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SYSTEMATICS OF EXCITATION FUNCTIONS  
FOR (n, charged particle) REACTIONS

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On the bases of evaporation model considering the preequilibrium emission under some approximations, the analytical expressions including two adjustable parameters have been derived for excitation functions of (n, charged particle) reactions. Fitting these expressions to the available measured data, these parameters have been extracted and the systematic behaviours of the parameters have been studied. More accurate predictions than before could be obtained by using these expressions and systematics parameters.

### Introduction

Charged particle producing data of neutron induced reactions are of great importance for design of fission and fusion reactor. Unfortunately, experimental data especially measured excitation functions are very scarce. The unmeasured energy regions and nuclei may be complemented by model theory calculation and systematics predictions. Generally, the latter is more efficient. All earlier work on systematics of (n,q) cross sections (q=p,d,t,  $^3\text{He}$  and  $\alpha$ ) except Pearlstein's<sup>(1)</sup> are carried out at  $E_n=14.5$  MeV<sup>(2-7)</sup>. The Pearlstein's work was performed by using a hybrid empirical-statistical model in region of proton number of target  $Z=20-83$ .

In the present work, we concerned that the neutron energy region is up to about 20 MeV and that target mass region is  $23 < A < 197$ . Charged particle p,d,t,  $^3\text{He}$  and  $\alpha$  emitted in (n,q) reactions are considered.

## Formulae

Based on evaporation model considering preequilibrium emission, the analytical expressions of excitation function for  $(n,q)$  reactions have been derived under some approximations. Some primary approximations are as follows:

1, The preequilibrium emission only occurs at the state of exciton number  $n=3$ .

2, There is only one competing reaction of  $(n,n')$ .

3, The  $(n,qn)$  reaction is primary channel for secondary particle emission and the neutron emission must follow if the rest energy in compound nucleus system is enough after the  $q$  emission.

4, Complex particle such as  $d, t, {}^3\text{He}$  and  $\alpha$  are regarded as excitons which are prior formed in target nucleus with a probability  $P_q$ . In this work, we took  $P_\alpha = 0.2^{(8)}$  and  $P_d, P_t, P_{{}^3\text{He}} \ll 1$ .

5, The penetration factor of rectangular-well potential

$$D_q(e_q) \approx \exp(-a_q(1 - e_q/E_c^q)) \quad (1)$$

$$a_q = 0.63772(A_q)^{\frac{1}{2}}(1 + A_q^{1/3})(E_c^q)^{\frac{1}{2}} \quad (2)$$

are used to describe the effects of coulomb barrier. In eq.(1) and eq.(2), The  $A$  and  $A_q$  are the mass number of target nucleus and emitted charged particle respectively. The  $e_q$  and  $E_c^q$  are the kinetic energy of particle  $q$  and the height of coulomb barrier respectively.

6, The energy level density of compound nucleus is taken in the form of constant temperature.

The details of the formulae deriving have been given in ref.(9). The ultimate expressions are given as follows:

$$\sigma_{n,q}(E_n) = C_q \left( G_1^q + \frac{\lambda_2}{L_n} \cdot \frac{G_2^q}{1 + \Gamma_n/\Gamma_q} \right) / (1 + \lambda_2/L_n + L_q/L_n) \quad (3)$$

for  $p$  and  $\alpha$  and

$$\sigma_{n,q}^t(E_n) = C_q \left( \frac{L_q/L_n}{\lambda_2/L_n} + \Gamma_q/\Gamma_n \right) \quad (4)$$

for  $d, t$  and  ${}^3\text{He}$ , where

$$\lambda_2/L_n = 0.035A(1 + S_n/E_n)^3 \quad (5)$$

$$G_1^q = \begin{cases} \theta_q h(t_0^q, t_0^q, t_2^q; a_q) & E_n \leq Q_{qn} \\ \theta_q (h(t_0^q, t_0^q, t_2^q; a_q) - h(t_0^q, t_1^q, t_2^q; a_q)) & E_n > Q_{qn} \end{cases} \quad (6)$$

$$G_2^q = \begin{cases} 1 \\ 1 - \alpha f_q(E_n + Q_{qn}) / f_\infty^q \end{cases} \quad (7)$$

$$L_q / L_n = \theta_q h(t_0^q, t_0^q, t_2^q; a_q) \quad (8)$$

$$\Gamma_n / \Gamma_q = u_q \beta (1 - (1 + E_n/T) \exp(-E_n/T)) f_\infty^q / f_q(E_n + Q_q) \quad (9)$$

$$h(z_0, z_1, z_2; a_q) = -\frac{6}{z_2^3} \times$$

$$\begin{cases} (z_0 + 2 + (z_0 z_1 - z_0 + 2z_1 - 2) \exp(z_1)) \exp(-a_q) & z_1 \leq a_q \\ (z_0 + 2) \exp(-a_q) + a_q z_0 - z_0 - a_q^2 + 2a_q - 2 + z_0 z_1^2 / 2 - z_1^3 / 3 - z_0 a_q^2 / 2 + a_q^3 / 3 & z_1 > a_q \end{cases} \quad (10)$$

$$f_q(z) = \begin{cases} b_q^2 (1 - (1 - z/(T b_q)) \exp(z/T b_q)) \exp(-a_q) & z \leq E_c^q \\ f_\infty^q - (1 + z/T) \exp(-z/T) & z > E_c^q \end{cases} \quad (11)$$

$$f_\infty^q = b_q^2 (1 - (1 - E_c^q / (T \cdot b_q)) \exp(z / (T \cdot b_q))) \exp(-a_q) + (1 + E_c^q / T) \exp(-E_c^q / T) \quad (12)$$

$$b_q = (a_q T / E_c^q - 1)^{-1} \quad (13)$$

$$a_q = 0.63772 (A_q)^{1/2} (1 + A^{1/3}) (E_c^q)^{1/2} \quad (14)$$

$$T = \begin{cases} 13A^{-1/2} & A \leq 165 \\ (0.0125A - 1.0625) \times 13A^{-1/2} & A > 165 \end{cases} \quad (15)$$

$$\begin{cases} t_0^q = (E_n + Q_q) a_q / E_c^q \\ t_1^q = (E_n + Q_{qn}) a_q / E_c^q \\ t_2^q = E_n a_q / E_c^q \end{cases} \quad (16)$$

$\alpha$  and  $\beta$  are two empirical parameters

$$\alpha = \begin{cases} 1 - A/130 & A \leq 130 \\ 0 & A > 130 \end{cases} \quad (17)$$

$$\beta = \begin{cases} 1 & A < 128 \\ 1.740 - 0.0063A & A > 128 \end{cases} \quad (18)$$

The values of  $\theta_q$  and  $u_q$  are given in table 1.

Tab.1 The values of  $\theta_q$  and  $u_q$

q	p	d	t	${}^3\text{He}$	
$\theta_q$	1	6	9	9	1.6
$u_q$	1	1/3	1/3	1/3	2.5

In eq.(3) to eq.(18),  $E_n$  is the incident neutron energy;  $S_n$  the neutron separation energy of compound nucleus system;  $T$  the nuclear temperature of target; the meanings of  $A$  and  $A_q$  are as the same as in eq.(2);  $Q_q$  and  $Q_{qn}$  are reaction energy for  $(n,q)$  and  $(n,qn)$  respectively. The  $E_n, S_n, T, Q_q$  and  $Q_{qn}$  are all in unit of MeV.

For the emissions of  $d, t$  and  ${}^3\text{He}$ , the process of secondary particle emission have been omitted so that the sum of cross sections

$$\sigma_{n,q}^t = \sigma_{n,q} + \sigma_{n,qn} + \sigma_{n,q2n} + \sigma_{n,qp} + \dots \quad (19)$$

are given as eq.(4).

In eq.(3) and eq.(4), there are two adjustable parameters  $C_q$  and  $E_c^q$ . The  $E_c^q$  represents the height of coulomb barrier and  $C_q$  a constant proportional to maximum of  $\sigma_{n,q}^t$  defined by eq.(19).

