analyze the temperature dependence of the linear response function and the influence of pairing correlations on the l -particle l -hole level density. Finally, a Taylor expansion method of the m -particle n -hole level densities is discussed.

^x) Present address : GSI, D-6100 Darmstadt, Germany; on leave from
Kernforschungszentrum, D-7500 Karlsruhe, Germany

1. INTRODUCTION

The accurate knowledge of level densities is important in nuclear reaction theory. The spreading width of giant resonances, for instance, involves the knowledge of the low energy tail of the density of two particle - two hole excitations. Level densities with equal number of particles and holes are also of interest for instance in the study of precompound reactions [1,2]. On the other hand, the level densities associated with unequal number of particles and holes find their application in the calculation of shell model optical potentials [3].

Most of the theoretical work [4-8] is based on statistical models to yield approximate level density formulas and some of them do not incorporate the Pauli exclusion principle. As such, they cannot be employed for low energies.

In this work, we describe a general scheme developed earlier [9] which permits to calculate exactly arbitrary m -particle n -hole excitations for any type of single particle Hamiltonians. We explicitly take into account the Pauli exclusion principle for fermions. The derivation of the scheme is discussed in Section 2.

In Section 3, we apply our scheme in the semiclassical Thomas Fermi (TF) approximation [10] to calculate all m -particle n -hole (mp - nh) level densities up to $m, n = 5$ of the three dimensional isotropic harmonic oscillator (HO) and compare them with the exact quantum results. The TF approximation has already been applied in Ref. [11] to the analytic calculation of 1-particle 1-hole ($1p$ - $1h$) and 2-particle 2-hole ($2p$ - $2h$) level densities and it has been found that the results reproduce precisely the average of the quantum calculation.

Furthermore, in Section 4, we calculate semiclassically the lp-1h level density of the Woods-Saxon potential. We investigate the influence of finite temperature and of pairing correlations on the lp-1h level density of the HO in Sections 5 and 6, respectively. In Section 7 a method is presented to Taylor expand the mp-nh level densities. This expansion may be important for practical purposes. Section 8 contains the conclusions.

2. DERIVATION OF AN ITERATION FORMULA FOR MANY-PARTICLE MANY-HOLE LEVEL DENSITIES

The definition of the m-particle n-hole level density for fermions as a function of the energy E is :

$$g_{mp \text{ } nh}(E) = \sum_{\substack{p_1 < \dots < p_m \\ h_1 < \dots < h_n}} \int (E - \sum_{p=p_1}^{p_m} \epsilon_p + \sum_{h=h_1}^{h_n} \epsilon_h) \quad (2.1)$$

where the m+n fold sum extends over all particle states (labelled by the indices p_1 through p_m) with the restriction $p_1 < \dots < p_m$, and similarly for the hole states h. The restrictions $p_1 < \dots < p_m$, and $h_1 < \dots < h_n$ are to avoid multiple counting of the same terms and to honor the Pauli exclusion principle. The ϵ_i are the eigenvalues of the single particle Hamiltonian H. Equation (2.1) can be rewritten as :

$$g_{mp \text{ } nh}(E) = \sum_{\substack{u_1 < \dots < u_m \\ v_1 < \dots < v_n}} \langle u_1, \dots, u_m, v_1, \dots, v_n | \prod_{u=u_1}^{u_m} \theta(H_u - \epsilon_F) \prod_{v=v_1}^{v_n} \theta(\epsilon_F - H_v) \delta(E - \sum_{u=u_1}^{u_m} \epsilon_u + \sum_{v=v_1}^{v_n} \epsilon_v) | u_1, \dots, u_m, v_1, \dots, v_n \rangle \quad (2.2)$$

By introducing the proper step functions, the particle states are guaranteed to lie above the Fermi energy ϵ_F , and the hole states below. We replaced the summation indices p_i and h_i by u_i and v_i to indicate that each of them runs now over all particle and hole states. We introduced

also the product eigenstates with the property

$$H_i |u_1 \dots u_m, v_1 \dots v_n\rangle = \epsilon_i |u_1 \dots u_m, v_1 \dots v_n\rangle \quad (2.3)$$

where i is within the set $\{u_1, \dots, u_m, v_1, \dots, v_n\}$.

A convenient representation of the level density is obtained by applying a Fourier transform ($E \rightarrow \epsilon$, with the definition $\beta = ic/\hbar$) to Eq. (2.2) :

$$G_{mp, nh}(\epsilon) \equiv \int_{-\infty}^{\infty} dE e^{-\epsilon E} g_{mp, nh}(E) = A_m(\beta) \cdot g_n(\beta) \quad (2.4)$$

i.e. the result factorizes into a particle component

$$A_m(\beta) \equiv \sum_{u_1 < \dots < u_m} \langle u_1, \dots, u_m | \tau e^{(H_U - \epsilon_F)} e^{-\beta \sum_{i=1}^m H_{U_i}} | u_1, \dots, u_m \rangle \quad (2.5)$$

and a hole component

$$g_n(\beta) \equiv \sum_{v_1 < \dots < v_n} \langle v_1, \dots, v_n | \tau e^{(\epsilon_F - H_V)} e^{+\beta \sum_{i=1}^n H_{V_i}} | v_1, \dots, v_n \rangle \quad (2.6)$$

Note that if the sums in Eqs. (2.5) and (2.6) were unrestricted, one could write A_m and g_n simply as traces. However, it is possible to express A_m and g_n in terms of traces, by observing the following. A restricted sum over two indices of a function f can be written as an unrestricted sum minus a sum over the diagonal terms and divided by two to take into account double counting. To simplify our notation, we define

$$f(u) \equiv \langle u | \theta(H_U - \epsilon_F) e^{-\beta H_U} | u \rangle \quad (2.7)$$

and note that Eq. (2.5) for two particles can then be written as

$$\sum_{u_1 < u_2} f(u_1) f(u_2) = \frac{1}{2} \left[\sum_{u_1, u_2} f(u_1) f(u_2) - \sum_{u_1} f^2(u_1) \right] \quad (2.8)$$

Similarly, in the case of three indices, we have

$$\sum_{u_1 < u_2 < u_3} f(u_1) f(u_2) f(u_3) = \frac{1}{3!} \left[\sum_{u_1, u_2, u_3} f(u_1) f(u_2) f(u_3) - 3 \sum_{u_1, u_2} f^2(u_1) f(u_2) + 2 \sum_{u_1} f^3(u_1) \right] \quad (2.9)$$

The sum with negative sign arises from the fact that we have to subtract from the unrestricted sum the terms with $\mu_1 = \mu_2$, $\mu_1 = \mu_3$ and $\mu_2 = \mu_3$. The last sum over terms with $\mu_1 = \mu_2 = \mu_3$ has to be added twice, since the three sums subtracted contains each these terms, but the unrestricted sum only contains them once. The factor $1/3!$ is due to compensation for overcounting, i.e. terms with $\mu_1 > \mu_2$ etc., which occur $3!$ times for three indices.

Note, that in Eqs. (2.8) and (2.9) the sums on the right hand side are now unrestricted and therefore can be written as traces. For an arbitrary number of indices we obtain from combinatorics the following iteration formulae for a new set of functions $A_m^{(\ell)}$ and $B_n^{(\ell)}$ defined as

$$A_m^{(\ell)} = m! \sum_{\mu_1 < \dots < \mu_m} \langle \mu_1, \dots, \mu_m | \pi \vartheta(H_\mu - \epsilon_F) e^{-\beta \sum_{\mu} H_\mu} e^{-\beta \mathcal{H}} | \mu_1, \dots, \mu_m \rangle \quad (2.10)$$

and similarly for $B_n^{(\ell)}$,

$$A_m^{(\ell)}(\beta) = A_1^{(\ell)}(\beta) \cdot (A_1^{(0)}(\beta))^{m-1} - \sum_{k=2}^m \binom{m}{k} A_{m-k+1}^{(k+\ell-1)}(\beta) \quad \text{for } m \geq 2 \quad (2.11)$$

$$B_n^{(\ell)}(\beta) = B_1^{(\ell)}(\beta) \cdot (B_1^{(0)}(\beta))^{n-1} - \sum_{k=2}^n \binom{n}{k} B_{n-k+1}^{(k+\ell-1)}(\beta) \quad \text{for } n \geq 2 \quad (2.12)$$

With the initial definitions

$$A_1^{(\ell)}(\beta) \equiv \text{Tr} \{ \vartheta(H - \epsilon_F) e^{-(\ell+1)\beta H} \} \quad (2.13)$$

$$B_1^{(\ell)}(\beta) \equiv \text{Tr} \{ \vartheta(\epsilon_F - H) e^{+(\ell+1)\beta H} \} \quad (2.14)$$

The Fourier transformed level density is then

$$G_{\text{mpnh}}(\beta) = A_m(\beta) \cdot \delta_n(\beta) = \frac{1}{m!} A_m^{(0)}(\beta) \cdot \frac{1}{n!} B_n^{(0)}(\beta) \quad (2.15)$$

The functions $A_m^{(2)}$ and $B_n^{(2)}$ are completely determined by the iteration procedure, Eqs. (2.10) through (2.14), as is the level density, Eq. (2.13). Note that level densities of the form g_{opnh} and g_{mpoh} are obtained by setting $\tilde{z}_0(z) = 1$ or $\tilde{z}_0(z) = 1$, respectively, as follows immediately from Eq. (2.1).

The above equations are valid for level densities of any (single particle) Hamiltonian. We emphasize that in the case $z = 0$, the sums which are subtracted in Eqs. (2.11) and (2.12) are the correction terms due to the Pauli exclusion principle (because they subtract terms with equal indices, compare with Eqs. (2.8) and (2.9)). We now proceed to apply these equations to a particular case.