#### Systematics of Local Parameters $\hat{C}_q$ and $\hat{E}_c^q$

For  $(n,p)$  and  $(n,\alpha)$  reactions in mass region  $23 < A < 197$  the measured excitation functions for about fifty nuclei have been collected in the light of ref.(10) and ref.(11). The least squares fits have been carried out for available experimental data by using eq.(3) and the fitting parameters (called local parameters) have been obtained. Before fitting,  $Q_q$  and  $Q_{qn}$  have been calculated from ref.(12) and  $S_n$  taken from ref. (10). The agreement between fitting curves and measured data is satisfactory. These results demonstrated that the contribution of the preequilibrium emission must be taken into account in the formulae. For middle weight nuclei, the portion of preequilibrium is about 30-50% at  $E_n = 20\text{MeV}$  (see fig.1).



The local parameters  $\hat{C}_q$  and  $\hat{E}_c^q$  for (n,p) and (n, $\alpha$ ) reactions can be expressed as a simple functions of neutron number N and proton number Z of target nucleus as following

$$\overline{E}_c^p = (0.6 + 0.25Z - 0.001Z^2 - 2\exp(-0.05(Z-28)^2)) \exp(29.6 \frac{N-Z}{A}), \text{MeV} \quad (20)$$

$$\overline{E}_c^\alpha = -3.4 + 0.57Z - 0.003Z^2 - 3\exp(-0.3(Z-28)^2), \text{MeV} \quad (21)$$

$$\overline{C}_p = (1 + A^{1/3})^2 \exp(5.88 - 33.7 \frac{N-Z}{A} - 16.8A^{-2/3}), \text{mb} \quad (22)$$

$$\overline{C}_\alpha = (1 + A^{1/3})^2 \exp(2.0 - 23.7 \frac{N-Z}{A} + 21.0A^{2/3}), \text{mb} \quad (23)$$

The parameters  $\overline{C}_q$  and  $\overline{E}_c^q$  calculated from above systematics are called regional parameters. The comparisons between local and regional parameters are given as fig.2 to fig.5. From fig.2 and fig.3, one can find that the shell effects exists at Z=28. And it can be illustrated empirically with a normal function. It is not clear for shell effect and odd-even effect of target nucleus on parameter  $\overline{C}_q$ .

The covarance matrix  $\overline{V}_q$  of regional parameters would be estimated in order to get the uncertainties of cross sections predicted with regional parameters. To combine the uncertainties such as negligence error, correlated error and uncertainties of the expression for excitation function and the systematics into  $\overline{V}_q$ ,  $\overline{V}_q$  was estimated by moment method<sup>(13)</sup>. Let

$$\overline{V}_q = \overline{P} \overline{M}_q \overline{P}_q \quad (24)$$

where

$$\overline{P}_q = \begin{pmatrix} \overline{C}_q & 0 \\ 0 & \overline{E}_c^q \end{pmatrix} \quad (25)$$

$$\overline{M}_q(1,1) = \frac{1}{m-1} \sum_{i=1}^m (t_i^q)^2 - \frac{1}{m(m-1)} \left( \sum_{i=1}^m t_i^q \right)^2 \quad (26)$$

$$\overline{M}_q(2,2) = \frac{1}{m-1} \sum_{i=1}^m (1_i^q)^2 - \frac{1}{m(m-1)} \left( \sum_{i=1}^m 1_i^q \right)^2 \quad (27)$$

$$\overline{M}_q(1,2) = \overline{M}_q(2,1)$$

$$= \frac{1}{m-1} \sum_{i=1}^m t_i^q 1_i^q - \frac{1}{m(m-1)} \sum_{i=1}^m t_i^q \sum_{i=1}^m 1_i^q \quad (28)$$

and

$$t_i^q = (\hat{C}_q(N_i, Z_i) - \overline{C}_q(N_i, Z_i)) / \overline{C}_q(N_i, Z_i) \quad (29)$$

$$1_i^q = (\hat{E}_c^q(N_i, Z_i) - \overline{E}_c^q(N_i, Z_i)) / \overline{E}_c^q(N_i, Z_i) \quad (30)$$

It follows that

$$\overline{M}_P = \begin{pmatrix} 0.30^2 & 0.11(0.30)(0.25) \\ 0.11(0.30)(0.25) & 0.25^2 \end{pmatrix} \quad (31)$$

$$\overline{M}_\alpha = \begin{pmatrix} 0.31^2 & -0.24(0.31)(0.14) \\ -0.24(0.31)(0.14) & 0.14^2 \end{pmatrix} \quad (32)$$

For (n,d), (n,t) and (n,<sup>3</sup>He) reactions, the fitting with two parameters  $C_q$  and  $E_c^q$  could not be carried out because of lack experimental data for excitation function. We had to replace  $E_c^d$  and  $E_c^t$  with  $\overline{E}_c^p$  and to replace  $E_c^{3He}$  with  $\overline{E}_c^\alpha$  so that there is one parameter  $C_q$  in eq.(4). Therefore, only one point cross section is needed to determine parameter  $\hat{C}_q$ . Taking into account the meaning of eq.(4) and status of experimental data for (n,d), (n,t) and (n,<sup>3</sup>He) reactions, the following experimental data have been selected:

(n,d) reaction: data measured at LLL with magnetic quadrupole spectrometer in neutron energy region  $E_n = 14-15\text{MeV}$  <sup>(14)</sup>.

(n,t) reaction: t emission cross sections measured at  $\overline{E}_n = 22.5\text{MeV}$  <sup>(15)</sup>.

(n,<sup>3</sup>He) reaction: cross section measured by activation method at  $\overline{E}_n = 22.5\text{MeV}$  <sup>(16)</sup>.

Assuming the shapes of  $C_d, C_t$  and  $C_{3He}$  versus Z all are the same, the systematics for  $\hat{C}_d, \hat{C}_t$  and  $\hat{C}_{3He}$  have been found as follows:

$$\overline{C}_d = 23(1 - 0.052Z + 0.00083Z^2), \text{mb} \quad (33)$$

$$\overline{C}_t = 5.81(1 - 0.052Z + 0.00083Z^2), \text{mb} \quad (34)$$

$$\overline{C}_{3He} = 2.9(1 - 0.052Z + 0.00083Z^2), \text{mb} \quad (35)$$

By the moment method, the relative errors have been estimated:

$$\begin{aligned} \Delta \overline{C}_d / \overline{C}_d &= 0.38 \\ \Delta \overline{C}_t / \overline{C}_t &= 0.35 \\ \Delta \overline{C}_{3He} / \overline{C}_{3He} &= 0.51 \end{aligned} \quad (36)$$

The comparison between local parameters and regional parameters are given in fig.6 to fig.8 for  $C_d, C_t$  and  $C_{3He}$  respectively.

### Discussion

With regional parameters, excitation functions of (n,p) and (n, $\alpha$ ) reactions have been calculated for about fifty nuclei in region  $23 < A < 197$ . The predicted cross sections are consistent with measured ones within errors calculated from  $\bar{V}_q$ . The results are shown in fig.A and fig.B. The excitation function of (n,t) reaction have also been predicted for several nuclei on which experimental data are available. The agreement between the predicted curves and experimental data are fair (see fig.9).