3. SEMICLASSICAL LEVEL DENSITIES OF THE HARMONIC OSCILLATOR

We consider the three dimensional isotropic Harmonic Oscillator Hamiltonian

$$H_i = \frac{p_i^2}{2m} + \frac{1}{2} m_0 \omega_0^2 r_i^2 \quad (3.1)$$

with m_0 the nucleon mass and ω_0 the oscillator constant.

The semiclassical (Thomas-Fermi) approximation in lowest order in \hbar [10] is obtained by replacing the operator H_i by its classical counterpart and the traces in Eqs. (2.11) - (2.15) by integrations over phase space, i.e.

$$\text{Tr}_i = d \cdot \left(\frac{1}{(2\pi\hbar)^3} \right) \int d^3 r_i \, d^3 p_i \quad (3.2)$$

where d denotes the spin degeneracy of fermions, $d = 2$. In this work, we do not take into account isospin degeneracy.

In the semiclassical approximation Eqs. (2.13) and (2.14) become

$$A_1^{(\ell)}(z) = \frac{d}{(2\pi\omega_0)^3} e^{-\frac{(\ell+1)8\epsilon_F}{2(\ell+1)z}} \left(\frac{\epsilon_F^2}{2(\ell+1)z} + \frac{\epsilon_F}{(\ell+1)^2 z^2} + \frac{1}{(\ell+1)^3 z^3} \right) \quad (3.3)$$

$$g_1^{(z)}(\beta) = \frac{d}{(\pi\omega_0)^2} \left[e^{+(z+1)\beta\epsilon_F} \left(\frac{\epsilon_F^2}{2(z+1)\beta} - \frac{\epsilon_F}{(z+1)^2\beta^2} + \frac{1}{(z+1)^2\beta^4} \right) - \frac{1}{(z+1)^2\beta^2} \right]. \quad (3.4)$$

Then the level densities $g_{mpnh}(E)$ are obtained with the help of iteration formulae Eqs. (2.11) and (2.12) and by an inverse Fourier transform of Eq. (2.15), which can be done analytically, since it involves only terms of the type $e^{-\beta\epsilon} F \beta^{-\nu}$ which transform to $\delta(E-\epsilon_F)(E-\epsilon_F)^{\nu-1}/(\nu-1)!$ (ν integers, $\nu > 0$, $\nu = 0, \pm 1, \dots$).

For the special cases of $1p-1h$ and $2p-2h$ excitations the level densities have been derived in Ref. [11]^{*} already. For higher order level densities the analytical expressions obtained from the iteration procedure become prohibitively lengthy to be calculated by hand. Therefore we made use of the computer code REDUCE [12], which allows for analytical treatment of complicated expressions.

3.1. Results

In the following we present some results and compare them with the exact quantum level densities. In order to derive the quantum values we rewrite Eq. (2.1) as

$$g_{mpnh}(E) = \frac{1}{m!n!} \sum_{\mu_1, \mu_2, \dots, \mu_m} \sum_{\nu_1, \nu_2, \dots, \nu_n} \prod_{\mu=\mu_1}^{\mu_m} D_p(\mu) \prod_{\nu=\nu_1}^{\nu_n} D_h(\nu) \delta\left(E - \sum_{\mu=1}^{\mu_m} E(\mu) + \sum_{\nu=1}^{\nu_n} E(\nu)\right) \quad (3.5)$$

where $\epsilon(\nu) = (\nu + \frac{1}{2})\hbar\omega_0$ is the energy associated with level ν , and the

^{*}In Ref. [11] g_{1p1h} is too small by a factor of 2 and g_{2p2h} by a factor of 4

arguments of the products are the degeneracy factors $D_p(\dots)$ and $D_h(\dots)$ of available particle and hole states. These are obtained by observing that e.g. the first particle created in level μ has $d_{\mu}(\dots)/2$ available positions, but the second one, if created in the same level, has only $d_{\mu}(\dots)/2 - 1$ left. Care has to be taken explicitly at the Fermi level, which might be only partially filled in the groundstate of a given system. Equation (3.5) is then evaluated numerically by representing each delta function by its integrated value to obtain the average behaviour of the exact level density. Due to the necessary multiple summations the computation time increases sharply with increasing particle-hole multiplicities.

The two examples depicted in Fig. 1, g_{2p2h} and g_3 (the latter is defined below) exhibit an excellent overall agreement of the semiclassical with the average exact results. The curves in this figure as in all following ones (with the exception of the Woods-Saxon potential) are calculated for a system of 40 fermions (no isospin considered). The Fermi energy for closed shell systems is given quantum mechanically by the relation

$$\left(\frac{\epsilon_F}{\hbar\omega_0}\right)^3 + 3\left(\frac{\epsilon_F}{\hbar\omega_0}\right)^2 + 2\left(\frac{\epsilon_F}{\hbar\omega_0}\right) = 3N \quad (3.6)$$

3.2. The Pauli Exclusion Principle

We now turn to the influence of the Pauli principle. In Fig. 2, the lower curve shows the complete level density g_{SpSh} as a function of the energy, whereas the upper curve represents the direct terms, i.e. is not corrected for the Pauli principle (see the remark below). The upper curve deviates by a factor of about 4 at $E/\hbar\omega_0 = 5$, and a factor of about 1.4 at $E/\hbar\omega_0 = 50$. The deviation gets smaller for higher energies. It is also smaller for lower order excitations (e.g. for

g_{2p2h} : it is 35% at $E/\hbar\omega_0 = 5$ and 17% at $E/\hbar\omega_0 = 50$, and for g_{2p2h} it is only 7% at $E/\hbar\omega_0 = 5$ and 3% at $E/\hbar\omega_0 = 50$). This behaviour is readily understandable, since for higher excitation energies more states are accessible, therefore the Pauli principle becomes less important for a given level density. On the other hand, if comparing different level densities, at the same energy, those with more particles or holes are more affected by the Pauli principle, since more particles (holes) have to be distributed among the available levels.

3.3. Level Densities with Equal Number of Particles and Holes

In Fig. 3 we show the level density g_{1p1h} (lowest curve) through g_{5p5h} (uppermost curve). As should be expected, the level densities differ by orders of magnitude, since the available phase space increases drastically with the order of particle-hole excitations. The energy range of g_{npnh} always extends from zero to infinity. Depositing a given amount of energy E in a system of many particles in a potential excites all $np-nh$ states which are above their respective thresholds. Therefore, we show the sum

$$g(E) = \sum_{i=1}^5 g_{ipih}(E) \quad (3.7)$$

in Fig. 4 and compare it to the well-known statistical formula [13]

$$g_{stat}(E) = \left(\frac{2\epsilon_F}{N}\right)^{3/4} \frac{1}{12} E^{-5/4} \exp(\sqrt{4\pi^2 N E/\epsilon_F}) \quad (3.8)$$

where we calculate the Fermi energy ϵ_F semiclassically from the particle number N in the HO,

$$\epsilon_F = (3N)^{1/3} \hbar\omega_0 \quad (3.9)$$

We want to point out that Eq. (3.8) has been derived from the grand canonical ensemble with fluctuating particle number, whereas our micro-canonical formalism conserves particle number.

Although the overall agreement in Fig. 4 of the sum (3.7) with the statistical formula (3.8) up to an energy of $\sim 5 \hbar\omega_0$ is excellent, the statistical prediction lies above the semiclassical one for higher excitations because we did not take into account excitations higher than $3p-3h$. Although the diagram shows only excitations below $6\hbar\omega_0$, the quantum mechanical threshold for $6p-6h$ excitations, the semiclassical $6p-6h$ excitation level density does have a contribution below $6\hbar\omega_0$, since it describes only an average behaviour. Therefore, g_{6p6h} should have been included in the sum of Eq. (3.7) even for energies below $6\hbar\omega_0$. For a finite nucleus with 5 particles, on the other hand, the summation up to $3p-3h$ excitations yields the correct semiclassical level density, and we see on Fig. 4 quite strong deviations from the statistical formula (3.8) for $E \gtrsim 5\hbar\omega_0$. These deviations come from finite size effects which are correctly treated in our theory.

3.4. Level Densities with Unequal Number of Particles and Holes

Level densities g_{mpnh} for $m \neq n$ are, for instance, used in calculating the optical potential [3], whereas multi-particle and multi-hole level densities, g_{np} and g_{nh} , are of interest for open shell nuclei. The latter are defined as

$$g_{np}(E) = \sum_{p_1, \dots, p_n} \delta(E - \sum_{p_i} E_{p_i}) \quad (3.10)$$

for the particles ($\epsilon_F \leq E < \infty$) and

$$g_{nh}(E) = \sum_{h_1, \dots, h_n} \delta(E - \sum_{h_i} E_{h_i}) \quad (3.11)$$

for the holes ($0 \leq E < \varepsilon_F$), where ε_p and ε_h have the same meaning as before. The sum of the particle and the hole densities,

$$g_n(E) = g_{np}(E) + g_{nh}(E) \quad (3.12)$$

can be expressed, using the definition, Eq. (2.1), as

$$g_n(E) = g_{npnh}(E) + g_{ohnp}(-E) \quad (3.13)$$

This sum is shown in Fig. 5 for $n = 2$ to 5. Each g_n decreases sharply at $E = n\varepsilon_p$, the matching point of the respective particle and hole densities, due to depletion of phase space at this point. The energy range of g_n extends from zero to infinity.

Finally, we show mp-nh level densities for $m \neq n$ ($m, n \neq 0$) in Fig. 6. Again, we include $g_{mpnh}(E)$ and $g_{nmpnh}(-E)$ in one curve. The results are defined in the range $-\infty < E < \infty$, and the level densities have again their minimum at the respective matching point, which lies at $(m-n)\varepsilon_F$.

3. ONE-PARTICLE ONE-HOLE LEVEL DENSITY IN THE WOODS-SAXON POTENTIAL

The iteration scheme derived in Section 2 is very convenient for level densities of the HO, since the results can be derived analytically. However, it is interesting to consider the more realistic case of a nuclear Woods-Saxon potential. Here, the calculations cannot be performed analytically, so we concentrate on one special case, the l th level density. We start directly from the definition, Eq. (2.2), by use of the spherical Woods-Saxon Hamiltonian for H_l ,

$$H_l = \frac{P_l^2}{2m} - V_0 \left(1 + \exp\left(\frac{r_l - R}{a}\right) \right)^{-1} + V_l \quad (4.1)$$

with

$$V_1 = V_0 (1 + \exp(-R/a))^{-1} \quad (4.2)$$

The constant V_1 is chosen for convenience to shift the bottom of the potential to the zero of the energy. We take a frequently used set of parameters [10] for the depth of the potential, $V_0 = 50$ MeV, the diffuseness, $a = 0.5$ fm, and the nuclear radius $R = A^{1/3} \times 1.2$ fm, where A is the nuclear mass number.

The Fermi energy ϵ_F in the step functions of Eq. (2.2) is calculated semiclassically from the single particle level density via the condition

$$A = \int_0^{\epsilon_F} dE' g_1^{WS}(E') \quad (4.3)$$

with

$$g_1^{WS}(E) = \frac{2\sqrt{2} m^{3/2} a^3}{h^3 \pi} \int_0^{\log(\frac{V_0}{V_0 - E} - 1) + \frac{R}{a}} dx \frac{1}{x^2 \sqrt{E - V_1 + V_0 (1 + e^{-R/a} x)^2}} \quad (4.4)$$

The semiclassical expression for the l p-th level density becomes, after integration over momenta and angles,

$$g_{l p l h}^{WS}(E) = \frac{m^3 \omega_0^2 d^2 a^6}{h^4 \pi^3} \int_0^{\infty} dx_1 x_1^2 \int_0^{\infty} dx_2 x_2^2 \Theta(\mathcal{G}_2 - \mathcal{G}_1) \left[\left[(46 + 2B) \sqrt{\mathcal{G}^2 + 8\mathcal{G}} - \mathcal{G}^2 \log(2\sqrt{\mathcal{G}^2 + 8\mathcal{G}} + 2\mathcal{G} + B) \right] \right]_{\mathcal{G}_1}^{\mathcal{G}_2} \quad (4.5)$$

where the bar after the square bracket denotes the difference of the quantity in square brackets at

$$\mathcal{G} = \mathcal{G}_2 = [E + \epsilon_F - V_1 + V_0 (1 + \exp(x_1 - R_0/a))^{-1}] / \hbar \omega_0 \quad (4.6)$$

minus its value at

$$\mathcal{G} = \mathcal{G}_1 = \max\{0; -B; \epsilon_F - V_1 + V_0 (1 + \exp(x_1 - R_0/a))^{-1}\} / \hbar \omega_0 \quad (4.7)$$

The quantity B is defined as

$$B = [V_0 (1 + \exp(x_2 - R_0/a))^{-1} - V_0 (1 + \exp(x_1 - R_0/a))^{-1} - E] / \hbar \omega_0 \quad (4.6)$$

and we use the standard value $\hbar \omega_0 = A^{-1/3} \times 41$ MeV in units of which we express the energies.

The maximum value contained in Eq. (4.7) results from the step functions and the integrated delta function of expression (2.2). The integral over x_1 in Eq. (4.5), however, diverges, as do all level densities of the Woods-Saxon potential. This is due to the infinite continuum contribution which we have to subtract. In practice, it is done as follows: we perform the x_1 integration up to a finite value of x_1 , say x_{\max} . Far outside of the potential well the integrand is not dependent on the depth of the well anymore, i.e. its contribution to the level density becomes identical to that obtained with $V_0 = 0$. We subtract therefore the level density calculated for a constant potential with $V_0 = 0$, integrating up to x_{\max} . From a certain x_{\max} on the subtracted level density becomes independent of x_{\max} . This is the quantity g_{1ph}^{us} presented in Fig. 7. To be consistent with the level densities shown above, we calculate g_{1ph}^{us} again for 40 fermions, say neutrons, for a nucleus with $A = 80$.

In contrast with the H0 case, the Woods-Saxon level density decreases for large energies due to the subtraction procedure, since the influence of the potential becomes less important for higher excitation energies. The maximum of the curve lies below the Fermi energy ϵ_F (indicated by an arrow in Fig. 7).

As a matter of fact, if one treated the excitation of np-nh levels as they come about in a nuclear reaction more accurately, no subtraction

procedure would be necessary; instead the finite range in momentum space of the nucleon-nucleon force would not allow for excitation too high into the continuum. For example, excitation of $2p-1h$ states in elastic nucleon scattering shows qualitatively a quite similar behaviour as the one given in Fig. 7 [3]. The subtraction procedure for finite potentials can therefore be viewed as an approximate account of two-body scattering processes.

5. TEMPERATURE DEPENDENCE OF THE NUCLEAR LINEAR RESPONSE FUNCTION

Evidently the one-particle one-hole level density is directly related to the free (non interacting) nuclear response function [10] which is needed for the calculation of collective nuclear states. The temperature dependence of these states seems to be accessible experimentally now

and it is therefore interesting to study the temperature dependence of the nuclear response function (more precisely of its imaginary part).

In a nucleus at finite temperature T the distribution of nucleons is not anymore confined to lie below the Fermi energy, but is "smeared out"; i.e. there is a certain probability for nucleons to be in a state above the Fermi level. Semiclassically, this effect is taken into account by replacing the step function distributions by Fermi distributions [14], defined as

$$\Theta_T(E) = (1 + \exp(-E/T))^{-1} \quad (5.1)$$

To calculate the linear response function at finite temperature one has now to include the graph which interchanges the role of particle and hole in the Fermi functions.

We write

$$g_{1p1h}^T(E) = \sum_{\mu\nu} \langle \mu\nu | (\Theta_T(H_\mu - \epsilon_F) \Theta_T(\epsilon_F - H_\nu) - \Theta_T(H_\nu - \epsilon_F) \Theta_T(\epsilon_F - H_\mu)) \delta(E - E_\mu + E_\nu) | \mu\nu \rangle \quad (5.2)$$

and remark that the subtracted quantity in Eq. (5.2) vanishes at $T = 0$.

In the semiclassical approximation, Eq. (5.2) can be rewritten for the HO as single integral

$$g_{1p1h}^T(E) = \left(\frac{d}{2\hbar^2 \omega_0^2} \right)^2 \int_0^\infty dt (t+E)^2 t^2 (\Theta_T(t+E-\epsilon_F) \Theta_T(\epsilon_F-t) - \Theta_T(t-\epsilon_F) \Theta_T(\epsilon_F-E-t)). \quad (5.3)$$

The temperature dependence of Eq. (5.3) up to $T = 20$ MeV is shown in Fig. 8 (the value of the oscillator constant $\hbar\omega_0$ is fixed as in the Woods-Saxon case). The response function increases monotonically with temperature due to increasing thermal fluctuations. A similar behaviour has been observed for the temperature dependence of the imaginary part of the optical potential [14], which contains $1p-2h$ and $2p-1h$ excitations.

6. THE INFLUENCE OF PAIRING CORRELATIONS ON THE ONE-PARTICLE ONE-HOLE LEVEL DENSITY

In the case of a pure pairing force, BCS theory [15,16] leads to "smeared out" occupation probabilities, as in the finite temperature case. The step functions in Eq. (2.2) have to be replaced by the corresponding BCS expressions. The $1p-1h$ level density thus becomes

$$g_{1p1h}^{BCS}(E) = \sum_{\mu\nu} \langle \mu\nu | \frac{1}{2} (u_\mu v_\nu - u_\nu v_\mu)^2 \delta(E - E_\mu - E_\nu) | \mu\nu \rangle \quad (6.1)$$

with

$$U_i^2 = \frac{E_i + H_i - E_F}{2E_i} \quad (6.2)$$

$$V_i^2 = \frac{E_i - H_i + E_F}{2E_i} \quad (6.3)$$

$$E_i = \sqrt{(H_i - E_F)^2 + \Delta^2} \quad (6.4)$$

where Δ is the pairing gap which we assume to take on the constant value 1 MeV.

In the limit $\Delta \rightarrow 0$, Eq. (6.1) reduces to Eq. (2.2) for $m = n = 1$.