### Acknowledgment

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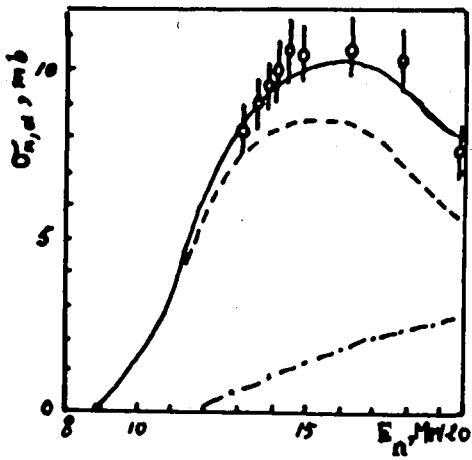


Fig. 1. Excitation function  
of  $^{92}\text{Zr}(n, \alpha)^{89}\text{Sr}$   
--- evaporation  
- · - pre-equilibrium  
— evaporation + pre-equilibrium

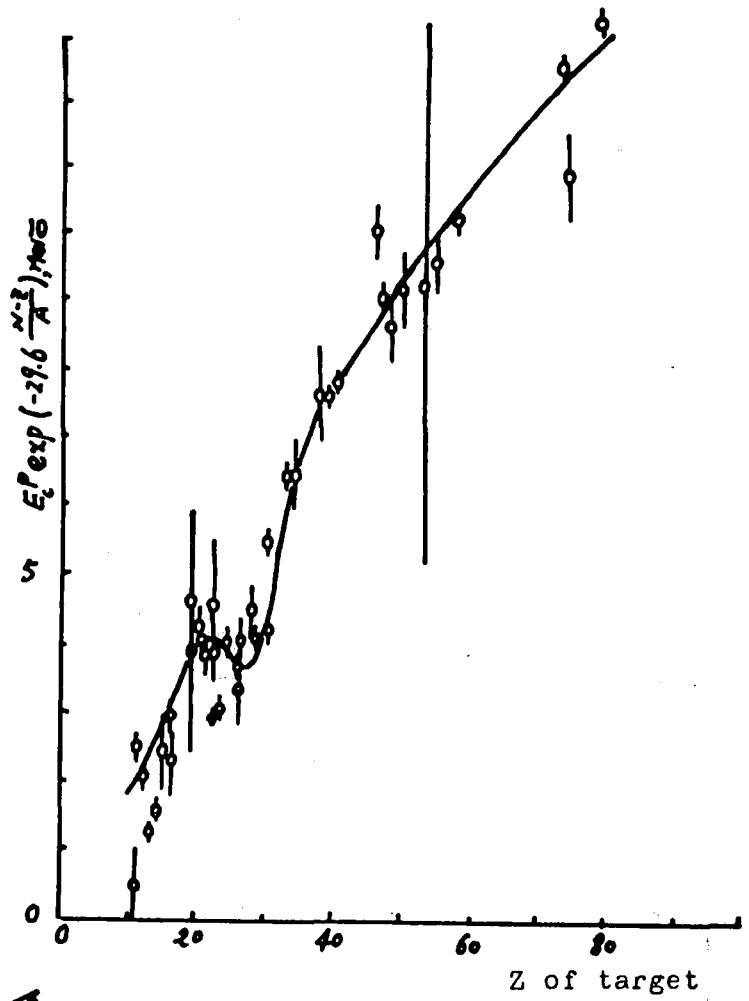


Fig. 2. Systematics of  $E_C^P$

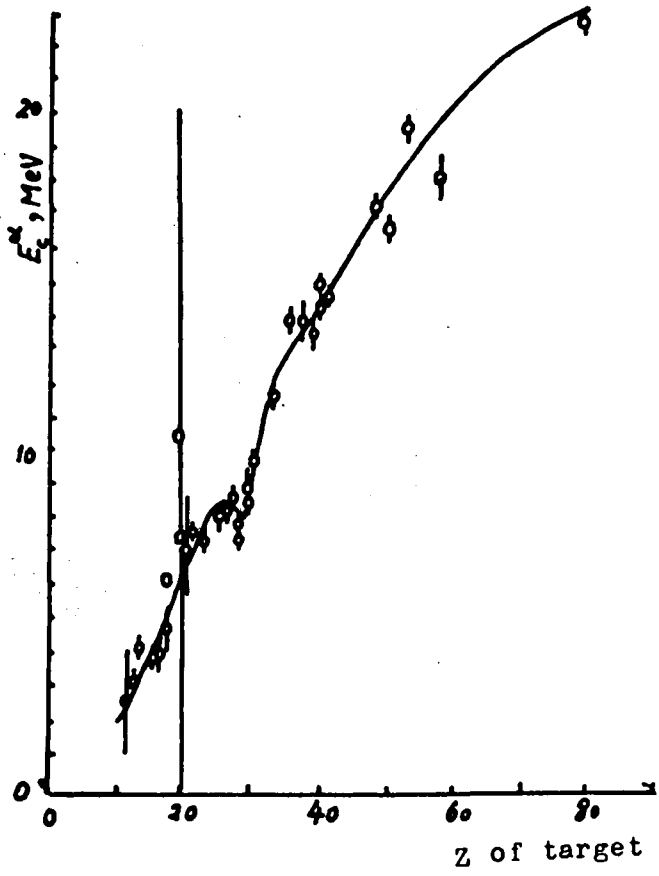


Fig. 3. Systematics of  $E_C$

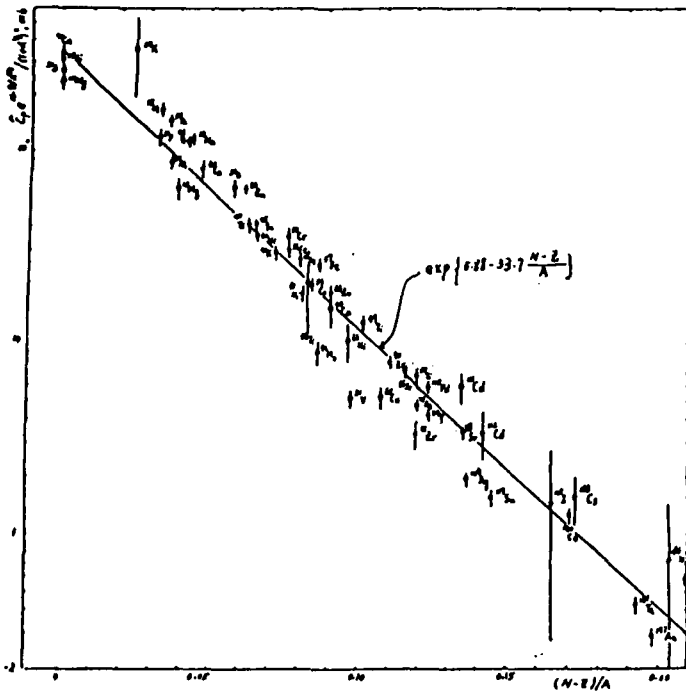


Fig. 4. Systematics of  $C_p$