All the integrations in Eq. (6.1) except one can be performed analytically in the case of a HO potential to yield

$$\begin{aligned} g_{\text{BCS}}^{\text{BCS}}(E) &= \left(\frac{d}{4u_0^2 b^2} \right)^2 \int_{\Delta}^{\infty} du \frac{\Theta(E - \Delta - u)}{a(u) b(u)} \left\{ [(E-u)(u+b(u)) \right. \\ &- a(u)u - a(u)b(u) - \Delta^2] (\epsilon_F + b(u))^2 (\epsilon_F + a(u))^2 + \Theta(\sqrt{\epsilon_F^2 + \Delta^2} + u - E) \\ &[(E-u)(u+b(u)) + a(u)u + a(u)b(u) - \Delta^2] (\epsilon_F + b(u))^2 (\epsilon_F - a(u))^2 \left. \right\} \\ &+ \int_{\Delta}^{\sqrt{\epsilon_F^2 + \Delta^2}} du \frac{\Theta(E - \Delta - u)}{a(u) b(u)} \left\{ [(E-u)(u-b(u)) - a(u)u + a(u)b(u) - \Delta^2] \right. \\ &(\epsilon_F - b(u))^2 (\epsilon_F + a(u))^2 + \Theta(\sqrt{\epsilon_F^2 + \Delta^2} + u - E) [(E-u)(u-b(u)) + a(u)u \\ &- a(u)b(u) - \Delta^2] (\epsilon_F - b(u))^2 (\epsilon_F - a(u))^2 \left. \right\} \end{aligned} \quad (6.5)$$

with

$$a(u) = \sqrt{(u-E)^2 - \Delta^2} \quad (6.6)$$

$$b(u) = \sqrt{u^2 - \Delta^2} \quad (6.7)$$

The comparison of Eq. (6.5) with the $1p-1h$ level density without pairing correlations is shown in Fig. 9. As expected, the curve with pairing lies below the one without pairing and sets in at the energy 2Δ (marked by an arrow).

7. A TAYLOR EXPANSION FOR THE SEMI-CLASSICAL $mp-nh$ LEVEL DENSITIES

In this section, we present a method for calculating the different power terms in a Taylor expansion of the semi-classical $mp-nh$ level densities. This can be of importance in solving actual problems.

In a representation independent way, the semi-classical $mp-nh$ level density at zero temperature is defined as follows :

$$g_{mpnh}(E) = T_{r_1} T_{r_2} \dots T_{r_m} T_{r_{m+1}} \dots T_{r_{m+n}} \left\{ \theta(H_1 - \epsilon_F) \dots \theta(H_m - \epsilon_F) \times \theta(\epsilon_F - H_{m+1}) \dots \theta(\epsilon_F - H_{m+n}) \delta(E - H_1 - \dots - H_m + H_{m+1} + \dots + H_{m+n}) \right\}, \quad (7.i)$$

where ϵ_F is the Fermi energy and H the single particle Hamiltonian.

A Taylor expansion of (7.1) is not straightforward : due to the presence of the theta and the delta functions, an artless attempt towards a Taylor expansion leads very early to unmanageable singularities; namely, to products of delta functions and/or their derivatives.

However, a proper Taylor expansion of the semi-classical level densities can be constructed when one takes the following steps :

a) One generalizes the definition for the level densities by introducing a new function $G_{mpnh}(E)$ such that

(7.2a)

$$G_{mpnh}[E - (m-n)\epsilon_F] = g_{mpnh}[E - (m-n)\epsilon_F] - g_{mpnh}[(m-n)\epsilon_F - E], \quad \text{when } m+n = \text{even}$$

and

$$G_{mpnh} [E - (m-n)\epsilon_F] = g_{mpnh} [E - (m-n)\epsilon_F] + g [(m-n)\epsilon_F - E], \text{ when } m+n = \text{odd.} \quad (7.2b)$$

The generalization (7.2) for the definition of the level densities is motivated by the close relation between the level densities and the free nuclear response function [10]; namely the level densities are the imaginary part of the corresponding free responses. In particular, for $m+n = \text{even}$, the level densities relate to boson-like responses, a property which determines the minus sign in the r.h.s. of (7.2a); on the other hand, when $m+n = \text{odd}$, the level densities relate to fermion-like responses, a property which establishes the plus sign in Eq. (7.2b).

When $E > (m-n)\epsilon_F$, the second g-term in the r.h.s. of (7.2) is zero; however, this term is essential for the cancellation of the intermediate singularities appearing when one expands each g-term separately.

b) For every single particle index, one makes the substitution

$$T_{r_i} \rightarrow d_i \int (2\pi\hbar)^{-3} d\vec{r}_i d\vec{p}_i \quad (7.3a)$$

and

$$H_i \rightarrow H_i^c = \frac{\vec{p}_i^2}{2m} + V(\vec{r}_i), \quad (7.3b)$$

where \vec{r}_i and \vec{p}_i are the position and momentum of the corresponding single particle.

$V(\vec{r}_i)$ is the nuclear mean field potential which, henceforth, will be approximated by an isotropic harmonic oscillator,

$$V(\vec{r}_i) = \frac{1}{2} m\omega_0^2 \vec{r}_i^2. \quad (7.3c)$$

Then one can integrate over the angle variables and express $g_{\text{mpnh}}(\mathcal{E})$ solely through the single particle energies ε_i . The integration over the angles is simplified by using the formula giving the surface area, S_{ν} , of a ν -dimensional sphere, i.e.

$$S_{\nu} = \frac{2\pi^{\nu/2}}{(\frac{\nu}{2}-1)!} \quad (7.4)$$

c) One introduces centre-of-mass variables. For example, for the 1p-1h case, one introduces new variables h and e such that

$$\left. \begin{aligned} \varepsilon_1 - \varepsilon_2 &= h \\ \varepsilon_1 + \varepsilon_2 &= 2e \end{aligned} \right\} \quad (7.5)$$

For the 2p-1h case, one introduces three new variables, h, e and a , such that

$$\left. \begin{aligned} \varepsilon_1 + \varepsilon_2 - \varepsilon_3 &= h + \varepsilon_F \\ \varepsilon_1 + \varepsilon_2 + \varepsilon_3 &= 4e \\ \varepsilon_1 - \varepsilon_2 &= a \end{aligned} \right\} \quad (7.6)$$

For the 2p-2h case, one has four variables, h, a, a and b , such that

$$\left. \begin{aligned} \varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4 &= h \\ \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 &= 4e \\ \varepsilon_1 - \varepsilon_2 &= a \\ \varepsilon_3 - \varepsilon_4 &= b \end{aligned} \right\} \quad (7.7)$$

d) After having completed step c), one sees that all the theta functions in Eq. (7.1) depend on the variable h and, as a result, successive differentiations with respect to h will yield additional undesirable singularities. However, by using the properties of the theta function, one can solve this difficulty. Specifically, one writes the product of N theta functions as a sum of N terms, each term being the product

of N theta functions among which only one depends on the variable h .

For example

$$\begin{aligned} \theta(h-A_1) \theta(h-A_2) \theta(h-A_3) = \\ \theta(h-A_1) \theta(A_1-A_2) \theta(A_1-A_3) + \\ \theta(h-A_2) \theta(A_2-A_1) \theta(A_2-A_3) + \\ \theta(h-A_3) \theta(A_3-A_1) \theta(A_3-A_2). \end{aligned}$$

(7.8)

e) One expands the delta function of Eq. (7.1) in a formal Taylor expansion; namely

$$\delta(E-h) = \delta(h) - E \delta'(h) + \frac{E^2}{2!} \delta''(h) - \frac{E^3}{3!} \delta'''(h) + \dots \quad (7.9)$$

Illustrative Examples

We will illustrate the previously described method by presenting several simple examples. Henceforth, for simplicity, we will set the degeneracy factor d_i equal to unity.

1st Example

As a first example, we calculate the first four terms in the l_p - l_h level density.

Using the definition (7.1) and step b), we find

$$g_{l_p, l_h}(E) = (\hbar \omega_0)^{-6} \int d\varepsilon_1 d\varepsilon_2 \varepsilon_1^2 \varepsilon_2^2 \theta(\varepsilon_1 - \varepsilon_7) \theta(\varepsilon_7 - \varepsilon_2) \delta(E - \varepsilon_1 + \varepsilon_2). \quad (7.10)$$

After the introduction of the centre-of-mass variables (7.5), eq. (7.10) can be written

$$g_{1plh}^{(0)}(E) = (\hbar\omega_0)^{-6} \int dh dk \left[(k + \varepsilon_F)^2 - \frac{\hbar^2}{4} \right]^2 \theta\left(\frac{\hbar}{2} + k\right) \theta\left(\frac{\hbar}{2} - k\right) \delta(E - \hbar)$$

$$= (\hbar\omega_0)^{-6} \int dh dk \left[(k + \varepsilon_F)^2 - \frac{\hbar^2}{4} \right]^2 \left\{ \theta\left(\frac{\hbar}{2} + k\right) \theta(-k) + \theta\left(\frac{\hbar}{2} - k\right) \theta(k) \right\} \delta(E - \hbar),$$

(7.11)

where the variable k is

$$k = e - \varepsilon_F. \quad (7.12)$$

We utilize now the expansion (7.9) for the delta function $\delta(E - \hbar)$, as well as the properties for the derivatives of the delta and the theta functions.

(i) For the zeroth order term, we find

$$g_{1plh}^{(0)}(E) = (\hbar\omega_0)^{-6} \int dh dk \left\{ \theta\left(\frac{\hbar}{2} + k\right) \theta(-k) + \theta\left(\frac{\hbar}{2} - k\right) \theta(k) \right\} \left[(k + \varepsilon_F)^2 - \frac{\hbar^2}{4} \right]^2 \delta(\hbar)$$

$$= (\hbar\omega_0)^{-6} \int dk (k + \varepsilon_F)^4 \left\{ \theta(k) \theta(-k) + \theta(-k) \theta(k) \right\} = 0. \quad (7.13a)$$

(ii) For the first order term, we find

$$g_{1plh}^{(1)}(E) = -\frac{E}{(\hbar\omega_0)^6} \int dh dk \left[(k + \varepsilon_F)^2 - \frac{\hbar^2}{4} \right]^2 \left\{ \theta\left(\frac{\hbar}{2} + k\right) \theta(-k) + \theta\left(\frac{\hbar}{2} - k\right) \theta(k) \right\} \delta'(\hbar)$$

$$= \frac{E}{2(\hbar\omega_0)^6} \int dk \delta(k) \left\{ \theta(-k) + \theta(k) \right\} (k + \varepsilon_F)^4$$

$$= \frac{E}{2(\hbar\omega_0)^6} \int dk \delta(k) (k + \varepsilon_F)^4 = \frac{\varepsilon_F^4}{(\hbar\omega_0)^6} \frac{E}{2}. \quad (7.13b)$$

(iii) The even power terms (≥ 2) in the expansion of $g_{1plh}^{(E)}$ exhibit singularities which, however, cancel when the corresponding contributions of $g_{1plh}^{(-E)}$ are subtracted (Eq. (7.2a) in step a)).