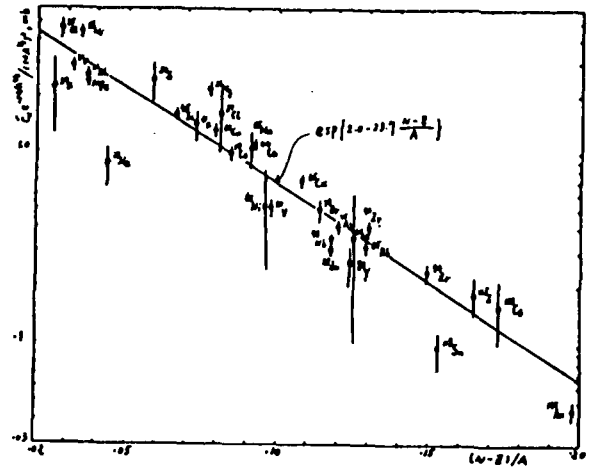


Fig. 5. Systematics of  $C$

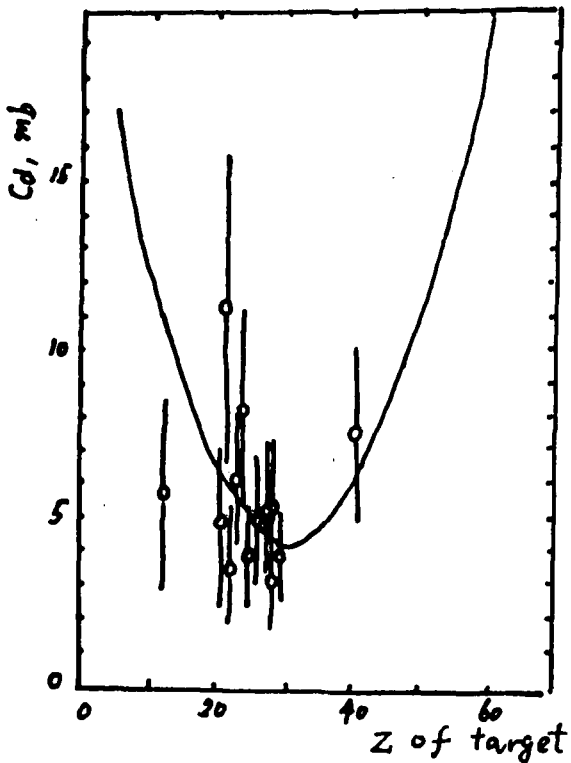


Fig. 6. Systematics of  $C_d$

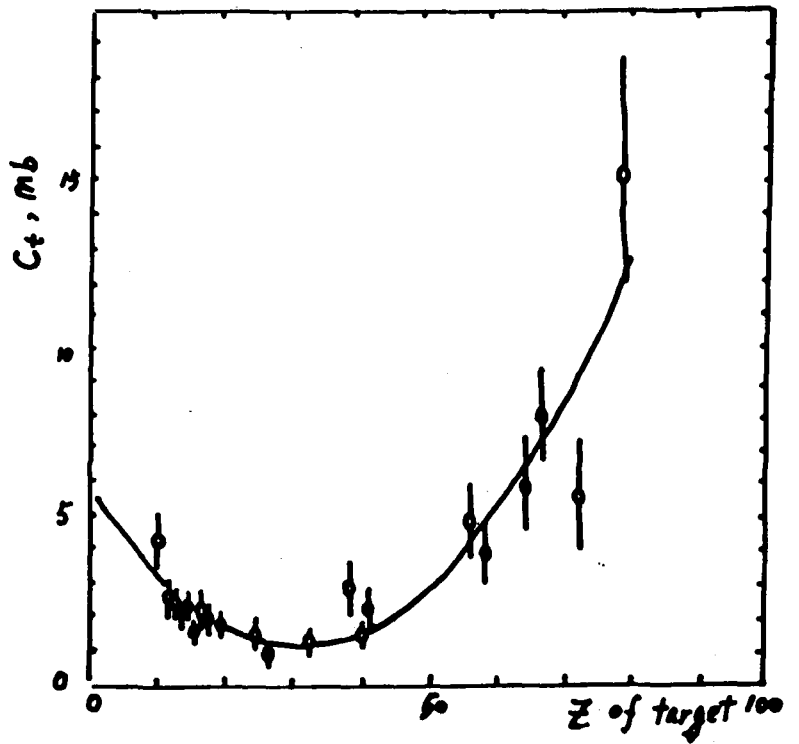


Fig. 7. Systematics of  $C_t$

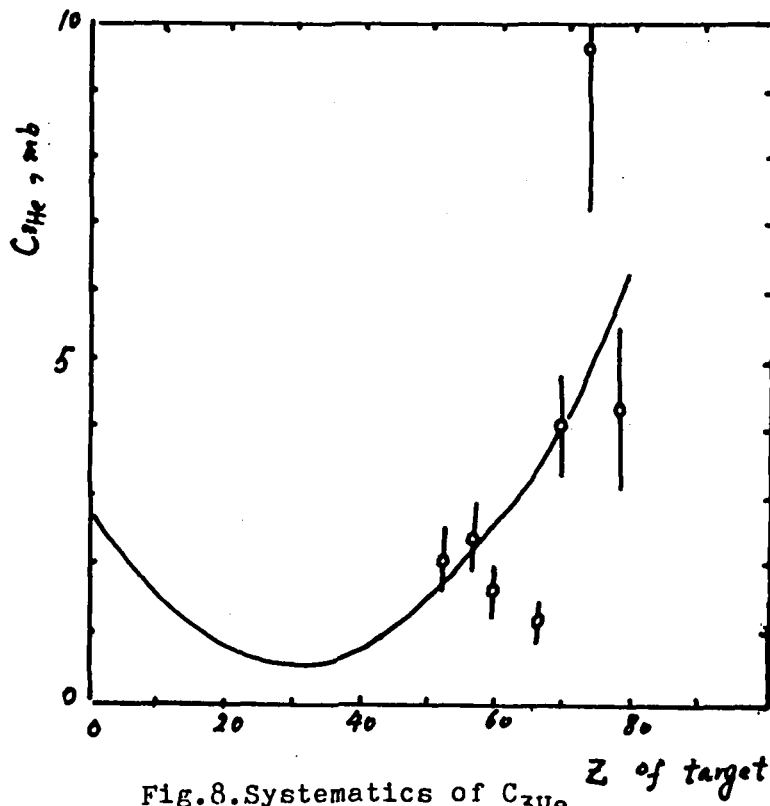


Fig.8. Systematics of  $C_3He$

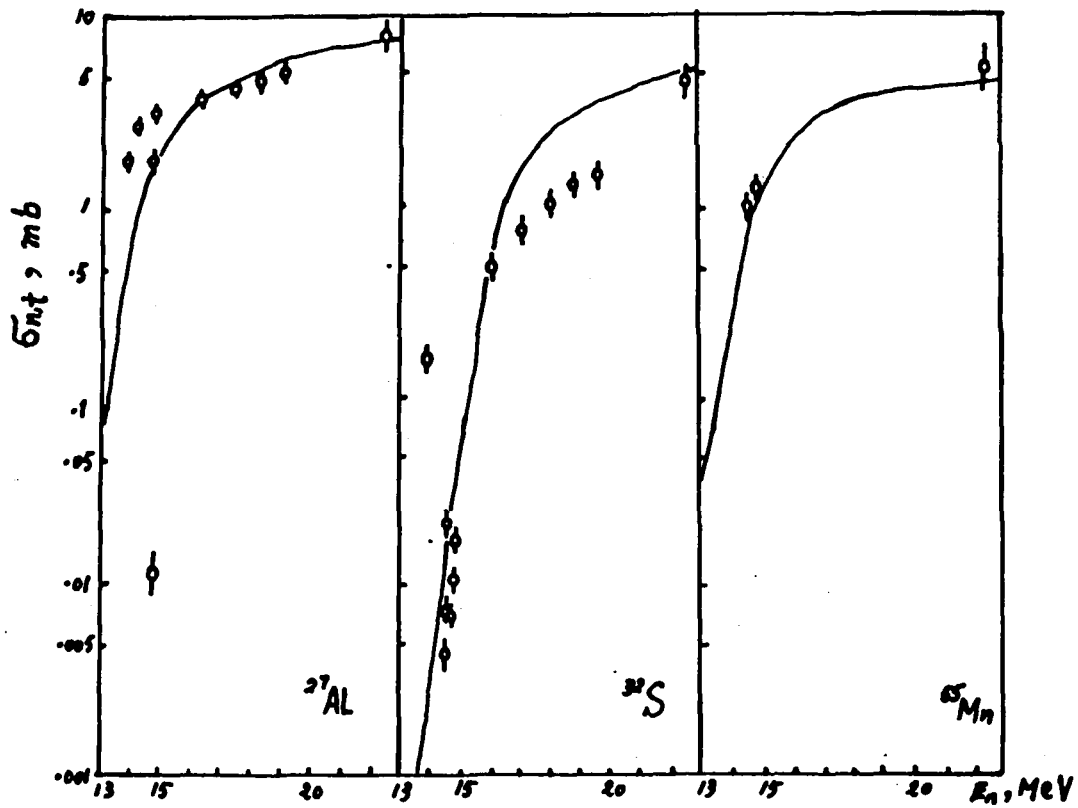


Fig.9. Excitation functions of  $(n,t)$  reaction for several nuclei  
 o experimental data  
 — predicted values by this systematics

Fig. A1-A51:  $\sigma_{n,p}$

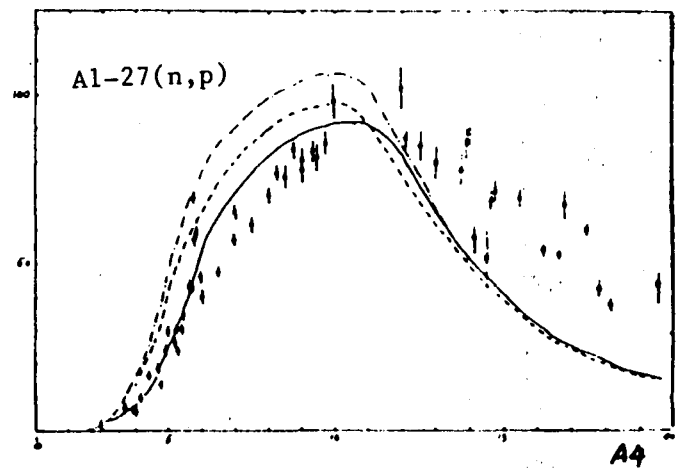
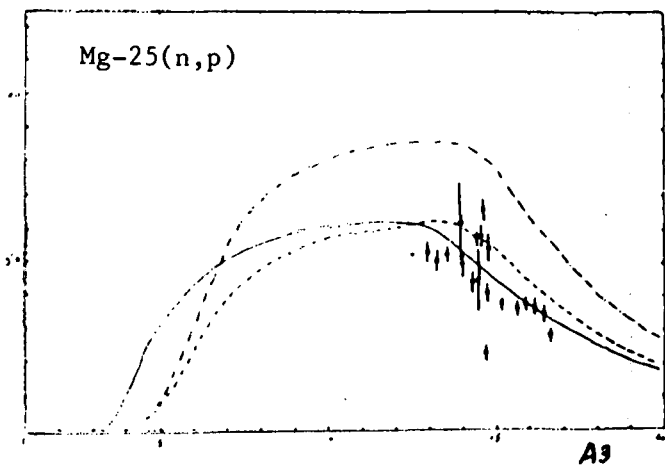
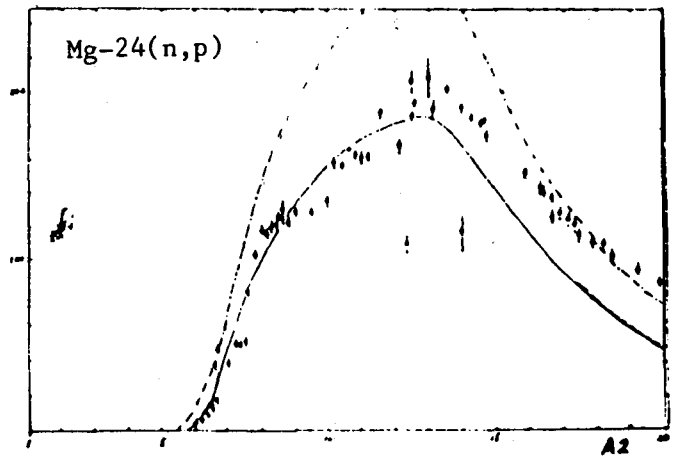
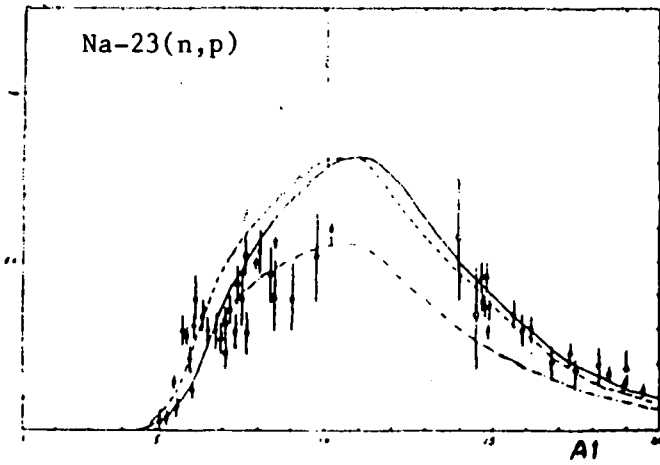
Fig. B1-B31:  $\sigma_{n,\alpha}$

———— the fitting curves;

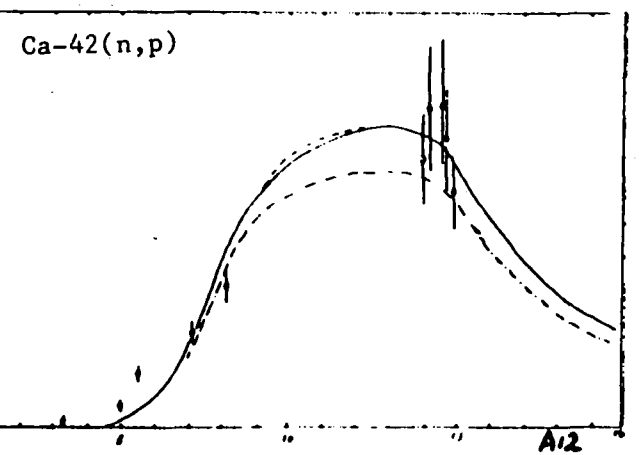
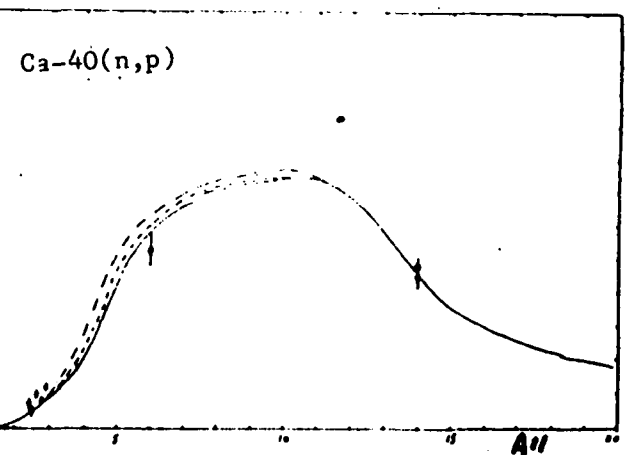
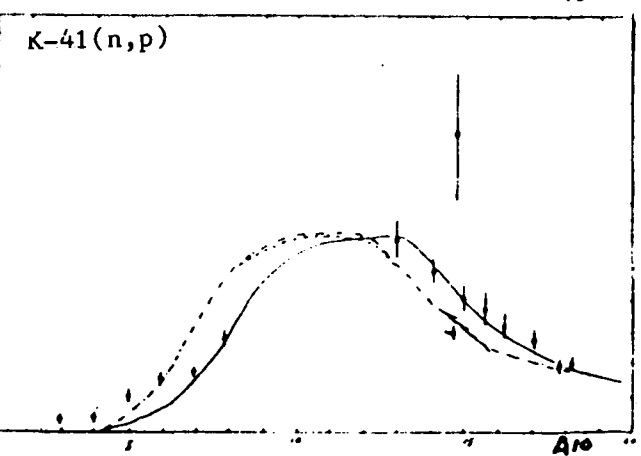
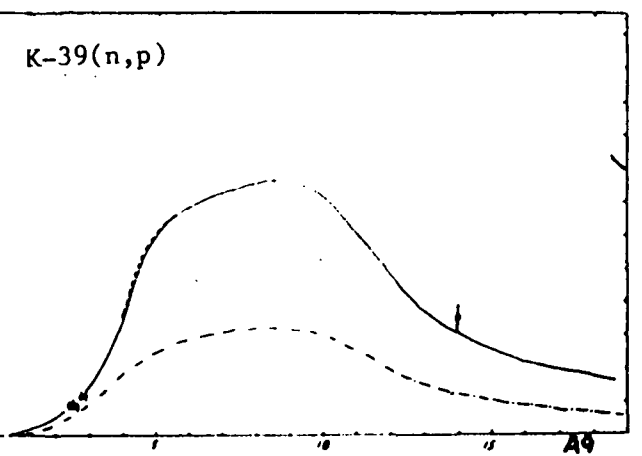
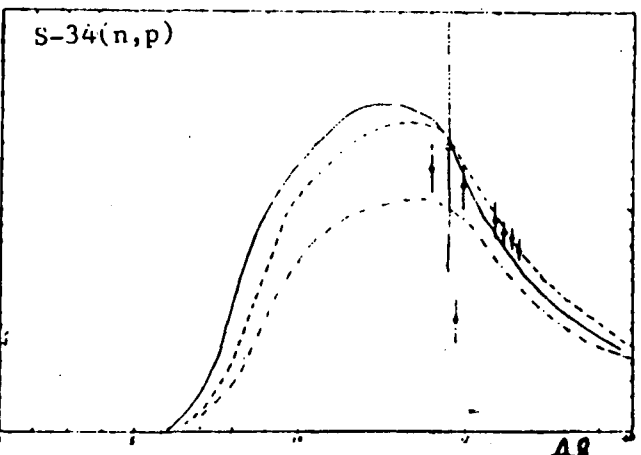
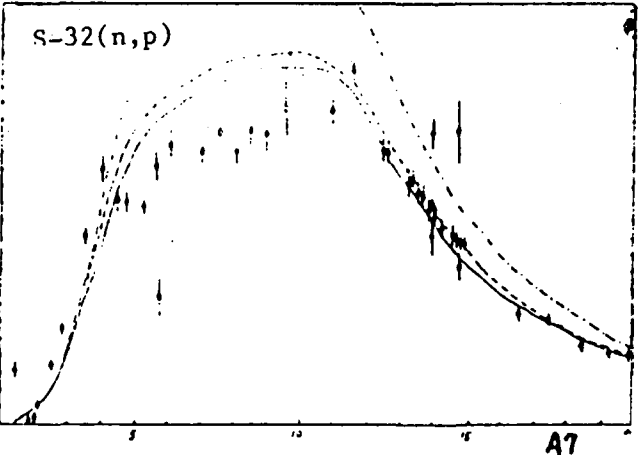
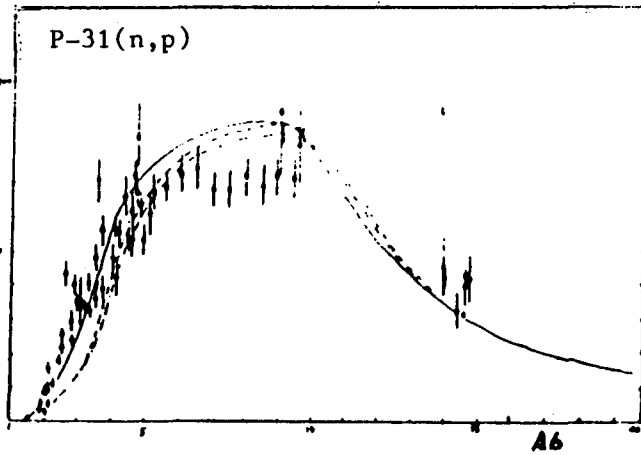
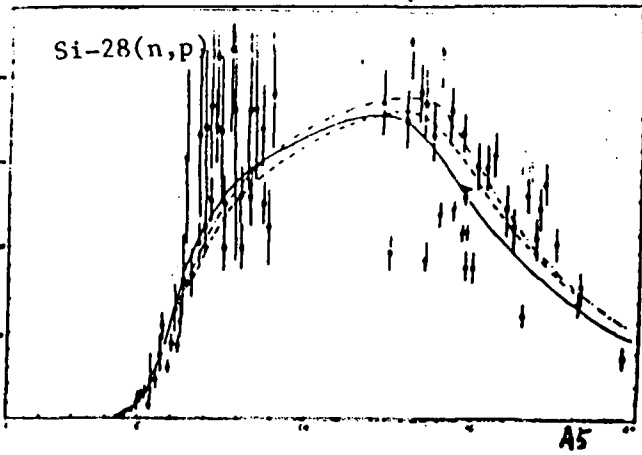
----- the values predicted by this work with parameters  $(\hat{C}_q, \overline{E}_c^q)$ ;