(iv) For the third order term, we have

$$\begin{aligned}
 g_{1p1h}^{(3)}(E) &= -\frac{E^3}{6(\hbar\omega_0)^6} \int dh dk \left[(k+\varepsilon_F)^2 - \frac{h^2}{4} \right]^2 \left\{ \theta\left(\frac{h}{2}+k\right)\theta(-h) + \theta\left(\frac{h}{2}-k\right)\theta(k) \right\} \delta'''(h) \\
 &= \frac{E^3}{6(\hbar\omega_0)^6} \int dk \left[-\frac{3}{2}(k+\varepsilon_F)^2 \delta(k) + \frac{1}{8}(k+\varepsilon_F)^4 \delta''(k) \right] \left\{ \theta(k) + \theta(-k) \right\} \\
 &= \left(-\frac{3}{2} \varepsilon_F^2 + \frac{3}{2} \varepsilon_F^2 \right) \frac{E^3}{6(\hbar\omega_0)^6} = 0.
 \end{aligned} \tag{7.13c}$$

The physical 1p-1h level density is given by Eq. (7.2a). Taking into consideration Eq. (7.13), one finds the final result

$$G_{1p1h}^{(0)}(E) = 0, \tag{7.14a}$$

$$G_{1p1h}^{(1)}(E) = g_{1p1h}^{(1)}(E) - g_{1p1h}^{(1)}(-E) = \frac{\varepsilon_F^4}{(\hbar\omega_0)^6} E, \tag{7.14b}$$

$$G_{1p1h}^{(2)}(E) = g_{1p1h}^{(2)}(E) - g_{1p1h}^{(2)}(-E) = 0, \tag{7.14c}$$

$$G_{1p1h}^{(3)}(E) = 0. \tag{7.14d}$$

The result (7.14) for the 1p-1h level density in an harmonic potential at zero temperature agrees with Eq. (5b) of Ref. [11].

2nd Example

As a second example, we calculate the $(E-\varepsilon_F)^2$ term in the 2p-1h level density.

After the integration over the angles, the definition (7.1) yields

$$g_{2p1h}(E-\varepsilon_F) = (\hbar\omega_0)^{-9} \int d\varepsilon_1 d\varepsilon_2 d\varepsilon_3 \varepsilon_1^2 \varepsilon_2^2 \varepsilon_3^2 \theta(\varepsilon_1-\varepsilon_F) \theta(\varepsilon_2-\varepsilon_F) \theta(\varepsilon_3-\varepsilon_3) \delta(E-\varepsilon_1-\varepsilon_2+\varepsilon_3) \tag{7.15}$$

The integral in Eq. (7.15) depends only on the parameters E and ϵ_F . Since the dimensions in energy of this integral are eight, the term $(E - \epsilon_F)^2$ must have a coefficient proportional to ϵ_F^6 . This factor ϵ_F^6 results by setting $(\epsilon_1 \epsilon_2 \epsilon_3)^2 = \epsilon_F^6$. Thus, when calculating the $(E - \epsilon_F)^2$ term, instead of the function $g_{2p1h}(E)$, one can use the function

$$\tilde{g}_{2p1h}(E - \epsilon_F) = \frac{\epsilon_F^6}{(\hbar\omega_0)^9} \int d\epsilon_1 d\epsilon_2 d\epsilon_3 \theta(\epsilon_1 - \epsilon_F) \theta(\epsilon_2 - \epsilon_F) \theta(\epsilon_F - \epsilon_3) \delta(E - \epsilon_F - h). \quad (7.16)$$

Introducing the new variables of Eq. (7.6), one can write the product of the three theta functions in (7.16) as follows (step d) :

$$\begin{aligned} &\theta(\epsilon_1 - \epsilon_F) \theta(\epsilon_2 - \epsilon_F) \theta(\epsilon_F - \epsilon_3) = \\ &\theta\left(k + \frac{a}{2} + \frac{h}{4} + \frac{\epsilon_F}{4}\right) \theta(-a) \theta\left(-k - \frac{a}{4} - \frac{\epsilon_F}{4}\right) \\ &+ \theta\left(k - \frac{a}{2} + \frac{h}{4} + \frac{\epsilon_F}{4}\right) \theta(a) \theta\left(-k + \frac{a}{4} - \frac{\epsilon_F}{4}\right) \\ &+ \theta\left(-k + \frac{h}{4} - \frac{\epsilon_F}{4}\right) \theta\left(k + \frac{a}{4} + \frac{\epsilon_F}{4}\right) \theta\left(k - \frac{a}{4} + \frac{\epsilon_F}{4}\right). \end{aligned}$$

(7.17)

Expanding the delta function $\delta(E - \epsilon_F - h)$ and using Eq. (7.17), one gets for the $(E - \epsilon_F)^2$ term from Eq. (7.16)

$$\begin{aligned} \tilde{g}_{2p1h}^{(2)}(E - \epsilon_F) &= \frac{\epsilon_F^6}{(\hbar\omega_0)^9} \frac{(E - \epsilon_F)^2}{2} \int dh da \theta(\epsilon_1 - \epsilon_F) \theta(\epsilon_2 - \epsilon_F) \theta(\epsilon_F - \epsilon_3) \delta''(h) \\ &= \frac{\epsilon_F^6}{(\hbar\omega_0)^9} \frac{(E - \epsilon_F)^2}{2 \cdot 4^2} \int dh da \left\{ \delta'\left(k + \frac{a}{2} + \frac{\epsilon_F}{4}\right) \theta(-a) \theta\left(-k - \frac{a}{4} - \frac{\epsilon_F}{4}\right) \right. \\ &\quad \left. + \delta'\left(k - \frac{a}{2} + \frac{\epsilon_F}{4}\right) \theta(a) \theta\left(-k + \frac{a}{4} - \frac{\epsilon_F}{4}\right) \right. \\ &\quad \left. + \delta'\left(-k - \frac{\epsilon_F}{4}\right) \theta\left(k + \frac{a}{4} + \frac{\epsilon_F}{4}\right) \theta\left(k - \frac{a}{4} + \frac{\epsilon_F}{4}\right) \right\} \\ &= \frac{\epsilon_F^6}{(\hbar\omega_0)^9} \frac{(E - \epsilon_F)^2}{4} \int da \{ \theta(-a) + \theta(a) \} \delta(a) = \frac{\epsilon_F^6}{(\hbar\omega_0)^9} \frac{(E - \epsilon_F)^2}{4}. \quad (7.18) \end{aligned}$$

We now use the definition (7.2b) which gives the physical 2ph level density. We find

$$G_{2ph}^{(d)}(E - \epsilon_F) = \tilde{g}_{2ph}^{(d)}(E - \epsilon_F) + \tilde{g}_{2ph}^{(d)}(\epsilon_F - E) = \frac{\epsilon_F^6}{(\hbar\omega_0)^9} \frac{(E - \epsilon_F)^2}{2}. \quad (7.19)$$

(Note that the result (7.19) agrees with Eq. (2.6) of Ref. 3). The setting of $(\tau_1 \epsilon_2 \epsilon_3)^2 = \epsilon_F^6$ in Eq. (7.15) was a short cut to avoid unnecessarily lengthy algebra here. The reader can check that the result (7.19) for the $(E - \epsilon_F)^2$ term is also found when the full expressions giving the single particle energies as functions of h, k and a are used.

3rd Example

As the third example, we show that no pure T^2 term contributes to the 1ph level density at finite temperatures.

The rules developed for the zero temperature limit apply as well to the case of finite temperature; however, instead of the theta functions, one now must use in (7.1) the corresponding Fermi distributions in conjunction with the Sommerfeld expansion, i.e.

$$f(\epsilon_F - \epsilon) = \theta(\epsilon - \epsilon_F) + \frac{\pi^2 T^2}{6} \delta'(\epsilon - \epsilon_F) + \dots \quad (7.20)$$

Instead of Eq. (7.10) for the zero temperature case, one has in the finite temperature case the following expression for the intermediary function $g_{1ph}(E, T)$:

$$g_{1ph}(E, T) = (\hbar\omega_0)^6 \int d\epsilon_1 d\epsilon_2 \epsilon_1^2 \epsilon_2^2 f(\epsilon_F - \epsilon_1) f(\epsilon_2 - \epsilon_F) \delta(\bar{\epsilon} - h). \quad (7.21)$$

Obviously, the same transformation to the centre of mass variables (7.5) will be used.