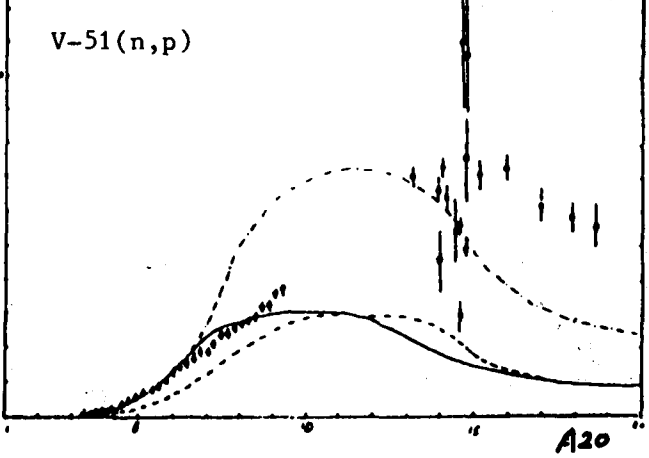
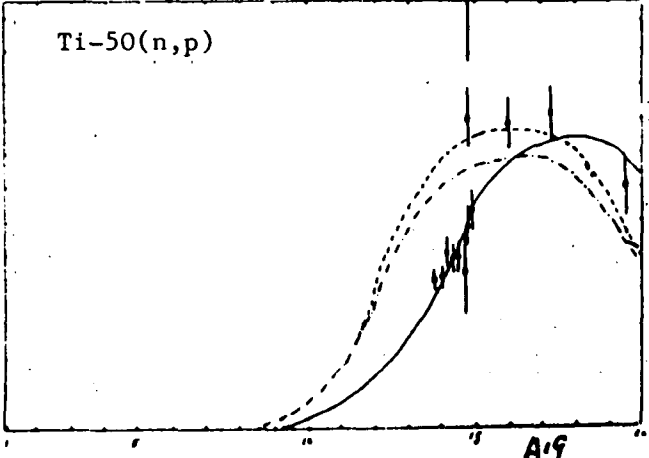
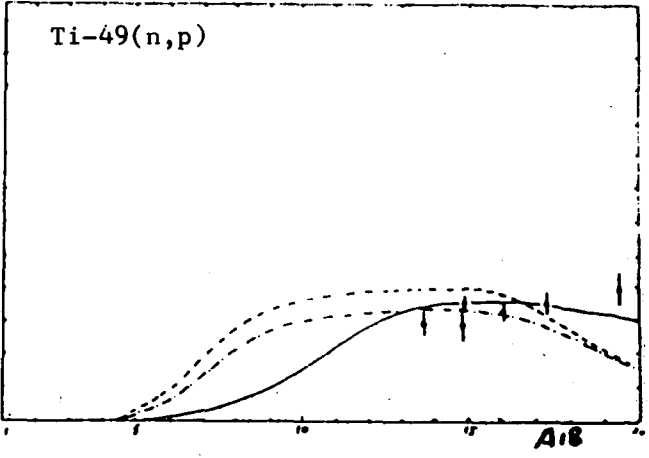
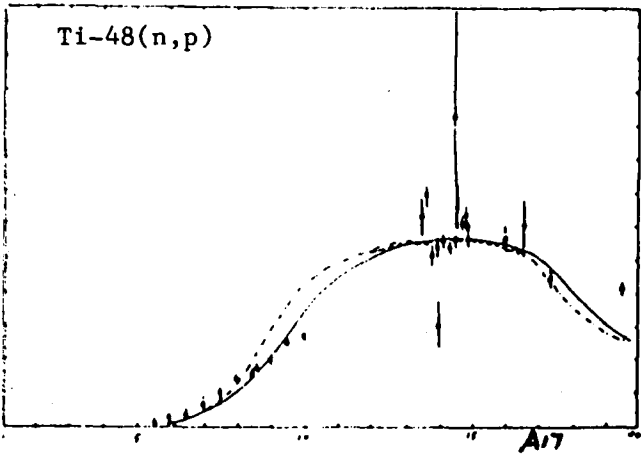
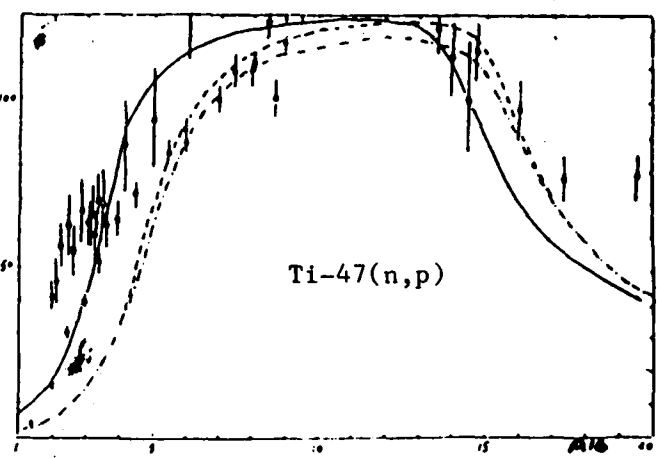
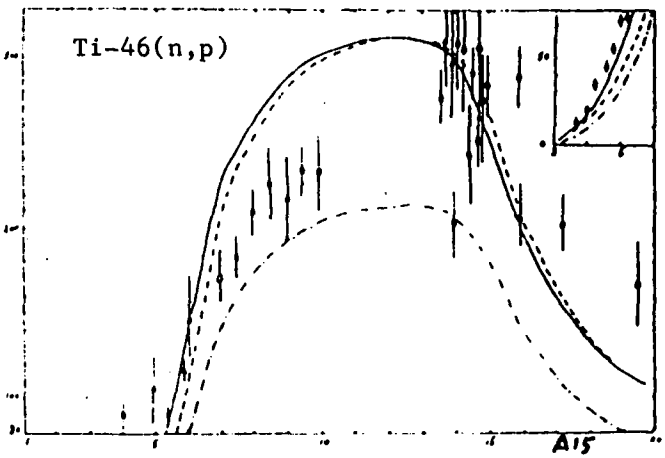
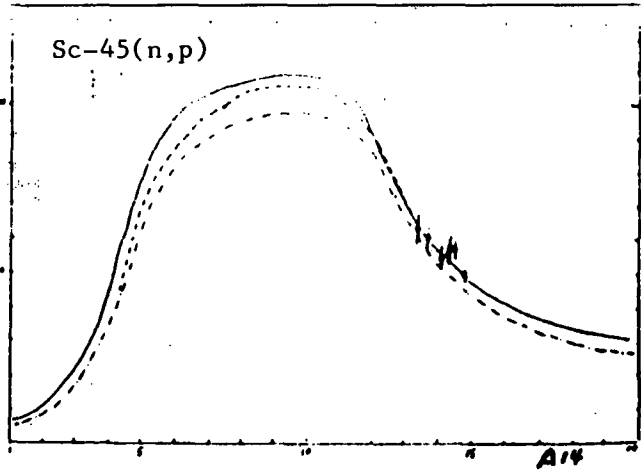
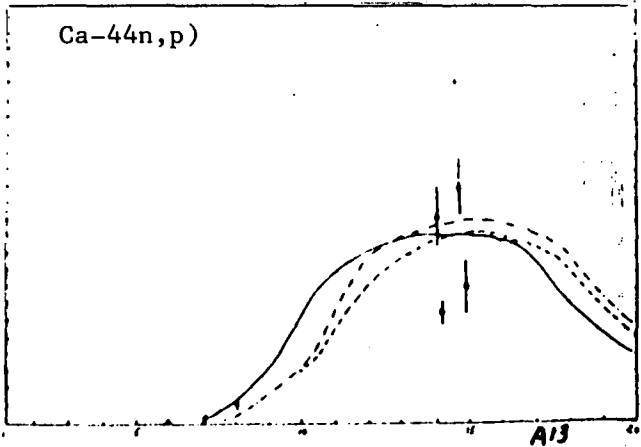
-·-·-· the values predicted by this work with parameters  $(\overline{C}_q, \overline{E}_c^q)$ ;