From Eq. (7.21), when using the Sommerfeld expansion (7.20), one gets for the pure T^2 contribution

$$g_{1p1h}^{(T^2)}(E, T) = \frac{\pi^2 T^2}{6} \frac{1}{(\hbar\omega_0)^6} \int dh dk \left[(k + \epsilon_F)^2 - \frac{\hbar^2}{4} \right]^2 \times \left[S'(k + \frac{\hbar}{2}) \theta(-k + \frac{\hbar}{2}) + S'(-k + \frac{\hbar}{2}) \theta(k + \frac{\hbar}{2}) \right] \delta(h) = \frac{\pi^2 T^2}{6} \frac{1}{(\hbar\omega_0)^6} \int dk (k + \epsilon_F)^4 \left[S'(k) \theta(-k) + S'(-k) \theta(k) \right]. \quad (7.22)$$

The r.h.s. of Eq. (7.22) exhibits a singularity. However, according to the rule (7.2a), the physical quantity $G_{1p1h}^{(T^2)}(E, T)$ giving the pure T^2 term in the $1p1h$ level density is finite and, in particular, zero; namely,

$$G_{1p1h}^{(T^2)}(E, T) = g_{1p1h}^{(T^2)}(E, T) - g_{1p1h}^{(T^2)}(-E, T) = 0. \quad (7.23)$$

Thus, one sees that there is no pure T^2 contribution in the $1p1h$ level density. Moreover, since, according to (7.2a), only odd powers of E can be different from zero and since the dimensions in energy of the integral in (7.21) are five, the lowest nonvanishing temperature contribution should be a term of the form $E^3 T^2$.

4th Example

As a fourth example, we calculate the E^3 -term in the $2p2h$ level density, a quantity important for the damping of giant resonances.

After the integration over the angles, the definition (7.1) yields

$$g_{2p2h}(E) = (\hbar\omega_0)^{-12} \int d\epsilon_1 d\epsilon_2 d\epsilon_3 d\epsilon_4 \epsilon_1^2 \epsilon_2^2 \epsilon_3^2 \epsilon_4^2 \theta(\epsilon_1 - \epsilon_F) \theta(\epsilon_2 - \epsilon_F) \theta(\epsilon_F - \epsilon_3) \theta(\epsilon_F - \epsilon_4) \delta(E - \epsilon_1 - \epsilon_2 + \epsilon_3 + \epsilon_4) \quad (7.2)$$

The integral in eq. (7.24) depends only on the parameters E and ϵ_F .

Since the dimensions in energy of this integral are eleven, the term E^3

must have a coefficient proportional to ϵ_F^8 . This factor ϵ_F^8 results by setting $(\epsilon_1 \epsilon_2 \epsilon_3 \epsilon_4)^2 = \epsilon_F^8$. Thus, when calculating the E^3 term, instead of the function $g_{2p,2h}(E)$, one can use the function

$$\tilde{g}_{2p,2h}(E) = \frac{\epsilon_F^8}{(h\nu_b)^2} \int d\epsilon_1 d\epsilon_2 d\epsilon_3 d\epsilon_4 \theta(\epsilon_1 - \epsilon_F) \theta(\epsilon_2 - \epsilon_F) \theta(\epsilon_3 - \epsilon_F) \theta(\epsilon_4 - \epsilon_F) \delta(E - h) \quad (7.25)$$

Introducing the new variables of eq. (7.7), one can write the product of the four theta functions in (7.25) as follows (step d) :

$$\begin{aligned} & \theta(\epsilon_1 - \epsilon_F) \theta(\epsilon_2 - \epsilon_F) \theta(\epsilon_3 - \epsilon_F) \theta(\epsilon_4 - \epsilon_F) = \\ & \left\{ \theta\left(\frac{h}{4} + k + \frac{a}{2}\right) \theta(-a) \theta\left(-k - \frac{a}{4} - \frac{b}{4}\right) \theta\left(-k - \frac{a}{4} + \frac{b}{4}\right) \right. \\ & + \theta\left(\frac{h}{4} + k - \frac{a}{2}\right) \theta(a) \theta\left(-k + \frac{a}{4} - \frac{b}{4}\right) \theta\left(-k + \frac{a}{4} + \frac{b}{4}\right) \\ & + \theta\left(\frac{h}{4} - k - \frac{b}{2}\right) \theta\left(k + \frac{a}{4} + \frac{b}{4}\right) \theta\left(k - \frac{a}{4} + \frac{b}{4}\right) \theta(b) \\ & \left. + \theta\left(\frac{h}{4} - k + \frac{b}{2}\right) \theta\left(k + \frac{a}{4} - \frac{b}{4}\right) \theta\left(k - \frac{a}{4} - \frac{b}{4}\right) \theta(-b) \right\}. \end{aligned} \quad (7.26)$$

At this point, one should notice that the Jacobian of the transformation (7.7) is 1/2.

Expanding the delta function $\delta(E-h)$ and using eq. (7.26), one gets four contributions for the E^3 term in eq. (7.25). Here, we calculate explicitly only the first contribution, since the other contributions, give identical results. We denote this contribution by C1. We have

$$\begin{aligned}
C_1 &= -\frac{\epsilon_F^8}{(\hbar\omega_0)^{12}} \frac{E^3}{3!} \frac{1}{2} \int dhdkdadb \left\{ \theta\left(\frac{h}{4} + k + \frac{a}{2}\right) \theta(-a) \theta\left(-k - \frac{a}{4} - \frac{b}{4}\right) \right. \\
&\quad \left. \times \theta\left(-k - \frac{a}{4} + \frac{b}{4}\right) \right\} \delta'''(h) \\
&= \frac{\epsilon_F^8}{(\hbar\omega_0)^{12}} \frac{E^3}{3!} \frac{1}{2 \cdot 4^2} \int dhdadb \delta''\left(k + \frac{a}{2}\right) \theta(-a) \theta\left(-k - \frac{a}{4} - \frac{b}{4}\right) \theta\left(-k - \frac{a}{4} + \frac{b}{4}\right) \\
&= \frac{\epsilon_F^8}{(\hbar\omega_0)^{12}} \frac{E^3}{3!} \frac{1}{2 \cdot 4} \int d\left(\frac{a}{4}\right) d\left(\frac{b}{4}\right) \theta\left(-\frac{a}{4}\right) \left\{ \delta'\left(\frac{a}{4} - \frac{b}{4}\right) \theta\left(\frac{a}{4} + \frac{b}{4}\right) \right. \\
&\quad \left. + \theta\left(\frac{a}{4} - \frac{b}{4}\right) \delta'\left(\frac{a}{4} + \frac{b}{4}\right) + 2\delta\left(\frac{a}{4} - \frac{b}{4}\right) \delta\left(\frac{a}{4} + \frac{b}{4}\right) \right\} \\
&= \frac{\epsilon_F^8}{(\hbar\omega_0)^{12}} \frac{E^3}{3!} \frac{1}{2 \times 4}. \tag{7.27}
\end{aligned}$$

Since there are four contributions like (7.27); we find after multiplying by four

$$\tilde{g}_{2p2h}^{(3)}(E) = \frac{1}{2} \frac{\epsilon_F^8}{(\hbar\omega_0)^{12}} \frac{E^3}{3!}. \tag{7.28}$$

We now use the definition (7.2b) which gives the physical $2p2h$ level density. We find

$$g_{2p2h}^{(3)}(E) = \tilde{g}_{2p2h}^{(3)}(E) - \tilde{g}_{2p2h}^{(3)}(-E) = \frac{\epsilon_F^8}{(\hbar\omega_0)^{12}} \frac{E^3}{3!}. \tag{7.29}$$

5th Example

As the last example, we calculate the $T^2 E$ term in the $2p-2h$ level density. This term represents the lowest order dependence on the temperature.

Using the Sommerfeld expansion (7.20), we find that the T^2 term, which corresponds to ^{the} definition (7.1), is

$$\begin{aligned}
g_{2p2h}^{(T^2)}(E, T) &= \frac{\epsilon_F^8}{(\hbar\omega_0)^{12}} \frac{\pi^2 T^2}{6} \int d\epsilon_1 d\epsilon_2 d\epsilon_3 d\epsilon_4 \times \\
&\quad \left[\delta'(\epsilon_1 - \epsilon_F) \theta(\epsilon_2 - \epsilon_F) \theta(\epsilon_F - \epsilon_3) \theta(\epsilon_F - \epsilon_4) \right. \\
&\quad + \delta'(\epsilon_2 - \epsilon_F) \theta(\epsilon_1 - \epsilon_F) \theta(\epsilon_F - \epsilon_3) \theta(\epsilon_F - \epsilon_4) \\
&\quad + \delta'(\epsilon_F - \epsilon_3) \theta(\epsilon_1 - \epsilon_F) \theta(\epsilon_2 - \epsilon_F) \theta(\epsilon_F - \epsilon_4) \\
&\quad \left. + \delta'(\epsilon_F - \epsilon_4) \theta(\epsilon_1 - \epsilon_F) \theta(\epsilon_2 - \epsilon_F) \theta(\epsilon_F - \epsilon_3) \right] [-E \delta'(h)].
\end{aligned}$$

There are four equal contributions in eq. (7.30). It is thus sufficient to consider only the first.

Introducing the variables (7.7), one has for the product of the three theta functions :

$$\begin{aligned} & \theta(\epsilon_2 - \epsilon_F) \theta(\epsilon_F - \epsilon_3) \theta(\epsilon_F - \epsilon_1) = \\ & \theta(k - \frac{a}{2} + \frac{h}{4}) \theta(-k + \frac{a}{4} - \frac{b}{4}) \theta(-k + \frac{a}{4} + \frac{b}{4}) \\ & + \theta(-k - \frac{b}{2} + \frac{h}{4}) \theta(k + \frac{b}{4} - \frac{a}{4}) \theta(b) \\ & + \theta(-k + \frac{b}{2} + \frac{h}{4}) \theta(k - \frac{b}{4} - \frac{a}{4}) \theta(-b). \end{aligned}$$

(7.31)

There are three terms in eq. (7.31) which all give equal contributions to the integral (7.30). We thus consider in detail only the second term in (7.31). All together there are $4 \times 3 = 12$ equal contributions to the integral (7.30).