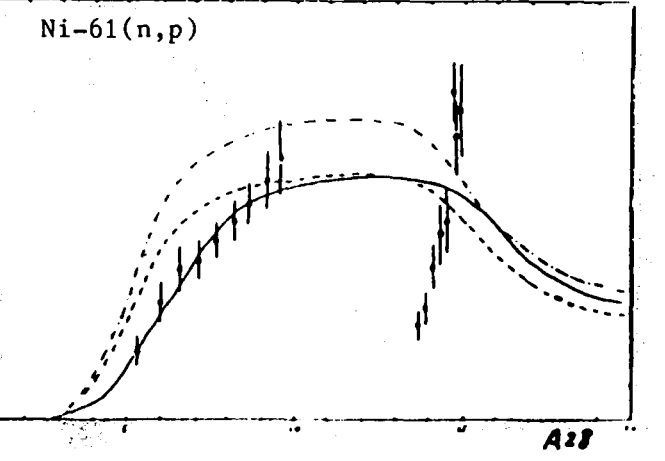
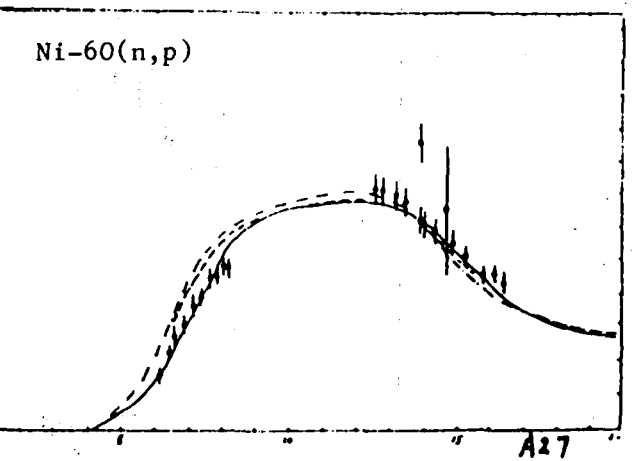
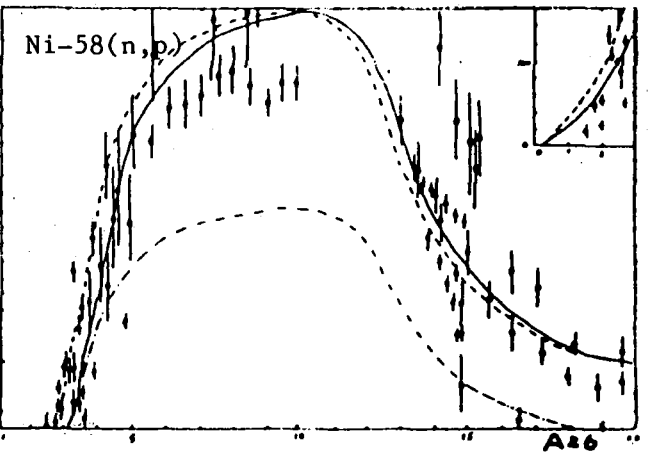
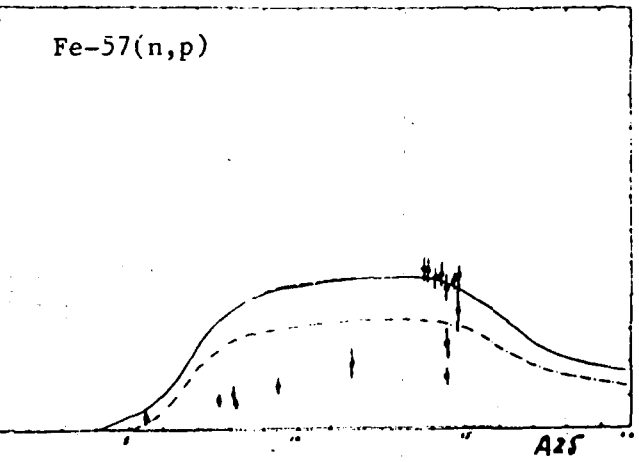
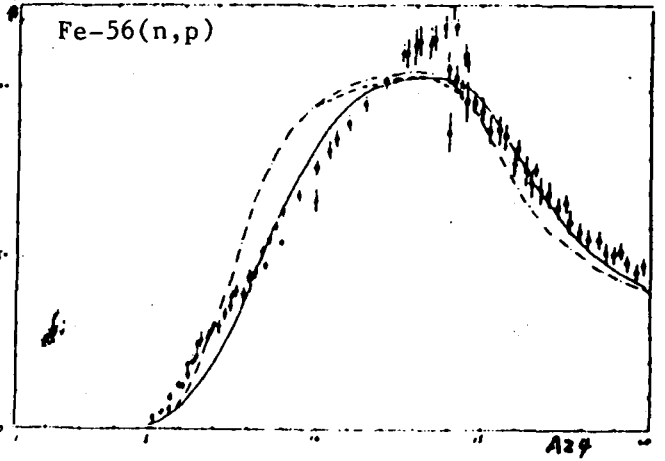
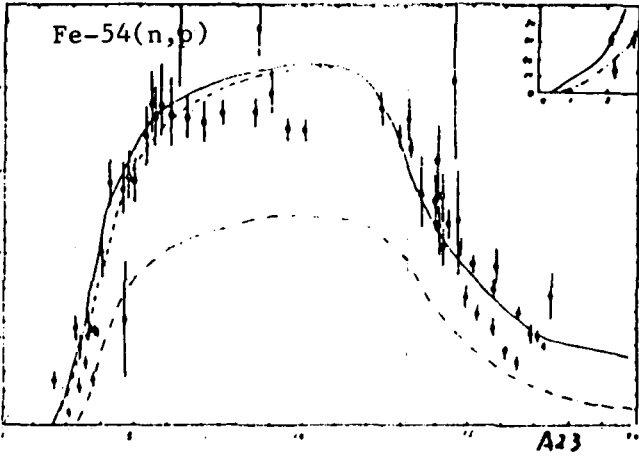
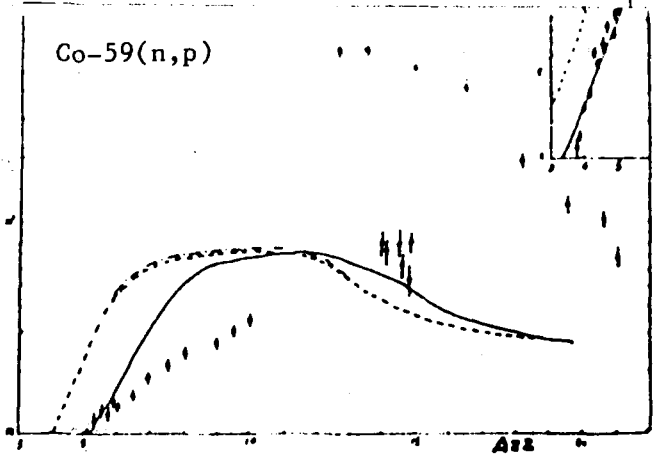
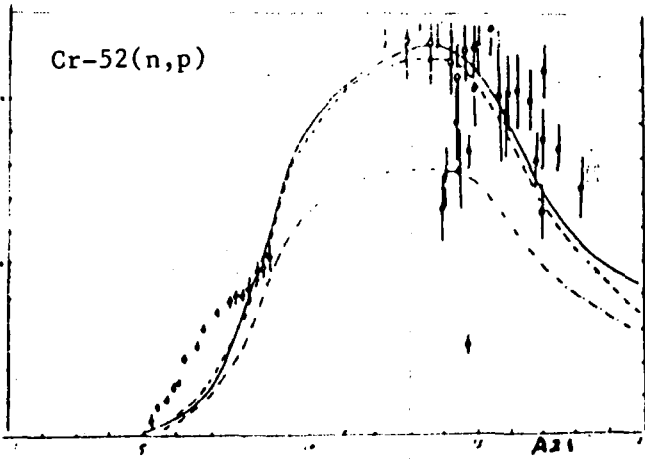
$E_n$  in MeV;  $\sigma_{n,q}$  in mb