We denote by G^1 the one chosen contribution. We have

$$\begin{aligned} G^1 &= - \frac{\epsilon_F^8}{(\hbar\omega_0)^{12}} \frac{\pi^2 T^2 E}{6} \int dk dh d(\frac{a}{2}) db \\ & \quad \times S'(k + \frac{a}{2} + \frac{h}{4}) \theta(-k - \frac{b}{2} + \frac{h}{4}) \theta(k + \frac{b}{4} - \frac{a}{4}) S'(h) \theta(b) \\ &= - \frac{\epsilon_F^8}{(\hbar\omega_0)^{12}} \frac{\pi^2 T^2 E}{6} \frac{1}{2} \int dk dh db \theta(-k - \frac{b}{2} + \frac{h}{4}) \theta(b) S(\frac{3k}{2} + \frac{b}{4} + \frac{h}{8}) S'(h) \\ &= \frac{\epsilon_F^8}{(\hbar\omega_0)^{12}} \frac{\pi^2 T^2 E}{3} \int dk [\frac{1}{4} S(k) \theta(-k) - \frac{1}{12} S(k) \theta(k)] \\ &= \frac{\epsilon_F^8}{(\hbar\omega_0)^{12}} \frac{\pi^2 T^2 E}{3} \frac{1}{12}. \end{aligned}$$

(7.32)

Since there are twelve contributions like (7.32), we find that eq. (7.30) yields

$$\tilde{g}_{2p2h}^{(T^2)}(E, T) = \frac{\epsilon_F^8}{(\hbar\omega_0)^2} \frac{\pi^2 T E}{3}. \quad (7.33)$$

We now use the definition (7.2b) which gives the physical 2p2h level density. We finally find

$$G_{2p2h}^{(T^2)}(E, T) = \tilde{g}_{2p2h}^{(T^2)}(E, T) - \tilde{g}_{2p2h}^{(T^2)}(-E, T) = \frac{\epsilon_F^8}{(\hbar\omega_0)^2} \frac{2\pi^2 T E}{3}. \quad (7.34)$$

Observe that the result (7.29) and (7.34) agrees with the corresponding result in ref. [16].

8. CONCLUSIONS

We present a general iteration procedure to calculate m-particle n-hole level densities of any single particle Hamiltonian, including the Pauli principle. We use this approach to derive analytical expressions for the semiclassical level densities up to five particles and five holes of the isotropic harmonic oscillator, in the Thomas-Fermi approximation.

We learn that the Pauli exclusion principle is more important at lower excitation energies and for higher particle-hole multiplicity, and

should not be neglected in the calculation of level densities. The available phase space at the Fermi surface is depleted for particle-hole excitations with $m \neq n$ (except for the single particle).

The comparison of the sum over n -particle n -hole densities with a statistical level density formula gives very good agreement up to the energy corresponding to the excitation of the highest np - nh state included in the sum.

In the case of a Woods-Saxon potential, the continuum contribution has to be subtracted from the level densities in order to obtain finite results, which leads to a maximum of the $1p$ - $1h$ level density instead of a monotonically increasing function as in the case of a HO potential.

The investigation on finite temperature effects shows an increase of the linear response function of the HO with temperature due to thermal fluctuations.

Pairing correlations are found to reduce the $1p1h$ level density, which sets in at an energy of twice the pairing gap.

Finally, a procedure is derived to Taylor expand mp - nh level densities properly. This can be of importance for practical problems. Several examples have been worked out explicitly.

ACKNOWLEDGEMENTS

We are grateful to Dr. C de Polignac of the Computer Centre of the Institut Laue-Langevin for his help with REDUCE.

References

- (1) G. Rohr, in Neutron-Capture Gamma-Ray Spectroscopy (1981), edited by T. von Egidy and F. Ginnenwein, IOP Conference Proceeding n° 62 (Institute of Physics, London, 1982), p. 322.
- (2) J. Winter and P. Schuck, in Time Dependent Hartree-Fock and Beyond, edited by K. Goeke and J. Reinhard, Lecture Notes in Physics Vol. 171 (Springer, New York, 1982), p. 190.
- (3) R.W. Hasse and P. Schuck, Nucl. Phys. A438 (1985) 157.
- (4) C. Bloch, Phys. Rev. 93 (1954) 1094.
- (5) T. Ericson, Adv. in Phys. 9 (1960) 425.
- (6) J.J. Griffin, Phys. Rev. Lett. 17 (1966) 479.
- (7) M. Söhning, Nucl. Phys. A152 (1970) 529.
- (8) F.C. Williams, Nucl. Phys. A166 (1971) 231.
- (9) A.H. Blin, B. Hiller, R.W. Hasse and P. Schuck, J. Phys. 45 (1984) C6-231.
- (10) P. Ring and P. Schuck, The Nuclear Many Body Problem (Springer, Berlin, 1980).
- (11) G. Ghosh, R.W. Hasse, P. Schuck, and J. Winter, Phys. Rev. Lett. 50 (1983) 1250.
- (12) A.C. Hearn, ed., Reduce User's Manual (The Rand Corporation, Santa Monica, California, 1983).
- (13) A. Bohr and B.R. Mottelson, Nuclear Structure (Benjamin inc., New York, 1969).
- (14) A.H. Blin, R.W. Hasse, B. Hiller and P. Schuck, to be published in Phys. Lett. B.
- (15) J. Bardeen, L.M. Cooper and J.R. Schrieffer, Phys. Rev. 108 (1957) 1175.
- (16) A.A. Abrikosov and I. M. Khalatnikov, Rep. Prog. Phys. ~~XIII~~ (1959) 329

Figure Captions

Figure 1 :

- a) The semiclassical 2-particle 2-hole level density g_{2p2h} (Solid line) and the average exact one (dots) as functions of energy, E , in units of oscillator constant $\hbar\omega_0$.
- b) The semiclassical 3-particle level density g_3 (Solid line) and the corresponding average exact one. Note that we use a linear scale here and still get good agreement.

Figure 2

The effect of the Pauli principle on the semiclassical 5-particle 5-hole level density g_{5p5h} . The upper (dashed) curve is calculated without taking Pauli exclusion into account, whereas the lower (solid) one is the full level density including the Pauli principle.

Figure 3

The excitation level density g_{1p1h} (lowest curve) through g_{5p5h} (uppermost curve).

Figure 4

The statistical excitation level density (see eq. (3.8)), dotted line, and the sum $\sum_{i=1}^5 g_{ip1h}$, full curve.

Figure 5

The n -particle densities g_2 through g_5 . The points on the energy axis with $n\epsilon_F$ are indicated by arrows (see text for discussion).

Figure 6

Level densities g_{mpnh} for $m \neq n$ and $m, n \neq 0$. Note the different energy scales in each graph.

Figure 7

The subtracted 1-particle 1-hole level density of the Woods-Saxon potential. The position of the Fermi energy ϵ_F is marked with an arrow.

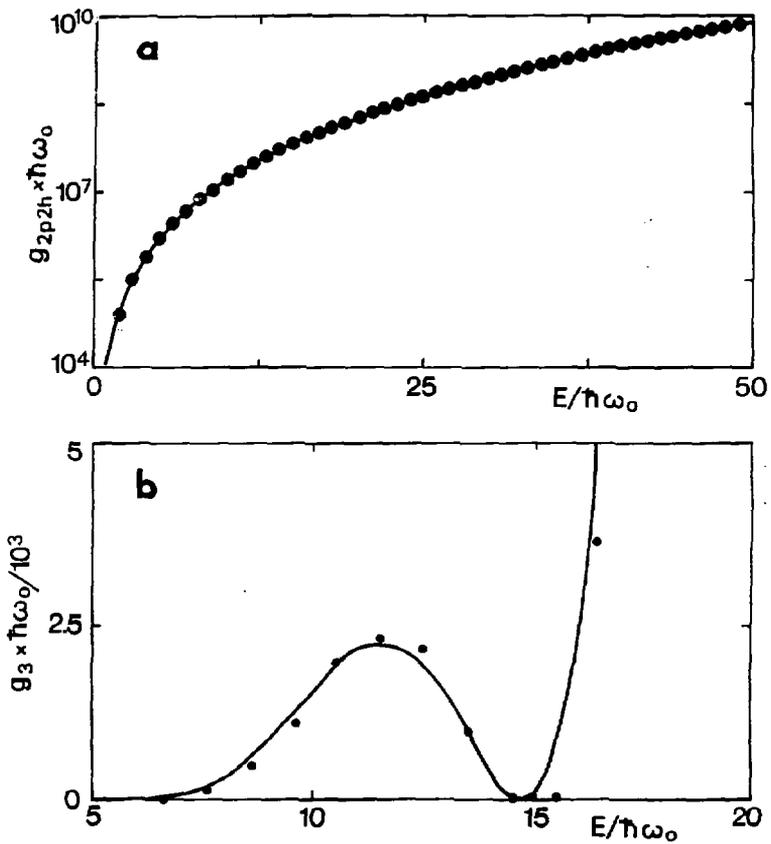
Figure 8

Temperature dependence of the linear response function χ_{ip}^I , for the harmonic oscillator in the temperature range $T = 0$ MeV until $T = 20$ MeV.

Figure 9

The influence of a pairing correlation on the 1-particle 1-hole level density for the harmonic oscillator. The curve with a pairing gap of $\Delta = 1$ MeV sets in at an energy of $2\Delta = 2$ MeV (see arrow).

Fig 1



147

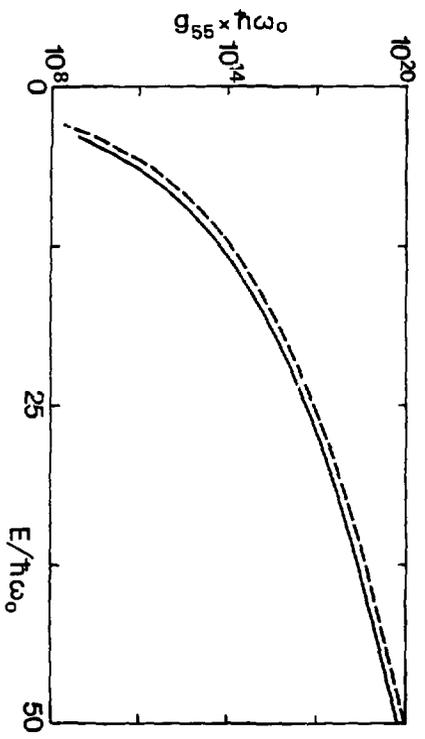


Fig.

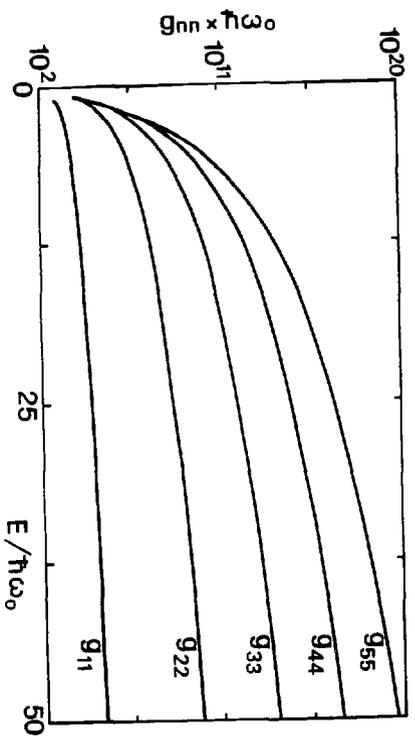


Fig. 9

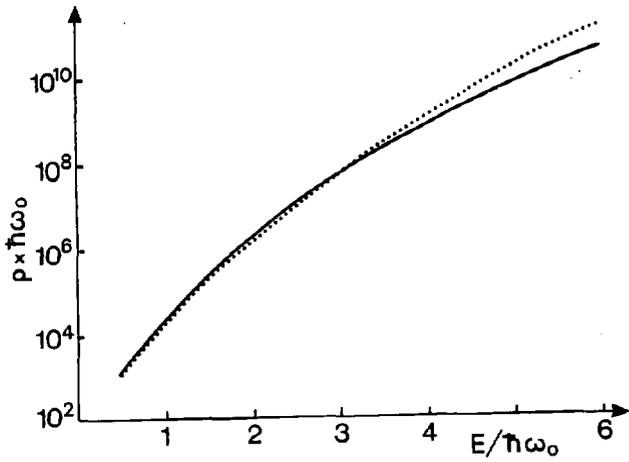
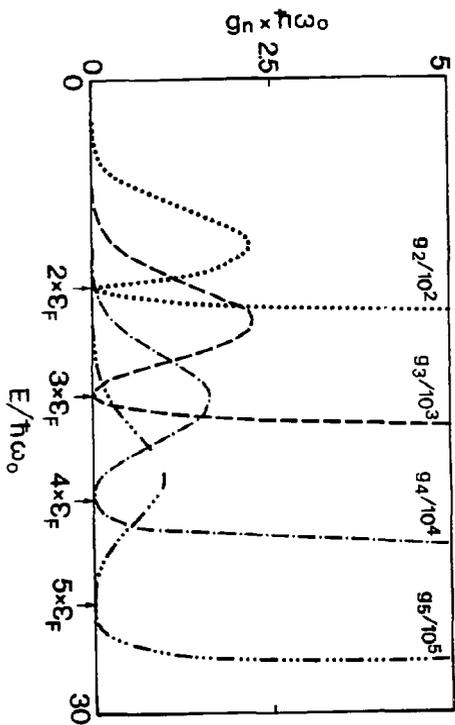


Fig. 5



Figs. 6

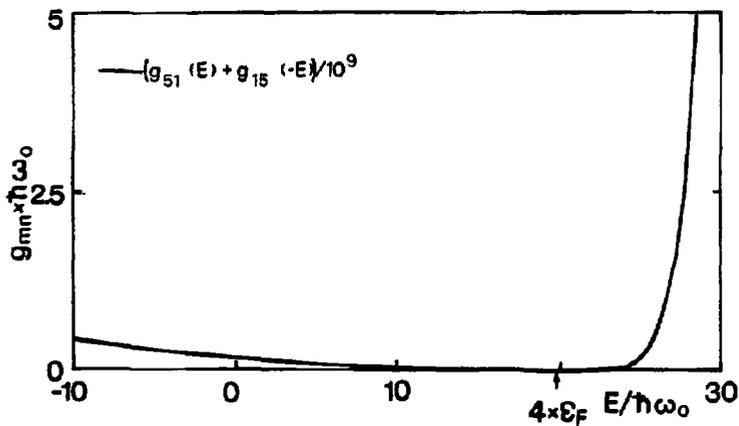
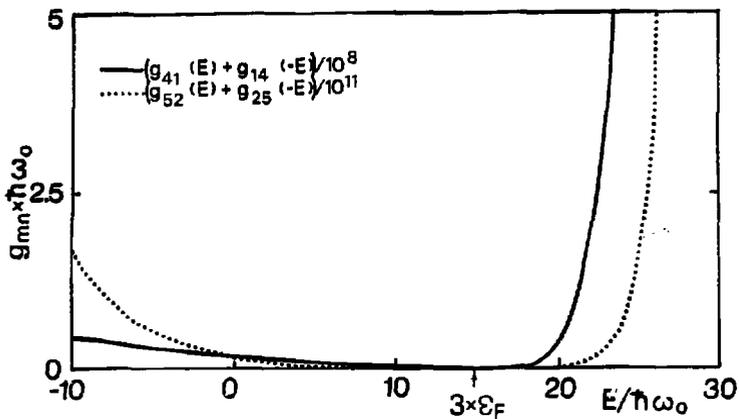
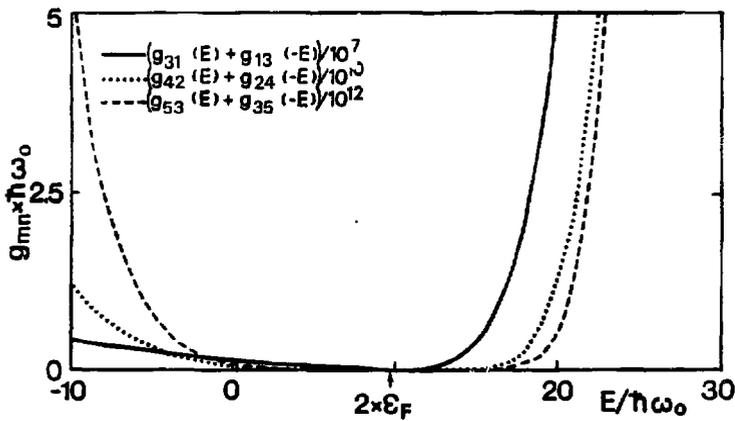
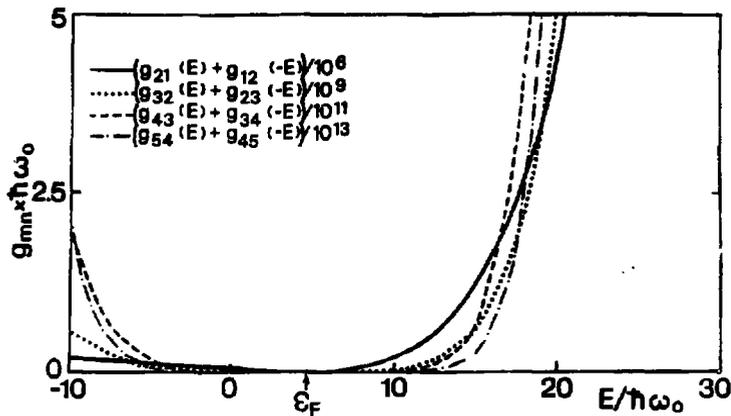


Fig. 6



11.7

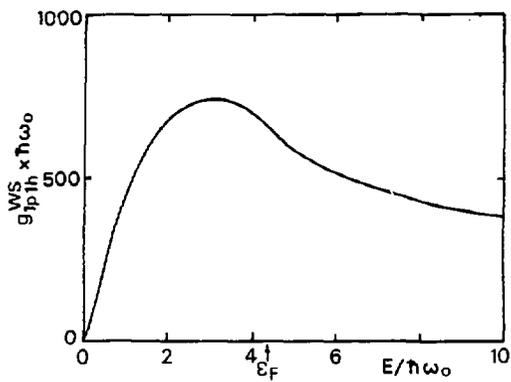


Fig 7

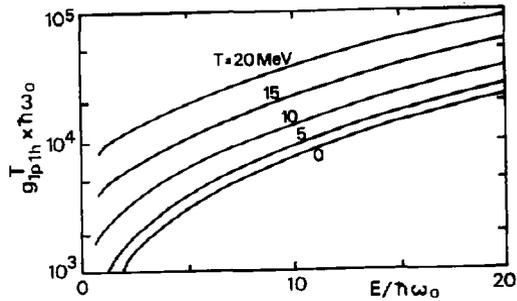


Fig. 3