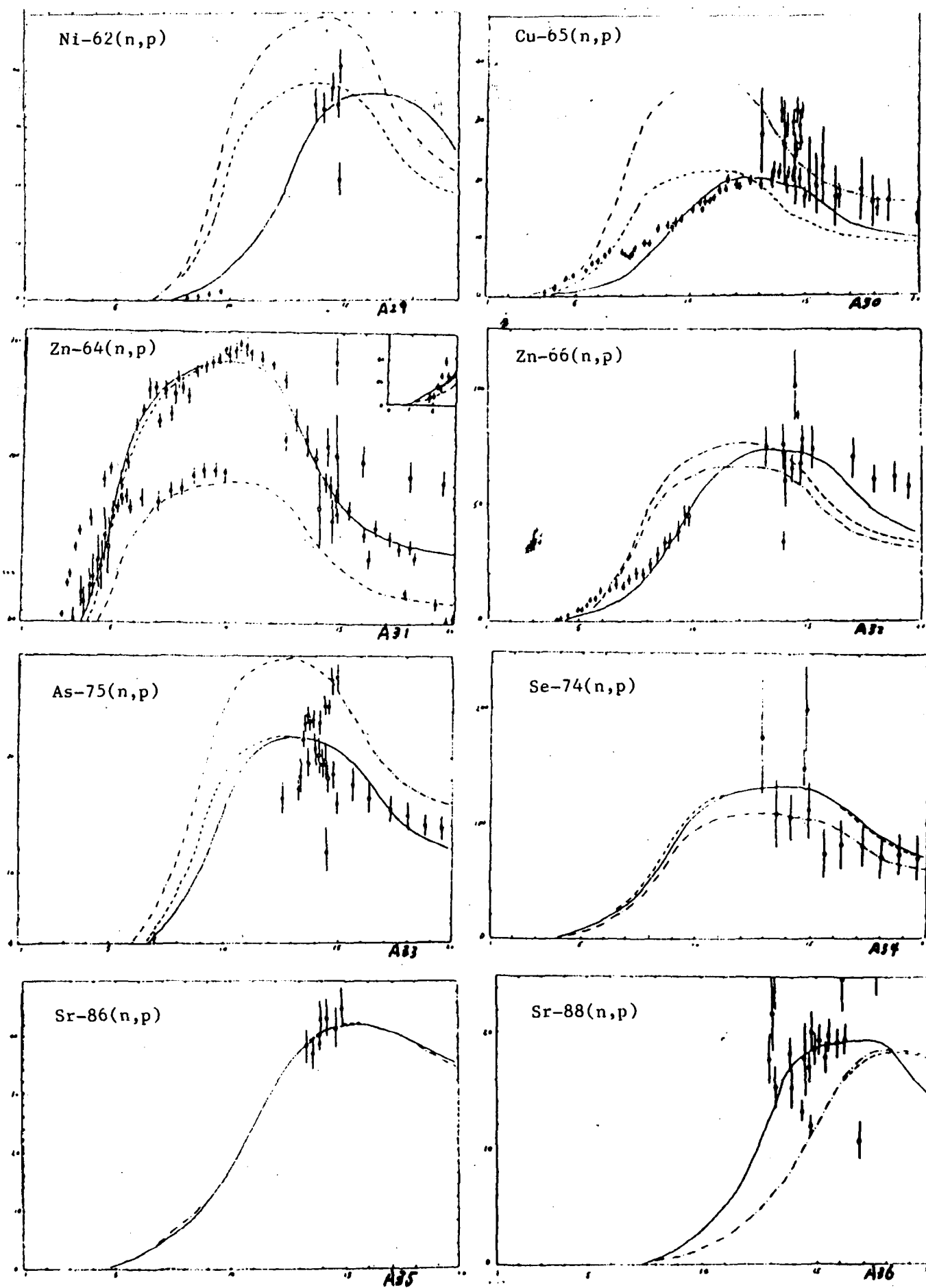












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